# Double $\beta$-decay nuclear matrix elements and lepton conservation*

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(Received 30 June 1975)

The nuclear matrix elements involved in the double $\beta$-decay of $^{48}Ca$, $^{130}Te$, and $^{128}Te$ were calculated using realistic nuclear interactions and shell model nuclear wave functions. The double doorway state is not appreciably mixed in the ground state of the final nuclei. So the ground state transitions contain a small fraction of the sum rule. A lepton nonconservation parameter $\eta \lessapprox 10^{-4}$ was deduced.

[NUCLEAR STRUCTURE Double $\beta$-decay nuclear matrix elements. Shell model, realistic interactions. Lepton conservation.]

## I. INTRODUCTION

Lepton conservation is a basic problem in the theory of weak interactions and it is well known that the double $\beta$ decay plays an essential role in this regard.[1,2] It has been shown[3] that if neutrinoless $\beta$ decay is absolutely forbidden, there must always be a way of assigning a lepton number $L_e$ which is conserved in weak interactions. Conversely,[4] the observation of neutrinoless $\beta$ decay

$$A(N,Z) \rightarrow A(N-2, Z+2) + e^- + e^- \quad (1)$$

definitely violates such assignments. On the other hand, the ordinary double $\beta$ decay

$$A(N,Z) \rightarrow A(N-2, Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e \quad (2)$$

obeys lepton conservation. Experimentally, one can distinguish between (1) and (2), even if one does not observe the emitted neutrinos, by measuring the sum of the energies of the two electrons. If the process is neutrinoless, this sum is fixed and equal to the available energy $\epsilon_0$. If, on the other hand, process (2) takes place, the energy $E_{e_1} + E_{e_2}$ will show a broad distribution centered around half the available energy. The nuclear systems $A(N,Z)$ must be such that the process

$$A(N,Z) \rightarrow A(N-1, Z+1) + e^- + \bar{\nu}_e \quad (3)$$

is energetically forbidden or hindered due to angular momentum mismatch.

The importance of the double $\beta$ decay in this connection was recognized a long time ago and experiments aimed at its observation were conducted[5,6] involving mass-spectroscopic examination of geologically old tellurium ores. These early experiments clearly established that double $\beta$ decay took place, and from the measured lifetimes ($T_{1/2} \sim 10^{21}$ yr) neutrinoless double $\beta$ decay was ruled out (expected lifetimes $\approx 10^{+15}$ yr). However, it was soon realized that this did not completely exclude neutrinoless double $\beta$ decay. The latter could still proceed at a reduced rate comparable to that of the lepton conserving process. In a further important discussion Pontecorvo pointed out that the ratio $R$ of the observed lifetime of $^{128}Te$ to that of $^{130}Te$, if one assumed that the nuclear matrix elements for these presumably similar nuclei are the same, is equal to the ratio at their decay phase spaces.[7] One finds in this way $R_2 = 1.4 \times 10^2$ and $R_4 = 8 \times 10^3$ for the two-lepton and four-lepton process, respectively.[2] The experimental value[8] $R_{exp} = (1.59 \pm 0.05) \times 10^3$ favors the four-lepton process. However, since mass-spectroscopic methods depend somewhat on assumptions about the history of the ores, another variety of experiments has been performed over the years, most recently by the Columbia group on $^{48}Ca$ and $^{82}Se$[9,10]; these experiments measure the sum of the electron energies in each decay directly, but are very difficult due to the small counting rates and background problems. From their analysis, lepton conservation is also favored and a limit is put on the lepton nonconservation parameter, i.e. $\eta \lessapprox 10^{-4}$.

From a theoretical point of view the value of the lepton nonconservation parameter, which is extracted from the experimental data, depends on the estimate of the nuclear matrix elements. These matrix elements have never before been reliably calculated, since the nuclei involved do not have very simple structure. Recently, however, it has become feasible to obtain reliable nuclear wave functions by diagonalizing Hamiltonians of realistic complexity. In the present paper we have calculated double $\beta$-decay nuclear matrix elements considering the process as a second-order Gamow-Teller $\beta$-decay transition. From these nuclear matrix elements the lepton nonconservation parameter $\eta$ is deduced.

