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Collective states in ^{233}U, ^{235}U, ^{237}Np, and ^{239}Pu excited by inelastic deuteron scattering*

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Energy spectra of inelastically scattered 16-MeV deuterons were measured at angles of 90° and 125° using enriched targets of ^{233}U, ^{235}U, ^{237}Np, and ^{239}Pu. The octupole band resulting from the ground-state coupling to the $K^\pi = 0^-$ octupole vibration of the core is identified in ^{233}U, ^{237}Np, and ^{239}Pu and is tentatively assigned in ^{235}U. Evidence for the observation of ground-state coupling to the higher lying octupole bands is discussed for ^{233}U, ^{237}Np, and ^{239}Pu. Comparison is made of the core vibrations of the odd-A nucleus with those observed in the adjacent even-even nuclei. The $K-2$ and $K+2$ γ bands built on the ground state of ^{235}U are seen.

[NUCLEAR REACTIONS ^{233}U, ^{235}U, ^{237}Np, ^{239}Pu(d,d'), $E=16$ MeV, measured $\sigma(E_{d'}, \theta=90°, 125°)$. ^{233}U, ^{235}U, ^{237}Np, ^{239}Pu deduced levels, J, K, π. Enriched targets.]

I. INTRODUCTION

Inelastic deuteron scattering preferentially excites the collective levels of nuclei. In the actinide region the (d,d') reaction populates to a significant extent the ground-state rotational band and the quadrupole and octupole vibrational bands. Earlier work[1-3] done in this laboratory has made a systematic study of the collective levels of even-even nuclei in the actinide region. The present experiment which includes the nuclei ^{233}U, ^{235}U, ^{237}Np, and ^{239}Pu is an attempt to extend this survey to odd-A nuclei which are one nucleon away from even-even nuclei studied previously by (d,d') experiments.[1-3]

The (d,d') reaction on these odd-A nuclei is expected to populate strongly members of the ground-state rotational band and the one-phonon states of the core coupled to the target ground state. The ground-state coupling to the one-phonon states gives rise to bands with quantum numbers $K^\pi = (K_{\text{g.s.}} \pm K_{\text{vib}})^{\pi_{\text{g.s.}} \cdot \pi_{\text{vib}}}$. In this region of the actinide nuclei the lowest collective vibration is the $K^\pi = 0^-$ octupole state which, coupled to the odd-A ground state, forms a single collective band. In ^{233}U, ^{237}Np, and ^{239}Pu we find this band strongly populated at approximately the energy of the $K^\pi = 0^-$ band in the adjacent even-even nuclei. And, in fact, the majority of the $K^\pi = 0^-$ core collectivity seems to be concentrated in this one band in contrast to being distributed widely among various single-particle states. It is apparent that the ground state coupling to the higher octupole bands ($K^\pi = 1^-, 2^-$) is also observed in these three nuclei, although specific assignments cannot be made.

The observation of quadrupole vibrations in the even-even nuclei of this region is limited to the $K^\pi = 2^+$ γ band. The $K^\pi = 0^+$ β vibration is populated, if at all, only weakly in deuteron inelastic scattering. The coupling of the ground state to the γ vibration is definitely observed in these experiments only in ^{235}U.

II. EXPERIMENTAL PROCEDURE

Using the tandem Van de Graaff accelerator of the University of Rochester Nuclear Structure Research Laboratory, deuteron inelastic scattering experiments were performed with isotopically enriched line targets of ^{233}U, ^{235}U, ^{237}Np, and ^{239}Pu.[4] The deuteron incident energy was 16 MeV and the scattered deuterons were momentum-analyzed in an Enge split-pole magnetic spectrograph and detected by Kodak NTB50 photographic plates. A NaI detector at 45° was used as a monitor in the scattering chamber. For each nucleus spectra were recorded at 90° and 125°. The 125° spectra for the ^{233}U(d,d')^{233}U, ^{235}U(d,d')^{235}U, ^{237}Np(d,d')^{237}Np, and ^{239}Pu(d,d')^{239}Pu reactions are shown in Figs. 1–4, respectively. No impurity groups were observed in these reactions. The resolution of the spectra varied from about 10 keV to 14 keV full width at half maximum.

The absolute cross sections for the ground states of the four nuclei studied were obtained by normalizing the measured ground-state intensities to the 16-MeV deuteron elastic scattering cross section calculated from a computer code such as DWUCK.[5]

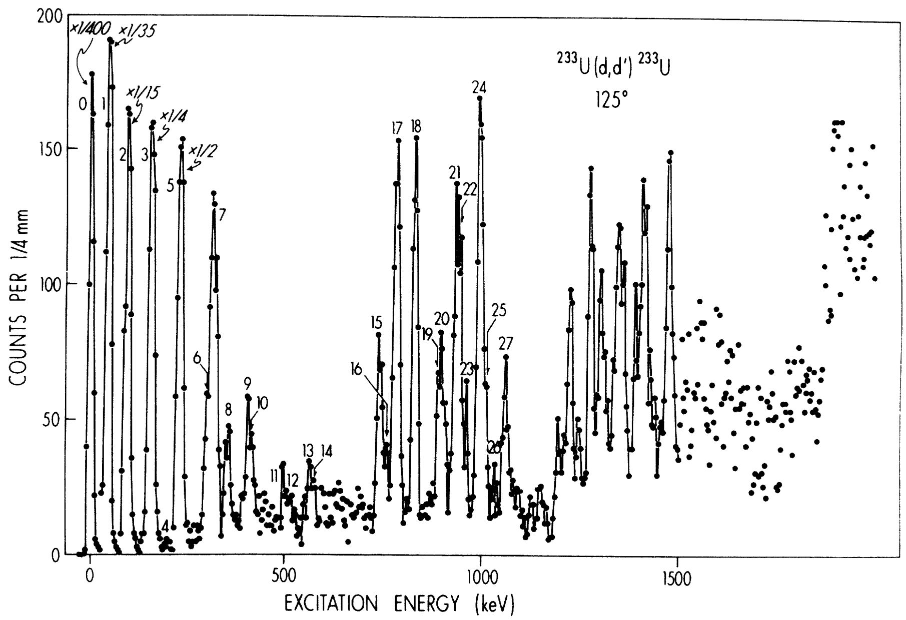

FIG. 1. Spectrum of the ^{233}U$(d, d')^{233}$U reaction at 125°.

