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Investigation of the $(d_{5/2})^2$ and $(d_{5/2}s_{1/2})$ two-particle configurations in ^{18}O using the ^{17}O$(d,p)^{18}$O reaction at 18 MeV†

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Differential cross sections of deuteron elastic scattering from ^{17}O and of the ^{17}O$(d,p)^{18}$O reaction have been measured at a deuteron bombarding energy of 18 MeV for transitions to 12 states in ^{18}O up to an excitation energy of 6.341 MeV. Absolute spectroscopic factors were extracted with the aid of distorted-wave–Born-approximation calculations. The results are compared with theoretical calculations and show a distinct preference for a weak-coupling model. Diagonal matrix elements of the effective neutron-neutron interaction were deduced from the data for the $(d_{5/2})^2{}_{0^+,2^+,4^+}$ and $(d_{5/2}s_{1/2})_{2^+,3^+}$ configurations. A theoretical estimate of the errors involved is in qualitative agreement with the deviations observed between experiment and theory.

[NUCLEAR REACTIONS ^{17}O(d,d), (d,p), $E=18.0$ MeV; measured $\sigma(\theta)$, deduced optical-model parameters; measured $\sigma(E_p,\theta)$, ^{18}O levels deduced l, S, effective interactions. Enriched target.]

I. INTRODUCTION

The nucleus ^{18}O occupies a central position in the study of nuclear structure. It was one of the first nuclei to be studied in a many-particle shell model,[1] and currently it is of interest in connection with attempts to calculate the shell-model effective interaction from the nucleon-nucleon force (e.g., Ref. 2). Furthermore, discovery of additional 0^+ and 2^+ levels not predicted by a simple two-particle model stimulated early work[3,4] on low-lying deformed particle-hole states in ^{18}O and in neighboring nuclei. It is thus surprising that the ^{17}O$(d,p)^{18}$O reaction, which provides information on the structure of ^{18}O, has been studied only at rather low bombarding energies[5,6] and that these data were analyzed only in the plane-wave Born approximation (PWBA).

The value of the ^{17}O$(d,p)^{18}$O reaction for a study of simple two-neutron configurations in ^{18}O is readily recognized. In a simple shell-model picture, stripping of a $1d_{5/2}$ neutron into an ^{17}O target ($J^\pi = \frac{5}{2}^+$) will excite the 0^+, 2^+, and 4^+ states of the $(1d_{5/2})^2$ configuration in ^{18}O. Stripping of a $2s_{1/2}$ neutron will lead to the 2^+ and 3^+ states of the $(1d_{5/2}2s_{1/2})$ configuration, and $1d_{3/2}$ stripping should excite the 1^+, 2^+, 3^+, and 4^+ states of the $(1d_{5/2}1d_{3/2})$ configuration. Furthermore, stripping to negative parity states in ^{18}O should give an indication of the extent to which the $1p$ shell is closed in the ground-state wave function of ^{17}O.

Of course, the simple configurations mentioned above are actually fragmented, and we will compare our results with recent calculations of Halbert *et al.*[7] in which all $(sd)^2$ configurations were considered beyond a closed ^{16}O core and in which the Kuo[2] interaction was used. As previously remarked, it has been found necessary to relax the restriction of a closed ^{16}O core in order to obtain a sufficient number of levels in ^{18}O. Two fairly recent calculations[8,9] of this type are also compared with the data obtained here. The first[8] assumes a closed ^{12}C core and allows the remaining particles to be distributed over the $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ orbitals without restriction. The second[9] is a weak-coupling model in which the basis was truncated by assuming that the most important correlations are those between particles in the same major shells. Thus particle-hole states were constructed by coupling together the separate eigenfunctions for the $(sd)^{n_1}$ and p^{n_2} problems. Since the particle-hole interaction does not strongly mix high- and low-lying states, it is possible to severely truncate the basis and finally diagonalize a rather small matrix containing the important low-lying states. The two types of calculation thus employ rather different truncation procedures; however, the results obtained are, broadly speaking, quite similar.

It is felt that a distorted-wave–Born-approximation (DWBA) analysis of data at deuteron energies higher than those of previous experiments would be useful to test the relative merits of these different models. This would seem particularly important if the one- and two-particle transfer data are used directly to infer the structure of the 0^+ states in mass 18, as has been done.[10] A point of particular interest lies in the inability of the two previous experiments[5,6] to detect an $l=2$ component in the transition to the 3.919-MeV state

because of a contamination by the $l = 2$ transition of the $^{16}O(d, p)^{17}O$(g.s.) reaction on targets of low ^{17}O enrichment. According to all model calculations[7-9] much of the $(1d_{5/2})^2_{2^+}$ and $(2s_{1/2}1d_{5/2})_{2^+}$ configurations is shared between the lower 2^+ states. One purpose of the present experiment is to determine the strength of the $l = 2$ component in the transition to the 3.919-MeV state. We may remark that the $l = 2$ component has been obtained[11] for the analog state in ^{18}F using the $^{17}O(^3He, d)$ reaction at 15 MeV. Unfortunately in that study there was some uncertainty in the absolute normalization, but we shall compare relative values of transition strengths to the low-lying states obtained in that work[11] with the present results.

We have studied the $^{17}O(d, p)^{18}O$ reaction at a deuteron bombarding energy of 18 MeV and have measured differential cross sections for transitions to 12 states in ^{18}O up to an excitation energy of 6.341 MeV. We have also measured differential cross sections for the elastic scattering $^{17}O(d, d)$ in order to help determine the $d + ^{17}O$ optical potential to be used in the DWBA analysis of the $^{17}O(d, p)$ data. In addition, some data on $d + ^{16}O$ elastic scattering have also been obtained. The experimental procedures and results are described in Sec. II, the extraction of spectroscopic factors from the data is discussed in Sec. III, and these are compared with theory in Sec. IV. In Sec. IV we also discuss the extraction of effective matrix elements, and in Sec. V our conclusions are presented.

II. EXPERIMENT AND RESULTS

The experiment was performed using an 18.0-MeV deuteron beam produced by the University of Minnesota MP tandem Van de Graaff accelerator. Targets[12] composed of ^{17}O-enriched tungsten oxide on carbon backings were prepared by first evaporating tungsten to a thickness of 100 to 150 $\mu g/cm^2$ onto a 30-$\mu g/cm^2$ carbon foil. This foil was then transferred to a small-volume vacuum system and a small amount of either natural or istopically enriched oxygen gas was admitted to the system. The tungsten was then oxidized by heating with the light from a projection lamp.