## II. BRIEF THEORY OF DOUBLE $\beta$ DECAY

In the present paper, following Primakoff and Rosen,[2] we take the point of view that the double $\beta$ decay is a second-order ordinary weak process involving the allowed $\beta$ decay of two nucleons. In this case the double $\beta$-decay matrix element can be written as

$$M_{\beta\beta} = \sum_n \frac{\langle \chi_f \psi_f | H_\beta | \psi_n \chi_n \rangle \langle \chi_n \psi_n | H_\beta | \psi_i \chi_i \rangle}{E_n - E_i}, \quad (4)$$

where $\psi_i, \psi_f, \psi_n, \chi_i, \chi_f, \chi_n$ are the initial, final, and intermediate nuclear and lepton wave functions, respectively [$\chi_i$ refers to a state of no leptons and $\chi_f$ refers to a state of two charged leptons and two or zero neutral leptons for processes (1) and (2), respectively]. $H_\beta$ is the conventional weak Hamiltonian with lepton current given by[4]

$$L_\lambda = \bar{\psi}_e \gamma_\lambda \frac{[(1+\gamma_5) + \eta(1-\gamma_5)]}{(1+\eta^2)^{1/2}} \frac{\psi_\nu + \xi\psi_{\bar{\nu}}}{(1+\xi^2)^{1/2}}. \quad (5)$$

It can be shown that the amplitude for neutrinoless double $\beta$ decay is proportional to $\xi\eta$. For neutrinoless double $\beta$ decay we will assume further that the neutrino is a Majorana particle ($\xi = 1$) so that limits can be set on the parameter $\eta$ from the experimentally measured lifetimes and the calculated nuclear matrix elements (ME). The processes involved are represented diagramatically in Fig. (1a) and (1b) for no neutrinos and two neutrinos in the final state, respectively.

The dependence of the matrix element $M_{\beta\beta}$ on the available energy $\epsilon_0$ and the bulk nuclear properties like $A$ and $Z$ is complicated. It has been approximately carried out by Primakoff and Rosen[1,2] and it will not be repeated here. We will only present their results which express the half-lives as follows:

$$T_{1/2}^{(2)} = \frac{f_2}{\eta^2} \frac{1}{|\mathrm{ME}|^2}, \quad (6a)$$

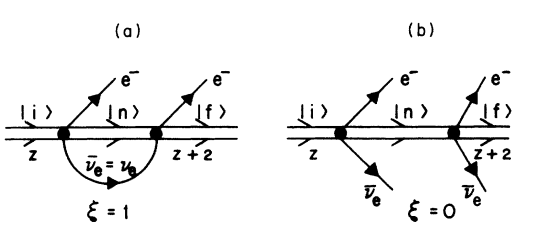


FIG. 1. (a) Neutrinoless double $\beta$ decay. The emitted virtual antineutrino is reabsorbed by another nucleon. $|i\rangle$, $|n\rangle$|, and $|f\rangle$ are the initial, intermediate, and final nuclear states. A summation over $n$ is understood. (b) Double $\beta$ decay with two neutrinos emitted; otherwise the notation is the same as Fig. 1(a).

$$T_{1/2}^{(4)} = \frac{f_4}{|\mathrm{ME}|^2}, \quad (6b)$$

for the two- and four-lepton final states, respectively. The effective phase space quantities $f_2$ and $f_4$ are

$$f_2 = \frac{10^{19.9}}{f(\epsilon_0)} \left(\frac{1}{2\pi\alpha Z}\right)^2 \times [1 - \exp(-2\pi\alpha Z)]^2 \left(\frac{A}{130}\right)^{2/3} \mathrm{yr},$$

$$f_4 = 6 \times 10^{18} \left(\frac{\bar{\Delta} + \frac{1}{2}\epsilon_0 + 1}{10}\right)^2 \left(\frac{60}{Z}\right)^2 \times [1 - \exp(-2\pi Z\alpha)]^2 \left(\frac{8}{\epsilon_0}\right)^{10} \mathrm{yr}, \quad (7)$$

