PHYSICAL REVIEW C VOLUME 13, NUMBER 6 JUNE 1976

Elastic pion-^{4}He scattering

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(Received 24 November 1975)

We have calculated differential cross sections for elastic π^+ and π^- scattering from ^{4}He at several incident pion energies from 24 to 110 MeV. A full multiple-scattering treatment incorporating all orders of scattering was used. A separable form for the pion-nucleon t matrix having off-shell cutoff form factors was assumed. Inclusion of an effective angle transformation, which takes into account the motion of the struck nucleon, along with binding and recoil factors in the pion-nucleon amplitude, leads to good over-all agreement with the experimental data.

[NUCLEAR REACTIONS ^{4}He($\pi^\pm$, $\pi^\pm$) elastic scattering, E = 24–110 MeV; multiple-scattering theory; separable pion-nucleon t matrices, angle transformation; $\sigma(\theta)$.]

I. INTRODUCTION

The elastic scattering of pions from ^{4}He has recently received noticeably increased theoretical attention.[1-9] The data,[10-14] which have a range of incident pion kinetic energy from 24 to 260 MeV, show one minimum at 75° independent of the pion energy plus secondary minima at larger angles which do exhibit the normal energy dependence characteristic of diffractive processes. There has been some difficulty in fitting these data, particularly in reproducing the fixed 75° minimum. Various attempts have been made, and several different theoretical approaches have been investigated. Approximations to the summation of the Watson multiple-scattering series were studied by Charlton and Eisenberg[1] in fitting the 153 MeV data. Optical model calculations were made using Kisslinger type potentials at energies of 24, 60, and 153 MeV.[2] Transformation to the pion-nucleon center-of-mass frame was later included in the optical model by Mach[6]; this gave a marked improvement over the earlier calculations. Glauber calculations, which attempted to include nucleon recoil effects, were performed[3-5] but, in view of the violation of the basic assumptions in the Glauber approximation, were unsuccessful except near 180 MeV. A different multiple-scattering calculation using the nonoverlapping potential approximation and including two-nucleon intermediate "reflection" scatterings was recently reported.[7] In contrast to the limited success of the above calculations in reproducing the primary features of the data, a recent calculation by Landau[8,9] does produce reasonable agreement. These more successful results come from an optical model in which (1) separable potentials with off-shell cutoff form factors are used and (2) the effects of nucleon motion and recoil, leading to an angle transformation and resulting mixing of the π-N partial wave amplitudes, are included.

From our study of the problem it is clear that an important aspect of achieving a realistic angular distribution in a reaction such as this is the inclusion of the initial- and final-nucleon momenta. This problem was first studied by Kowalski and Feldman in elastic nucleon-deuteron scattering.[15] It was further examined by Mach[6,16] who found that by combining such kinematic transformations with a Kisslinger potential an improved agreement with low energy π^--^{12}C scattering data was obtained. Adelberg and Saperstein used the methods of Ref. 15 in the construction of an optical potential to calculate neutron-nucleus scattering and obtained good fits to n-^{12}C cross sections and polarizations.[17] In a pion-nucleus calculation by Phatak, Tabakin, and Landau,[18] nucleon motion effects were included in a different manner: The π-N t matrix was evaluated at a reasonable value of the average nucleon momentum in the fixed-nucleon approximation and a KMT-type optical potential was then constructed. An important feature of this work was the mixing of the π-N partial waves due to the nucleon motion. Kujawski and Lambert later demonstrated the importance of including nucleon recoil, where the nucleon recoils with the target as a whole, in their three-body calculations of neutron-deuteron scattering.[19] Other authors have also noted the need to include the kinematics of the struck nucleon; the essential result is to produce an angle transformation in the π-N amplitude and mix the partial waves. Landau, Phatak, and Tabakin demonstrated the improvement in fits to π^--^{12}C elastic scattering when such effects are included in the optical potential[20]; the same model was also applied to inelastic pion-nucleus scattering calculations.[21] A coordinate space optical potential incorporating the same angle transformation as Ref. 20 and separable π-N t matrices was used by Kisslinger and Tabakin[22] to obtain good fits to π^--^{12}C data between 120–280 MeV. Further study of the use of an angle transformation in optical potentials was made by Kujawski and Miller[23] and by Miller.[24] In addition to the above scattering effects, Koltun and Nalcioglu[25] showed that including the momentum of the struck nucleon in distorted-wave impulse-approximation (DWIA) inelastic pion-nucleus scattering calculations can lead to excitation of nuclear electric dipole states, whose experimental observation at small angles might help in the determination of the degree of nonlocality in the π-N interaction. Finally, the recent work of Wallace[26] stressed the importance of including the nucleon recoil terms in any attempt to relate the Glauber and Watson multiple-scattering formalisms.