The reaction products were momentum analyzed in an Enge split-pole spectrograph and detected by use of six position-sensitive solid-state detectors. Three of these detectors have a depletion depth of 1.0 mm and the other three have a depletion depth of 0.5 mm. All the detectors have a sensitive area 10 mm high by 30 mm wide. They

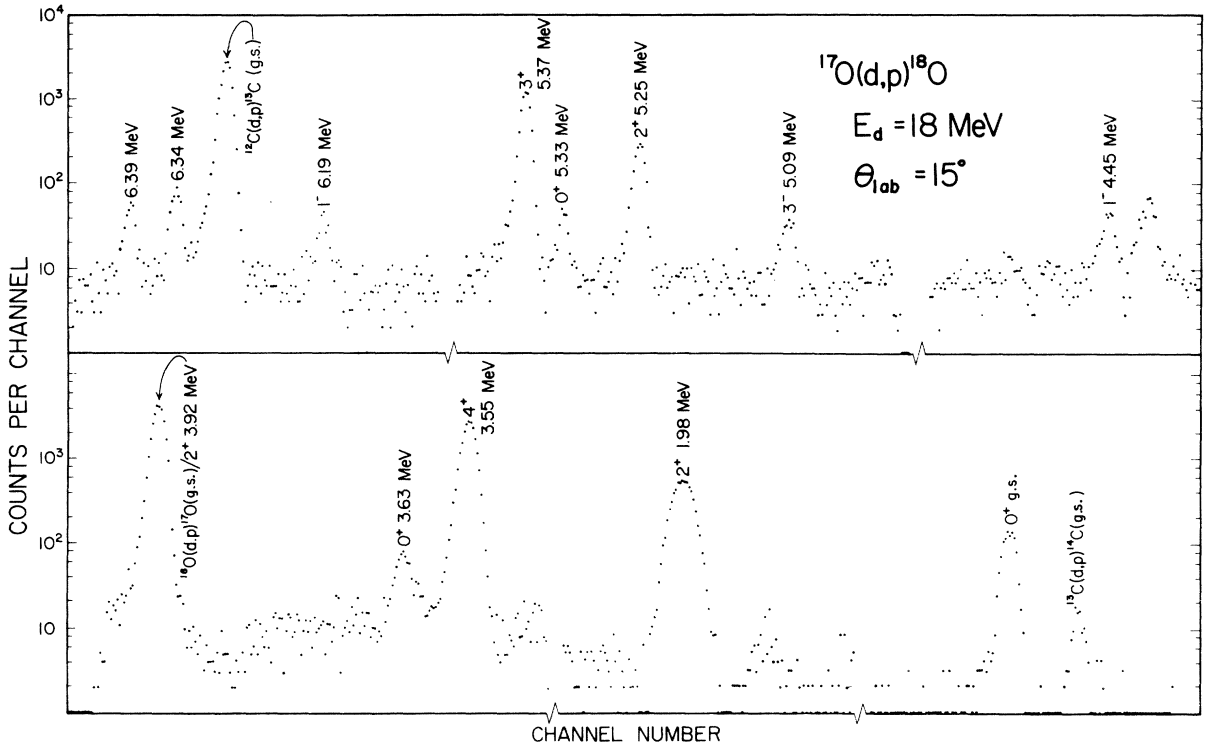

FIG. 1. Proton spectrum from the $^{17}O(d, p)^{18}O$ reaction measured at the spectrograph focal surface by six position-sensitive detectors.

were placed along the spectrograph focal surface with about 12-mm spacings between sensitive areas. The position of this surface was set to compensate for the kinematics of the $^{17}O(d,p)$ or $^{17}O(d,d)$ reactions. Therefore, reaction products from the tungsten in the target were detected off focus and generated a broad background in the spectra.

Polyethylene films 0.6 mm thick were placed immediately in front of the detectors in order to increase the magnitude of the detector signals produced by protons which passed through the depletion region. The two parameters, energy and position multiplied by energy, which are measured by each of the six detectors, were digitized and routed to a CDC 3100 computer where divisions were performed to obtain the two parameters, energy and position. The computer system allows energy thresholds to be selected to permit separate accumulation of position spectra for those different types of particles which produce different energy losses in the detectors. The dead time of the system was continuously monitored and found to be negligible except for angles $\leq 12.5°$ where they mounted to less than 8%.

In Fig. 1 are shown the combined position spectra for protons obtained by the six focal-surface detectors when the spectrograph was set at $\theta_{lab} = 15°$. Dead spaces between detectors are indicated by breaks in the horizontal axes. Except for the transition to the state at 5.517 MeV, proton groups for all transitions to ^{18}O states up to $E_x = 6.341$ MeV fell on the sensitive areas of the detectors. The narrowest peaks in Fig. 1 have a width of about 16 keV full width at half maximum. The low-level background is mainly from (d,p) reactions on the tungsten. Proton groups are also seen from the two principal contaminant reactions $^{16}O(d,p)$-^{17}O(g.s.) and $^{12}C(d,p)^{13}C$(g.s.).