where $\alpha$ = fine structure constant, $f(\epsilon_0) = \epsilon_0^4(\epsilon_0^3 + 13\epsilon_0^2 + 77\epsilon_0 + 70)$, and $\epsilon_0$ is the energy release in units of the electron rest mass. $\bar{\Delta}$ is the average energy difference between the initial and intermediate nuclear states. $f_2$ is independent of $\bar{\Delta}$ because in the energy denominator of Fig. (1a) the neutrino energy is assumed to dominate. Using standard Racah algebra, the relevant nuclear matrix element can be expressed in terms of reduced matrix elements as

$$|\mathrm{ME}|^2 = \frac{\bar{\Delta}}{2J_i + 1} \sum_x F(k,x) [Q(J_i, J_f, x)]^2, \quad (7a)$$

$$F(k,x) = \sum_\lambda \left( \sum_\alpha \langle k\alpha k\lambda - \alpha | x\lambda \rangle \right)^2,$$

$$Q(J_i, J_f, x) = \sum_n \begin{Bmatrix} J_i & k & J_n \\ k & J_f & x \end{Bmatrix} \frac{\langle J_f \| T^k \| J_n \rangle \langle J_n \| T^k \| J_i \rangle}{\Delta_n}. \quad (7b)$$

In the present case $k = 1$, $x = 0, 1, 2$, and $T^k = \vec{Y}^+ = \sum_i \vec{\sigma}(i) t_+(i)$. In the special case $J_i = J_f = 0^+$ we obtain

$$|\mathrm{ME}|^2 = \left( \sum_n \frac{\bar{\Delta}}{3\Delta_n} \langle 0 \| \vec{Y}^+ \| 1_n^+ \rangle \langle 1_n^+ \| \vec{Y}^+ \| 0 \rangle \right)^2. \quad (7c)$$

The reduced matrix elements are as defined by Edmonds[11]; they are reduced in $J$ space but not in isospin space.

For experiments done in the laboratory[8-10] the count rate and the sum of the energies of the two leptons are measured.

If $E_{e_1} + E_{e_2}$ is sharply peaked at energy $\epsilon_0$, then one concludes that a neutrinoless double $\beta$ decay occurs and the lepton nonconservation parameter $\eta$ can be obtained from Eq. (6a) once the nuclear matrix element is known. If $E_{e_1} + E_{e_2}$ is a broad function centered at $\frac{1}{2}\epsilon_0$ the final state contains four leptons, which share the energy $\epsilon_0$, and the lifetime can be compared with the predictions of Eq. (6b). For experiments involving mass-spectroscopic examination of geologically old ores[5-7] the electrons are not detected. In this case the measured lifetime can be written[12]

$$\frac{1}{T_{1/2}} = \left(\frac{\eta^2}{f_2} + \frac{1}{f_4}\right)|\mathrm{ME}|^2 . \quad (8)$$

From this equation the parameter $\eta$ can be obtained.

## III. CALCULATION OF THE NUCLEAR MATRIX ELEMENTS

The nuclei which are of great experimental[5,10] interest are $A(N,Z) = {}^{48}$Ca, $^{130}$Te, $^{128}$Te, $^{82}$Se. In the present paper we will discuss the first three. Calculations involving the nucleus $^{82}$Se are also under way.

### A. Nuclear matrix element of $^{48}$Ca $\to$ $^{48}$Ti

$^{48}$Ca is perhaps the most interesting candidate for the study of double $\beta$ decay since it is believed to be a good closed shell nucleus. It has been extensively studied by Lazarenko and Lukyanov, by der Mateosian and Goldhaber, by Shapiro *et al.*[13] and most recently by the Columbia group.[9]

