Deuteron-carbon elastic scattering at 650 MeV

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The Glauber theory of nucleus-nucleus scattering is used to calculate the elastic differential cross section for the scattering of deuterons off carbon nuclei using two-particle scattering data as input. We obtain qualitatively better agreement with the experimental measurements of Dutton *et al.* than their calculations using proton-carbon optical potentials.

[NUCLEAR REACTIONS $^{12}C(d,d)$, $E = 650$ MeV; calculated $\sigma(E,\theta)$ for $0° < \theta_{lab} < 13°$ and $\sigma_T(E)$ using p-p, p-n scattering parameters. Compared with experimental measurements of $\sigma(E,\theta)$ and $\sigma_T(E)$.]

INTRODUCTION

We report a calculation of the elastic differential cross section for deuteron-carbon scattering in the Glauber formalism, this being one of the few processes of the type "two-on-many" for which data are available, permitting a comparison of theory with experiment.

After considerable success in the application of the Glauber model of the multiple scattering of strongly interacting particles at high energy to a variety of particle-nucleus scattering processes,[1] the formalism was extended to the study of nucleus-nucleus scattering by Franco who applied it in particular to deuteron-deuteron scattering.[2] Several generalizations and limiting expressions of the model have also been obtained by Kofoed-Hansen[3] as well as by Czyz and Maximon[4] for nucleus-nucleus collisions; however, for lack of data, no comparison between theory and experiment has been attempted, with the exception of the work of Alberi, Bertocchi, and Bialkowski[5] in deuteron-deuteron scattering.

Starting from the usual formulation of the Glauber theory for nucleus-nucleus collisions, we calculate the elastic differential cross section for deuteron-carbon scattering at 650 MeV where the measurements of Dutton *et al.*[6] are available. There also exist data on d-Be and d-C scattering at 420 MeV[7] for which fits have been attempted using suitable optical potentials in the Glauber model.[8] Since these data are at an even lower energy where the small angle approximation is expected to be less valid and the multiple scattering series to be more slowly convergent, we have attempted no fits to them. In the present calculation we use two-particle nucleon-nucleon amplitudes suitably parametrized as input and we use shell model product harmonic oscillator wave functions to describe the carbon nucleus and retain terms up to the 4th order in the multiple scattering series. We obtain better qualitative agreement with the observed angular distribution than was obtained by Dutton *et al.*[6] using proton-carbon optical potentials.

The motivation for the application of the Glauber model to nucleus-nucleus scattering is of course the hope that such processes may be understood purely in terms of two-particle processes, thus establishing a connection between the seemingly remote fields of particle and nuclear physics. The present work may be taken as an indication that this "grand design" may turn out to be feasible.

DEUTERON-CARBON SCATTERING AMPLITUDE

Our starting point is the expression for the elastic scattering amplitude for a deuteron off a target with A nucleons (neglecting for the moment the effect of center of mass motion) given by:

$$F(\vec{\Delta}) = \frac{iK}{2\pi}\int e^{i\vec{\Delta}\cdot\vec{b}}\,|\varphi(\vec{r})|^2\,|\psi(\vec{r}_1,\ldots,\vec{r}_A)|^2 \times\Gamma(\vec{b},\vec{s},\vec{s}_1,\ldots,\vec{s}_A)\,d^2\vec{b}\,d^3\vec{r}\prod_{j=1}^{A} d^3\vec{r}_j\,, \tag{1}$$

where $\vec{\Delta}$ is the momentum transfer, K is the magnitude of the incident deuteron momentum, $\varphi(\vec{r})$ and $\psi(\vec{r}_1,\ldots,\vec{r}_A)$ are the deuteron and the nuclear wave functions, respectively, and $\Gamma(\vec{b},\vec{s},\vec{s}_1,\ldots,\vec{s}_A)$ is the so called profile function. The position of the A nucleons which make up the target nucleus are given by the vectors $\vec{r}_i$, $i=1,\ldots,A$, and $\vec{s}_i$ are the projections of these vectors in the plane of the impact vector $\vec{b}$; while $\vec{r}$ is the relative position vector of the two nucleons in the deuteron and $\vec{s}$ is its projection on the impact parameter plane.