With this in mind we were led to examine a method of including nucleon momentum and recoil in our formalism, in particular for low energy pion scattering where such effects can be significant. In Sec. II we describe our multiple-scattering formalism and the modification that we have made to include the effects of nucleon recoil and binding. Our numerical results and comparison with the data between 24 and 110 MeV are presented in Sec. III. Our conclusions are summarized in Sec. IV.

II. FORMALISM

We have employed a formalism which avoids both the small-angle approximation that is basic to the Glauber approximation and the low density expansion approximation that is inherent in lowest-order optical potential theories. A complete description of the formalism can be found in Ref. 27. The primary results are summarized below.

We have assumed a separable s- and p-wave pion-nucleon t matrix of the form[28-31]

$$\langle \vec{q}\,' | t(\omega) | \vec{q} \rangle = \lambda_0(\omega) v_0(q) v_0(q') + \lambda_1(\omega) \vec{q}\cdot\vec{q}\,' v_1(q) v_1(q'), \qquad (1)$$

where $\omega = (\kappa^2 + \mu^2)^{1/2}$ is the pion energy in the π-N center-of-mass frame. The factor

$$\lambda_l(\omega) = \frac{2l+1}{2i\kappa} \frac{\exp[2i\delta_l(\omega)] - 1}{k^{2l}}, \qquad (2)$$

where k is the pion laboratory momentum, ensures that the t matrix has the right on-shell dependence.[32] The form factors

$$v_l(q) = \frac{k^2 + \alpha_l^2}{q^2 + \alpha_l^2} \qquad (3)$$

describe the off-shell extension of the π-N t matrix and go to unity on shell ($q \to k$). The parameters α_l have been determined by fits to π-deuteron absorption to be approximately[33]: $\alpha_0 = 500$ MeV/c and $\alpha_1 = 300$ MeV/c. Since there is negligible sensitivity to α_0, we have used $\alpha_0 = \alpha_1 = 300$ MeV/c in this work. This t matrix is then inserted into a set of self-consistent coupled equations which describe multiple scattering in the fixed-nucleon approximation:

$$G_i(\vec{k}, \vec{q}) = f_i(\vec{k}, \vec{q}) e^{i\vec{k}\cdot\vec{x}_i} + \sum_{j\neq i} \int f_i(\vec{p}, \vec{q}) G_0(k, p) e^{i\vec{p}\cdot\vec{x}_j} G_j(\vec{k}, \vec{p}) \frac{d^3p}{(2\pi)^3}. \qquad (4)$$

Here f_i is the free pion-nucleon scattering amplitude, G_0 is the pion propagator, and G_i is an amplitude which describes the multiple scattering to all orders, the last scattering taking place on the ith nucleon, and which contains the dependence on all A nucleons. The total pion-nucleus scattering amplitude is related to the set of the G_i by

$$F(\vec{k}, \vec{q}) = \sum_{j=1}^{A} e^{i\vec{q}\cdot\vec{x}_j} G_j(\vec{k}, \vec{q}). \qquad (5)$$

By applying partial wave expansions to the quantities in Eq. (4), using the separable t matrix of Eq. (1), and applying standard matrix inversion techniques, one can solve the resulting coupled system of equations exactly to obtain the G_i called for in Eq. (5). Thus, one can calculate multiple scattering to all orders. The amplitude $F(\vec{k}, \vec{q})$ must then be averaged over the initial and final nuclear wave functions; in practice this is done by Monte Carlo techniques, and the details are described in Ref. 27.