The reliability of this method was checked by taking a short exposure at 20° with the ^{237}Np target, where the elastic scattering cross section is very close to the Rutherford value.[1] Relating this to the monitor elastic scattering cross section at 45°, it was found that the cross section calculated by DWUCK agrees with experiment within the experimental uncertainty. The absolute cross sections of the excited levels were subsequently obtained by normalizing their intensities to the ground-state cross section.

It was necessary to make three exposures for

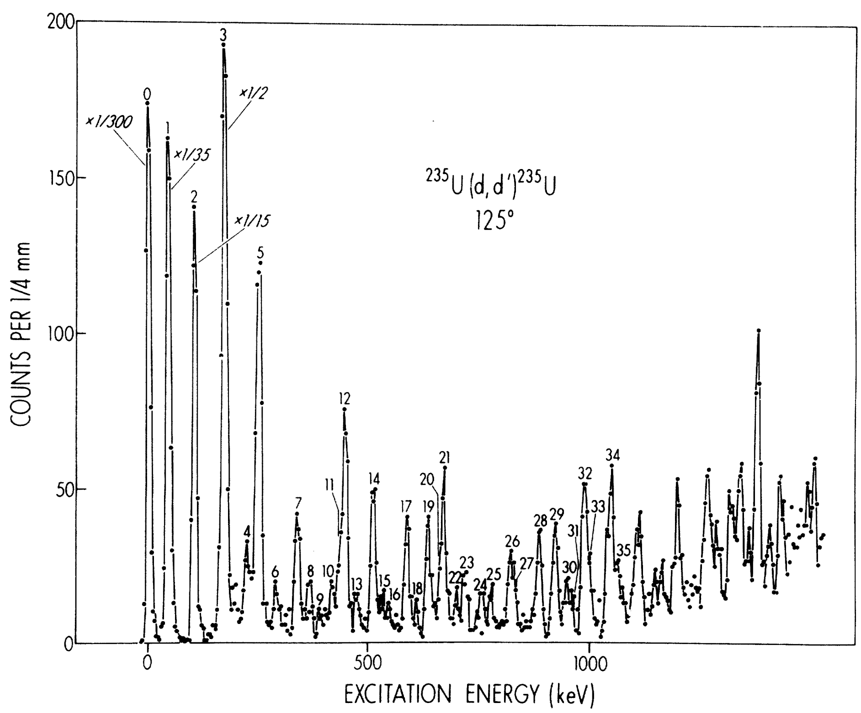

FIG. 2. Spectrum of the ^{235}U$(d, d')^{235}$U reaction at 125°.

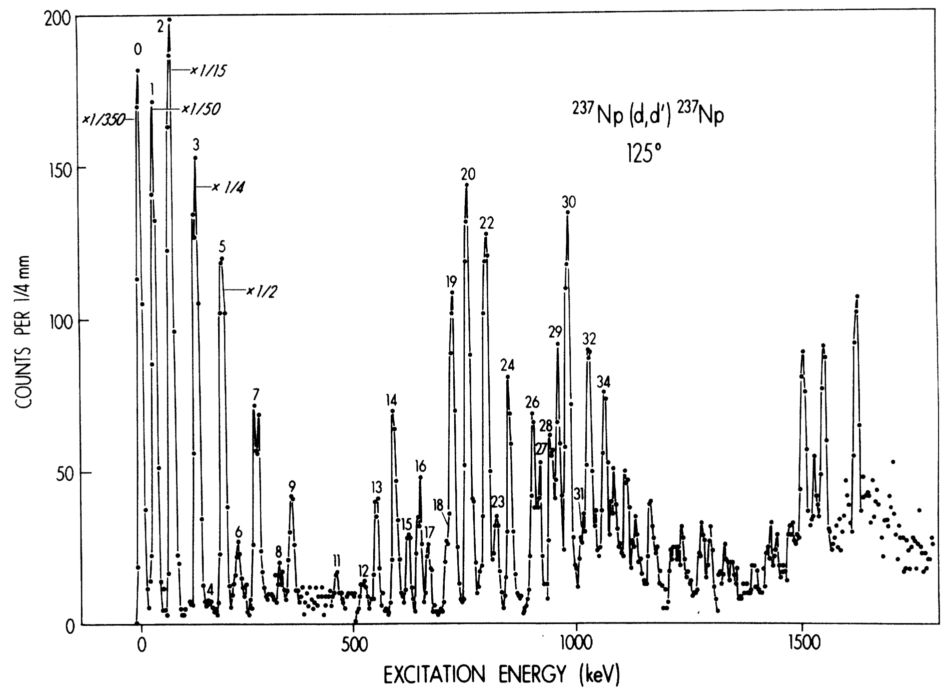

FIG. 3. Spectrum of the ^{237}Np$(d, d')^{237}$Np reaction at 125°.

each angle in order to compare the weakly excited levels with the elastic group. A short run properly exposed the elastic group, a long run enabled the weak levels to be observed, and a medium run showed ground-band rotational members which could be seen in both the long and short runs.

Tables I–IV present the level energies and differential cross sections of the four odd-A nuclei studied. The absolute cross section errors are approximately 20%.