In the Glauber approximation, the assumption of the additivity of the phase shifts leads to the composition law of the profile function given by

$$\Gamma(\vec{b},\vec{s},\vec{s}_1,\ldots,\vec{s}_A)=1-\prod_{j=1}^{A}[1-\Gamma_{pj}(\vec{b}+\tfrac{1}{2}\vec{s}-\vec{s}_j)]\times[1-\Gamma_{nj}(\vec{b}-\tfrac{1}{2}\vec{s}-\vec{s}_j)]\,, \tag{2}$$

where Γ_{xj} are the two-particle profile functions which are related to the two-particle amplitudes f_{xj}, corresponding to the scattering of particle x in the deuteron off the jth nucleon in the target, by an inverse Fourier transform

$$\Gamma_{xj}=\frac{1}{2\pi iK}\int e^{-i\vec{q}\cdot\vec{b}}f_{xj}(\vec{q})d^2\vec{q}\,. \tag{3}$$

We assume that the ground state of the nucleus can be described by an independent particle model, so that neglecting all position correlations between the nucleons we can write ψ as a product wave function. In terms of the single-particle densities $\rho_j(\vec{r}_j)$, we have

$$|\psi(\vec{r}_1,\ldots,\vec{r}_A)|^2=\prod_{j=1}^{A}\rho_j(\vec{r}_j)$$

with the normalization condition

$$\int\rho_j(\vec{r}_j)d^3\vec{r}_j=1\,.$$

The nuclear form factors are related to the single-particle densities by

$$S(\vec{q})=\int e^{-i\vec{q}\cdot\vec{r}_j}\rho_j(\vec{r}_j)d^3\vec{r}_j\,. \tag{4}$$

Using nuclear density functions corresponding to a harmonic oscillator potential[16] we have

$$S(\vec{q})=(1-a^2q^2/9)e^{-a^2q^2/4}\,. \tag{5}$$

On using Eqs. (2)–(4) in Eq. (1), we have the following expressions for the deuteron-carbon elastic scattering amplitude:

$$F(\vec{\Delta})=\frac{iK}{2\pi}\int e^{i\vec{\Delta}\cdot\vec{b}}d^2\vec{b}\,|\varphi(\vec{r})|^2\,d^3\vec{r}\times[1-\{1-F_p(\vec{b}+\tfrac{1}{2}\vec{s})-F_n(\vec{b}-\tfrac{1}{2}\vec{s})+G(\vec{b},\vec{s})\}^{12}]\,, \tag{6}$$

where

$$F_x(\vec{b})=\frac{1}{2\pi ik}\int e^{-i\vec{q}\cdot\vec{b}}f_{px}(\vec{q})S(\vec{q})d^2\vec{q},\quad x=p,n$$

corresponds to single scattering of either the incident proton or neutron off one of the target nucleons, while

$$G(\vec{b},\vec{s})=\frac{1}{(2\pi ik)^2}\int e^{-i\vec{q}\cdot(\vec{b}+\vec{s}/2)-i\vec{q}'\cdot(\vec{b}-\vec{s}/2)}\times f_{pp}(\vec{q})f_{pn}(\vec{q}')S(\vec{q}+\vec{q}')d^2\vec{q}d^2\vec{q}'$$

corresponds to the special kind of double scattering in which both the incident proton and neutron scatter off the same nucleon within the target.

In order that the integrals in Eq. (6) may be carried out analytically, we choose a Gaussian wave function for the deuteron:

$$\varphi(x)=(2\pi B^2)^{-3/4}e^{-x^2/4B^2}$$

so that the expression for the deuteron form factor is given by:

$$T(\vec{q})\equiv\int e^{i\vec{q}\cdot\vec{r}}|\varphi(\vec{r})|^2d^3\vec{r}=e^{-B^2q^2/2}\,. \tag{7}$$

We parametrize the two-particle amplitudes in the form:

$$f_{px}(\vec{q})=\frac{1}{4\pi}(i+\alpha_x)k\sigma_{px}e^{-\beta_x^2q^2/2}\,. \tag{8}$$