The isotopic content of the targets was deter-

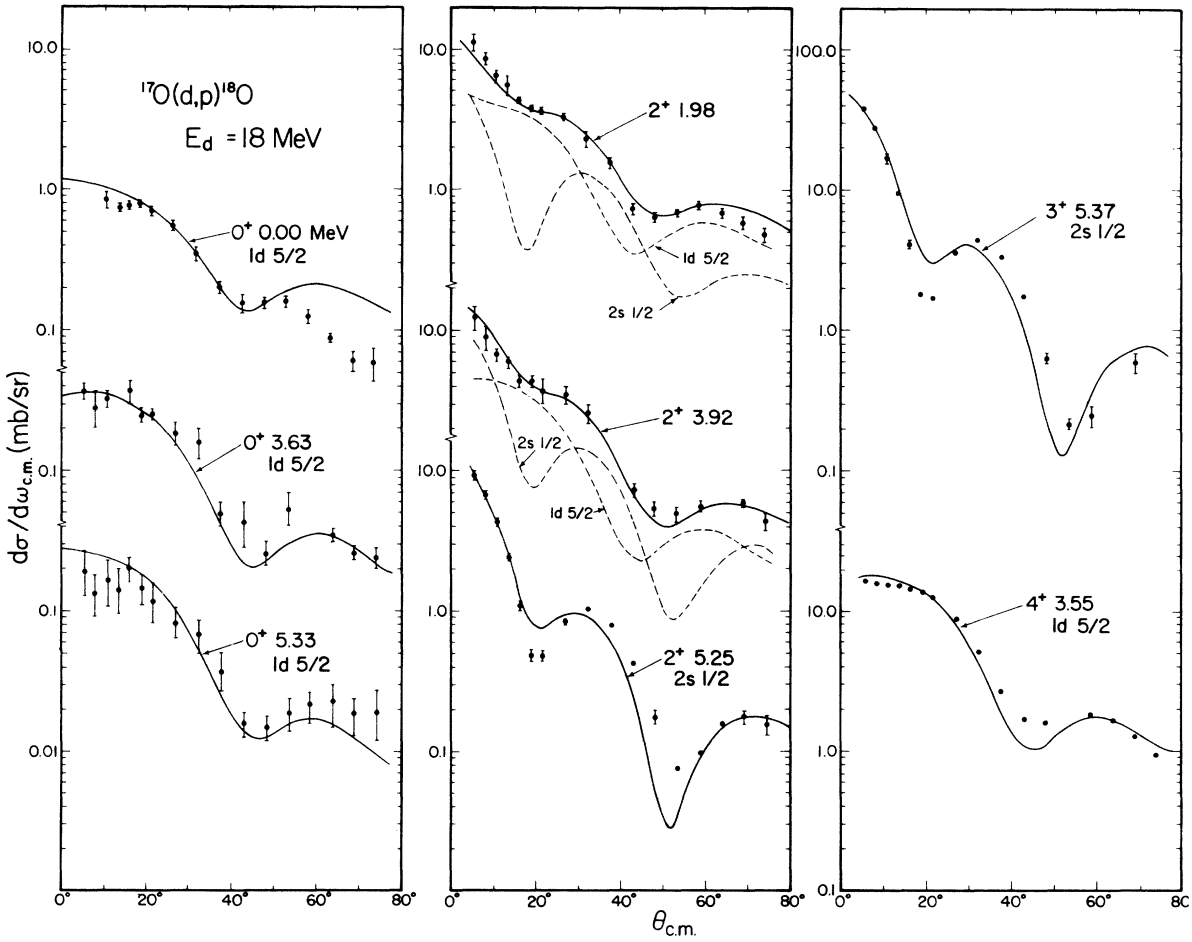

FIG. 2. Differential cross sections of the $^{17}O(d,p)$ reaction leading to positive-parity states of ^{18}O. The curves show the results of DWBA calculations using the optical-potential parameters $P1$-$D1$ of Table I. The dashed curves indicate the separate DWBA contributions from $1d_{5/2}$ and $2s_{1/2}$ neutron transfer.

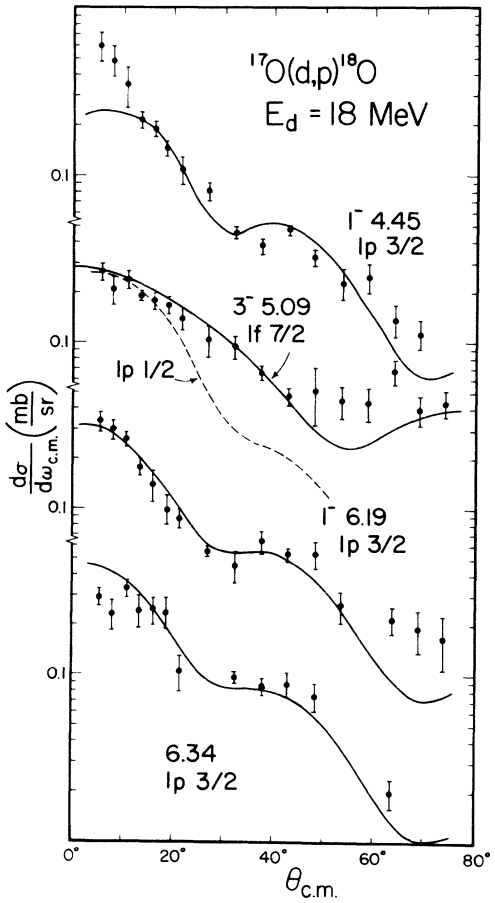

FIG. 3. Differential cross sections of the ^{17}O(d, p) reaction leading to negative-parity states of ^{18}O. The curves show the results of DWBA calculations using the optical-potential parameters $P1$-$D1$ of Table I.

mined during a different experiment[12] by measuring the yield of 25-MeV ^{3}He particles elastically scattered from the different nuclei in the targets. The amount of ^{16}O in the targets was obtained by using values for the ^{16}O(^{3}He, ^{3}He) differential cross section at 25 MeV, which were measured with a gas target system. The ^{17}O content was deduced by using an optical-model calculation to obtain an estimate of the ratio of the desired ^{17}O(^{3}He, ^{3}He) cross section to the measured ^{16}O(^{3}He, ^{3}He) cross section. The target ultimately used in the present experiment was determined to have an ^{17}O areal density of 17 μg/cm^2, which amounted to 65% of the total oxygen content. Further details are given in Ref. 12. Two other additional sources of information on the target content are available. First, upon performing a series of optical-model fits to our 18-MeV ^{16,17}O(d, d) measurements, we found that a change in the normalization of the data by as much as 15% often resulted in improved fits. Second, at 15.8 MeV a value of 33 ± 5 mb/sr has been reported[13] for the ^{16}O(d, d) differential cross section at the relative maximum near 47° (c.m.). When we use an optical-model calculation to correct approximately for the energy dependence of this cross section we predict a value of 20 ± 3 mb/sr at 18 MeV; our measured value is 24 mb/sr. All these considerations have led us to assess an error of ± 15% in our scale of absolute cross section.

Differential cross sections were measured for the reaction ^{17}O(d, p)^{18}O leading to 12 states in ^{18}O in the excitation-energy range from 0 to 6.341 MeV. These measurements were made in the angular range θ_{lab} = 5° to 70° in steps of 2.5° in the forward angular region and in steps of 5° in the remaining angular region. The angular acceptance of the spectrograph was set to Ω = 1.24 msr, $\Delta\theta$ = 2° for $\theta_{lab} \geq 12.5°$, to Ω = 0.57 msr, $\Delta\theta$ = 1° at 7.5° and 10°, and to Ω = 0.21 msr, $\Delta\theta$ = 0.5° for θ_{lab} = 5°. At several angles the proton group from the reaction ^{17}O(d, p)^{18}O(3.919 MeV) could not be resolved from the proton group from the reaction ^{16}O(d, p)^{17}O(g.s.) (see Fig. 1 for example). At these angles a separate measurement was made of the ^{16}O(d, p)^{17}O(g.s.) reaction using a tungsten-oxide target fabricated with natural oxygen. A determination of the ratio of the ^{16}O content of this target to that of the ^{17}O-enriched target allowed a subtraction to be made to determine the ^{17}O(d, p)-^{18}O(3.919 MeV) cross section.

Figures 2 and 3 show the results of our measurements of the ^{17}O(d, p) differential cross sections together with the results of DWBA calculations, which are discussed below. The indicated errors are relative standard deviations and are composed

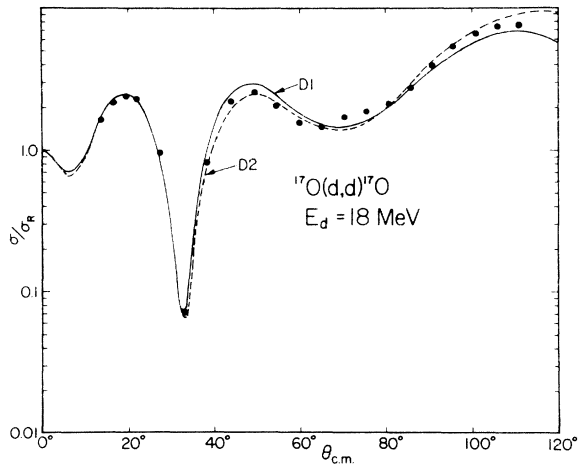

FIG. 4. ^{17}O(d, d) differential cross sections σ plotted as the ratio to the Rutherford cross section σ_R. The lines show the results of the optical-model calculations using the indicated parameter sets of Table I.

TABLE I. Optical-model parameters. The combination $P1$-$D1$ was used in the DWBA calculations of this work.