The initial ground state is fairly well described by a closed shell wave function with the $1f_{7/2}$ harmonic oscillator neutron shell completely full. The ground state of the final nucleus $^{48}$Ti is slightly more complicated. To a first approximation, it can also be described as six neutrons and two protons in the $1f_{7/2}$ shell. Thus, the ground state wave function is obtained by diagonalizing the nuclear Hamiltonian in a shell model basis given by $|1f_{7/2}{}^6(n)J;1f_{7/2}{}^2(p)J;0^+\rangle$, $J=0, 2, 4, 6$. One of these eigenstates is completely independent of the specific interaction used, provided that it conserves isospin; this is the double analog $|AA\rangle$ of $^{48}$Ca with $T=4$, $T_z=2$ which is given explicitly by

$$|AA\rangle = \left[\frac{2}{2j(2j+1)}\right]^{1/2} \sum_{J=\text{even}} (2J+1)^{1/2} |j^6(n)J;j^2(p)J;0^+\rangle ,$$

with $j=1f_{7/2}$. The other three states, one of which is the ground state, are characterized by $T=2$, $T_z=2$, and their exact nature depends on the details of the interaction. In the present calculations two nuclear interactions were used: one constructed with the bare $G$-matrix elements of Kuo-Lee-Brown,[14] the other a phenomenological[15] set of particle-particle and particle-hole matrix elements deduced from the energy levels of appropriate nuclei around the closed shell.[15,16] The latter takes into account to some extent the effect of shells other than $1f_{7/2}$ shell. The intermediate states need not be eigenstates of the nuclear Hamiltonian. In the present calculation one intermediate state was sufficient, namely the doorway state defined by $|d\rangle = N\vec{Y}^+|\psi_i\rangle$; $N$=normalization constant. In the simple model[17] described above $|d\rangle = |f_{7/2}^7(n)f_{7/2}(p); 1^+\rangle$. The energy $E(d)$ of this state was found to be 4.3 MeV above the ground state, using the Kuo-Brown interaction, which yields $\bar{\Delta} = \Delta_n = 4.0$ MeV. The results of our calculation are presented in Table I. From Table I it is evident that the transition to the $^{48}$Ti ground state is extremely weak. The strongest transition is the one to the third excited $0^+$ $T=2$ state (89.4% of the total strength) and some in the transition to the double analog (8.3%). Both such transitions are, however, energetically forbidden. We also note that the above results do not seem to be crucially dependent on the interaction. The strongest state has an overlap of 90% with the double doorway state defined as follows:

$$|dd\rangle = N\,\mathcal{A}\vec{Y}^+\vec{Y}^+\,|^{48}\mathrm{Ca}\rangle ,$$

where $N$ is a normalization constant and $\mathcal{A}$ is the antisymmetrization operator (the state $|dd\rangle$ is restricted within the $1f_{7/2}$ shell). Strictly speaking, the operator $\mathcal{A}$ is redundant. It is used, however, to emphasize that the wave function $|dd\rangle$ is not just a product of the 1p-1h states $|d\rangle$ introduced earlier. Further examination of the three $^{48}$Ti $0^+$ $T=2$ states indicates that the strong state is composed predominantly (86%) of one of the two seniority $v=4$, $T=2$ states for all interactions used. This $v=4$, $T=2$ state also has an overlap of 0.96 with the highest eigenstate of the schematic Hamiltonian $H_{sc}=\sum_{i<j}(\vec{t}_i\cdot\vec{t}_j)(\vec{\sigma}_i\cdot\vec{\sigma}_j)$, and thus carries the bulk of the strength of the Gamow-Teller operator. Thus it seems that this component of the effective interaction is responsible for pushing up the strength of the double $\beta$-decay Gamow-Teller operator for all calcium isotopes which contain a $v=4$ state within the $1f_{7/2}$ shell (see Table II). This is valid not only for $0^+$ final states, but for $2^+$ as well. As we shall see below in the case of the Te isotopes, the admixture of the double doorway state $|dd\rangle$ in the ground state is very small even when neutrons occupy a number of shells which are not occupied by protons. Even though in this case the schematic interaction mentioned above is not expected to dominate, the small admixture is easily understood. The Gamow-Teller operator cannot change the spatial quantum numbers of the orbits. If the first unfilled proton shell is $j=l+\frac{1}{2}$, the corresponding neutron shell in this case will be deeply bound. If the $j=l-\frac{1}{2}$ neutron shell is also fairly deeply bound (as is the case in Te isotopes)

