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Theoretical investigations of the yrast states of even erbium isotopes

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An angular momentum projection formulation in conjunction with the many-body Nilsson-BCS variational wave function is presented for the pairing plus quadrupole interaction Hamiltonian. The theory is applied to calculate the energies and half-lives of the recently observed high spin yrast states in ^{156,158,160,162}Er and ^{156}Dy. The results of the calculation are in good agreement with the corresponding experimental data.

[NUCLEAR STRUCTURE Er isotopes; calculated energy levels, $B(E2)$. Variation after projection method. Pairing plus quadrupole interaction. Derived simplified analytic expressions.]

I. INTRODUCTION

During the last few years, there has been a growing interest in a systematic γ-ray decay study of high spin states populated in the heavy ion reactions in heavier nuclei, especially in the rare-earth nuclei. In some of the investigations, the $B(E2)$ values for the cascading γ transitions are also measured besides their energies.[1] There is a large amount of accurate experimental data available in nuclei of this mass region. One knows that neither the many-body theory nor the nuclear forces are understood well enough so as to calculate, from first principles, the nuclear energy levels within an accuracy of a hundred keV. In such circumstances, it is but natural that the first attempts would be based on phenomenological macroscopic approaches aimed at fitting the observed energy spectra. The broad characteristic of these spectra is the increase of the moment of inertia $\mathcal{I}$ with the angular momentum J and, in particular, the stiff increase in $\mathcal{I}$ at the critical angular momentum $J_c \approx 14$. This anomalous behavior of $\mathcal{I}$ with J was first predicted by Mottelson and Valatin[2] as a phase transition from the superfluid to the normal nuclear state. In this phase transition approach, however, the value of J_c comes out to be much larger than the observed value. The phenomenological variable moment of inertia model[3] gives a quantitative agreement with a part of the data. However, this model does not clearly bring out the physics behind the observed "back-bending" phenomenon. The anomalous back-bending phenomenon was qualitatively explained, in an alternative approach, in terms of a band crossing[4] in which the band intersecting the ground band becomes the yrast band for higher spin states. Other attempts based on a semimicroscopic deformed nuclear model[5] with pairing interaction, were quite successful in explaining the general features of the yrast levels in a few selected nuclei. However, in almost all the above attempts, one could not quantitatively explain the observed data over a wide range of rare-earth nuclei without fitting a few relevant parameters in each nucleus. The recent attempts based on variational many-body methods resort to simplifying approximations[6] regarding the angular momentum conservation. These approximations can easily introduce errors of the same order of magnitude as the observed energy differences to be explained. The many-body variational calculations with good angular momentum, reported so far, are very few.[7,8] We have, however, not noticed any published work on the calculation of $B(E2)$ transition probabilities (except that in Ref. 8) which would provide an additional test for the validity of the theory. The purpose of this paper is to present the simplified formulas for the relevant physical quantities in the microscopic theory and to find out whether it can successfully explain the observed spectra and the $B(E2)$ values of the electromagnetic transitions between the high spin yrast states. This may provide some insight into our understanding of the mechanism responsible for the back-bending effect. The success would enhance our confidence in this variational approach and may develop further interest in extending such investigations to still heavier nuclei.

In Sec. II we derive the mathematical formulas in their most suitable form for the purpose of numerical calculations. The application of this theory to the even erbium isotopes and ^{156}Dy is pre-

sented in Sec. III. The discussion of the results is also given in the same section. The conclusions are summarized in Sec. IV.

II. DERIVATION OF FORMULAS

In heavier nuclei where the level spacing is smaller, one has to treat many nucleons dynamically active in a large enough configuration space. One can certainly cope with such a situation within the constrained Hartree-Fock (HF) variational approach[6] wherein the angular momentum is not conserved. The construction of good angular momentum states by the projection method is met with numerical complexities[9] and this is the main reason why such calculations are not carried out in heavy nuclei. In what follows, we will first briefly review a general formulation of the angular momentum projection method and then indicate the subsequent simplifications possible in the case of a simpler nucleon-nucleon interaction. Let the Hamiltonian of the system be

$$H=\sum(\alpha|T|\beta)a_\alpha^\dagger a_\beta+\frac{1}{4}\sum V_{\alpha\beta\gamma\delta}a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma\,, \tag{1}$$