Channel	Set	V_R (MeV)	r_R (fm)	a_R (fm)	$4W_D$ (MeV)	W_s (MeV)	r_I (fm)	a_I (fm)	V_{so} [a] (MeV)	r_{so} (fm)	a_{so} (fm)	r_c (fm)
Proton	$P1$ [b]	51.5	1.04	0.67	32.4	1.7	1.17	0.52	6.2	1.01	0.75	1.30
	$P2$ [c]	50.1	1.10	0.74	22.4	0	1.30	0.66	4.25	0.90	0.52	1.30
Deuteron	$D1$ [d]	85.18	1.18	0.72	20.8	0	1.73	0.75	4.58	0.75	0.18	1.30
	$D2$ [d]	85.14	1.15	0.74	14.8	0	1.55	1.04	4.53	0.92	0.74	1.30

[a] The spin-orbit well depth V_{so} must be multiplied by a factor of 4 for protons and by a factor of 2 for deuterons before being entered into DWUCK 4.

[b] See Ref. 19.

[c] See Ref. 20.

[d] This work.

of the statistical error in the number of detected protons and an uncertainty in the subtraction of the background beneath the proton peak. For most of the data, this latter uncertainty is the dominant contribution to the relative error.

The measurements of the differential cross sections for the elastic scattering of deuterons from ^{16}O and ^{17}O were made between $\theta_{lab} = 10°$ and $105°$. Here the angular acceptance was set to $\Omega = 0.21$ msr, $\Delta\theta = 0.5°$, for $\theta_{lab} \leq 55°$ and to $\Omega = 1.84$ msr, $\Delta\theta = 3°$, for $\theta_{lab} \geq 55°$. The relative errors of these measurements are almost all less than 3%. The points in Fig. 4 show the ratio of the measured ^{17}O(d, d) differential cross section to the Rutherford differential cross section, and the curves indicate optical-model fits to the data, which are described below. No correction was applied for the finite angular acceptance of the spectrograph.

III. DATA ANALYSIS

A. Optical-model analysis

In order to obtain optical-model fits to our ^{16}O(d, d) and ^{17}O(d, d) differential-cross-section data, some initial parameter searches were performed with the code RAROMP.[14] This code treats the spin-orbit term properly only for spin $\frac{1}{2}$ particles. In these searches, starting values for the parameters were taken from the work of Haight.[15] Some attempt was made at first to fit the data with the optical spin-orbit strength V_{so} set equal to zero. However, even though it was rather easy to fit the ^{16}O(d, d) data with $V_{so} = 0$, we were not able to obtain a satisfactory fit to the ^{17}O(d, d) data with $V_{so} = 0$. The difficulty seemed to be caused by a larger peak to valley ratio in the ^{17}O(d, d) differential cross section in the angular range 30° to 50° (c.m.). A satisfactory fit to the ^{17}O(d, d) data was obtained with RAROMP when a value for V_{so} of about 4 MeV was used.

The best-fit parameters obtained with RAROMP were then used as starting values for parameter searches with the code SNOOPY5,[16] which treats the spin-orbit term properly for spin-1 particles. Two sets of optical-model parameters were determined for $d + ^{17}$O. They are listed in Table I as Sets $D1$ and $D2$. Both sets were obtained with the same parameter starting values, but a different procedure was followed in the variation of the parameters. For Set $D2$ the combinations of parameters varied during each search was chosen so as to avoid the very small values in the radius and the diffuseness of the spin-orbit potential of Set $D1$. Therefore r_{so} and a_{so} were kept constant at 1.11 and 0.74 fm until the final search in which r_{so} decreased to 0.92 fm and the other parameters varied only very little. The main result of this different search procedure was a significant difference in the absorptive potential of $D2$ compared to $D1$, in addition to the intended difference in the spin-orbit geometry. Figure 4 shows that good fits to our ^{17}O(d, d) data were obtained by both optical parameter sets $D1$ and $D2$.

B. DWBA analysis

A DWBA analysis of the ^{17}O$(d, p)^{18}$O data was carried out with the code DWUCK4.[17] A Hulthén finite-range correction was used in the calculations, but no nonlocality correction was used. The neutron bound-state wave function was generated from a Woods-Saxon potential of radius $r = 1.25(17)^{1/3}$ fm, diffuseness $a = 0.65$ fm, and a spin-orbit potential of 25 times the Thomas-Fermi term. The potential depth was adjusted to yield the neutron separation energy of the appropriate ^{18}O final state. Shapes of DWBA cross sections for several ^{17}O(d, p) transitions were investigated when calculated with a variety of combinations of optical-potential sets. Four such sets are given in Table I. Set $P1$, based on the proton parameters of Becchetti and Greenlees[18] (with some modification of the central geometry), was used by Fleming *et al.*[19] to analyze the ^{18}O(p, t) reaction at 20 MeV.

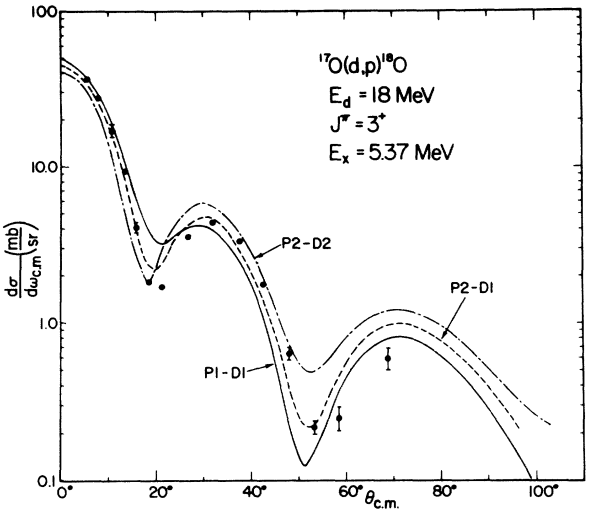

FIG. 5. Comparison of DWBA curves for the pure l = 0 transition ^{17}O(d, p) ^{18}O(5.372 MeV) using the indicated combinations of the optical-potential parameter sets of Table I.

Set $P2$ was used by Escudie *et al.*[20] to analyze the elastic scattering of protons by ^{18}O at 24.5 MeV. Sets $D1$ and $D2$ were discussed in the previous section. Other sets of proton parameters[15,18,21] and deuteron parameters[13,15,22] were also studied for their effects on the DWBA shapes. It was found, however, that the potential sets of Table I best reproduce the experimental shapes, especially for l = 0 transfer. Figure 5 illustrates the changes produced in the DWBA shape of a pure l = 0 transition by the use of some different combinations of the optical-potential parameters of Table I. A comparison of the curve produced by use of the combination $P2$-$D1$ with that produced by use of the combination $P2$-$D2$ illustrates the change in DWBA shape which can result when two different deuteron parameter sets are used, each of which fits the d + ^{17}O elastic scattering data (see Fig. 4.). Although the use of the combination $P2$-$D1$ gives reasonable DWBA shapes for l = 0 transitions, the l = 2 angular distributions are less well reproduced. We therefore used the combination $P1$-$D1$, which gives somewhat poorer l = 0 shapes (see Fig. 5) but yields the best over-all agreement with all the experimental data. The results of the DWBA calculations are shown in Figs. 2 and 3. The dashed curves in Fig. 2 indicate the separate contributions from $1d_{5/2}$ and $2s_{1/2}$ neutron transfer. The spectroscopic factors S extracted from these calculations are listed in Table II. They were obtained by normalizing the cross section σ_{DW} calculated with DWUCK4 to the experimental cross section σ_{exp} through the equation

$$\frac{\sigma_{exp}}{\sigma_{DW}} = NC^2S\frac{(2J+1)}{(2J_0+1)(2j+1)}, \quad (1)$$

where J_0 and J are the total angular momenta of the target and residual nucleus, respectively, j is the total angular momentum of the transferred neutron, the constant N is taken as[17] N = 1.53, and C^2 = 1 for this reaction.