Finally the expression for the elastic scattering amplitude given by Eq. (6) should contain a δ function in the integral to take into account momentum and energy conservation which imposes a constraint on the nuclear center of mass. The correct nuclear scattering amplitude taking this constraint into account can be written as[9]

$$\mathfrak{F}(\vec{\Delta})=R(\vec{\Delta})F(\vec{\Delta})\,, \tag{9}$$

where the explicit form of the correction factor for the case of harmonic oscillator densities for the carbon nucleus is[10]

$$R(\vec{\Delta})=e^{a^2\Delta^2/48}\,. \tag{10}$$

DISCUSSION AND RESULTS

Equations (6) and (10) form the basis of our numerical work. For the particular choice of the deuteron wave function and the proton-nucleon amplitudes given by Eqs. (7) and (8), respectively, all integrals up to and including fourth order scattering in the expansion of the amplitude can be evaluated analytically. The energy of the interaction is not high enough to warrant the assumption of charge independence of the nucleon-nucleon scattering amplitudes. However, for the sake of simplicity we have taken the two-particle parameters to be the mean of the proton-proton and the proton-neutron values. We have checked by explicit calculation up to terms of the second order that this gives results which are not significantly different from those obtained by keeping track of the separate p-p and p-n contributions. The results we report include all contributions up to the

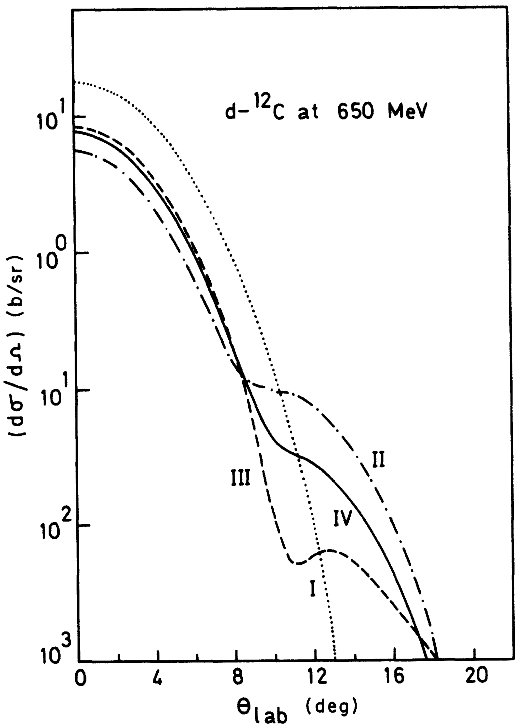

FIG. 1. d-^{12}C elastic differential cross section at 650 MeV as a function of the laboratory scattering angle. The curves I, II, III, and IV are the results of retaining terms up to the first, second, third, and fourth order scattering, respectively, in the Glauber expansion.

fourth order scattering terms.

The values of the parameters used in the present calculation are:

$a = 1.64$ fm

in Eq. (5) corresponding to the root-mean-square radius of the carbon nucleus equal to 2.40 fm[11];

$B^2 = 3.25\ \text{fm}^2$

in Eq. (7) corresponding to $\langle 1/r^2 \rangle = 0.308\ \text{fm}^{-2}$ for the deuteron—the value obtained for the Gartenhaus wave function with the D wave included[12]; and the nucleon-nucleon parameters are taken to be the mean of the proton-proton and proton-nucleon values at 325 MeV incident energy:

$$\sigma_{pp} = 24.3 \pm 1.0\ \text{mb},\quad \sigma_{pn} = 32.5 \pm 4.0\ \text{mb (Ref. 13)},$$

$$\alpha_{pp} = 0.8,\quad \alpha_{pn} = 0.05 \pm 0.2\ \text{(Ref. 14)},$$

$$\beta_p^2 = 0.66\ (\text{GeV}/c)^{-2},\quad \beta_n^2 = 5.4\ (\text{GeV}/c)^{-2}\ \text{(Ref. 5)}.$$

Using the experimental estimates of these parameters without error bars, we display in Fig. 1 the calculated elastic differential cross section as a function of the laboratory scattering angle when terms up to and including the first, second, third, and fourth order in the multiple scattering expansion are retained. Higher order contributions were not evaluated, firstly because both the number of terms to be considered and the multiple integrals involved become unmanageable and secondly because the behavior of the third and fourth order terms would lead us to expect that the higher order scattering contributions would not materially affect the fit for $\theta_{\text{lab}} \leq 9°$, up to which angle the Glauber series seems to have converged. In Fig. 2 we compare the results of the present calculation, when the contribution of terms up to the fourth order scattering are included, with the experimental points and it is evident that they fit the data exceedingly well for $\theta_{\text{lab}} \leq 9°$. Although we have displayed curves also for $\theta_{\text{lab}} > 9°$, the fits beyond 9° should not be taken seriously since higher order terms will be important in this region. Further, the values of the input parameters used in Fig. 2 lead to the total cross section for deuteron-carbon scattering given by