where T is the kinetic energy operator and $V_{\alpha\beta\gamma\delta}$ is the antisymmetrized matrix element of the nucleon-nucleon interaction. Quite extensive investigations of the ground-state properties of even-even heavy nuclei are carried out by Kumar and Baranger[10] by employing a simpler quadrupole plus pairing interaction between nucleons. The parameters of the Hamiltonian of such a many-body nuclear system are well studied by them. In the numerical calculations presented in this paper, we will resort to their Hamiltonian in the form

$$\begin{aligned}H=&\sum\epsilon_\alpha a_\alpha^\dagger a_\alpha\\&-\frac{1}{2}\chi\sum(\alpha|q^{2\mu}|\gamma)(\delta|q^{2\mu}|\beta)a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma\\&-\frac{1}{2}G\sum(-1)^{j_\alpha-m_\alpha+j_\gamma-m_\gamma}a_\alpha^\dagger a_{\bar\alpha}^\dagger a_{\bar\gamma}a_\gamma\,,\end{aligned} \tag{2}$$

where $q^{2\mu}$ is the quadrupole operator and χ and G are the strengths of the quadrupole and pairing interactions, respectively. The subscript α in Eqs. (1) and (2) denotes all the quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ necessary for the specification of a spherical single-particle state. The sums in Eqs. (1) and (2) run over the entire configuration space considered in the calculations. In actual numerical investigations, we will employ the same configuration space as that in Ref. 10.

We consider only axially symmetric intrinsic shapes since the rare-earth nuclei under investigation are found[10,11] to prefer axially symmetric equilibrium deformations. Besides this point, the numerical complexity is a great hurdle in including other degrees of freedom. The trial variational nuclear wave function is the projected good angular momentum (J) state obtained from the Nilsson-BCS intrinsic state $\Phi_K(\beta, \Delta_p, \Delta_n)$ of the A-nucleon system. The deformation parameter β and the proton and neutron pairing gaps Δ_p and Δ_n, respectively, are the variational parameters for each J. For axially symmetric intrinsic states of even-even nuclei, the band quantum number is $K=0$. In the second quantized notation, one can express the Nilsson-BCS intrinsic state for even-even axially symmetric nuclei by

$$\Phi_0(\beta,\Delta_p,\Delta_n)=\prod_{i+}(u_i+v_i b_{i+}^\dagger b_{i-}^\dagger)|0\rangle\,. \tag{3}$$

The gap parameters Δ_p and Δ_n are related to u_i and v_i, where $v_i^2=1-u_i^2$ is the occupation probability for the ith deformed single-particle state and the fermion operator $b_i^\dagger$ is obtained from the corresponding spherical state operators $a_\alpha^\dagger$ by the transformation

$$b_{i\pm}^\dagger=\sum(\alpha\pm|C|i\pm)\,a_{\alpha\pm}^\dagger\,. \tag{4}$$

It should be noted that the basis states are divided into two subsets that are connected by a time-reversal operator T:

$$T|\alpha+\rangle=(-1)^{l_\alpha+j_\alpha-m_\alpha}|\alpha-\rangle\,, \tag{5}$$

where

$$|\alpha\pm\rangle=|a\pm m_\alpha\rangle=|n_\alpha, j_\alpha, l_\alpha, \pm m_\alpha\rangle\,.$$

The matrix representation of the operator T is

$$\begin{aligned}&\langle\alpha\pm|T|\beta\pm\rangle=0\,,\\&\langle\alpha\pm|T|\beta\mp\rangle=(-1)^{l_\beta+j_\beta\pm m_\beta}\delta_{\alpha\beta}\,.\end{aligned} \tag{6}$$

The transformation coefficients in Eq. (4) satisfy the following relations.

$$\begin{aligned}&(\alpha-|C|i-)=(-1)^{l\alpha+j_\alpha-m_\alpha}(\alpha+|C|i+)\,,\\&C\tilde C=\tilde C C=1\,,\end{aligned} \tag{7}$$

where $\tilde C$ is the transpose of matrix C.