The approximations inherent in DWBA calculations are often assumed to contribute an uncertainty of about ±20% to the spectroscopic factors extracted from experimental data. If we combine such an uncertainty with the ±15% uncertainty in our absolute-cross-section measurements, we obtain an uncertainty of ±25% in our extracted spectroscopic factors.

IV. DISCUSSION

A. Absolute spectroscopic factors

In Table II we compare our spectroscopic factors S with those from the previous ^{17}O(d, p) experiments of Moreh and Daniels[5] at 5.56 MeV and of Wiza, Middleton, and Hewka[6] at 10 MeV. The S values from these experiments had been obtained through PWBA analyses, and we have normalized them to the theoretical weak-coupling value of 1.35 for the transition to the ground state of ^{18}O. In addition, the values from Ref. 6 have been modified[9] to incorporate the energy dependence of the single-particle reduced widths suggested by Macfarlane and French.[23] Such an energy dependence has already been included in Ref. 5 and was found to be important. We also compare with results[11] of the ^{17}O(^{3}He, d) reaction to the T = 1 states of ^{18}F. These data were analyzed in DWBA, but since there was some uncertainty in the absolute cross section, we again normalize to the weak-coupling ground-state prediction.

As regards theoretical predictions, we show in Table II the $(sd)^2$ results of Halbert *et al.*,[7] labeled K-dsd (see also Ref. 8). We also show results obtained from two calculations, mentioned in the Introduction, which allow breaking of the ^{16}O closed shell. For the first, due to McGrory and Wildenthal,[8] we have listed the results obtained with the set of matrix elements they label F-pds. These were obtained by fitting the energies of selected levels in nuclei with A = 13 to 22. A second set of matrix elements, labeled Z-pds, was also employed in Ref. 8. With these matrix elements the fit to the ^{18}O energy spectrum was not as good as given by the F-pds matrix elements; however, the spectroscopic factors are very similar for the two calculations. The second calculation employed the weak-coupling model and we list in Table II the spectroscopic factors for each

TABLE II. Spectroscopic factors.

E_x (MeV)	J^π	nlj [a]	S(theory) K-dsd [b] Total	S(theory) F-pds [c] Total	S(theory) Weak-coupling model [d] $j=l+\frac{1}{2}$	S(theory) Weak-coupling model [d] $j=l-\frac{1}{2}$	S(theory) Weak-coupling model [d] Total	S(experiment) $^{17}O(d,p)^{18}O$ This work 18 MeV	S(experiment) $^{17}O(d,p)^{18}O$ Previous work 10 MeV [e]	S(experiment) $^{17}O(d,p)^{18}O$ Previous work 5.56 MeV [f]	S(experiment) $^{17}O(^3He,d)^{18}F(T=1)$ 15 MeV [g]
0.0	0^+	$1d_{5/2}$	1.7	1.6	1.35	···	1.35	1.22	1.35 [h]	1.35 [h]	1.35 [h]
1.982	2^+	$2s_{1/2}$	0.4	0.5	0.29	···	0.29	0.21	0.13	0.19	0.19
		$1d_{5/2}$	1.1	0.7	1.02	0.01	1.02	0.83	0.78	0.93	0.86
3.553	4^+	$1d_{5/2}$	1.9	1.8	1.55	0.07	1.63	1.57	1.03	1.33	1.14
3.632	0^+	$1d_{5/2}$	0.2	0.5	0.10	···	0.10	0.28	0.18	<0.3	
3.919	2^+	$2s_{1/2}$	0.6	0.3	0.46	···	0.46	0.35	0.28		0.25
		$1d_{5/2}$	0.9	1.1	0.63	2×10^{-3}	0.63	0.66			0.57
4.449	1^-	$1p_{3/2}$			2×10^{-4}	···	2×10^{-4}	0.03			
5.090	3^-	$1p_{1/2}$		0	0.01	0.02	0.02	≤0.02			
		$1f_{7/2}$						0.03			
5.250	2^+	$2s_{1/2}$		0.2	0.05	···	0.05	0.35	0.19	0.27	
		$1d_{5/2}$		0	3×10^{-3}	2×10^{-3}	5×10^{-3}	~0		<0.13	
5.329	0^+	$1d_{5/2}$		0	0.14	···	0.14	0.16		<0.20	
5.372	3^+	$2s_{1/2}$	1.0	0.9	0.84	···	0.84	1.01	0.64	0.83	
		$1d_{3/2}$	0	0	0	6×10^{-4}	6×10^{-4}	~0			
6.191	1^-	$1p_{3/2}$						0.03			
6.341	$\leq3^{(-)}$	$1p_{1/2}$		0 [i]	0.01	3×10^{-3}	0.01 [i]	0.03 [j]			
7.114	4^+	···	(1.1)		0.01	0.16	0.17		0.13		
$\sum\frac{2J+1}{2J_0+1}S(l=2)$ [k]			4.8	4.6	3.95	0.12	4.07	3.87	2.45	<3.20	3.13
$\sum\frac{2J+1}{2J_0+1}S(l=0)$			2.0	1.9	1.65	···	1.65	1.94	1.25	1.36	(0.37)

[a] n,l,j values assumed for DWBA analysis of this work.
[b] See Ref. 7, see also Ref. 8 for further discussion.
[c] See Ref. 8.
[d] See Ref. 9.
[e] Values quoted in Ref. 9, based on data of Ref. 6.
[f] Values quoted in Ref. 9, based on data of Ref. 5.
[g] See Ref. 11.
[h] Normalized to theoretical value of Ref. 9.
[i] Making a tentative association with the first calculated 2^- state.
[j] $S=0.04$ if $j=\frac{3}{2}$ is assumed.
[k] None of the summed $l=2$ strengths include the 7.114-MeV 4^+ state.

value $j = l \pm \frac{1}{2}$ of the total angular momentum transfer j. The present experiment does not, of course, distinguish between the different possible j values for a given l transfer.

On comparing in Table II our DWBA-deduced spectroscopic factors with the PWBA-deduced ones of Refs. 5 and 6 we note the following points. There is good agreement between our value of 0.68 for the ratio $S(l = 2, 1.982\ \text{MeV})/S(\text{g.s.})$ and that of 0.69 from Ref. 5, 0.58 from Ref. 6, and 0.64 from Ref. 11. Our values for the ratios $S/S(\text{g.s.})$ for other transitions are generally about 25% higher than the values of Ref. 5 and are about 45% larger than the values of Ref. 6. Thus, our relative spectroscopic factors are in better agreement with those of Ref. 5 than those of Ref. 6, although this may not be significant given the inherent uncertainties in the PWBA.