TABLE I. Nuclear matrix elements leading to the various $0^+$ states of $^{48}$Ti. Since these are off-diagonal matrix elements the signs are not significant. The phenomenological force is isospin violating, so the fourth eigenstate is not exactly the state $|AA\rangle$ defined in the text.

| $^{48}$Ti states | Kuo-Brown force $E_x$ | ME | Phenomenological force $E_x$ | ME |
|---|---|---|---|---|
| $0^+$ g.s. | 0 | −0.130 | 0 | −0.081 |
| $0_2^+$ | 2.3 | 0.311 | 6.1 | 0.157 |
| $0_3^+$ | 6.6 | 2.122 | 8.3 | 2.124 |
| $AA$ | 7.10 | 0.648 | 9.2 | 0.737 |

it costs a lot of energy to change a neutron into a proton and such components are not present in the ground state. If the $j=l-\frac{1}{2}$ neutron shell is not deeply bound, the state $|dd\rangle$ may be somewhat mixed in the ground state. The investigation of this latter case, however, is beyond the aims of the present work.

It seems, therefore, that due to large cancellations among the various components of the $^{48}$Ti wave functions, the double $\beta$ decay of the $A=48$ system is greatly hindered. A similar result has also been found by Khodel,[18] who used Migdal's theory of finite nuclear systems.[19] Since both the Kuo-Brown and the phenomenological effective interaction used in the present paper predict a small nuclear matrix element ($|\mathrm{ME}|^2 \approx 1.7\times 10^{-2}$) we expect this smallness to persist even if the shell model is enlarged. Its precise value, however, cannot be very reliably calculated as is always the case with quantities which are the results of large cancellations. Therefore, contrary to our expectations, if the precise value of the nuclear matrix is essential, the nucleus $^{48}$Ca may not be the best candidate for the study of lepton nonconservation.

### B. Nuclear matrix element for $^{130}\mathrm{Te}\to{}^{130}\mathrm{Xe}$ and $^{128}\mathrm{Te}\to{}^{128}\mathrm{Xe}$

The above nuclei are quite far away from the closed shell nucleus $^{132}$Sn and an exact shell model calculation is out of the question. Therefore one will have to devise an approximation scheme. Since the above two sets of nuclei differ by a pair of neutrons, the two nuclear matrix elements involved are not expected to be very different. As it was mentioned earlier, the above equality of the nuclear matrix elements of the $A=130$ and $A=128$ systems has been used by Primakoff and Rosen[2] to make the analysis of double $\beta$ decay independent of the nuclear structure. One of the main purposes of the present paper is to examine the validity of the above assertion. The shell model appropriate for this problem consists of protons occupying the $2d_{5/2}$, $1g_{7/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ shells and neutron holes in the above orbitals. The number of neutron holes was $n=4, 6, 6, 8$ for $^{130}$Te, $^{128}$Te, $^{128}$Xe, and $^{130}$Xe, respectively. The number of active protons was two for the Te isotopes and four for the Xe isotopes.

The shell model Hamiltonian was constructed from the Kuo-Brown[14] bare $G$-matrix elements for the Zr region and appropriate single particle energies. The single particle energies were determined from the experimental levels of the odd nuclei around the $^{132}$Sn closed shell[20]; i.e., $\epsilon(j_p)=0.0$, 0.30, 3.00, 2.68, and 1.71 for $2d_{5/2}$, $1g_{7/2}$, $1d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ proton orbitals and −3.08, −2.6, −0.58, −0.99, and 0.0 for the neutron hole orbitals. The proton single particle energies were modified due to their interaction with the $n$ neutron holes in the $j'=1h_{11/2}$ shell by an additional energy calculated by

TABLE II. The nuclear double $\beta$-decay matrix element for the calcium isotopes. The last row was obtained using the phenomenological interaction described in the text.

