PHYSICAL REVIEW C VOLUME 13, NUMBER 2 FEBRUARY 1976

Pion charge exchange on ^{3}H†

A. T. Hess and B. F. Gibson
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545
(Received 8 September 1975)

We have examined the reaction $^3\mathrm{H}(\pi^+, \pi^0)^3\mathrm{He}$ using separable pion-nucleon t matrices in a formalism which incorporates multiple scattering to all orders. Spin-flip effects are shown to be important. The total cross section has a maximum of about 7 mb in the neighborhood of the (3, 3) resonance.

[NUCLEAR REACTIONS $^3\mathrm{H}(\pi^+, \pi^0)$, $E=50$–250 MeV; multiple-scattering theory; separable pion-nucleon t matrices; spin flip; $\sigma(\theta)$ and $\sigma(E)$.]

I. INTRODUCTION

Recently there has been considerable interest in the pion-nucleus charge exchange reaction as a means of examining details of nuclear structure.[1-8] One such reaction which has merited extensive study[3-8] is $^{13}\mathrm{C}(\pi^+,\pi^0)^{13}\mathrm{N}$, a pure isobaric analog transition. The difficulty in achieving reasonable agreement between theory and experiment has led several authors[9,10] to consider a second analog transition: $^3\mathrm{H}(\pi^+,\pi^0)^3\mathrm{He}$ [or the isospin-reflected $^3\mathrm{He}(\pi^-,\pi^0)^3\mathrm{H}$]. This reaction, unlike the ^{13}C reaction in which activation techniques were used in the absence of a π^0 detector, permits one to obtain angular distributions by observing the recoiling residual nucleus. The additional experimental information contained in the angular distributions should help clarify some of the questions that relate to charge-exchange analog transitions.

Previous calculations of the $^3\mathrm{H}(\pi^+,\pi^0)^3\mathrm{He}$ reaction were carried out using the Glauber multiple-scattering formalism.[11] Application of this approximation to charge-exchange reactions is questionable, due to the lack of strong forward peaking in the basic pion-nucleon charge-exchange amplitude, an assumption inherent in the Glauber formalism.

In the present calculation, we use a formalism free from the small-angle, forward-peaked Glauber approximations. This formalism is described briefly in Sec. II. Our numerical results for both the differential and total cross sections are presented and discussed in Sec. III. Our conclusions are summarized in Sec. IV.

II. FORMALISM

A complete description of Gibb's multiple-scattering theory can be found in Ref. 12. We review the relevant results here. The pion-nucleon t matrix is assumed to have the form[13-16] in the laboratory frame

$$\langle \vec{q}\,'|t(\omega)|\vec{q}\rangle = \lambda_0(\omega)v_0(q)v_0(q') + \lambda_1(\omega)\vec{q}\cdot\vec{q}\,'v_1(q)v_1(q'), \quad (1)$$

where $\omega = (\kappa^2+\mu^2)^{1/2}$ is the pion energy in the π-N center of mass frame and μ is the pion mass. The factor

$$\lambda_l(\omega) = \frac{2l+1}{2i\kappa}\,\frac{\exp[2i\delta_l(\omega)]-1}{k^{2l}} \quad (2a)$$

where k is the pion laboratory momentum, ensures that the t matrix has the right on-shell dependence.[17] The functions

$$v_l(q) = \frac{k^2+\alpha_l^{\,2}}{q^2+\alpha_l^{\,2}} \quad (2b)$$

describe the off-shell extension of the t matrix and go to unity on shell ($q\to k$) as they should. The parameters α_l have been determined previously from fits to π-deuteron absorption to be approximately[18] $\alpha_0 = 500$ MeV/c, $\alpha_1 = 300$ MeV/c. This t matrix is then inserted into the self-consistent set of multiple scattering equations of the form

$$\mathcal{G}_i(\vec{k},\vec{q}) = f_i(\vec{k},\vec{q})e^{i\vec{k}\cdot\vec{x}_i} + \sum_{j\neq i}\int\frac{d^3p}{(2\pi)^3}f_i(\vec{p},\vec{q})G_0(k,p)e^{i\vec{p}\cdot\vec{x}_j}\mathcal{G}_j(\vec{k},\vec{p}). \quad (3)$$