The (^{3}He, d) results of Ref. 11 agree with the present work except for the $l = 0$ transfer to the 3.919-MeV, $J^\pi = 2^+$ level and the $l = 2$ transfer to the 3.553-MeV, $J^\pi = 4^+$ level where our S values are significantly larger. For the latter case theory strongly favors the larger value.

We will now compare our experimental S values and the different theoretical predictions for transitions to the positive-parity levels of ^{18}O. The S values from the F-pds calculation are often quite similar to those from the K-dsd calculation, whereas the S values from the weak-coupling calculation in a number of cases disagree with those from the other two calculations. The calculation which yields the best over-all agreement with the spectroscopic factors of the present experiment is that of

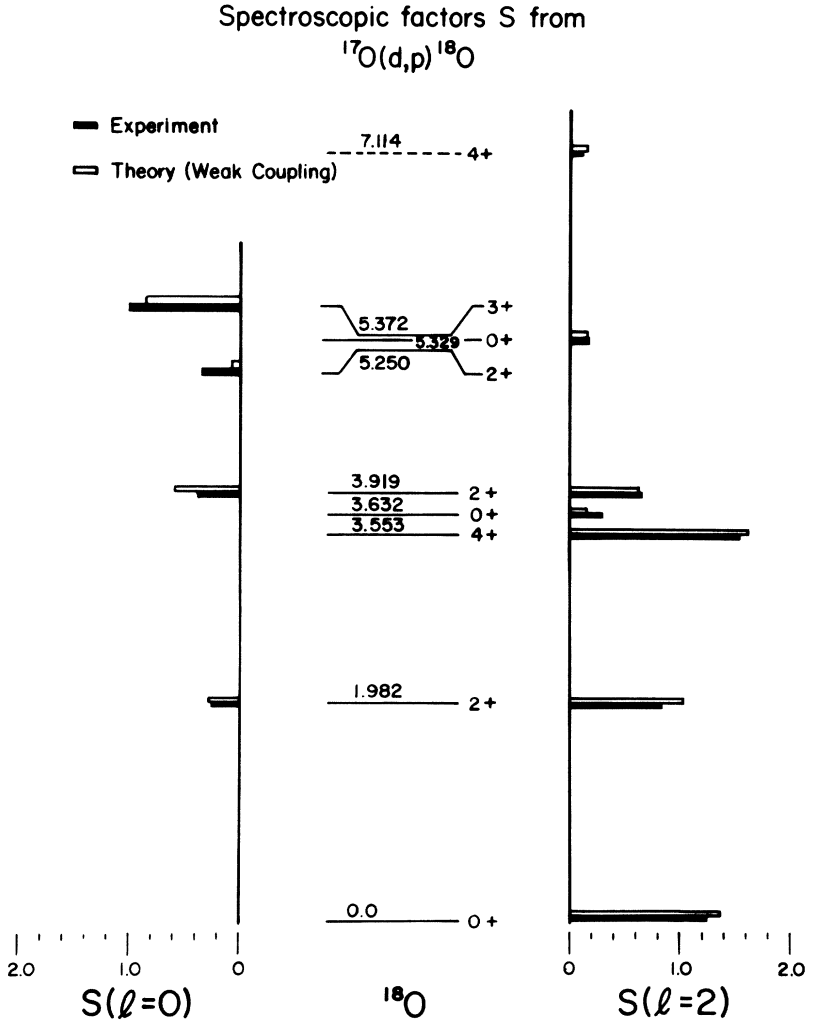

FIG. 6. Spectroscopic factors S from the ^{17}O(d, p)^{18}O reaction compared with the theoretical predictions of the weak-coupling model (Ref. 9).

the weak-coupling model. This point is emphasized further in Fig. 6. Here the only strong deviation of the calculation from experiment occurs for the $l = 0$ transition to the 5.250-MeV, 2_3^+ state in which the calculated S value is 7 times too small. This level is dominantly of four-particle–two-hole (4p-2h) character and the small value of S results from destructive interference between the spectroscopic amplitudes for the zero- and two-hole contributions. Hence the calculated S value for this transition is rather sensitive to the small components in the wave functions, but it would appear difficult to raise the calculated S value sufficiently through minor adjustments in the weak-coupling model. The McGrory-Wildenthal (F-pds) calculation[8] does give an S value in better agreement with experiment for this transition; however, the value is still two times too small.

It is particularly interesting to compare S values for transitions to the 0^+ states. The F-pds calculation gives an S value for the 0_2^+ (3.632 MeV) level 2 times the experimental value and gives an S value of essentially zero for the 0_3^+ (5.329 MeV) level, whereas the experimental value of S for the 0_3^+ state is 0.16. This disagreement is likely to be connected with the F-pds assignment of a predominant 2p-0h structure to the 0_2^+ state and a 4p-2h structure to the 0_3^+ state. The weak-coupling model, however, reverses these two assignments. This reversal yields somewhat better agreement with experiment, although the weak-coupling S value for the 0_2^+ state is too small, and again destructive interference between the zero- and two-hole components is responsible. The weak-coupling assignment of a predominant 4p-2h structure to the 0_2^+ state is also supported by other evidence—for example by γ-decay data,[4,9] by the observed[24] Coulomb energy shifts between ^{18}Ne and ^{18}O, and by an analysis[10] of the existing one- and two-particle transfer data.

With regard to the pure $(sd)^2$ calculation K-dsd, it of course suffers from the deficiency of producing too few states. In addition the calculated spectroscopic factors appear somewhat too large, a fact which suggests a need for the inclusion of p-h admixtures in the ^{17}O ground state.

In Table II we have also listed the $1d_{3/2}$ spectroscopic factors (column labeled $j = l - \frac{1}{2}$) from the weak-coupling calculation. They are uniformly small except for the transition to the 7.114-MeV, 4^+ state; the calculation yields a sizable $(d_{5/2}d_{3/2})$ component (although the 4p-2h admixtures are dominant) for this state. The transition to this state was not investigated in the present experiment; however, the calculated spectroscopic factor is in agreement with that from a PWBA analysis.[6,9] It thus appears incorrect to associate this level with the $(sd)^2$ 4_2^+ state of Ref. 7, as we have done in Table II for comparison purposes.

We now discuss the S values for transitions to the negative-parity levels of ^{18}O. The spectroscopic factors extracted from the data are all very small. This is to be expected, because the transferred neutron must fill $1p$ holes in the ^{17}O ground state. The calculated values are also small and may not be reliable since they are sensitive to small components in the calculated wave functions. Furthermore, for such weak processes multistep reaction mechanisms are often important, and therefore caution should be exercised in drawing conclusions about these weakly excited states. For the 4.449-MeV, 1^- state the spectroscopic factor from the weak-coupling calculation is much smaller than the experimental value, and the F-pds calculation gives $S = 0$ for this transition since it involves $1p_{3/2}$ transfer. For the 5.090-MeV, 3^- state the weak-coupling calculation agrees with experiment for $1p_{1/2}$ transfer; however, the angular distribution is better fitted under the assumption of $1f_{7/2}$ transfer. The S value for this transition is essentially zero with the F-pds interaction; however, with the Z-pds interaction a value of 0.2 was obtained. The correspondence of the 6.191- and 6.341-MeV states with the theoretical energy levels is at present uncertain. The 6.341-MeV state has[25] $J \leq 3$, and the present experiment suggests that it has negative parity. We make a tentative association of this state with the theoretical 2^- level, which then yields fair agreement between the calculated and experimental S values. (Note, however, that 2^- has also been suggested for the 5.517-MeV state.[25]) The weak-coupling calculation yields a very small spectroscopic factor for the transition to the theoretical 1_2^- level; however, we do not associate this 1_2^- level with the 6.191-MeV state because a study[26] of the ^{19}F(d, ^{3}He)-^{18}O reaction showed a large transition strength to the unresolved 6.191- and 6.341-MeV states, and this strength is not reproduced by the weak-coupling calculation.