III. RESULTS

The spectra are analyzed by using three sources of information: (1) the $J(J+1)$ rotational energy

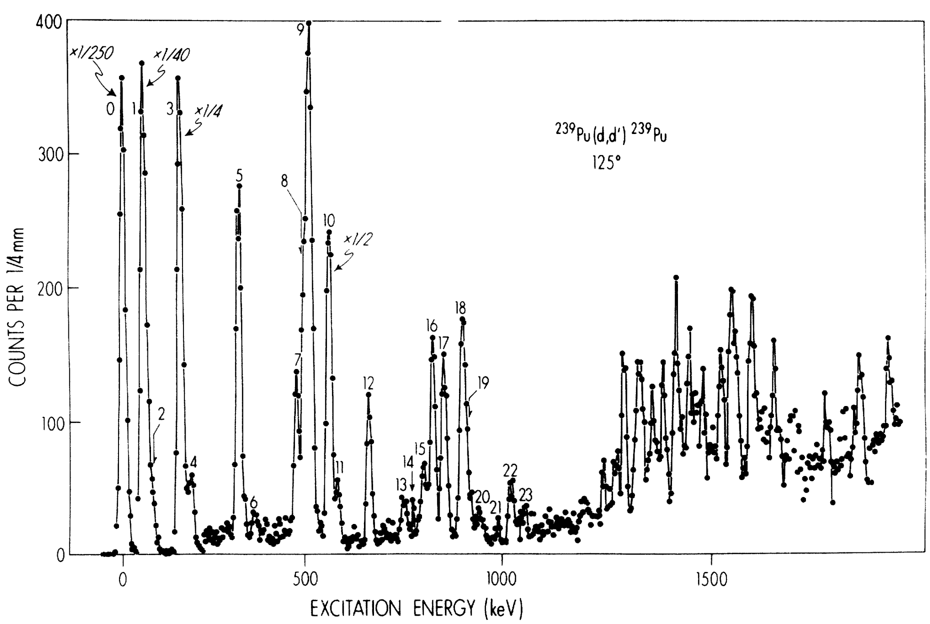

FIG. 4. Spectrum of the ^{239}Pu$(d, d')^{239}$Pu reaction at 125°.

TABLE I. ^{233}U.

Level	Energy (keV)	Cross section (μb/sr) 90°	125°	Assignment [a]
0	0	44 300	9300	$\frac{5}{2}, \frac{5}{2}^+[633]$
1	39(1)	3755	1378	$\frac{7}{2}, \frac{5}{2}^+[633]$
2	91(2)	1740	588	$\frac{9}{2}, \frac{5}{2}^+[633]$
3	153(2)	374	142	$\frac{11}{2}, \frac{5}{2}^+[633]$
4	197(4)	8	4	
5	228(2)	152	70	$\frac{13}{2}, \frac{5}{2}^+[633]$
6	298(3)	25	14	$(\frac{5}{2}, \frac{5}{2}^-[752])$
7	318(3)	61	27	$\frac{15}{2}, \frac{5}{2}^+[633] + \frac{7}{2}, \frac{5}{2}^-[752]$
8	353(2)	27	11	$\frac{9}{2}, \frac{5}{2}^-[752]$
9	403(4)	11	12	$(\frac{11}{2}, \frac{5}{2}^-[752])$
10	425(4)	28	5	$(\frac{17}{2}, \frac{5}{2}^+[633])$
11	500(2)	11	6	$\frac{7}{2}, \frac{7}{2}^-[743]$
12	521(3)	8	5	
13	560(4)	15	5	$\frac{9}{2}, \frac{7}{2}^-[743]$
14	575(4)	...	6	
15	748(2)	36	15	$\frac{5}{2}, \frac{5}{2}^-[633\downarrow \otimes K^\pi = 0^-]$
16	766(3)	15	7	
17	790(2)	80	34	$\frac{7}{2}, \frac{5}{2}^-[633\downarrow \otimes K^\pi = 0^-]$
18	838(2)	85	30	$\frac{9}{2}, \frac{5}{2}^-[633\downarrow \otimes K^\pi = 0^-]$
19	898(3)	38	13	
20	914(3)	34	12	$\frac{11}{2}, \frac{5}{2}^-[633\downarrow \otimes K^\pi = 0^-]$
21	940(3)	58	13	
22	952(4)	46	23	
23	968(3)	22	8	
24	1001(2)	71	34	
25	1017(4)	48	10	
26	1046(4)	18	6	
27	1071(3)	43	15	

[a] Only dominant component given.

TABLE II. ^{235}U.

Level	Energy (keV)	Cross section (μb/sr) 90°	125°	Assignment [a]
0	0	44 400	9300	$\frac{7}{2}, \frac{7}{2}^-[743]$
1	45(1)	2865	1241	$\frac{9}{2}, \frac{7}{2}^-[743]$
2	103(2)	770	436	$\frac{11}{2}, \frac{7}{2}^-[743]$
3	170(1)	106	81	$\frac{13}{2}, \frac{7}{2}^-[743]$
4	223(2)	10	7	$\frac{9}{2}, \frac{5}{2}^+[622]$
5	247(1)	28	27	$\frac{15}{2}, \frac{7}{2}^-[743]$
6	288(2)	5	4	$\frac{11}{2}, \frac{5}{2}^+[622]$
7	338(1)	7	9	$\frac{17}{2}, \frac{7}{2}^-[743]$
8	366(2)	4	4	$\frac{7}{2}, \frac{5}{2}^+[633]$
9	392(3)	4	2	$\frac{3}{2}, \frac{3}{2}^+[631]$
10	414(3)	4	4	$\frac{9}{2}, \frac{5}{2}^+[633]$
11	430(4)	4	4	$\frac{5}{2}, \frac{3}{2}^+[631]$
12	444(2)	13	13	$(\frac{7}{2}, \frac{7}{2}^+[743\uparrow \otimes K^\pi = 0^-])$
13	474(2)	6	3	
14	510(3)	15	11	$(\frac{9}{2}, \frac{7}{2}^+[743\uparrow \otimes K^\pi = 0^-])$
15	537(5)	4	~4	
16	554(5)	2	~1	
17	585(1)	11	8	$(\frac{11}{2}, \frac{7}{2}^+[743\uparrow \otimes K^\pi = 0^-])$
18	608(4)	3	2	
19	636(3)	16	7	$\frac{3}{2}, \frac{3}{2}^-[743\uparrow \otimes K^\pi = 2^+_\gamma]$
20	663(4)	3	2	$\frac{5}{2}, \frac{3}{2}^-[743\uparrow \otimes K^\pi = 2^+_\gamma]$
21	670(2)	16	11	$\frac{7}{2}, \frac{5}{2}^-[752]$
22	698(2)	4	3	$\frac{7}{2}, \frac{5}{2}^-[743\uparrow \otimes K^\pi = 2^+_\gamma]$
23	719(2)	6	5	$\frac{9}{2}, \frac{5}{2}^-[752]$
24	749(4)	3	...	
25	774(4)	4	4	$\frac{11}{2}, \frac{5}{2}^-[752]$
26	818(3)	9	5	$\frac{9}{2}, \frac{9}{2}^-[734]$
27	829(3)	3	3	
28	886(2)	14	7	$\frac{11}{2}, \frac{9}{2}^-[734]$
29	921(2)	16	9	$\frac{11}{2}, \frac{11}{2}^-[743\uparrow \otimes K^\pi = 2^+_\gamma]$
30	947(4)	6	3	
31	960(4)	3	2	$\frac{13}{2}, \frac{9}{2}^-[734]$
32	986(3)	10	11	
33	1001(3)	7	3	
34	1041(3)	17	11	
35	1060(3)	6	5	