Here f_i is the free pion-nucleon scattering amplitude, G_0 is the pion propagator, and $\mathcal{G}_i$ is an amplitude which describes the multiple scattering to all orders, the last scattering taking place on the ith nucleon, and it depends on all A nucleons. The total pion-nucleus amplitude is related to the $\mathcal{G}_i$ by

$$F(\vec{k},\vec{q}) = \sum_{j=1}^{A} e^{-i\vec{q}\cdot\vec{x}_j}\mathcal{G}_j(\vec{k},\vec{q}). \quad (4)$$

This total pion-nucleus amplitude for elastic scattering must then be averaged over the nuclear wave function, the $3A$ dimensional integral being done by Monte Carlo techniques. By applying partial wave expansions to the quantities in Eq. (3), using the separable t matrix of Eq. (1), and applying matrix inversion techniques, one can solve the resulting coupled system of equations exactly to obtain the $\mathcal{G}_i$ called for in Eq. (4). Thus, one calculates multiple scattering to all orders in the fixed nucleon approximation.

For elastic scattering of an incident π^+ we can write Eq. (3) in the shorthand form

$$\mathcal{G}_i^+ = f_i^{++} + f_i^{++} \sum_{j\neq i} \mathcal{G}_j^+ \tag{5a}$$

By allowing for charge exchange we can construct a set of coupled equations, similar in form to Eq. (5), but containing amplitudes for charge exchange as well as elastic scattering[8]:

$$\mathcal{G}_i^+ = f_i^{++} + f_i^{++} \sum_{j\neq i} \mathcal{G}_j^+ + f_i^{+0} \sum_{j\neq i} \mathcal{G}_j^0 ,$$
$$\mathcal{G}_i^0 = f_i^{0+} + f_i^{00} \sum_{j\neq i} \mathcal{G}_j^0 + f_i^{0+} \sum_{j\neq i} \mathcal{G}_j^+ \tag{5b}$$

Matrix techniques can then be used to solve for the total charge exchange amplitudes $\mathcal{G}_i^0$, as well as elastic scattering amplitudes, $\mathcal{G}_i^+$.

We have examined a method of accounting for the fact that the successive nucleons which are struck are not at rest but have some initial momentum as well as a final recoil momentum. The $\vec{q}\cdot\vec{q}'$ factor in Eq. (1) is replaced by the approximately Galilean invariant form

$$\vec{q}_r\cdot\vec{q}_r' = \left(\vec{q} - \frac{\omega}{M}\vec{P}_i\right)\cdot\left(\vec{q}' - \frac{\omega}{M}\vec{P}_f\right), \tag{6}$$

where M is the nucleon mass and $\vec{P}_i$ and $\vec{P}_f$ are the initial and final nucleon momenta, respectively. Since each nucleon is struck successively in the various terms in the expansion of Eq. (3), we can use this effective angle transformation in the entire multiple scattering solution. Using conservation of momentum in replacing $\vec{P}_f$ in Eq. (6) by the quantity $\vec{q} - \vec{q}' + \vec{P}_i$, and retaining only terms of order (ω/M), we obtain

$$\vec{q}_r\cdot\vec{q}_r' = -\left(\frac{\omega}{M}\right)q^2 + \left(1 + \frac{\omega}{M}\right)\vec{q}\cdot\vec{q}', \tag{7a}$$

where we have dropped terms linear in $\vec{P}_i$ since their average over the nucleus tends to zero. This angle transformation therefore modifies the s- and p-wave pion-nucleon amplitudes; in particular, a sizable effective s-wave amplitude is induced from the basic π-N p-wave amplitude. (A complete relativistic treatment of this correction introduces in addition an effective d-wave component.) A similar