A comparison of appropriate sums of extracted transition strengths with theoretical sum-rule limits can sometimes indicate whether or not gross errors exist in the method used to extract spectroscopic factors. Here, for example, we have from Table II $\sum[(2J+1)/(2J_0+1)]\,S(l=2) = 3.87$, where the sum is over all measured $l = 2$ transitions, and $\sum[(2J+1)/(2J_0+1)]\,S(l=0) = 1.94$, where the sum is over all measured $l = 0$ transitions. The sum-rule limits are 5 and 2, respectively, assuming ^{17}O to be a $1d_{5/2}$ neutron beyond a closed ^{16}O core. Actually, there will be particle-hole admixtures in the ^{17}O ground state so that smaller values for these sums should be obtained, as is found. Com-

parison with the theoretical calculations which allow core breaking shows agreement to within 20% for the summed strengths. The contributions from $1d_{3/2}$ transfer, which are included in the sum over our experimental values below 7 MeV, are expected to be small, i.e., 3 to 4% of the $1d_{5/2}$ contributions according to the weak-coupling model. The total experimental $2s_{1/2}$ strength is perhaps a little too large. Additional evidence for an overestimate of the $2s_{1/2}$ strength lies in the value of S for the transition to the 3^+ state at 5.372 MeV; we find $S = 1.01$. Most models interpret this state as an almost pure $(1d_{5/2}2s_{1/2})_{3^+}$ configuration, for which $S = 1$ if the ^{17}O ground state were a pure $1d_{5/2}$ state. However, particle-hole admixtures in the ground state of ^{17}O will reduce the strength and a value $S = 0.8$ to 0.9 for the transition to the 3^+ state would appear more realistic. In any case, it seems that the estimate of a 25% uncertainty in our absolute S values may be somewhat conservative.

The present work also has implications for analyses of results of (^{18}O, ^{17}O) neutron-transfer experiments. The value of the single-neutron-transfer spectroscopic factor for $^{18}\text{O} \rightarrow {}^{17}\text{O}$ is needed in such analyses, and our work indicates that a value of about 1.2 should be used for this rather than a value of 2, which would be correct if ^{17}O and ^{18}O were pure $1d_{5/2}$ and $(1d_{5/2})^2$ neutron states, respectively.

B. Matrix elements of the effective interaction

Appropriate sums over the ^{18}O excitation-energy spectrum weighted by the single-neutron spectroscopic factors can be related to diagonal matrix elements $\langle (d_{5/2}, lj)J|V|(d_{5/2}, lj)J\rangle$ of the effective neutron-neutron interaction V in the (sd) shell. Specifically, the diagonal matrix element H_{ii} of the Hamiltonian H with respect to a basis state $|i\rangle$ is

$$H_{ii} = \sum_m E_m |a_i^m|^2, \tag{2}$$

where the sum is over all states m of ^{18}O of a given J^π, and a_i^m is the amplitude of configuration i in state m. We take the energies E_m of the states of ^{18}O with respect to the ground-state energy of ^{16}O, and therefore H_{ii} includes the matrix elements of the two-neutron interaction and the single-particle energies $\epsilon(d_{5/2})$ and $\epsilon(lj)$ of both neutrons. If we note that the configuration probability $|a_i^m|^2$ is essentially the spectroscopic factor $S_j(m)$, where the configuration i is $(d_{5/2}, lj)$ and lj are the quantum numbers of the transferred neutron, then Eq. (2) becomes

$$\begin{aligned}\langle (d_{5/2}, lj)J|V|(d_{5/2}, lj)J\rangle &= \frac{\sum_m E_m S_j^m}{\sum_m S_j^m} - \epsilon(d_{5/2}) - \epsilon(lj) \\ &= \frac{\sum_m E_{nn}^m S_j^m}{\sum_m S_j^m} .\end{aligned} \tag{3}$$

In Eq. (3) E_{nn}^m is the effective neutron-neutron interaction energy in state m. We obtain the single-particle energies from the $\frac{5}{2}^+$ ground state and $\frac{1}{2}^+$ first excited state of ^{17}O, thus

$$\begin{aligned}\epsilon(d_{5/2}) &= -4.143 \text{ MeV}, \\ \epsilon(s_{1/2}) &= -3.272 \text{ MeV}.\end{aligned} \tag{4}$$

For application of Eq. (3) it is convenient to write E_{nn} for each state m of ^{18}O in terms of the excitation energy E_x,

$$E_{nn} = E_x - \epsilon(lj) - 8.046 \text{ MeV}. \tag{5}$$

In applying Eq. (3) to the data we assume that an observed $l = 2$ transfer corresponds to $j = \frac{5}{2}$. This seems reasonable in view of the weak-coupling

TABLE III. Matrix elements $\langle (d_{5/2})^2 J \mid V \mid (d_{5/2})^2 J \rangle$ of the effective neutron-neutron interaction in ^{18}O.

E_x (MeV)	J^π	$S(l=2)$	E_{nn}	Matrix element Exp. [a]	Theory Kuo [b]	Theory F-pds [c]	Theory PW [d]
0.0	0^+	1.22	−3.90				
3.632	0^+	0.28	−0.27	−2.77	−2.44	−1.69	−2.12
5.329	0^+	0.16	+1.43				
1.982	2^+	0.83	−1.92				
3.919	2^+	0.66	+0.02	−1.06	−1.04	−0.82	−1.23
5.250	2^+	~0	−1.35				
3.553	4^+	1.57	−0.35	−0.35	−0.05	−0.32	+0.16

[a] From Eq. (3). We assume here $S(l=2) \approx S_{5/2^+}$ for $E_x < 7$ MeV.
[b] See Ref. 2.
[c] See Ref. 8.
[d] See Ref. 27.

TABLE IV. Matrix elements $\langle (d_{5/2}s_{1/2})J \mid V \mid (d_{5/2}s_{1/2})J \rangle$ of the effective neutron-neutron interaction in ^{18}O.