As we noted in Sec. I, the inclusion of nucleon momenta and recoil effects was found previously to be important in obtaining good fits to the π-^{4}He angular distributions. We have taken these effects into account in our calculations.

We begin by replacing the $\vec{q}\cdot\vec{q}\,'$ factor in Eq. (1) by the Galilean invariant form

$$\vec{q}_r\cdot\vec{q}_r\,' = [\vec{q} - (\mu/M)\vec{P}_i]\cdot[\vec{q}\,' - (\mu/M)\vec{P}_f], \qquad (6)$$

where μ and M are the pion and nucleon masses and $\vec{P}_i$ and $\vec{P}_f$ are the initial and final nucleon momenta. Since each nucleon is struck successively in the various terms in the expansion of Eq. (4), we can use this effective angle transformation in the entire multiple-scattering solution. Using conservation of momentum by replacing $\vec{P}_f$ in Eq. (6) by $\vec{q}-\vec{q}\,'+\vec{P}_i$ and retaining terms of order (μ/M) we obtain

$$\vec{q}_r\cdot\vec{q}_r\,' = -q^2(\mu/M)+(1+\mu/M)\vec{q}\cdot\vec{q}\,'\,. \tag{7}$$

Note that we have dropped terms linear in $\vec{P}_i$ since these tend to average to zero. In our calculations with medium energy pions, we must use a relativistic extension of Eq. (6), which has the form

$$\vec{q}_r\cdot\vec{q}_r\,' = \frac{[\vec{q}-(\omega/M)\vec{P}_i]\cdot[\vec{q}\,'-(\omega/M)\vec{P}_f]}{[1-(\vec{q}\cdot\vec{P}_i/\omega M)][1-(\vec{q}\,'\cdot\vec{P}_f/\omega M)]}\,, \tag{8}$$

where ω is the total pion energy. The denominators in Eq. (8) come from the usual relativistic velocity transformation expressions. Again setting $\vec{P}_f=\vec{q}-\vec{q}\,'+\vec{P}_i$, expanding and retaining terms of order (ω/M) while dropping terms linear in $\vec{P}_i$ gives:

$$\vec{q}_r\cdot\vec{q}_r = -\frac{\omega}{M}q^2+\vec{q}\cdot\vec{q}\,'\left(1-\frac{q'^2}{\omega M}+\frac{\omega}{M}\right)+\frac{(\vec{q}\cdot\vec{q}\,')^2}{\omega M}\,. \tag{9}$$

This expression, in addition to modifying the s- and p-wave amplitudes, introduces an effective d-wave component.

Two points must be mentioned here. As is explained in Ref. 27, the time required to solve the system of coupled equations while obtaining satisfactory convergence of the Monte Carlo integration grows nonlinearly with the number of π-N partial waves. Since the d-wave component which comes from the angle transformation of Eq. (9) is relatively small for the energies considered, it has been treated in a single scattering, or impulse approximation. Also, in evaluating the angle transformation of Eq. (9) we have used the on-shell momentum relations $q^2=q'^2=k^2$ for evaluating the s- and d-wave pieces as well as the p-wave coefficient. While this is not strictly correct for off-shell scattering contributions, we assume that corrections are of higher order and can be ignored. This leads to new π-N amplitudes of the form

$$\begin{aligned} f_0' &= f_0+\left(\frac{k^2}{3\omega M}-\frac{\omega}{M}\right)f_1\,,\\ f_1' &= f_1\left(1+\frac{\mu^2}{\omega M}\right), \\ f_2' &= \frac{2k^2}{3\omega M}f_1\,, \quad \text{where } f_l=e^{2i\delta_l}-1\,. \end{aligned} \tag{10}$$