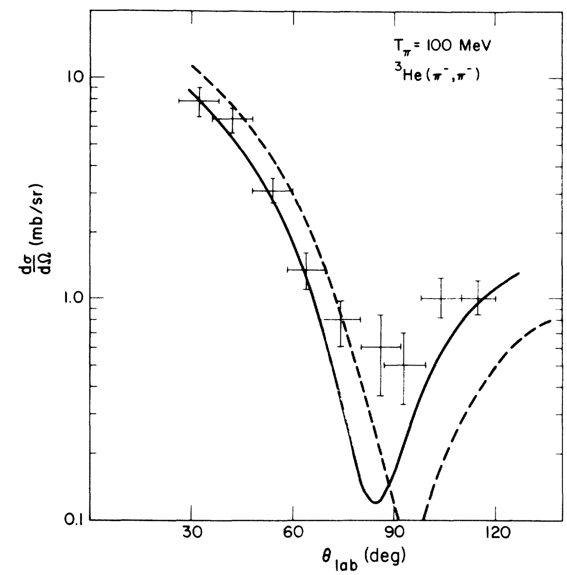

FIG. 1. Elastic cross section for ^{3}He$(\pi^-, \pi^-)^3$He at 100 MeV. The solid curve includes the angle transformation. Data are from Ref. 18.

modification to the spin-flip amplitude corresponding to Eq. (1) leads to

$$\vec{\sigma}\cdot(\vec{q}_r \times \vec{q}_r') = (1 + \omega/M)\,\vec{\sigma}\cdot(\vec{q}\times\vec{q}'). \tag{7b}$$

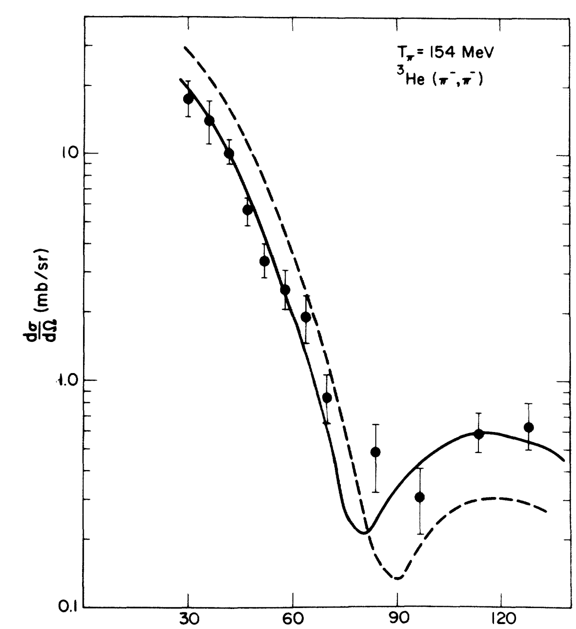

FIG. 2. Elastic cross section for ^{3}He$(\pi^-, \pi^-)^3$He at 154 MeV. The solid curve includes the angle transformation. Data are from Ref. 19.

III. RESULTS

While no data presently exist for pion charge exchange on ^{3}He or ^{3}H, there are some data on elastic π^- scattering from ^{3}He. In Figs. 1 and 2 we plot these data[19,20] at 100 and 154 MeV along with the results of our calculations. (Except where noted, we use a Gaussian form with an rms radius of 1.68 fm to describe the nuclear states.) The dashed curve in each case depicts the calculation without the angle transformation of Eq. (7). The solid curves describe the results including the angle transformation. The inclusion of the initial and final nucleon momenta through the use of Eq. (7) provides a distinct improvement in comparison with the data, particularly at back angles. Spin-flip on the unpaired nucleon is included and amounts to 10–20% of the elastic differential cross section in the region of the minimum. We note that for the variation in α_0 and α_1 allowed by the πd capture work reported in Ref. 18, the resulting differences in the elastic and charge-exchange cross sections lie in the Monte Carlo noise.

In Fig. 3 we present our results for the ^{3}H(π^+, π^0)^{3}He differential cross section at incident pion kinetic energies of 50, 100, 154, 200, and 250 MeV. [For other than extreme forward angles the ^{3}He(π^-, π^0)^{3}H cross sections differ little from ^{3}H(π^+, π^0)^{3}He values.] Spin-flip transitions and the angle transformation of Eq. (7) are includ-