[a] Only dominant component given.

spacing, (2) the relative population of the levels in the (d,d') reaction, and (3) previous J^π assignments from other experiments.

The (d,d') population of a particular collective level is determined by

$$\frac{d\sigma}{d\Omega}(\theta) = \beta_l^2 \langle J_T l K_T \Delta K | J K\rangle^2 \sigma_l(\theta),$$

where J_T and K_T refer to the target states, $\Delta K = K - K_T$, l is the transferred orbital angular momentum, $\sigma_l(\theta)$ is the differential cross section calculated with the distorted-wave Born-approximation code DWUCK,[5] and β_l is a normalization parameter which can be related to the nuclear deformation. The distribution of the inelastic scattering strength among the rotational members of a particular collective band is determined by a set of Clebsch-Gordan coefficients. In inelastic

TABLE III. ^{237}Np.

Level	Energy (keV)	Cross section (μb/sr) 90°	125°	Assignment [a]
0	0	47 200	10 000	$\frac{5}{2},\frac{5}{2}^{+}[642]$
1	32(1)	3339	1357	$\frac{7}{2},\frac{5}{2}^{+}[642]$
2	75(1)	1452	508	$\frac{9}{2},\frac{5}{2}^{+}[642]$
3	131(1)	164	95	$\frac{11}{2},\frac{5}{2}^{+}[642]$
4	160(3)	22	4	$\frac{9}{2},\frac{5}{2}^{-}[523]$
5	193(1)	72	40	$\frac{13}{2},\frac{5}{2}^{+}[642]$
6	227(2)	12	4	$\frac{11}{2},\frac{5}{2}^{-}[523]$
7	268(1)	23	12	$\frac{15}{2},\frac{5}{2}^{+}[642]$
8	323(3)	6	3	
9	348(2)	7	7	
10	362(3)	5	···	$(\frac{17}{2},\frac{5}{2}^{+}[642])$
11	452(3)	4	3	
12	516(2)	6	3	$\frac{3}{2}^{-}[521]$ [b]
13	545(1)	12	6	$\frac{5}{2}^{-}[521]$ [b]
14	588(1)	21	10	$\frac{7}{2}^{-}[521]$ [b]
15	618(2)	12	4	
16	644(1)	14	6	$\frac{9}{2}^{-}[521]$ [b]
17	666(2)	7	4	
18	705(3)	6	4	$\frac{11}{2}^{-}[521]$ [b]
19	721(2)	30	16	$\frac{5}{2},\frac{5}{2}^{-}[642\uparrow\otimes K^{\pi}=0^{-}]$
20	755(2)	35	22	$\frac{7}{2},\frac{5}{2}^{-}[642\uparrow\otimes K^{\pi}=0^{-}]$
21	768(4)	19	<5	
22	797(1)	41	22	$\frac{9}{2},\frac{5}{2}^{-}[642\uparrow\otimes K^{\pi}=0^{-}]$
23	823(3)	17	5	
24	853(1)	16	11	$\frac{11}{2},\frac{5}{2}^{-}[642\uparrow\otimes K^{\pi}=0^{-}]$
25	863(4)	6	···	
26	906(2)	17	10	
27	920(3)	9	5	
28	946(2)	16	10	
29	963(2)	25	11	
30	984(2)	37	20	
31	1013(3)	10	5	
32	1030(3)	20	16	
33	1040(4)	16	~5	
34	1066(3)	29	13	

[a] Only dominant component given.
[b] See text and Table VI.

TABLE IV. ^{239}Pu.

Level	Energy (keV)	Cross section (μb/sr) 90°	125°	Assignment [a]
0	0	50 000	10 500	$\frac{1}{2}+\frac{3}{2},\frac{1}{2}^{+}[631]$
1	57(2)	6138	1896	$\frac{5}{2},\frac{1}{2}^{+}[631]$
2	76(2)	b	~183	$\frac{7}{2},\frac{1}{2}^{+}[631]$
3	164(2)	209	253	$\frac{9}{2},\frac{1}{2}^{+}[631]$
4	194(2)	23	40	$\frac{11}{2},\frac{1}{2}^{+}[631]$
5	317(2)	36	48	$\frac{13}{2},\frac{1}{2}^{+}[631]$
6	360(2)	8	7	$\frac{15}{2},\frac{1}{2}^{+}[631]$
7	470(2)	10	24	$\frac{1}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
8	491(4)	~9	~39	$\frac{3}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
9	505(2)	93	60	$\frac{5}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
10	555(2)	119	93	$\frac{7}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
11	583(3)	8	13	$\frac{9}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
12	659(3)	10	19	$\frac{11}{2},\frac{1}{2}^{-}[631\downarrow\otimes K^{\pi}=0^{-}]$
13	750(3)	14	10	
14	779(3)	9	5	
15	800(3)	13	13	
16	824(2)	32	28	
17	854(2)	34	27	
18	900(2)	39	32	
19	915(3)	5	10	
20	948(3)	<5	7	
21	993(3)	<5	4	
22	1027(2)	7	11	
23	1062(2)	7	7	