E_x (MeV)	J^π	$S(l=0)$	E_{nn}	Matrix element Exp. [a]	Theory Kuo [b]	Theory F-pds [c]	Theory PW [d]
1.982	2^+	0.21	−2.79				
3.919	2^+	0.35	−0.85	−0.79	−1.29	−1.84	−0.85
5.250	2^+	0.35	+0.48				
5.372	3^+	1.01	+0.60	+0.60	+0.17	+1.64	+0.78

[a] From Eq. (3).
[b] See Ref. 2.
[c] See Ref. 8.
[d] See Ref. 27.

results listed in Table II. We omit, however, the 4_2^+ (7.114 MeV) state because it is expected that this state would be reached mainly by $j = \frac{3}{2}$ transfer. The results for the matrix elements obtained from the experimentally observed levels are given in Tables III and IV. The columns labeled Kuo and F-pds list values of the matrix elements used in Refs. 7 and 8, respectively. An early version of the Kuo[2] matrix elements was used in the weak-coupling work.[9] These calculations have been discussed in Sec. IV A. The column labeled PW is from a calculation[27] which fits energy levels of nuclei with A = 18 to 22 by taking the active orbitals $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ outside an inert ^{16}O core. The matrix elements of Kuo[2] were used for configurations involving the $1d_{3/2}$ orbital and the other matrix elements were allowed to vary as free parameters.

A major problem when determining the matrix elements from experimental spectroscopic factors and excitation energies is the uncertainty as to whether or not all states that contain significant fractions of a given configuration have been included in the sum of Eq. (3). A weak transition to a highly excited state could make a significant contribution to the sum. If we were to miss any transitions to higher-lying states then our matrix elements would be too attractive. Another problem is the assumption made in Eq. (3) that the ground state and first excited state of ^{17}O are pure $1d_{5/2}$ and $2s_{1/2}$ single-particle states, respectively. It is therefore appropriate to discuss estimates of the magnitudes of the errors which might be involved. At first we discuss theoretical estimates for a calculation where the ^{16}O core is closed and $(sd)^1$ and $(sd)^2$ configurations are allowed in ^{17}O and ^{18}O. These are the (sd) basis states of Ref. 9, and the calculation is essentially the same as the K-dsd calculation.[7] We take the calculated energies and spectroscopic factors for the two lowest 0^+ and 2^+ levels and the lowest 4^+ level and derive matrix elements according to Eq. (3). These can then be compared with the exact values used in the calculations. Thus the error introduced by omitting the 0_3^+, 2_3^+, 2_4^+, 2_5^+, and 4_2^+ levels is derived. Note that to be consistent with the experimental procedure, we actually use the sum of the calculated $d_{5/2}$ and $d_{3/2}$ spectroscopic factors, even though we are interested in $d_{5/2}$ transfer. This, in fact, hardly affects the results, because the states considered do not have significant $d_{3/2}$ strength. In this way the largest error, about 1 MeV, is found for the $(d_{5/2})^2_{0+}$ matrix element. This error arises from the mixing of an 8% $(d_{5/2})^2$ component into the 2p-0h state of predominant $(d_{3/2})^2$ structure predicted at E_x = 15 MeV. The calculated error seems to be in accord with Table III, where the experimental value differs significantly from both the Kuo and PW values (the F-pds case is discussed later), but the differences are less than estimated. For the $(d_{5/2})^2_{2+}$ matrix element the calculated error is smaller, 0.13 MeV, since little strength is pushed up to high energy. The values in Table III bear out this small figure. For the $(d_{5/2})^2_{4+}$ case the lowest predicted 4^+ excitation energy is used. This gives a matrix element which differs from that of the calculation by 0.36 MeV. This difference is due to an 8% $(d_{5/2})^2$ admixture in the theoretical $(sd)^2$, 4_2^+ level.

For the $(d_{5/2}s_{1/2})_{2+}$ matrix elements, the predicted error is 0.2 MeV, which is not unreasonable from the comparison in Table IV. The lowest 3^+ state is essentially a pure $(d_{5/2}s_{1/2})$ configuration so that the results in Table IV simply reflect differing predictions of the energy of this state.

Overall we conclude that the errors in the experimental matrix elements caused by the omission of high-lying unobserved states will be, at best, a few tenths of an MeV. They may be much larger, however, so that to have confidence in the extracted experimental values additional knowledge of the strength in higher-lying states is required.

We now discuss the case where the ^{16}O core is broken, and p-h components are present in ^{17}O and ^{18}O. We do this using the results of the weak-coupling model. If we use the calculated spectro-

scopic factors and excitation energies for the three lowest 0^+ and 2^+ levels and the first 4^+ level we derive through Eq. (3) virtually the same matrix elements as obtained previously with the pure $(sd)^2$ model. This is in fact surprising since in the weak-coupling calculation 0.4 MeV was added to all the diagonal $(sd)^2$ matrix elements in order that the mixing with the 4p-2h states would give the correct ground-state binding energy of ^{18}O. The reason this additional 0.4 MeV present in the two-particle matrix elements is not regained by using Eq. (3) is twofold. First, Eq. (2) requires that the probabilities $|a_i^n|^2$ be used for a given configuration i. These are not proportional to the spectroscopic factors S in the weak-coupling calculation since this quantity involves a coherent sum over zero-hole and two-hole contributions, for say $d_{5/2}$ transfer. As remarked this tends to make the states appear purer 2p-0h (or 4p-2h) because of the constructive (or destructive) interference between the zero- and two-hole components. Second, as a result of the mixing of the 2p-0h and 4p-2h configurations, small fragments of the $(d_{5/2})^2$ and $(d_{5/2}s_{1/2})$ configurations occur at fairly high excitation energy. If these two points were to be properly taken into account, the 0.4 MeV shift would be obtained. Therefore we conclude that if the spectroscopic factors for a few low-lying levels are used in Eq. (3) to extract matrix elements, the results should be compared with effective matrix elements designed for a simple $(sd)^2$ calculation. These differ, of course, from the matrix elements designed for a calculation involving a different basis. This view is confirmed by the comparison between the experimental and *F-pds* matrix elements in Tables III and IV, where there is seen to be substantial disagreement.

The above discussion indicates that the errors estimated in the various cases, while roughly consistent with the data in Tables III and IV, are model dependent. They are also rather difficult to calculate reliably since the mixing of small fragments of $(sd)^2$ strength into high-lying states is involved. We have therefore not attempted to apply any corrections to the experimental results given in Tables III and IV.

V. CONCLUSIONS

We have measured ^{17}O$(d, p)^{18}$O differential cross sections at 18 MeV and have analyzed the results in the DWBA. The extracted absolute spectroscopic factors show significant differences from earlier work[5,6] using the PWBA; however, our spectroscopic factors are mostly in reasonable agreement with those from[11] a DWBA analysis of ^{17}O$(^3$He$, d)^{18}$F$(T = 1)$. Comparisons with theoretical calculations for transitions to positive-parity states in ^{18}O show a distinct preference for the weak-coupling model,[9] although some deviations are observed for transitions to states of largely 4p-2h structure. This preference for the weak-coupling calculation suggests that correlations involving the $1d_{3/2}$ and $1p_{3/2}$ orbitals are important.

Matrix elements of the effective neutron-neutron interaction were extracted from our data. By appealing to theoretical calculations, it was suggested that these extracted matrix elements should be compared with those required in $(sd)^2$ calculations. Such a comparison showed agreement that was not unreasonable, and the differences were understood through analyzing the results of an $(sd)^2$ calculation.[9] It was found, however, that the predicted differences between theory and experiment were larger than actually observed.

As regards transitions to the negative-parity states in ^{18}O, our extracted spectroscopic factors are small, as expected, and are in fair agreement with theory. Negative parity was suggested for the 6.341-MeV state on the basis of an observed $l = 1$ angular distribution.

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