| Transition \ $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $A=42$ $(0^+\to 0_n^+)$ | −0.857 | ... | ... | ... | ... | ... | ... | ... | ... |
| $A=44$ $(0^+\to 0_n^+)$ | −0.116 | −0.290 | 0.505 | 0.350 | ... | ... | ... | ... | ... |
| $A=46$ $(0^+\to 0_n^+)$ | −0.137 | −0.045 | −0.658 | −0.784 | −1.552 | 0.332 | ... | ... | ... |
| $A=48$ $(0^+\to 0_n^+)$ | −0.137 | +0.311 | 2.122 | 0.648 | ... | ... | ... | ... | ... |
| $A=48$ $(0^+\to 2_n^+)$ | 0.555 | 0.000 | 0.641 | 0.000 | 0.336 | 1.000 | 3.120 | 0.000 | 12.570 |
| $A=48$ $(0^+\to 2_n^+)$ | 0.000 | 0.726 | 0.618 | 0.000 | 4.338 | 1.790 | 3.125 | 0.000 | 12.259 |

$$\epsilon(j) = -\frac{n}{2(2j+1)(2j'+1)} \times \sum_{J,T} \langle (jj')JT| V |(jj')JT\rangle (2J+1),$$

using the Kuo-Brown interaction.

Even within the above model space the problem is too complex to tackle and some further approximations are necessary. The following approximations were adopted. (i) The basis states are products of a proton wave function and a neutron wave function. (ii) The neutron wave functions were of the form $1h_{11/2}{}^n (J_n = 0, v = 0)$. The zero seniority approximation tremendously simplifies the calculation and it is expected to be good since it was found that the Kuo-Brown interaction does not cause substantial seniority mixing of this configuration ($\leqslant 3\%$).

With the above approximations the following proton wave functions were obtained for the initial Te nuclei:

$$^{128}\mathrm{Te} \quad \psi_p = 0.9361|2d_{5/2}{}^2\rangle + 0.1798|1g_{7/2}{}^2\rangle + 0.1563|2d_{3/2}{}^2\rangle + 0.0711|3s_{1/2}{}^2\rangle - 0.2489|1h_{11/2}{}^2\rangle ,$$

$$^{130}\mathrm{Te} \quad \psi_p = (0.9315,\ 0.2245,\ 0.1587,\ 0.0701,\ -0.2274),$$

in the above order.

The proton wave functions for the final nuclear states are much more complex. However, to an extremely good approximation (98% of the wave function) they can be represented as follows:

$$^{128}\mathrm{Xe} \quad \psi_p \simeq -0.8928|2d_{5/2}{}^4\rangle + 0.1753|2d_{5/2}{}^2\,1g_{7/2}{}^2\rangle + 0.1879|2d_{5/2}{}^2\,1d_{3/2}{}^2\rangle + 0.0880|2d_{5/2}{}^2 3s_{1/2}{}^2\rangle$$
$$- 0.3166|2d_{5/2}{}^2\,1h_{11/2}{}^2\rangle - 0.0549|1g_{7/2}{}^2\,1h_{11/2}{}^2\rangle - 0.0527|2d_{3/2}{}^2 1h_{11/2}{}^2\rangle ,$$

$$^{130}\mathrm{Xe} \quad \psi_p \simeq (-0.9013,\ 0.2054,\ 0.1923,\ 0.0882,\ -0.2752,\ -0.0552,\ -0.0463),$$

in the previous order.

The intermediate states in the I isotopes need not be eigenstates of the nuclear Hamiltonian. In the present calculation we found it convenient to work with a single state $|d\rangle$, doorway state, which exhausts the entire sum rule for the Gamow-Teller operator. It was defined by $|d\rangle \equiv N\mathcal{A}\vec{Y}^+|\psi_i\rangle$ (the $1h_{9/2}$ proton shell at 8.9 MeV was also included). Again, $N$=normalization and $\mathcal{A}$ is the antisymmetrizer. The energy of this state was found to be $E(d) = 11$ MeV above the ground state of the initial nucleus both for $A = 130$ and 128. The inclusion of the state $|d\rangle$ at $\overline{\Delta} = 11$ MeV is equivalent to using the complete set of intermediate states.