[a] Only dominant component given.
[b] Obscured by level 1.

deuteron scattering only one l transfer contributes to the single-step population of a particular vibrational band, viz., $l=3$ populates octupole bands and $l=2$ populates β- and γ-vibrational bands. Hence, for one-step reactions, which predominate in the cases studied, the population of the levels of a collective band has a unique pattern which is determined by the l transfer and the K quantum number of the band. In every case where the (d,d') reaction populates bands with known J,K^{π} assignments the cross sections are consistent with the predicted signature pattern of a single l transfer. Only in the case of ^{239}Pu, where the $K^{\pi}=\frac{1}{2}^{-}$ octupole band is strongly excited, is there evidence of populating additional levels by multistep processes. Nevertheless, even in this case the states populated strongly by $l=3$ transfer follow quite well the predicted single-step signature.

For the observed levels which have not been previously assigned a spin and parity, care must be taken in making assignments based solely upon energy spacings and cross sections. The only firm level assignments we felt confident in making in this fashion were the very strongly populated octupole bands in ^{233}U, ^{237}Np, and ^{239}Pu formed by coupling the ground state to the $K^{\pi}=0^{-}$ octupole core state.

TABLE V. Odd-*A* octupole levels.

Nucleus	Ground-state Configuration	K^π	Levels populated in (d,d') by $l=3$ transfer	Octupole $K=0^-$ core state coupled to ground state: Calculated distribution of strength $\langle J_{g.s.} 3 K_{g.s.} 0 \mid JK\rangle^2$	Experimental cross section [a]	Energy (keV)	Energy of the $J, K^\pi=3, 0^-$ level in adjacent even-even nucleus
^{233}U	$\frac{5}{2}^+[633]n$	$\frac{5}{2}^-$	$\frac{5}{2}^-$	0.119	0.16	748	
			$\frac{7}{2}^-$	0.381	0.37	790	
			$\frac{9}{2}^-$	0.379	0.32	838	740 [b]
			$\frac{11}{2}^-$	0.121	0.15	898	
^{237}Np	$\frac{5}{2}^+[642]p$	$\frac{5}{2}^-$	$\frac{5}{2}^-$	0.119	0.22	721	
			$\frac{7}{2}^-$	0.381	0.31	755	
			$\frac{9}{2}^-$	0.379	0.31	797	704 [c]
			$\frac{11}{2}^-$	0.121	0.16	853	
^{239}Pu	$\frac{1}{2}^+[631]n$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	0.43	0.39	505	
			$\frac{7}{2}^-$	0.57	0.61	555	655 [d]

[a] Normalized such that their sum is unity.
[b] Average of ^{232}U and ^{234}U, Refs. 2 and 20.
[c] Average of ^{236}U and ^{238}Pu, Refs. 1 and 2.
[d] Average of ^{238}Pu and ^{240}Pu, Refs. 1 and 3.

A. ^{233}U

The $\frac{5}{2}^+[633]$ ground-state band of ^{233}U has previously been observed up through the $J^\pi=\frac{17}{2}^+$ member at 415 keV.[6] The (d,d') results corroborate the previous energy assignments. The (d,d') energy of the $J^\pi=\frac{17}{2}^+$ level is 425 ± 4 keV as compared with the previous value[7] of 415 keV. It is entirely possible that much of this rotational level is obscured by the 403 ± 4-keV level with which it overlaps in the (d,d') spectra and does not imply a discrepancy with previous assignments. Using rotational parameters[7] of $\hbar^2/2\mathcal{I}=5.8$ keV and $B=-1.458\times10^{-3}$ keV, the $J^\pi=\frac{19}{2}^+$ band member should occur at 513 keV. Strength observed at 521 ± 3 keV is possibly due to this level.

The vibrational levels that are expected to be populated in a (d,d') reaction are members of the bands which are formed by coupling the single-nucleon target configuration to the one-phonon vibrations of the core. These core vibrations should be closely approximated by the vibrational states of the adjacent even-even nuclei, ^{232}U and ^{234}U. Earlier work on ^{234}U using the (d,d') reaction[2] has shown that the low-energy collective excitations are dominated by the $l=3$ transfer to the $K^\pi=0^-$, 1^-, and 2^- octupole bands and the $l=2$ transfer to the $K^\pi=2^+$ γ band.

The octupole band which is formed by the coupling of the ground state to the $K^\pi=0^-$ band has been clearly identified in ^{233}U. The information on the octupole bands in ^{233}U, ^{237}Np, and ^{239}Pu is summarized in Table V. To the extent that the vibrating core of the odd-*A* nucleus corresponds to the adjacent even-even nucleus, one can, using the same incident deuteron energy, compare the ^{234}U(d,d')^{234}U cross sections with those of the ^{233}U(d,d')^{233}U reaction. The total reaction cross section populating the $K^\pi=0^-$ band in ^{234}U is expected to be the same as the summed cross section to all of the levels in ^{233}U which involve the $J^\pi=\frac{5}{2}^+$ ground state coupled to the $K^\pi=0^-$ core state. The $K^\pi=0^-$ strength in ^{234}U at 90°, using 16-MeV incident deuterons, has been reported[2] to be 197 μb/sr. In ^{233}U the summed strength of the levels we have assigned as the $J^\pi=\frac{5}{2}^-$, $\frac{7}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ members of the octupole band at 748 keV is 235 μb/sr at 90°. From this one surmises that a majority of the $K^\pi=\frac{5}{2}^-$ octupole band strength in ^{233}U is contained in the 748-keV band. Little collective strength is admixed into the surrounding $K^\pi=\frac{5}{2}^-$ singleparticle bands. The level at 353 ± 2 keV has been previously shown in (p,t) experiments[8] to be the $J^\pi=\frac{9}{2}^-$ member of the $\frac{5}{2}^-$ [752] single-neutron band. This band is strongly Coriolis coupled with the $\frac{7}{2}^-[743]$ band observed[8] at 502 ± 1.5 keV. For this reason the (d,d') reaction populates these two bands through their coupling with both the $K^\pi = 0^-$ and $K^\pi = 1^-$ core vibrations. Using the excitation energies of the $J^\pi = \frac{7}{2}^-$ and $\frac{9}{2}^-$ levels of the $\frac{5}{2}^-[752]$ band, a rotational parameter $\hbar^2/2\mathscr{I} = 3.8$ keV is obtained. The $J^\pi = \frac{11}{2}^-$ member of the $\frac{5}{2}^-[752]$ band is then predicted to have an approximate energy of 394 keV and this level may be partially responsible for the (d,d') strength at 403 keV. The excitation energy of the $J^\pi = \frac{5}{2}^-$ bandhead relative to the higher rotational levels depends upon the extent of the Coriolis compression of the $\frac{5}{2}^-[752]$ band. A possible candidate for this $J^\pi = \frac{5}{2}^-$ level is level 6 at 298 ± 3 keV. Since this band has a mixture of $K = \frac{5}{2}$ and $\frac{7}{2}$ spin projections, the cross section distribution of the $l = 3$ strength will depend upon a coherent sum of Clebsch-Gordan coefficients which involves the amplitudes of the wave function components. Incomplete knowledge of the latter quantities prevents an accurate quantitative prediction of the relative cross sections of these levels, although their experimental cross sections are not inconsistent with the above assignments.