The expectation value $E^J(\beta,\Delta_p,\Delta_n)$ of the Hamiltonian in Eq. (1) with respect to the projected wave function of angular momentum J,

$$\Psi_M^J(\beta,\Delta_p,\Delta_n)=P_{M0}^J\Phi_0(\beta,\Delta_p,\Delta_n)\,, \tag{8}$$

can be expressed as

$$E^J(\beta,\Delta_p,\Delta_n)=h^J(\beta,\Delta_p,\Delta_n)/p^J(\beta,\Delta_p,\Delta_n)\,, \tag{9}$$

where

$$h^J(\beta, \Delta_p, \Delta_n) = \langle \Psi^J_M | H \Psi^J_M \rangle = \frac{(2J+1)}{8\pi^2} \int D^J_{00}(\Omega) \langle \Phi_0 | H | R(\Omega) \Phi_0 \rangle \tag{10}$$

and $p^J(\beta, \Delta_p, \Delta_n)$ is obtained from Eq. (10) by replacing H by 1. It can be easily seen that Eq. (10) is obtained from Eq. (8) by expressing[12] the angular momentum projection operator P^J_{M0} in terms of the rotation operator $R(\Omega)$ corresponding to the rotation through Euler angles $\Omega = (\Omega_1, \Omega_2, \Omega_3)$. The calculation of E^J thus essentially reduces to the evaluation of the matrix elements $\langle \Phi_0 | H | R\Phi_0 \rangle$ and $\langle \Phi_0 | R\Phi_0 \rangle$. It can be shown[13] that

$$\langle \Phi_0 | R\Phi_0 \rangle = A^2 [\det(1+M)]^{1/2}, \tag{11}$$

where

$$A = \prod_{i+} u_i, \quad M = F\tilde{f}^*. \tag{12}$$

The matrices f and F are given by

$$f = CO\tilde{C}\tilde{T}, \quad F = Df\tilde{D}. \tag{13}$$

The diagonal matrix O in Eq. (13) has elements $O_{ij} = (v_i/u_i)\delta_{ij}$ in the deformed basis. The matrices f, F, and D are defined in the spherical basis with the rotation matrix

$$(\alpha | D | \beta) = \delta_{ab} D^{j\alpha}_{m_\alpha m_\beta}(\Omega). \tag{14}$$

It can easily be verified that

$$\tilde{f} = -f, \quad \tilde{F} = -F$$
$$f^*\left(\frac{1}{1+M}\right) = \left(\frac{\tilde{1}}{1+M}\right) f^*, \tag{15}$$
$$F\left(\frac{1}{1+M}\right) = \left(\frac{\tilde{1}}{1+M}\right) F.$$

After carrying out some algebra with the help of the relations in Eqs. (12)–(15), one finally obtains

$$\langle \Phi_0 | R | \Phi_0 \rangle = [\det W]^{1/2}, \tag{16}$$

where the matrix W is given by

$$W = U\tilde{C}\tilde{D}^*CU + V\tilde{C}\tilde{D}CV. \tag{17}$$

Here U and V are the diagonal matrices with

$$U_{ij} = u_i \delta_{ij}, \quad V_{ij} = v_i \delta_{ij}. \tag{18}$$

In order to calculate $\langle \Phi_0 | H | R\Phi_0 \rangle$, one has to evaluate the matrix elements $\langle \Phi_0 | a^\dagger_\alpha a^\dagger_\beta | R\Phi_0 \rangle$ and $\langle \Phi_0 | a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma | R\Phi_0 \rangle$. Let us introduce a shorthand notation

$$\langle \hat{O} R \rangle = \langle \Phi_0 | \hat{O} | R\Phi_0 \rangle / \langle \Phi_0 | R\Phi_0 \rangle \tag{19}$$

for any operator $\hat{O}$. For the fermion operators, it can be proved that

$$\langle a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma R \rangle = \langle a^\dagger_\alpha a^\dagger_\beta R \rangle \langle a_\delta a_\gamma R \rangle + \langle a^\dagger_\alpha a_\gamma R \rangle \langle a^\dagger_\beta a_\delta R \rangle - \langle a^\dagger_\alpha a_\delta R \rangle \langle a^\dagger_\beta a_\gamma R \rangle. \tag{20}$$