Another kinematic correction which we have included is that due to the nucleon binding and recoil which alters the effective energy at which the π-N phase shifts are evaluated. The energy is lowered by the average nucleon binding, which for ^{4}He was taken to be 7 MeV. Furthermore, to the resultant energy we must add the kinetic energy of the struck nucleon. We have utilized a prescription previously employed in some optical model calculations[34] to take this into account. Using conservation of momentum, we can write the average nucleon momentum as $\frac{1}{2}(\vec{P}_i+\vec{P}_f)=\frac{1}{2}(\vec{q}-\vec{q}\,')+\vec{P}_i$. Neglecting the term linear in $\vec{P}_i$ and converting to a kinetic energy, we obtain a $Q^2/8M$ correction to the effective interaction energy, where $\vec{Q}=\vec{q}-\vec{q}\,'$ and where we have used the on-shell momentum k for the magnitude of $\vec{q}$ and $\vec{q}\,'$. Therefore, we evaluate the π-N phase shifts an at energy $E-E_{\text{binding}}+Q^2/8M$. Expanding the amplitudes about $E-E_{\text{binding}}+k^2/4M$, using the fact that $Q^2=2k^2\times(1-\cos\theta)$ on shell, and retaining terms of order $(1/M)$ we find the s- and p-wave amplitudes modified and mixed and a newly generated effective d-wave amplitude, which is again included as a single-scattering term. The resulting amplitudes are

$$\begin{aligned} f_0' &= f_0\left(E-E_{\text{binding}}+\frac{k^2}{4M}\right)+\tfrac{1}{3}f_1\left(E-E_{\text{binding}}\right)\\ &\quad+\tfrac{1}{3}f_1\left(E-E_{\text{binding}}+\frac{k^2}{4M}\right),\\ f_1' &= f_1\left(E-E_{\text{binding}}+\frac{k^2}{4M}\right)+f_0\left(E-E_{\text{binding}}\right)\\ &\quad+f_0\left(E-E_{\text{binding}}+\frac{k^2}{4M}\right),\\ f_2' &= \tfrac{2}{3}f_1\left(E-E_{\text{binding}}\right)-\tfrac{2}{3}f_1\left(E-E_{\text{binding}}+\frac{k^2}{4M}\right). \end{aligned} \tag{11}$$

In practice, this evaluation of the amplitudes is done prior to making the kinematic π-N angle transformation discussed previously. In the remainder of this paper, references to the angle transformation describe the kinematic partial wave mixing of Eq. (10); binding and recoil effects describe those of Eq. (11).

III. RESULTS

We have calculated elastic π^+ and π^- scattering for incident pion energies from 24 to 110 MeV, using the formalism developed in the preceding section. A Gaussian nuclear density was assumed for the ^{4}He target having an oscillator parameter $\alpha=1.31$ fm. This corresponds to an rms radius of 1.6 fm. The McKinley π-N phase shift parametrization was used.[32]

In addition to the strong interaction scattering amplitude, Coulomb effects were included by adding a Coulomb-nuclear amplitude containing the nuclear charge form factor and an appropriate relative Coulomb-nuclear phase.[35-38]

Figure 1 shows our calculated results for π^+ and for π^- at 24 MeV. We note the poor agreement with the experimental data, especially at the back angles where we are low by a factor of about 2.

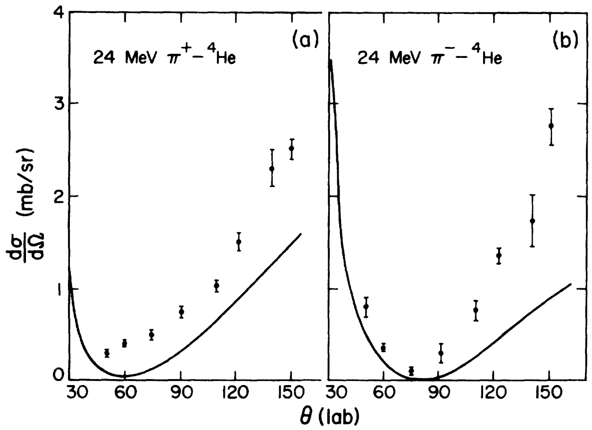

FIG. 1. Elastic cross sections for 24 MeV pions on ^{4}He. The curves shown include the angle transformation of Eq. (8) and the shift in the energy due to nucleon binding and recoil. Data are from Ref. 11. Fig. 1(a) π^+; (b) π^-.