Levels at 502 ± 1.5 and 567 ± 2 keV were previously reported[8] to be the $\frac{7}{2}^-$ and $\frac{9}{2}^-$ members, respectively, of the $\frac{7}{2}^-[743]$ band. We interpret the groups at 500 ± 2 and 560 ± 4 keV as these levels, which in the (d,d') reaction are excited through their $\frac{7}{2}^-[633\downarrow \otimes K^\pi = 1^-]$ admixture and through the Coriolis admixed $\frac{5}{2}^-[633\downarrow \otimes K^\pi = 0^-]$ configuration.

Levels at 399 and 416 keV have been identified in the β^- decay of ^{233}Pa (Ref. 9) and at 398 ± 2 and 415 ± 2 keV in the ^{234}U(d,t) ^{233}U reaction[10] as the $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ members of the $\frac{1}{2}^+[631]$ band, respectively. There is a possibility that some of the (d,d') strength of levels 9 and 10 of Fig. 1 can be contributed by these positive-parity levels. They would be observed through an admixture of the $\frac{1}{2}^+[633\downarrow \otimes K^\pi = 2^+_\gamma]$ γ band.

Apparent in the ^{233}U(d,d') ^{233}U spectrum is a second envelope of strength centered about an excitation energy of 1 MeV. These levels may be associated with the ground-state coupling to the $K^\pi = 2^-$ octupole core state, the $J^\pi = 3^-$ level of which has been observed[2] at 1023 keV in ^{234}U. Some of the levels around an excitation of 1 MeV in ^{233}U may also result from the ground-state coupling to the γ vibration of the core. The $J^\pi = 2^+$ level of the γ band in ^{234}U has been seen[2] in inelastic deuteron scattering at 929 keV. The 90° cross section for 16-MeV deuteron inelastic population of the γ band and the $K^\pi = 2^-$ octupole band in ^{234}U has a total of 186 μb/sr. In ^{233}U the summed cross section of levels 26 through 33 is 341 μb/sr at 90°. These strengths are consistent with the supposition that a significant fraction of the strength of the ^{233}U(d,d') levels around 1 MeV results from the ground state coupled to both the γ vibration and the $K^\pi = 2^-$ octupole vibration of the core. A cautionary remark should be kept in mind when directly comparing inelastic cross sections of an odd-A nucleus with those of the adjacent even-even nuclei. The collective core states in the odd-A nucleus are not identical to the observed collective states in the adjacent even-even nucleus. In particular, the unpaired ground-state configuration of the odd-A nucleus is blocked from participating in the microscopic composition of the core states. The effect of this on the collective strengths and energy positions of the core states, while probably small, may not be negligible.

B. ^{235}U

The ground band of ^{235}U, the $\frac{7}{2}^-[743]$ neutron configuration, is seen in the (d,d') up through the $J^\pi = \frac{17}{2}^-$ member.[11]

The ground state coupling to the $K^\pi = 0^-$ core, which is clearly seen as a single band in the other nuclei studied in the present work, is not as easily identified in ^{235}U. The $\frac{7}{2}^-[743]$ band is strongly Coriolis coupled to the other deformed configurations which form part of the $j_{15/2}$ spherical configuration. As a result of the Coriolis admixtures in the ground state, the $l = 3$ cross sections of the levels of the $\frac{7}{2}^-[743\uparrow \otimes K^\pi = 0^-]$ band will have a pattern which is determined by a coherent sum of Clebsch-Gordan coefficients and unknown wave function amplitudes. Consequently, a firm assignment for this octupole band cannot be made in this experiment. However, a 509-keV level in ^{235}U has been tentatively assigned as the $J^\pi = \frac{9}{2}^+$ member of the $\frac{7}{2}^+[624]$ band in (d,p) and (d,t) experiments.[12] Deuteron inelastic scattering would primarily populate this $K^\pi = \frac{7}{2}^+$ band through its admixture of the $\frac{7}{2}^+[743\uparrow \otimes K^\pi = 0^-]$ octupole band. The (d,d') reaction strongly populates a level at 510 ± 3 keV. Assuming a $J(J+1)$ energy spacing and a rotational parameter of $\hbar^2/2\mathscr{I} = 6.82$ keV, the energies for the $J^\pi = \frac{7}{2}^+$ through $\frac{13}{2}^+$ members of this band are 449, 510, 585, and 674 keV, respectively. The (d,d') reaction populates strongly collective levels at 444 ± 2, 510 ± 3, and 585 ± 1 keV, which can be identified with the $J^\pi = \frac{7}{2}^+$, $\frac{9}{2}^+$, and $\frac{11}{2}^+$ members of this octupole band. The $J^\pi = \frac{13}{2}^+$ member may contribute part of the strength of the intense level at 670 ± 2 keV. On the basis that these levels are among the largest in the ^{235}U(d,d') spectra one can safely conclude that their wave functions include an appreciable collective component. The fact that the (d,p) and (d,t) reactions observe only the 510-keV level, and that only weakly, supports this. The assignment of these levels as the ground state coupled to the $K^\pi = 0^-$ core vibration seems plausible in light of the available information, although the complications introduced by the strong

TABLE VI. The $\frac{3}{2}^-[521]+\frac{3}{2}^-[642\uparrow\otimes K^\pi=1^-]$ band in ^{237}Np.

