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Covariant calculation of pion-^{4}He elastic scattering in the (3, 3) resonance region*

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Differential cross sections for π^--^{4}He elastic scattering were calculated at energies between 110 and 260 MeV. The analysis is completely novel since we have used a covariant multiple scattering theory, and a complete integration over the motion of the target particles has been carried out (instead of invoking the fixed-scatterer approximation). The full dynamical effects of nuclear binding and Fermi motion are considered as well as some modification of the free πN interaction. Theoretical results are in good agreement with the data.

[NUCLEAR REACTIONS Covariant optical potential, π^--^{4}He elastic scattering from 110 to 260 MeV.]

Pion-nucleus elastic scattering in the energy region near the (3, 3) resonance has been studied extensively in recent years. The data for π-^{4}He elastic scattering are of particular interest. The relative simplicity of the ^{4}He wave function makes the analysis of π-^{4}He elastic scattering a useful tool for studying pion-nucleus dynamics.

Recently, Binon *et al.*[1] have measured the differential cross sections at energies between 110 and 260 MeV. In this energy domain, the kinetic energy of the pion is comparable to or greater than its rest mass. It is therefore necessary to use a relativistic theory to describe the pion-nucleus system. The covariant multiple scattering theory which we have developed[2,3] does not presuppose the "fixed-scatterer" or eikonal approximations. Binding effects and general off-shell effects can be treated unambiguously. This feature is particularly important since the fundamental pion-nucleon amplitude is strongly energy dependent.

If we denote the strong-interaction potential by K_N and the Coulomb interaction by K_C we may write the total interaction as $K=K_N+K_C$. The interaction K satisfies a *covariant* three-dimensional integral equation[3] which takes the form:

$$\langle \vec{k}' | \overline{M}_6(W) | \vec{k}\rangle = \langle \vec{k}' | \overline{K}_6(W) | \vec{k}\rangle + \int d\vec{k}'' \langle \vec{k}' | \overline{K}_6(W) | \vec{k}''\rangle R_6(\vec{k}'')[2W - (M_\pi^2+\vec{k}''^2)^{1/2} - (M_A^2+\vec{k}''^2)^{1/2} + i\epsilon]^{-1} \langle \vec{k}'' | \overline{M}_6(W) | \vec{k}\rangle . \quad (1)$$

In Eq. (1), the quantity $2W$ is the total energy of the system. The subindices of M, K, and R refer to the specific covariant reduction scheme used in the multiple scattering theory.[3]

We have performed two sets of covariant calculations. In the first case, we carried out a full dynamical calculation, with[3]

$$\langle \vec{k}' | \overline{K}_{N,6}(W) | \vec{k}\rangle = \sum_{nILs's''} \int d\vec{Q} (M_{nI}/E_{nI,\vec{Q}}) \bar{u}^{s''}(P'-Q)[(2\pi)^3 \langle p', P'-Q | M_{\pi N}(s) | p, P-Q\rangle] u^{s'}(P-Q) \times \rho^{n(I1/2)L}_{s's''}(\vec{Q}_R, \vec{Q}'_R) . \quad (2)$$

Here, the product $\bar{u}Mu$ is the invariant off-shell πN amplitude and the quantity ρ represents an invariant density matrix. The quantity $E_{nI,\vec{Q}} = (\vec{Q}^2 + M_{nI}^2)^{1/2}$ is the energy of the spectator nucleus. The $\vec{Q}_R$ and $\vec{Q}'_R$ are the momenta of the struck nucleon in the rest frame of the ^{4}He, before and after the πN collision. Calculations found in the literature often invoke the fixed-scatterer approximation[4] (FSA). For comparison, we also performed calculations using a (covariant) fixed-scatterer approximation (CFSA), described previously.[2] In this approximation, Eq. (2) factorizes and becomes:

$$\langle \vec{k}' | \overline{K}_{N,6}(W) | \vec{k}\rangle \simeq \sum_{s's''} \bar{u}^{s''}(P'-\tilde{Q})[(2\pi)^3 \langle p', P'-\tilde{Q} | M_{\pi N}(\tilde{s}) | p, P-\tilde{Q}\rangle] u^{s'}(P-\tilde{Q}) F_{s's''}[(\vec{k}-\vec{k}')^2] , \quad (3)$$

where F now represents a nuclear form factor. The quantities $\tilde{Q}$ and $\tilde{s}$ are the spectator-nucleus momentum and the square of the πN collision energy. The choice of these quantities serves to define the particular factorization scheme used for the CFSA. (We refer to Ref. 2 for the comparison between the CFSA and the usual FSA.)