$$\sigma_{\text{total}} = 476\ \text{mb}.$$

This is to be compared with the experimental value[6] of

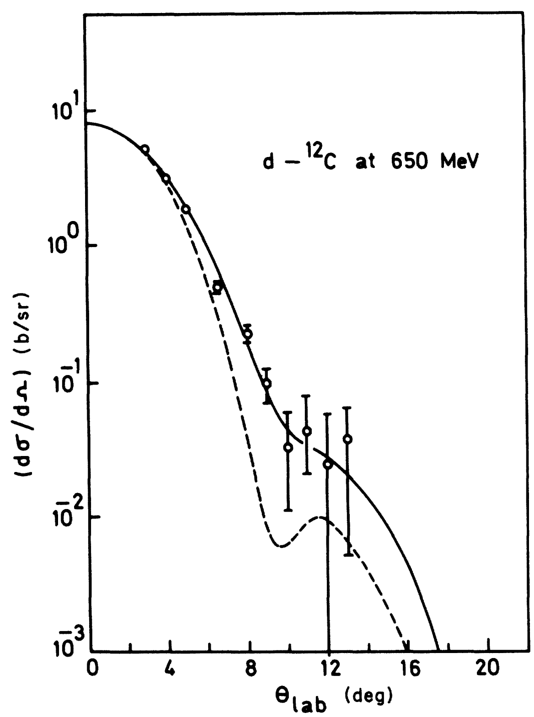

FIG. 2. The solid curve represents our results obtained by including terms up to the fourth order in the Glauber series; the dashed curve is the result of using a p-^{12}C optical potential and the experimental points are from Ref. 6.

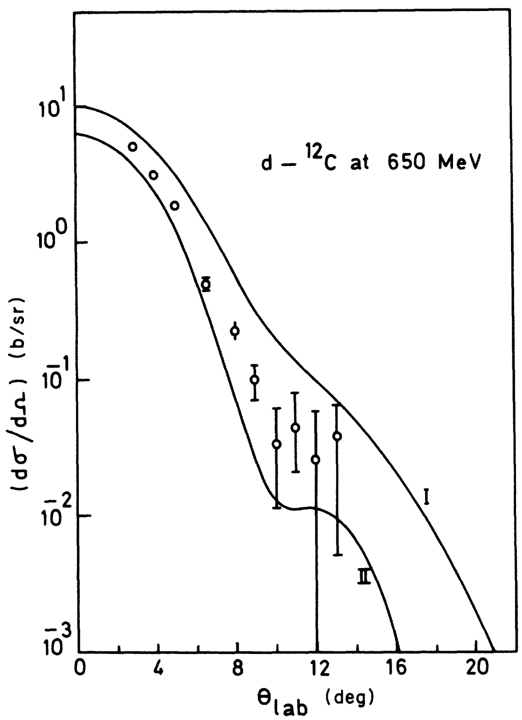

FIG. 3. Curve I is the differential cross section resulting from the choice of the maximum experimentally allowed values of σ and α and the minimum value of β^2, i.e., σ=30.9 mb, α=0.525, β^2=2.7 (GeV/c)$^{-2}$. The complementary set of values, i.e., σ=25.9 mb, α=0.325, and β^2=3.3 (GeV/c)$^{-2}$, leads to curve II.

$$\sigma_{\text{total exp}} = 456 \pm 18 \text{ mb}.$$

As reported in Ref. 6 the value obtained in the impulse approximation is 565 ± 10 mb and the use of proton-carbon optical potentials in the Glauber approximation yields the value of 505 ± 10 mb. Therefore both for the differential as well as the total cross section, the present calculation seems to give a better fit to the data than the original calculation using a proton-carbon optical potential

This is not to claim that the present scheme of calculation is superior to the optical potential approach. The latter is a more comprehensive formalism, taking into account as it does the effects of spin and isospin and at lower energies providing a detailed fit to the experimental data. Although the present work is, in principle, a zero parameter fit to the deuteron-carbon data (all quantities being determined from information on either two-particle scattering or from the structure of the deuteron and the carbon nucleus) the inaccuracy in our knowledge of the two-particle scattering parameters at the energy we are interested in results, unfortunately, in a degree of imprecision in the quantitative fits to the experimental data. We have therefore investigated the sensitivity of our fits to variations in the input two-particle parameters and we observe the following broad features:

(i) The fit is sensitive to variations in the values of two-particle total cross sections. With an increase in $\sigma = \frac{1}{2}(\sigma_{pp} + \sigma_{pn})$ the differential cross section increases at all angles.