The generalized density matrices appearing in Eq. (20) can easily be expressed in terms of the matrices f, F, and M.

$$\rho_{\beta\alpha} = \langle a^\dagger_\alpha a_\beta R \rangle = [M(1+M)^{-1}]_{\beta\alpha},$$
$$\sigma_{\alpha\beta} = \langle a^\dagger_\alpha a^\dagger_\beta R \rangle = [f^*(1+M)^{-1}]_{\alpha\beta}, \tag{21}$$
$$\sigma'_{\beta\alpha} = \langle a_\alpha a_\beta R \rangle = [(1+M)^{-1}F]_{\beta\alpha}.$$

After carrying out a straightforward but rather laborious algebra, one finally obtains for the Hamiltonian operator in Eq. (1),

$$\langle HR \rangle = {\sum_i}' \{ [2(T + \tfrac{1}{2}X)\rho]_{i+i+} - [Y'\sigma]_{i+i+} \}, \tag{22}$$

where the prime on the summation indicates that the sum in Eq. (22) is over only one subset of states and

$$X_{ij} = \sum_{kl} V_{ikjl} \rho_{lk},$$
$$Y'_{ij} = \frac{1}{2} \sum_{kl} V_{ijkl} \sigma'_{lk}. \tag{23}$$

The generalized density matrices in the deformed basis are given by

$$\rho_{ji} = \langle b^\dagger_i b_j R \rangle = \langle j | 1 - UW^{-1}U\tilde{C}\tilde{D}^*C | i \rangle,$$
$$\sigma_{ij} = \langle b^\dagger_i b^\dagger_j R \rangle = \langle i | \tilde{C}D^*CU\tilde{W}^{-1}V\tilde{C}\tilde{T}C | j \rangle, \tag{24}$$
$$\sigma'_{ji} = \langle b_i b_j R \rangle = \langle j | UW^{-1}V\tilde{C}\tilde{D}C\tilde{C}\tilde{T}C | i \rangle.$$

The matrix elements of the density matrices can be easily evaluated to obtain the following relations necessary in numerical computations.

$$\rho_{j+i+} = \rho_{j-i-} = \delta_{ij} - \langle j+ | Z_1 | i+ \rangle,$$
$$\rho_{j+i-} = -\rho_{j-i+} = \langle j+ | Z_1 | i- \rangle,$$
$$\sigma_{i+j+} = \sigma_{i-j-} = \langle j- | Z_2 | i+ \rangle,$$
$$\sigma_{i+j-} = -\sigma_{j-i+} = \sigma_{j+i-} = \langle j+ | Z_2 | i+ \rangle, \tag{25}$$
$$\sigma'_{j+i+} = \sigma'_{j-i-} = \langle j- | Z_3 | i+ \rangle,$$
$$\sigma'_{j+i-} = -\sigma'_{i-j+} = \sigma'_{i+j-} = \langle j+ | Z_3 | i+ \rangle.$$

Here, the matrices Z_1, Z_2, Z_3 are given by

$$Z_1 = UW^{-1}U\tilde{C}\tilde{D}^*C,$$
$$Z_2 = VW^{-1}U\tilde{C}\tilde{D}^*C, \tag{26}$$
$$Z_3 = UW^{-1}V\tilde{C}\tilde{D}C.$$

It can be easily checked that, for a special case $R(\Omega=0) = 1$, one obtains the usual Nilsson-BCS results for ρ, σ, σ', and $\langle HR \rangle$.

In the case of the nuclear Hamiltonian of Eq. (2) with quadrupole and pairing interactions, the expression for $\langle HR\rangle$ in Eq. (22) can be considerably simplified if one makes the usual assumptions[10] of neglecting the contribution of the pairing interaction to the field producing potential X in Eq. (23), the contribution of quadrupole interaction to the pairing potential Y' in Eq. (23), and the contribution from the exchange term of the quadrupole interaction. These assumptions were examined in detail and the validity of the three approximations was found[14] to be quite good. With these approximations, the expression for $\langle HR\rangle$ simplifies to

$$\langle HR\rangle = 2\sum_i{}' \epsilon_i \rho_{i+i+} - G\eta^2 - \tfrac{1}{2}\chi[(Q_+^0)^2 + 2(Q_-^1)^2 + 2(Q_-^2)^2], \tag{27}$$

where

$$Q_\pm^\mu = \sum_{kl}{}' (q^{2\mu}_{l+k\pm} + q^{2\mu}_{k\pm l+})\rho_{l+k\pm},$$
$$\eta = \sum_k{}' \sigma_{k+k-}. \tag{28}$$