J^π	(keV)	$E_{(d,d')}$ [a] (keV)	E_{calc} [b] (keV)	$\frac{d\sigma}{d\omega}(90°)$ (μb/sr)	$\frac{d\sigma}{d\omega}(90°)$ [c] (Normalized)	$\langle\frac{5}{2}3\frac{5}{2}-1\mid J\frac{3}{2}\rangle^2$
$\frac{3}{2}^-$	514	516(2)	514	6	0.10	0.07
$\frac{5}{2}^-$	545	545(1)	545	12	0.20	0.29
$\frac{7}{2}^-$	589	588(1)	589	21	0.36	0.38
$\frac{9}{2}^-$	···	644(1)	645	14	0.24	0.22
$\frac{11}{2}^-$	713	705(3)	713	6	0.10	0.05

[a] See Ref. 13.
[b] Calculated using $\hbar^2/2\mathcal{I}=6.22$ keV.
[c] Normalized such that their sum is unity.

Coriolis coupling of the ground state makes the assignment only tentative.

The ^{235}U(d,d') ^{235}U reaction also populates levels which were identified in Coulomb excitation[11] as $K-2$ and $K+2$ γ bands built on the ground state. The $K^\pi=\frac{11}{2}^-[743\uparrow\otimes K^\pi=2^+]$ has been firmly established in the Coulomb excitation with the $J^\pi=\frac{11}{2}^-$ bandhead at 920.7 keV. The (d,d') reaction should populate only the bandhead of this band with a direct $l=2$ transfer. The 921 ± 2-keV level seen in the 90° spectrum with 16 μb/sr is identified as the $J^\pi=\frac{11}{2}^-$ level. The $K^\pi=\frac{3}{2}^-[743\uparrow\otimes K^\pi=2^+]$ band is tentatively assigned to levels with a bandhead energy of 637.9 keV in Coulomb excitation. The present (d,d') experiment populates levels at 636 ± 3, 663 ± 4, and 698 ± 2 keV which are consistent with a $l=2$ transfer to the $K^\pi=\frac{3}{2}^-$ γ band and having spin assignments of $J^\pi=\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$, respectively. The level density in this nucleus makes accurate cross section totals somewhat uncertain; however, the sum of levels 19, 20, and 22 gives a total (d,d') strength of 23 μb/sr at 90°. The total cross section to the γ bands in ^{235}U is, then, approximately 39 μb/sr at 90°. The total (d,d') strength populating[2] the $J^\pi=2^+$ level in ^{234}U is 67 μb/sr and in ^{236}U is 104 μb/sr at 90°. This result supports the findings of Stephens *et al.*[11] in Coulomb excitation that the $B(E2)$ strength of the γ bands in ^{235}U is well below those of the γ bands in ^{238}U and indicates significant single-particle admixture.

Levels previously assigned to the $\frac{3}{2}^+[631]$, $\frac{5}{2}^+[633]$, and $\frac{5}{2}^+[622]$ bands below 500 keV are weakly excited in the (d,d') spectra and are thus only weakly collective in nature.

The levels at 670 ± 2, 719 ± 2, and 774 ± 4 keV are interpreted as the $\frac{7}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ members of the $\frac{5}{2}^-[752]$ band on the basis of previous assignments.[11] Similarly, the groups at 818 ± 3, 886 ± 2, and 960 ± 2 keV are identified as the $\frac{9}{2}^-$, $\frac{11}{2}^-$, and $\frac{13}{2}^-$ levels of the $\frac{9}{2}^-[734]$ configuration. These two bands as well as the $\frac{7}{2}^-[743]$ ground state are part of the $j_{15/2}$ spherical configuration and, as a result, are strongly Coriolis coupled. The (d,d') reaction directly populates the $\frac{5}{2}^-[752]$ and $\frac{9}{2}^-[734]$ bands through their Coriolis admixtures with the ground state.

C. ^{237}Np

The (d,d') spectrum of ^{237}Np is an interesting analog to that of ^{233}U. Both nuclei have a ground state of $J^\pi=\frac{5}{2}^+$; and consequently, their (d,d') spectra share many similarities which result from the angular momentum coupling to their collective core states.

The ground band is the proton configuration $\frac{5}{2}^+[642]$. The rotational members of this band have been observed up through the $J^\pi=\frac{13}{2}^+$ level in the (^{3}He,d) reaction[13] and in the α decay[14] of ^{241}Am. The present experiment observes this band up through the $J^\pi=\frac{15}{2}^+$ level at 268 ± 1 keV and some evidence for the $J^\pi=\frac{17}{2}^+$ member may be seen in the (d,d') strength around 360 keV.

The excited levels of ^{237}Np are similar to ^{233}U in that the characteristic pattern of the collective $K^\pi=\frac{5}{2}^-[642\uparrow\otimes K^\pi=0^-]$ octupole band is seen at 721 ± 2 keV. The $J^\pi=\frac{7}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ members of this octupole band are at 755, 797, and 853 keV, respectively. The excitation energy of this band is similar to the $J,K^\pi=3,0^-$ level energies in the adjacent even-even nuclei ^{236}U and ^{238}Pu which are 746 keV[2] and 661 keV[15], respectively.