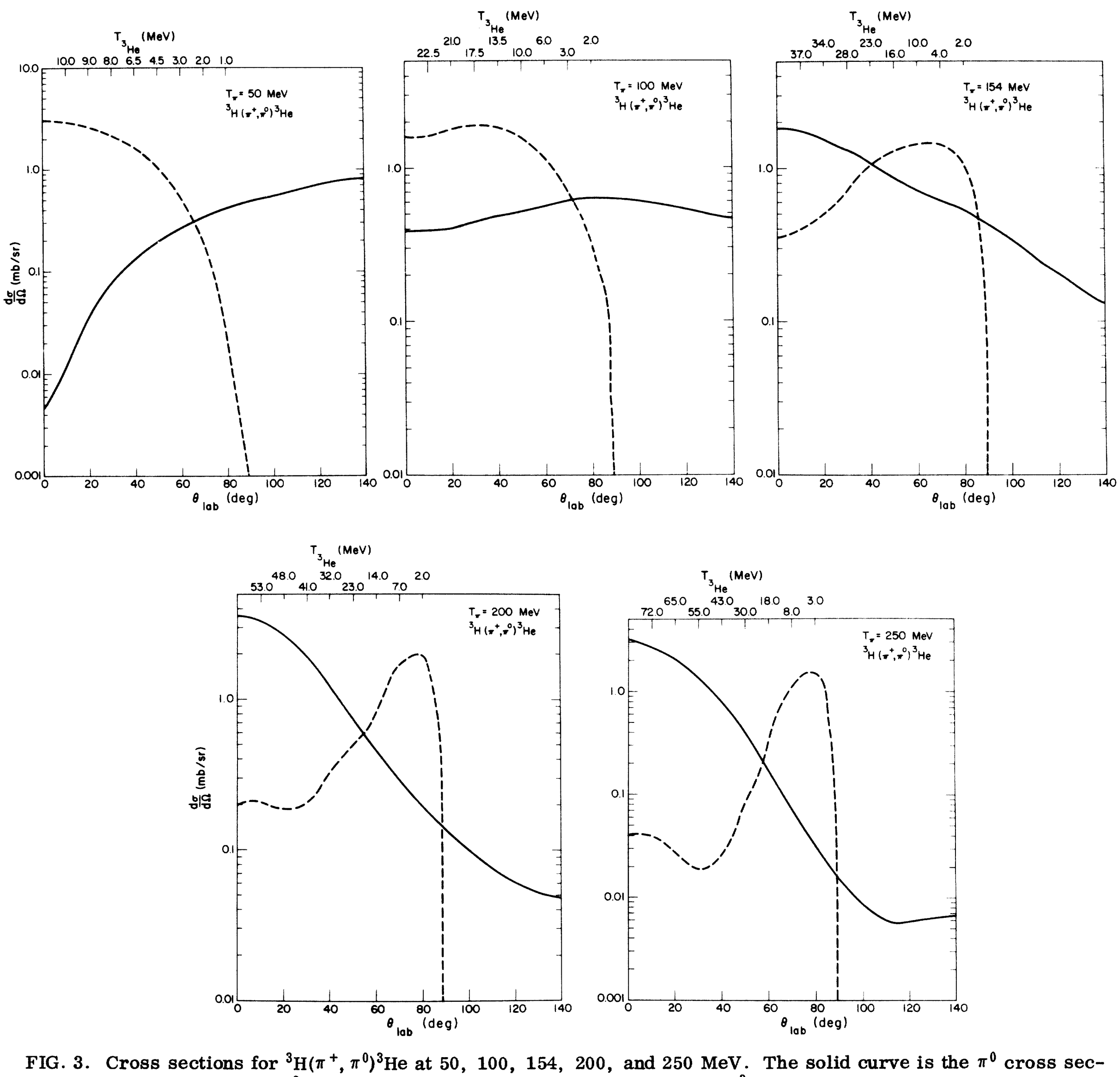

FIG. 3. Cross sections for ^{3}H(π^+, π^0)^{3}He at 50, 100, 154, 200, and 250 MeV. The solid curve is the π^0 cross section. The dashed curve is the ^{3}He cross section. The energy of the recoiling ^{3}He is tabulated at top of graph.

ed. Since the analog charge-exchange reaction is nearly elastic, the effect of the angle transformation on the charge-exchange differential cross section is similar to that found for elastic scattering. Both the angular distribution for the outgoing π^0 and the recoiling residual nucleus are given. Of experimental interest are the observations that: (1) below 100 MeV the angular distribution of recoiling nuclei is forward peaked; (2) above 150 MeV the angular distribution of recoiling nuclei is strongly peaked around 90°; (3) as the incident energy increases, the energy of the recoil peak decreases; (4) at pion energies above the resonance, there may be difficulty in extracting the recoil nucleus from the target.