Using the above wave functions and Eq. (7c) we obtained ME = − 0.311 and − 0.355 for $A = 130$ and 128, respectively. The above approximations must be improved, however, before one compares with experiment. Clearly the completely filled neutron shells cannot be completely neglected, since the Gamow-Teller operator can promote such a neutron into a proton. Such effects were taken into account in perturbation theory. For a given initial state $|\psi_i\rangle$ all the double $\beta$ decay strength lies in the state $|dd\rangle$ defined by

$$|dd;0^+\rangle = N\mathcal{A}\vec{Y}^+\vec{Y}^+|\psi_i\rangle .$$

The state $|dd\rangle$, even with the above approximations on the state $|\psi_i\rangle$, is too complex to present here explicitly. It is almost orthogonal to the Xe states listed above and lies at excitation energy $E_x(dd) = 23$ and 24 MeV for $A = 128$ and 130, respectively. It can be connected to the above Xe states via the nuclear interaction. Hence, to first order in perturbation theory,

$$\psi_f \simeq \psi_p \otimes h_{11/2}{}^n(J=0\ \ V=0) + \alpha|dd\rangle ,$$

$$\alpha = \frac{\langle dd|V|\psi_p \otimes h_{11/2}{}^n(J=0\ \ V=0)\rangle}{E_x(dd)} .$$

$\alpha$ was found to be − 0.0120 and − 0.0151 for $^{130}$Xe and $^{128}$Xe, respectively. Although these admixtures are small, their contribution to the double $\beta$ decay is substantial because of the large Gamow-Teller matrix element involved. In fact, their contribution changes the ME values quoted above to ME = − 0.496 and − 0.568 for $A = 130$ and 128, respectively. Thus, our calculation confirms the near equality of the nuclear matrix elements which has been previously asserted.[2,7] These nuclear matrix elements for $A = 130$ and $A = 128$ are much larger than those predicted for the $A = 48$. However, even in the present case, the ground state transitions exhaust only a small fraction of the sum rule (0.10% and 0.15% for $A = 130$

TABLE III. Nuclear matrix elements, life times and lepton nonconservation parameter $\eta$ (for notation see text).

| $A$ | $\epsilon_0$ ($m_e c^2$) | $f_2$ (yr) | $f_4$ (yr) | $T_{1/2}^{\exp}$ (yr) | $\lvert ME \rvert^2$ | $\eta$ |
|---|---|---|---|---|---|---|
| 48 | 8.6 | $4.0 \times 10^{12}$ | $7.94 \times 10^{18}$ | $\geq 2 \times 10^{21}$ [a] | $1.2 \times 10^{-2}$ | $\leq 4 \times 10^{-4}$ |
| 130 | 5.0 | $3.16 \times 10^{13}$ | $6.64 \times 10^{20}$ | $2.0 \times 10^{21}$ | 0.25 | $2.8 \times 10^{-4}$ |
| 128 | 1.7 | $4.0 \times 10^{15}$ | $4.10 \times 10^{24}$ | $1.5 \times 10^{24}$ | 0.32 | $0.86 \times 10^{-4}$ |

[a] For neutrinoless mode.

and $A = 128$, respectively). The state $|dd\rangle$, which carries the bulk of the strength, is energetically inaccessible.

## IV. LEPTON NONCONSERVATION IN DOUBLE $\beta$ DECAY

As was mentioned earlier, mass-spectroscopic examination of geologically old ores has shown rather convincinbly that double $\beta$ decay takes place. Of the experiments which attempted to measure the sum of the two charged leptons in coincidence, we mention again the work of the Columbia group in the double $\beta$ decay of $^{48}$Ca.[9] Even in their experiment, the most refined executed so far, the counting rate was extremely low and, due to the background radioactivity, only lower limits for the half-lives could be obtained, viz., $T_{1/2}^{(2)} > 2 \times 10^{21}$ yr for neutrinoless double $\beta$ decay (at the 80% confidence level), and $T_{1/2}^{(4)} > 3.6 \times 10^{19}$ yr for the two neutrino mode of $^{48}$Ca double $\beta$ decay. In the case of Te isotopes, a number of mass-spectroscopic measurements performed in geologically old telluride ores exist.[5,6,8] The measured half-lives are $T_{1/2}(130) = 1.0 \times 10^{21}$ yr and $T_{1/2}(128)$[6] $= 1.5 \times 10^{24}$ yr[8], while the measured value of the ratio of the half-life of $^{128}$Te to that of $^{130}$Te is $R = (1.59 \pm 0.05) \times 10^3$.[8]