The $J^\pi=\frac{5}{2}^-$ and $\frac{7}{2}^-$ levels at 721 and 755 keV have previously been interpreted[16] to be members of the β band built upon the $\frac{5}{2}^-[523]$ single-particle state. The present data clearly indicate, however, that this collective band is coupled directly to the ground state and is populated by $l=3$ transfer which implies an octupole character. There is a second envelope of collective strength centered around an excitation energy of 1 MeV which perhaps corresponds to the ground-state coupling to the $J^\pi = 2^+$ γ bandhead at 959 keV and the 1037-keV $J^\pi = 3^-$ octupole state in ^{236}U observed in inelastic scattering.[2] Unfortunately, the level density and experimental resolution are such that the levels cannot be separated into rotational bands.

The $\frac{3}{2}^-[521]$ single-proton band, which had been previously observed[13] at 514 keV, illustrates the degree to which the (d,d') reaction populates the collective properties of a band with only a single l transfer. Five members of this particular single-particle band are seen through the admixed component of the $K^\pi = \frac{3}{2}^-[642\uparrow \otimes K^\pi = 1^-]$ octupole state. Table VI presents the energy and cross section data for this band. It is apparent that the $l = 3$ population of the octupole admixture to this band adequately explains its cross section pattern.

There is also evidence for the inelastic population of the $\frac{5}{2}^-[523]$ band at 60 keV. This band should be seen through its $K^\pi = \frac{5}{2}^-[642\uparrow \otimes K^\pi = 0^-]$ octupole admixture. One should thus observe the four levels of $J^\pi = \frac{5}{2}^-$, $\frac{7}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$. However, the intense ground band rotational members obscure all but the $J^\pi = \frac{9}{2}^-$ and $\frac{11}{2}^-$ levels at 160 ± 3 and 227 ± 2 keV, respectively. These two levels have the expected cross-section ratio. If one calculates the cross sections for the obscured $J^\pi = \frac{5}{2}^-$ and $\frac{7}{2}^-$ levels from the Clebsch-Gordan distribution of the $l = 3$ transfer, the total (d,d') cross section to this band is found to be 73 μb/sr at 90°. This strength added to the 122-μb/sr total of the $K^\pi = \frac{5}{2}^-$ collective band at 721 keV is in reasonable agreement with the (d,d') cross section of the $K^\pi = 0^-$ band in ^{236}U, which at 90° is 246 μb/sr.[2]

D. ^{239}Pu

The ground band of ^{239}Pu is the $\frac{1}{2}^+[631]$ neutron configuration. This band is populated in the present (d,d') reaction up through the $J^\pi = \frac{15}{2}^+$ rotational member at 360 ± 2 keV. The level assignments for the $J^\pi = \frac{1}{2}^+$ through $\frac{11}{2}^+$ made in earlier ^{243}Cm α-decay studies[17] are corroborated and the $J^\pi = \frac{13}{2}^+$ and $\frac{15}{2}^+$ levels are identified for the first time. The extracted rotational parameter is $\hbar^2/2\mathcal{I} = 6.25$ keV and the experimental decoupling parameter is $a = -0.565$.

The $J^\pi = \frac{1}{2}^+$ target spin and parity causes all of the directly populated octupole strength to be concentrated in the $J^\pi = \frac{5}{2}^-$ and $\frac{7}{2}^-$ levels of the respective collective bands. The $K^\pi = \frac{1}{2}^-[631\downarrow \otimes K^\pi = 0^-]$ octupole band has been identified in earlier deuteron inelastic scattering,[18] in which work the $J^\pi = \frac{5}{2}^-$ and $\frac{7}{2}^-$ levels were observed at 505 and 558 keV, respectively. The present (d,d') experiment corroborates this assignment. The $J^\pi = \frac{5}{2}^-$ and $\frac{7}{2}^-$ levels are very strongly populated at 505 ± 2 and 555 ± 2 keV, respectively. In Table V we show the close agreement of the (d,d') strength ratio of these two levels and the strength distribution predicted for pure $l = 3$ transfer by the Clebsch-Gordan coefficients. In addition to these strong levels the remaining members of this band are observed up through the $J^\pi = \frac{11}{2}^-$ level. The levels $J^\pi = \frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ all have approximately the same cross section and are likely populated by multistep processes. The extracted rotational parameters for this $K^\pi = \frac{1}{2}^-$ band are $\hbar^2/2\mathcal{I} = 5.13$ keV and $a = +0.393$. By considering the microscopic composition of the collective bands in the actinide region, Malov and Soloviev[19] have calculated the decoupling parameters of the collective $K = \frac{1}{2}$ bands in this mass region. For this particular band in ^{239}Pu their calculation indicated $a = +0.25$ in reasonable agreement with the present experimental result. The summed (d,d') cross section of the $J^\pi = \frac{1}{2}^-$ through $\frac{11}{2}^-$ levels for 125° is 248 μb/sr. This is in agreement with what one would expect in light of the comparable (d,d') population[3] of the $K^\pi = 0^-$ band in ^{240}Pu with 206 μb/sr.

The strongly collective levels between 800 and 900 keV are probably composed of the $\frac{1}{2}^+[631]$ ground state coupled to the octupole band seen in ^{240}Pu at about 1 MeV.[3] The strongly collective ^{240}Pu level at 1001 keV has $J^\pi = 3^-$ and is a member of either the $K^\pi = 1^-$ or 2^- octupole band. The γ-band strength in ^{240}Pu appears to be divided among several $K^\pi = 2^+$ bands[3] and, as a result, should not contribute a great deal of collective strength to any one level in ^{239}Pu. The 16-MeV (d,d') cross section to the 1-MeV octupole band in ^{240}Pu is 96 μb/sr at 125°. The corresponding strength of the levels between 800 and 900 keV is 100 μb/sr. One could expect to make a definite K assignment to the 1001-keV level in ^{240}Pu from the distribution of the $l = 3$ strength observed in ^{239}Pu. However, the levels in ^{239}Pu between 800 and 900 keV seem to be sufficiently modified by Coriolis coupling to make unambiguous J, K^π assignments impossible.

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