In Fig. 4 we present results from our investigation of the sensitivity of the charge exchange reaction to the particular form of the ground state radial density employed and the importance of including spin-flip in the charge exchange reaction. The lower dashed curve corresponds to a simple Gaussian density with an rms radius of 1.87 fm and no spin-flip. In contrast, the solid curve corresponds to a density arising from the Fourier transform of the elastic electron scattering charge form factor having the same rms radius.[21] (This radius does not correspond properly to a mass density, but for the comparison made in this figure the difference is insignificant.) The maximum difference occurs at large pion angles where the momentum transfer is largest; however, the differences do not appear to be experimentally significant at the present time. In contrast, the dot-dashed curve corresponds to a calculation with the latter density in which spin-flip is included. Clearly spin-flip fills in the minimum around 60°. Comparison with the solid curve shows that the spin-flip transition cannot be neglected in the charge exchange reaction.

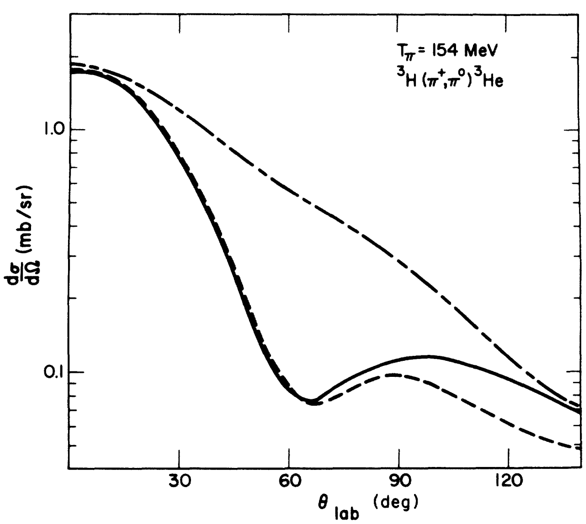

FIG. 4. Cross section for ^{3}H(π^+, π^0)^{3}He at 154 MeV. The dashed curve is for the Gaussian density. The solid curve is for the Fourier transformed charge form factor. The dot-dashed curve includes the spin-flip in a calculation using the same density as for the solid curve.

Our total pion charge-exchange cross section is shown in Fig. 5. The dashed curve describes the non-spin-flip result; the dot-dashed curve gives the spin-flip contribution. The solid curve is the sum of these two, our complete cross section. Note that our non-spin-flip curve has a slight minimum near the resonance, the result of absorption due to the multiple scattering. However, the spin-flip cross section is strongly peaked in this energy region. Thus, the total charge-exchange cross section shows a maximum rather than a minimum between 150 and 200 MeV incident pion kinetic energy of about 7 mb.

As noted above, previous theoretical treatment of this reaction has been limited to the Glauber approximation, the assumptions of which are in doubt for pion charge exchange. The Gaussian parameterization of the Glauber amplitudes ensures that the angular distributions of Refs. 9 and 10 fall well below our results at larger angles; the total cross section weights the region around 90° heavily, so that the Glauber result is guaranteed to be smaller. The nonforward peaking of the elementary π-N charge exchange amplitude forces the Glauber total cross section to very small values at small pion kinetic energy in basic disagreement with our result and simple impulse limits. However, such qualitative features as the importance of the spin-flip contribution discussed in Ref. 10 and the slight minimum in the non-spin-flip cross section discussed in Ref. 9 and 10 are confirmed by our calculations.

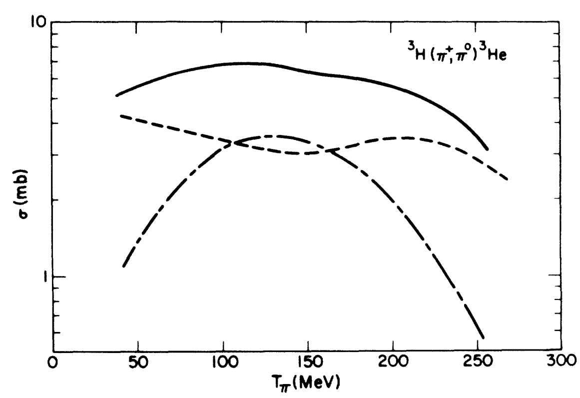

FIG. 5. Total charge exchange cross section for ^{3}H-($\pi^+\pi^0$)^{3}He. The dashed curve is the non-spin-flip cross section. The dot-dashed curve is the spin-flip result. The solid curve is the complete cross section.