The form factor, $F\{(\vec{k}-\vec{k}')^2\}=F(q^2)$ can be derived from the charge form factor and the proton form factor, using the relation $F(q^2)=F_{ch}(q^2)/F_p(q^2)$. We have used the *modified*-Gaussian charge form factor determined experimentally by Frosch *et al.*[5] We also assumed the identity of proton and neutron matter distributions and used $F_p(q^2)=\exp(-a_p^2q^2/4)$. (The value $a_p=0.66$ fm was used.[6]) The momentum-space wave function needed for the computation of the density matrix, $\rho(\vec{Q}_R,\vec{Q}_{R'})=\phi(\vec{Q}_R)\phi^*(\vec{Q}_R')$, can be determined from $F(q^2)$ by assuming pure s-state relative motion in ^{4}He; then,

$$F(q^2)=\int \phi^*(\vec{Q}_R')\phi(\vec{Q}_R)d\vec{Q}\,. \tag{4}$$

Owing to the large mass of the nucleus, the momenta $\vec{Q}_R$ and $\vec{Q}_R'$ may be calculated using nonrelativistic kinematics. It can be shown that, with $\eta=(A-1)/A=0.75$,

$$F(q^2)=\int e^{i\eta\vec{q}\cdot\vec{x}}\,|\phi(\vec{x})|^2 d\vec{x}\,, \tag{5}$$

where $|\phi(\vec{x})|^2$ is the matter density. It follows that the wave function $\phi(\vec{x})$ is given by

$$\phi(\vec{x})=\left[(2\pi^2)^{-1}\int_0^\infty j_0(\eta q x)\eta^3F(q^2)q^2dq\right]^{1/2}. \tag{6}$$

The momentum-space wave function, $\phi(\vec{k})$, is obtained by Fourier transformation.

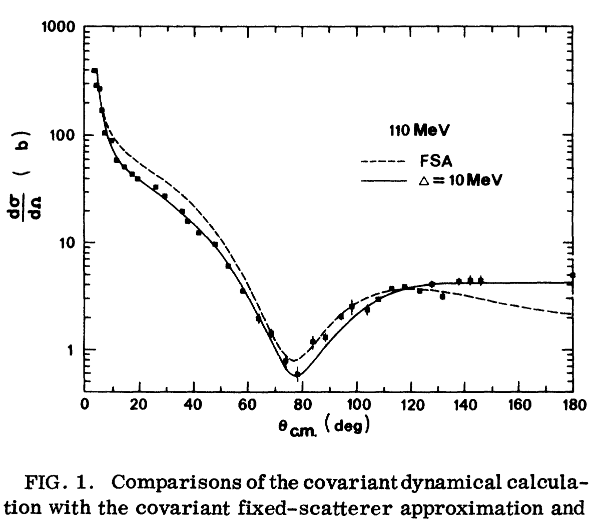

FIG. 1. Comparisons of the covariant dynamical calculation with the covariant fixed-scatterer approximation and data at 110 MeV.

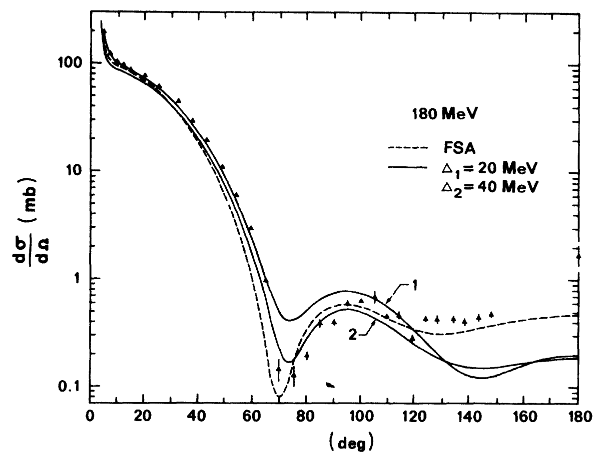

FIG. 2. Comparisons of the covariant dynamical calculation with the covariant fixed-scatterer approximation and data at 180 MeV.