The expression for the $B(E2; J_i \to J_f)$ value of the electromagnetic transition from a state with angular momentum J_i to that of angular momentum J_f can also be easily derived[15] and in the case of even-even axially symmetric nuclei, one finally obtains

$$B(E2; J_i \to J_f) = \frac{(2J_f+1)}{(2J_i+1)}(p^{J_i} p^{J_f})^{-1}[Q(J_i \to J_f)]^2, \tag{29}$$

where

$$Q(J_i \to J_f) = \sum_{\gamma=0}^{2} (2 - \delta_{\gamma 0})(J_i 0, 20|J_f 0) Q_\gamma^{J_i}. \tag{30a}$$

Here $(J_i 0, 20|J_f 0)$ is the Clebsch-Gordan coefficient and

$$Q_\gamma^J = (2J+1)\int_0^{\pi/2} d^J_{-\gamma 0}(\theta)(Q_+^\gamma + Q_-^\gamma) \times [\det W(\theta)]^{1/2} \sin\theta\, d\theta. \tag{30b}$$

The necessary quantities $Q_\pm^\gamma$ entering into this expression are given by Eq. (28).

III. RESULTS AND DISCUSSION

The nuclear structure calculations in even-even rare-earth nuclei reported in this paper are performed by employing the variational wave functions projected from the Nilsson-BCS intrinsic state $\Phi_0(\beta, \Delta_p, \Delta_n)$ and minimizing the projected energy $E^J(\beta, \Delta_p, \Delta_n)$ obtained from Eqs. (9), (10), (16), and (27) by varying the deformation parameter β and proton and neutron pairing gaps Δ_p and Δ_n for each angular momentum state J. We have employed the Kumar-Baranger[10] nuclear Hamiltonian H with the long-range quadrupole interaction and the short-range pairing interaction between nucleons. The same configuration space as specified by them has been considered in the present calculations. It should be mentioned here that the inert core[10] of $Z=40$ and $N=70$ necessitates the introduction of the core contribution to the nucleon charges and the moment of inertia $\mathcal{I}$. As is the standard practice,[10,15] we replace bare nucleon charges by effective charges to simulate core polarization. In the same manner, core-polarization effects are incorporated in $\mathcal{I}$ by introducing a multiplicative constant

$$f = E^J_{\text{correct}}/E^J_{\text{out}},$$

where E^J_{out} is the energy computed by considering only the valence nucleons outside the inert core

TABLE I. The calculated and experimental values of the γ-transition energy $(E^J - E^{J-2})$ (in MeV) are displayed for ^{156,158,160,162}Er and ^{156}Dy. The tabulated experimental energies are accurate only up to the second decimal place.

	^{156}Er		^{158}Er		^{160}Er		^{162}Er		^{156}Dy	
J	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory
2^+	0.34	0.20	0.19	0.16	0.13	0.12	0.10	0.11	0.14	0.14
4^+	0.45	0.42	0.34	0.36	0.26	0.26	0.23	0.24	0.27	0.29
6^+	0.54	0.56	0.44	0.50	0.38	0.40	0.34	0.34	0.37	0.39
8^+		0.63	0.52	0.63	0.46	0.48	0.43	0.45	0.45	0.52
10^+		0.72	0.58	0.52	0.53	0.55	0.51	0.49	0.51	0.43
12^+		0.79	0.61	0.55	0.58	0.58	0.56	0.52	0.56	0.51
14^+			0.51	0.42	0.59	0.56	0.58	0.55	0.60	0.57
16^+			0.47	0.34	0.53	0.54	0.55	0.53	0.61	0.60
18^+					0.56	0.54				0.67
20^+										

TABLE II. The content p^J of the angular momentum state J in the intrinsic Nilsson-BCS state Φ_0 is displayed for all the nuclei under consideration.