Turning off the angle transformation and energy shift in the amplitudes has a negligible effect, because, at 24 MeV, we are seeing only the s-wave part of the π-N amplitude. The angle transformation modifies the strength of the s-wave piece by adding to it a p-wave contribution. However, at very low energies, the p-wave π-N amplitude is small.

Figure 2 shows our results for 51 MeV π^+ and π^-. For π^+ [Fig. 2(a)], we do quite well at forward and back angles although our minimum is too deep. For π^- [Fig. 2(b)], we tend to be a bit low for angles beyond the minimum. We note the good agreement at forward angles (30°) where Coulomb interference effects are important.

Similar descriptions apply to our results at 60 MeV (Fig. 3), 68 MeV (Fig. 4), and 75 MeV (Fig.

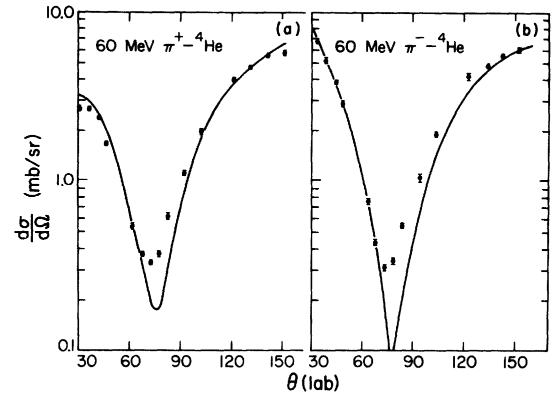

FIG. 3. Elastic cross sections for 60 MeV pions on ^{4}He. Kinematics as in Fig. 1. Data are from Ref. 12. Figure 3(a) π^+; (b) π^-.

5). The agreement at forward angles is excellent; the minimum is at the correct position; and as the energy increases, our results improve at angles beyond the minimum.

All of the calculations illustrated contain the angle transformation and the recoil and binding energy shift in the effective π-N interaction energy. We note that the minimum at 75° is well reproduced. If we turn off these kinematic effects in our calculations, the minimum is moved out to about 85°–90°.

In Fig. 6 we show our results for π^--^{4}He scattering at 110 MeV. We present three curves. The solid curve includes all of the kinematic transformations. The dashed curve includes the angle transformation, but not the recoil and binding shift in the energy. The dot-dashed curve results from turning off the angle transformation as well. It is apparent that the angle transformation is important in getting the minimum at the correct posi-

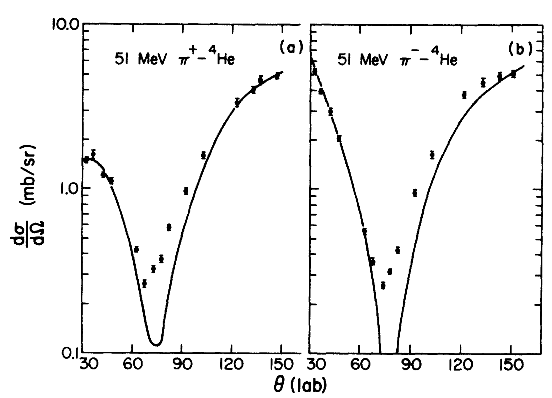

FIG. 2. Elastic cross sections for 51 MeV pions on ^{4}He. Kinematics as in Fig. 1. Data are from Ref. 12. Figure 2(a) π^+; (b) π^-.

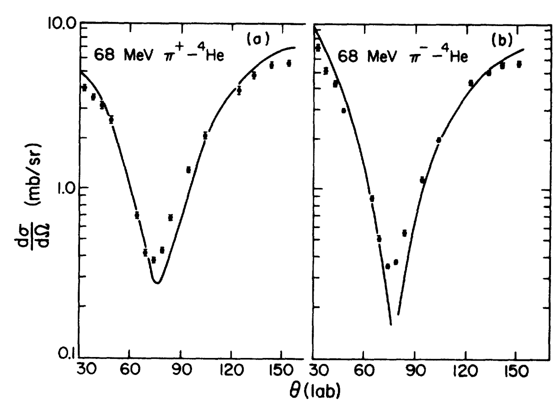

FIG. 4. Elastic cross sections for 68 MeV pions on ^{4}He. Kinematics as in Fig. 1. Data are from Ref. 12. Figure 4(a) π^+; (b) π^-.