IV. CONCLUSIONS

We have studied the isobaric analog transition $^3H(\pi^+, \pi^0)^3He$ in a multiple-scattering formalism using realistic pion-nucleon amplitudes in which off-shell effects are incorporated by means of a separable approximation. A sizable contribution from spin-flip on the charge-exchanged nucleon was found. Little sensitivity to details of the nuclear density was noted for incident pion kinetic energies examined, providing the same rms radius was maintained. Our theoretical results indicate a flat maximum of about 7 mb between 150 and 200 MeV, some 25% below the impulse theory result due to absorption arising from the multiple scattering. However, both differential and total cross sections are yet to be measured. Thus, a detailed test of the calculation awaits the availability of such data; such a comparison should then increase our understanding of the pion-nucleus charge-exchange mechanism.

The authors thank Dr. W. R. Gibbs for his critical comments and Dr. W. B. Kaufmann and Dr. G. J. Stephenson, Jr., for useful and informative discussions.

---

†Work performed under the auspices of the U.S. Energy Research and Development Administration.

[1] G. A. Miller and J. E. Spencer, Phys. Lett. 53B, 329 (1974).

[2] E. Rost and G. W. Edwards, Phys. Lett. 37B, 247 (1971).

[3] L. A. Charlton, J. M. Eisenberg, and W. B. Jones, Nucl. Phys. A171, 625 (1971).

[4] M. Zaider, J. Alster, D. Ashery, N. Auerbach, S. Cochavi, M. A. Moinester, J. Warszawski, and A. I. Yavin, in *High-Energy Physics and Nuclear Structure*, edited by G. Tibell (North-Holland, Amsterdam, 1974).

[5] D. Tow and J. M. Eisenberg, Nucl. Phys. A237, 441 (1975).

[6] A. Reitan, Nucl. Phys. B68, 387 (1974).

[7] A. T. Hess, Ph.D. thesis, University of Virginia, 1974 (unpublished).

[8] W. B. Kaufmann, J. C. Jackson, and W. R. Gibbs, Phys. Rev. C 9, 1340 (1974).

[9] J. M. Eisenberg and V. B. Mandelzweig, Phys. Lett. 53B, 405 (1975).

[10] D. A. Sparrow, Phys. Lett. 58B, 309 (1975).

[11] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Britten and B. W. Downs (Interscience, New York, 1959), Vol. I.

[12] W. R. Gibbs, A. T. Hess, and W. B. Kaufmann, Phys. Rev. C (to be published).

[13] L. L. Foldy and J. D. Walecka, Ann. Phys. (N.Y.) 54, 447 (1969).

[14] R. H. Landau and F. Tabakin, Phys. Rev. D 5, 2746 (1972).

[15] W. R. Gibbs, Phys. Rev. C 10, 2166 (1974).

[16] J. T. Londergan, K. W. McVoy, and E. J. Moniz, Ann. Phys. (N.Y.) 86, 147 (1974).

[17] J. M. McKinley, Rev. Mod. Phys. 35, 788 (1963).

[18] W. R. Gibbs, in *The Investigation of Nuclear Structure by Scattering Processes at High Energies*, in Proceedings of the International School of Nuclear Physics, Erice, 1974, edited by H. Schopper (North-Holland, Amsterdam, 1975).

[19] I. V. Falomkin, C. Georgescu, M. M. Kulyukin, V. I. Lyashenko, A. Mihul, F. Nichitiu, G. Pontecorvo, Y. A. Scherbakov, R. Garfagnini, and G. Piragino, Lett. Nuovo Cimento 5, 1121 (1972).

[20] I. V. Falomkin, M. M. Kulyukin, V. I. Lyashenko, G. B. Pontecorvo, Y. A. Scherbakov, M. Albu, T. Angelescu, A. Mihul, F. Nichitiu, R. Garfagnini, and G. Piragino, Nuovo Cimento 24, 93 (1974).

[21] J. S. McCarthy, I. Sick, R. R. Whitney, and M. R. Yearian, Phys. Rev. Lett. 25, 884 (1970).