J	^{156}Er	^{158}Er	^{160}Er	^{162}Er	^{156}Dy
0^+	0.0485	0.0348	0.0245	0.0238	0.0284
2^+	0.2068	0.1559	0.1134	0.1104	0.1297
4^+	0.2597	0.2173	0.1710	0.1675	0.1899
6^+	0.2182	0.2117	0.1875	0.1852	0.1991
8^+	0.1412	0.1641	0.1691	0.1689	0.1691
10^+	0.0743	0.1066	0.1313	0.1328	0.1222
12^+	0.0329	0.0597	0.0899	0.0922	0.0776
14^+	0.0127	0.0294	0.0551	0.0573	0.0442
16^+	0.0042	0.0129	0.0306	0.0322	0.0228
18^+		0.0051	0.0155	0.0114	0.0105
20^+		0.0019	0.0050	0.0049	0.0040

and E^J_{correct} is the energy corrected for the core deformation caused by outer nucleons. One can get a good estimate of f from

$$f=\frac{\mathcal{I}_{\text{out}}}{\mathcal{I}_{\text{out}}+\mathcal{I}_{\text{core}}}\approx 1-\frac{\mathcal{I}_{\text{core}}}{\mathcal{I}_{\text{out}}},$$

where $\mathcal{I}_{\text{out}}$ is the moment of inertia calculated with only the "outer" nucleons and $\mathcal{I}_{\text{core}}$ ($\mathcal{I}_{\text{core}}<\mathcal{I}_{\text{out}}$) is the contribution of the core to the moment of inertia. In the numerical computations reported in this paper, we have assumed that f is constant for all angular momentum states and its value is fixed by renormalizing the calculated energy E^J_{calc} with the corresponding experimental energy E^J_{expt} for a particular J. In particular, we have chosen

$$f\approx(E^{J=4}-E^{J=0})_{\text{expt}}/(E^{J=4}-E^{J=0})_{\text{calc}}$$

so as to approximately reproduce the excitation energy of $J=4$ state.

It may be stated here that, at the minimum of energy for each angular momentum state, the exact conservation of nucleon number does not change the results obtained from the BCS wave functions provided, the chemical potentials are so calculated as to conserve the average nucleon numbers to a high degree of accuracy. The fluctuations from the actual nucleon numbers are found to be very small (less than 0.01%) at the energy minimum for each angular momentum state.

The calculated energy spectra of ^{156,158,160,162}Er and ^{156}Dy are displayed in Table I. A comparison of the experimental and calculated energies in Table I indicates that the accuracy in calculated energy is quite good, the deviations in almost all cases being within 100 keV. As mentioned above, we have employed an empirical numerical factor f, constant for all J states, to simulate the effect of the core on the energy spectrum. The value of f goes on decreasing from $f=1.00$ in ^{156}Er, $f=0.95$ in ^{158}Er, $f=0.80$ in ^{160}Er to $f=0.72$ in ^{162}Er, as the number of neutrons and consequently the deformation β increases from $\beta=0.23$ in ^{156}Er to $\beta=0.32$ in ^{162}Er. This decrease in the value of f with the increase in deformation β can be qualitatively understood in view of the increase in $\mathcal{I}_{\text{core}}$ due to the addition of outer nucleons which enhance the deformation of the core. The calculated energy spectra in ^{156}Dy and ^{160,162}Er agree very well with the corresponding experimental data. This good agreement can be correlated to the fact that the different yrast states in these three nuclei have, as seen from Table II, an appreciable content P^J in the intrinsic Nilsson-BCS state from which they are projected. With the reduction in deformation β in ^{156,158}Er, however, the single Nilsson-BCS state ceases to be a good variational state for the yrast levels, particularly for those with high angular momentum. This can be seen from Table II, by observing a rapid decrease of the content p^J of these high J states and consequently, the agreement in ^{156,158}Er is not as good as in ^{160,162}Er and ^{156}Dy.