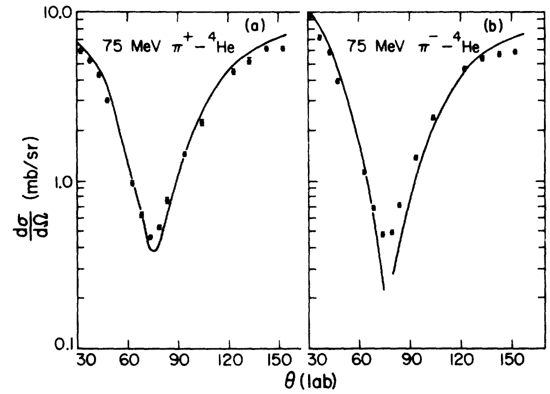

FIG. 5. Elastic cross sections for 75 MeV pions on ^{4}He. Kinematics as in Fig. 1. Data are from Ref. 12. Figure 5(a) π^+; (b) π^-.

tion and that the energy shift modified the size of the cross section particularly at large angles.

Calculations were also done at higher energies. However, the effective d-wave amplitude which comes from the p-wave π-N interaction due to the kinematic transformations becomes much more important as we approach the (3, 3) resonance region. Treating the d-wave term in a single-scattering approximation is no longer sufficiently accurate, for significant multiple-scattering effects are being left out. Consequently, we do not believe that those results warrant publication.

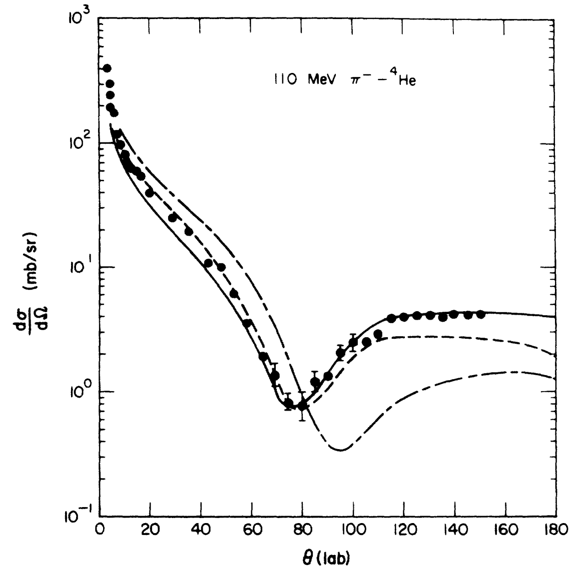

FIG. 6. Elastic cross section for 110 MeV π^- on ^{4}He. The solid curve contains both the angle transformation [Eq. (8)] and the energy shift due to nucleon binding and recoil. The dashed curve includes the angle transformation only. The dot-dashed curve has neither kinematic correction. Data are from Ref. 14.

IV. CONCLUSIONS

In conclusion, the multiple-scattering formalism of Ref. 27 has yielded good fits to the low energy π-^{4}He scattering data, similar to those of Landau.[8,9] We have made use of the fixed-nucleon approximation supplemented by two modifications: (1) the nucleon binding and kinetic energy additions to the effective energy at which the π-N scattering amplitudes are evaluated, and (2) the angle transformation which follows from incorporating the nucleon motion before and after the π-N scattering. Both of these modifications alter the s- and p-wave amplitudes as well as introduce a d-wave amplitude. We have found that the energy modification tends to increase the cross section, particularly at back angles, while the angle transformation produces the correct angular position for the stationary minimum at 75°. The extension of this application of our multiple-scattering theory to higher pion energies awaits our ability to solve the system of coupled equations for more than s and p waves in the effective π-N amplitudes.

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*Work performed under the auspices of the U.S. Energy Research and Development Administration.

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