(ii) Variations in the ratio of the real to the imaginary part of the two-particle amplitude $\alpha = \frac{1}{2}(\alpha_p + \alpha_n)$ affects the differential cross section primarily in the region of interference, the value in the forward direction not being too sensitive to α; the interference minimum becomes filled up with the increase in α.

(iii) Variations in $\beta^2 = \frac{1}{2}(\beta_p^2 + \beta_n^2)$ affects the slope of the fit-larger values of β^2 leading to more rapidly falling differential cross sections.

In Fig. 3 we present the "worst possible" fits to the differential cross section when the two-particle input parameters are allowed to assume the extremes of their experimentally permitted values. As it is difficult to estimate the experimental error bars on the parameter β^2, we have allowed β^2 to vary by 10% about its mean value of 3 (GeV/c)$^{-2}$. Curve I is the result of using the maximum allowed values of σ and α, and the minimum value of β^2. Notice that curve I lies everywhere above the experimental points. The corresponding value of the total cross section is 540 mb. Curve II is the result of using the minimum allowed values of σ and α, and the maximum value of β^2. Except in the interference region curve II lies below the experimental points, the corresponding value of the total cross section being 420 mb. Although the variations are large we would like to stress that a choice of the median values of the two-particle parameters does lead to reasonable fits both to the differential and the total cross sections, and that if consistently higher (or lower) values of the parameters are chosen then they lead to consistently higher (or lower) fits to both the differential and total cross sections.[15] However, it is evident that before more than qualitative success can be claimed for the Glauber fit to the deuteron-carbon data, the required two-particle information needs to be known more accurately.

In addition to the variations arising from uncertainties in the two-particle data, there are two other criticisms that can be leveled against the present calculation. The first has to do with the energy of the interaction not being sufficiently high to warrant the use of the Glauber model. Although it is true that the Glauber model is a good approximation at high energies, where the multiple series expansion is expected to be rapidly convergent, an examination of Fig. 1 reveals that the differential cross section for $\theta_{\text{lab}} \leq 9°$ does not change appreciably with the inclusion of fourth order scattering terms, indicating that the Glauber series seems to have converged up to this angle at the present energy. The second criticism has to do with the parametrization of the nucleon-nucleon amplitude as a simple Gaussian. In justification of our choice, we can appeal only to the resulting simplicity and draw attention to its use even in the application of the Glauber model to much simpler scattering processes. A sum of Gaussians, for example, would obviously be a better representation of the nucleon-nucleon amplitude at 325 MeV, but such a calculation was not attempted because in our opinion the additional effort involved would be worthwhile only if the two-particle data were known more accurately.

Finally, we note that the carbon nucleus is known to be strongly deformed and various attempts at fitting p-^{12}C data have shown that neither the product wave functions of the kind used in the present calculation, nor suitably antisymmetrized versions thereof are enough to fit the data at large angles.[16] A calculation due to Lesniak and Lesniak[17] seemed to show that it was necessary to take the ^{12}C nucleus to be oblately deformed to fit the large angle p-^{12}C data. However, it was found later[18] that this same deformed nucleus model was unable to explain high energy electron scattering data. Therefore, keeping in mind that these more sophisticated models for the deformation of the ^{12}C nucleus affect the angular distribution only at large angles (beyond the secondary maximum), we have for the present calculation felt it quite adequate to describe the ^{12}C nucleus by a simple product wave function in the shell model.

On the basis of the present work we can conclude that where data on elastic nucleus-nucleus scattering is available, the Glauber model is able to provide a reasonable explanation of the experimental results. The answer to whether or not it will be able to explain all nucleus-nucleus scattering data must await further experiments.

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