The result of our calculations are presented in Figs. 1–4. The solid and dashed curves represent the full dynamical calculations and the results obtained with the CFSA. In both cases, the Coulomb interaction was treated exactly.[7] A separable-interaction model was used for the πN scattering amplitudes.[8] The CFSA results exhibit significant quantitative disagreements with the data at all energies. In particular, the forward cross sections are too high at the lower energies and too low at higher energies. This discrepancy suggests that the πN amplitudes in the CFSA may be evaluated with inappropriate energy parameters. We recall that in the FSA the $\tilde{s}$ of Eq. (3) is calculated for the collision between the incoming pion and a *fixed free* nucleon, as is cus-

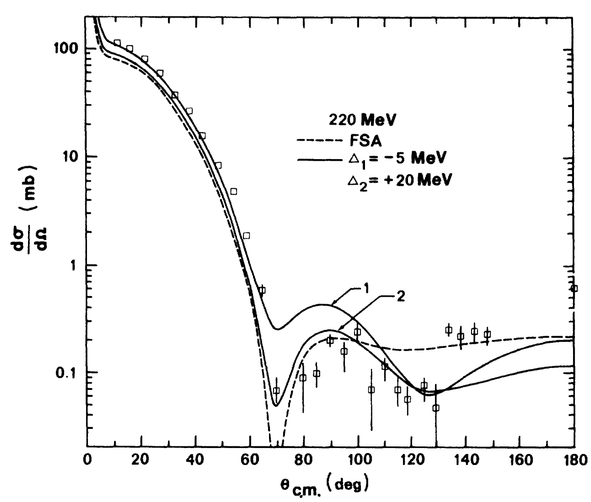

FIG. 3. Comparisons of the covariant dynamical calculation with the covariant fixed-scatterer approximation and data at 220 MeV.

tomary.[2] In the dynamical calculation, however, the πN amplitude is calculated with the actual collision energy $s^{1/2}$ calculated for the scattering from a bound particle.[2,3] We then have $s < \bar{s}$ due to the binding effect. Further, s depends on the detailed off-shell dynamics[3] and can be driven down to the unphysical region, where $s < (M_\pi + M_N)^2 < \bar{s}$. Consequently, the average of those s values which contribute appreciably to the cross section is much less than $\bar{s}$. In addition to this feature we must also take into account the modification of the free πN amplitude in the nuclear medium. This modification produces a change in both the real and the imaginary parts of the amplitude.[9] In the present analysis, we neglect the modification of the imaginary part of the πN amplitude, and evaluate the energy dependent denominator $D(s)$ of the separable πN interaction[8] at $\sqrt{s_{\text{eff}}} = \sqrt{s} + \Delta$. (We point out that the introduction of the energy shift Δ may be justified in a full dynamical calculation based upon a detailed microscopic analysis.[9]) In Figs. 2 and 3 the results for two different values of Δ are indicated. One value is chosen to yield the correct magnitude for the amplitude at small angles (and the total cross section) while the other produces a better "overall" angular distribution. More elaborate modification of the free amplitude should await detailed numerical studies of the *effective* πN interaction. In most cases the effect of the nuclear medium is to increase the *effective* s value for the calculations which use the free interaction. This upward displacement of the s value compensates, in part, the downward displacement arising from the off-shell dynamics. This is perhaps the reason that a FSA calculation which employs the standard specification of s is often able to provide a qualitative fit to the data. On the other hand, only a covariant dynamical calculation can provide insight into the various aspects of the off-shell pion-nucleus dynamics and the effective πN interaction.

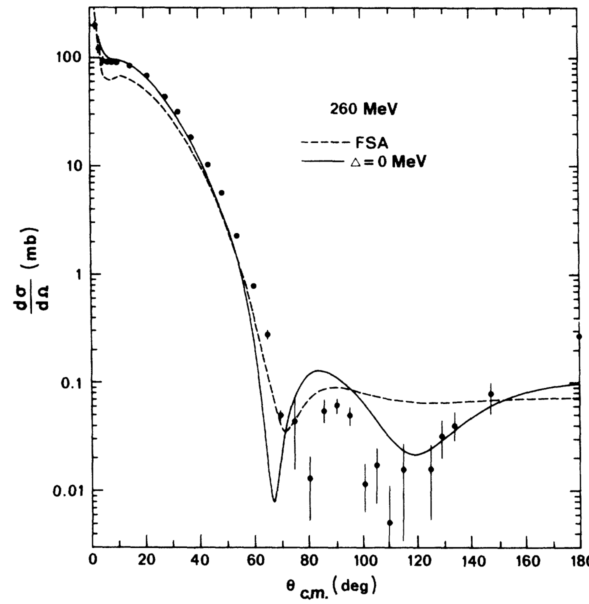

FIG. 4. Comparisons of the covariant dynamical calculation with the covariant fixed-scatterer approximation and data at 260 MeV.

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