It may be stressed here that the deformation parameter β changes very slowly with J except at $J_c\approx 14$ where it changes abruptly from 0.27 to 0.24 in ^{158}Er, from 0.32 to 0.30 in ^{160,162}Er, and from 0.30 to 0.34 in ^{156}Dy. This characteristic feature of a sudden change of β at $J_c\approx 14$ may be correlated to the experimentally observed[1] back bending in ^{158,160,162}Er and upward bending in ^{156}Dy at around the same critical angular momentum. Similar observation was made in our earlier investigation of Dy isotopes.[8] We also find that the pairing gaps Δ_p and Δ_n decrease with increasing J. They, however, do not vanish at $J_c\approx 14$. This behavior indicates that the back-bending phenomenon at $J_c\approx 14$ in these nuclei is not due to a phase transition from the superfluid to the normal intrinsic state of the nucleus. The neutron gap Δ_n vanishes at $J_c\approx 20$ in ^{158,160,162}Er and this Mottelson-Valatin phase transition may be correlated with the experimentally observed[1] forward bending in ^{162}Er at around this critical angular momentum $J_c\approx 20$.

The $B(E2)$ values given by Eq. (29) are calculated from the wave functions projected from the intrinsic state which minimizes the energy of the $J=0$ ground state of a nucleus under consideration. The only parameters used in the calculation of reduced transition matrix elements are the effective charges, e_p for protons and e_n for neutrons. In the calculations presented here, we have used the effective charges $e_p=1.53e$ and $e_n=0.53e$ which are slightly less than those quoted by Kumar and

TABLE III. The calculated and experimental half-lives (in psec) for γ transitions between the states $J \rightarrow J-2$ are exhibited.

	^{156}Er		^{158}Er		^{160}Er		^{162}Er		^{156}Dy	
J	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory	Expt.	Theory
2^+	33.2 ± 1.7	23.2	300 + 15	318.5	2156 ± 46	2141	···	5352	···	1623
4^+	5.42 ± 0.66	4.31	14.4 ± 0.72	13.7	34.5 ± 1.7	39.8	···	71.5	···	42.5
6^+	1.14 ± 0.66	1.62	2.8 ± 0.47	3.05	5.39 ± 0.47	6.82				
8^+		1.64	1.21 ± 0.47	1.25	2.16 ± 0.47	2.51				
10^+			0.8 ± 0.4	0.72	1.24 ± 0.47	1.18				
12^+			< 0.7	0.55		0.54				
14^+			2.1 ± 0.5	1.29						
16^+			1.7 ± 0.6	1.83						
18^+			< 1.5	0.73						

Baranger.[10] For comparison with the experimental data, we have, in Table III, displayed the half-lives $\tau_{1/2}$ of various states in the nuclei under consideration. The corresponding $B(E2)$ values can be obtained from the relation

$$B(E2) = 568/(\tau_{1/2} E_\gamma^{\,5}),$$

where $\tau_{1/2}$ is in psec, E_γ is in MeV, and $B(E2)$ in $e^2\,\mathrm{fm}^4$. As seen from Table III, the calculated half-lives of the nuclear states are in good agreement with the corresponding experimental values[1] except for the 2^+ state in ^{156}Er. The tabulated experimental half-life of the 2^+ state in ^{160}Er is extracted from the experimental[16] $B(E2)$ value since the reported[1] experimental half-lives include decay processes other than electromagnetic transitions.

IV. CONCLUSION

An explicit formulation of the angular momentum projection calculations in connection with the many-body Nilsson-BCS variational wave function is presented. The essential simplification of the results in the case of quadrupole plus pairing interactions is worked out. The theory is applied to calculate the energies and half-lives of the nuclear states in ^{156}Dy and ^{156,158,160,162}Er. The results of our calculations are in good agreement with the corresponding experimental data. The characteristic feature of a sudden change in the deformation parameter β at the critical angular momentum $J_c \approx 14$ in ^{158,160,162}Er may be correlated to the experimentally observed[1] back bending in these nuclei at around the same critical angular momentum. The pairing gaps Δ_p and Δ_n, though decreasing with increasing J, do not vanish at $J_c \approx 14$, thereby indicating that the observed back bending is not due to a phase transition from the superfluid to the normal intrinsic state in these nuclei. The calculated pairing gap Δ_n vanishes at $J_c \approx 20$ in ^{162}Er and this may be associated with the experimentally observed[1] forward bending in ^{162}Er at around the same critical angular momentum.

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