From the above experimental information and the previously calculated nuclear matrix elements the lepton nonconservation parameter $\eta$ can be obtained from Eq. (6b) in the case of $^{48}\text{Ca} \to {}^{48}\text{Ti}$ double $\beta$ decay, or from Eq. (8) in the case of Te → Xe. The effective phase space quantities $f_2$ and $f_4$ were obtained from the work of Primakoff and Rosen[2,12], explicitly, $f_2(48) = 4.0 \times 10^{12}$ yr, $f_4(48) = 7.94 \times 10^{18}$ yr, $f_2(128) = 4 \times 10^{15}$ yr, $f_4(128) = 4.10 \times 10^{24}$ yr, $f_2(130) = 3.16 \times 10^{13}$ yr, and $f_4(130) = 6.64 \times 10^{20}$ yr. The $f_4$ functions were somewhat modified from those of Primakoff[12] to take into account the average energy of the intermediate states as predicted by our calculation. The values of $\eta$ obtained are $2.8 \times 10^{-4}$, $0.86 \times 10^{-4}$, and $\leq 4 \times 10^{-4}$ for $A = 130$, 128, and 48, respectively. Thus, if neutrinoless $\beta$ decay does take place, it proceeds via a relatively small lepton nonconserving amplitude in the weak Hamiltonian.

The calculated nuclear matrix element of the Gamow-Teller operator involved in the double $\beta$ decay of the Te isotopes is fairly reliable. Unlike the case of $^{48}$Ca, all contributions to the nuclear double $\beta$-decay matrix elements, arising from the different pieces of the wave functions, are of the same sign. The ratio of the nuclear matrix elements, i.e., $|ME(128)/ME(130)|^2 = 1.3$, is expected to be even more accurate. The parameter $\eta$ can be obtained more reliably from this ratio and the experimental ratio of the half-lives. The result is $\eta = 0.45 \times 10^{-4}$, which is in excellent agreement with the values obtained above. This gives a further indication that the nuclear matrix elements are reliable. The results of our calculations are summarized in Table III.

## V. CONCLUSIONS

We have calculated the nuclear matrix elements for the double $\beta$ decay of $^{48}$Ca, $^{130}$Te, and $^{128}$Te considered as a second-order weak process involving the allowed $\beta$ decay of two nucleons. From these nuclear matrix elements and the experimentally observed lifetimes a lepton nonconservation parameter $\eta \lesssim 10^{-4}$ was found. Although the smallness of $\eta$ is rather well established, its precise value is somewhat uncertain. This is due partly to the uncertainties involved in the nuclear matrix elements and partly to the approximation of the effective phase space quantities $f_4$ of Primakoff and Rosen.[2] The latter arises because $f_4/|ME|^2$ is close to the experimentally observed half-lives and, therefore, the results are sensitive to the difference between them.

We also note that, in general, the contributions of the various forbidden $\beta$ decays of the two nucleons, proceeding via intermediate states in the $(N-1, Z+1)$ system, may have to be considered.

However, in the nuclei considered here such contributions are expected to be small. All the above effects cannot change our basic conclusions about the smallness of $\eta$, which implies that lepton conservation is consistent with the experimental information thus far. Finally, the smallness of $\eta$ seems to imply that lepton nonconservation need not have anything to do with $CP$ violation. Such relationship, conjectured by Primakoff and Sharp,[21] implies $\eta \simeq 10^{-3}$.

The author is greatly indebted to Professor Henry Primakoff for encouragement, suggestions, useful discussions, and a careful reading of the manuscript.

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*Work supported in part by the U. S. Energy Research and Development Administration.

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