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Rescattering corrections to the coherent photoproduction of π^0 on ^{4}He, ^{12}C, and ^{16}O at low energies

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Coherent photoproduction of π^0 on ^{4}He, ^{12}C, and ^{16}O is studied in the energy region $E_\gamma = 200$–400 MeV, with particular emphasis on the questions: where is rescattering important, what structure does it have, and what approximate scheme can be applied for calculating it? For this purpose, the second-order scattering term has been calculated in a model which incorporates the Fresnel correction to the Glauber theory, and takes into account the spin and the charge exchange effects. It is noted that near the threshold the correction would be very small. However, where rescattering is important, the spin and charge exchange effects and also Fresnel corrections are found to be important. This means that only in specific cases will a distorted-wave approach or a Glauber expansion provide the correct higher-order contributions for these reactions.

[NUCLEAR REACTIONS ^{4}He, ^{12}C, ^{16}O(γ,π^0); $E=200-400$ MeV; rescattering terms are calculated to lowest order.]

I. INTRODUCTION

Photoproduction of pions on nuclei at low energies was suggested to be a favorable reaction for investigating nuclear structure.[1] Previously[2] many authors studied the photoproduction of pions from heavier nuclei at low energies. The effects of higher-order correlations in the nuclear wave functions were discussed[3,4] in connection with charged pion photoproduction on ^{16}O in the energy region $E_\gamma = 170-350$ MeV. However, the above studies were made without considering the rescattering of the produced pions.

Rescattering effects at low energies were studied by some authors[5] in different approaches for lighter nuclei. For heavier nuclei only Saunders[6] has done calculations for the coherent photoproduction of π^0 mesons. Saunders took the final state interaction between the produced pion and the residual nucleus into account in an optical potential model. However, his results underestimate the experimental results very much and he ascribes this fact to the poor knowledge of the parameters of the optical potential used. Guy and Eisenberg[7] studied the rescattering effects in the case of the radiative pion absorption on nuclei, which is the inverse of the charged pion photoproduction process and which should essentially reflect the features of the threshold photoproduction. Their calculation, which is effectively equivalent to the optical potential approach, shows that the rescattering contribution would be about 40% for ^{16}O.

In this situation we are concerned with the question of rescattering in view of the Glauber model, whose applicability was ensured in the case of pion-nucleus scattering[8] in the region of the (3, 3) resonance, and to discuss the validity of the impulse approximation which was taken for granted in the previous studies concerning nuclear structure.

Photoproduction of pions is in this energy region peaked at non-forward angles. This means that the longitudinal momentum transfer cannot be approximated by its value in the forward direction (given by the pion mass and the photon momentum), but that a more realistic approximation must be used. We therefore consider the Fresnel correction to the Glauber theory.

The coherent production of π^0 on spin-zero nuclei is theoretically the simplest case, so we have investigated this process. As our target nuclei we have chosen ^{4}He, ^{12}C, ^{16}O, and we consider photon energies in the range 200–400 MeV. For nuclei like ^{12}C and ^{16}O there are considerable experimental difficulties in studying coherent π^0 photoproduction. But from the experiments done recently at Stanford[9] it appears feasible to perform such studies, which increases the theoretical interest in these processes.

Considering the full spin and isospin dependence of the amplitudes, the description becomes very complex. For practical reasons, we consequently had to limit ourselves to second-order terms. However, in most cases considered, higher-order terms are clearly also important.

There are two easy ways of including higher-order terms, but unfortunately they will in general both fail in the present case:

(i) A distorted-wave approach will fail, due to the importance of spin- and charge-exchange terms.

(ii) A Glauber summation will fail, since Fresnel corrections are very large.

Even the calculated second-order terms are mainly of qualitative validity, for the following reasons.

(i) All amplitudes have been taken at their on-shell values. Although this is probably a good approximation for not too dense nuclei, it might be questionable for the tightly bound ^{4}He nucleus.

(ii) Pion-nucleon non-spin-flip amplitudes have been parametrized as Gaussians in the momentum transfer. This leads to simple integrals, and is supposed to be a good approximation since the nuclear form factors effectively suppress large-angle scattering.

(iii) Nucleon recoil energies are neglected. This is questionable at the highest energies and the largest angles considered.

(iv) We consider only photon states leading to s- and p-wave pion-nucleon states. At the highest energies considered, d waves might also be important.

Our main concern is to determine where rescattering is important and what structure it has. In order to display its structure, we discuss explicitly the importance of exchange terms, and the Fresnel corrections.

II. THEORY

The differential cross section for coherent photoproduction of π^0 on closed shell nuclei is for unpolarized photons given by

$$\frac{d\sigma}{d\Omega}=\frac{p}{k}\overline{\sum}\,|T_{ii}|^2\exp\left(\frac{q^2R^2}{2A}\right) \tag{1}$$

in the lab system, where bar indicates the average over photon polarizations, and $\vec{k}$ and $\vec{p}$ are the momenta of the incident photon and the outgoing pion, respectively. The exponential takes care of the c.m. correlation, where $\vec{q}$ is the momentum transfer, R is the harmonic oscillator size parameter, and A the nucleon number. Further,

$$T_{ii}=\langle i|T|i\rangle. \tag{2}$$

The initial state of the nucleus is denoted by $|i\rangle$. Following the method indicated in Appendix A we write

$$T=\frac{ik}{2\pi}\langle\gamma|\int d^2b\exp(i\vec{q}_\perp\cdot\vec{b})\,\Gamma\,|\pi\rangle, \tag{3}$$

where

$$\Gamma=\sum_{i=1}^{A}\Gamma_i^{\gamma\pi}-\sum_{i=1}^{A}\sum_{j\neq i}^{A}\Gamma_i^{\gamma\pi}\Gamma_j^{\pi\pi}\theta(z_j-z_i) \tag{4}$$

and

$$\Gamma_i^{\gamma\pi}=(2\pi ik)^{-1}\int d^2q'_\perp\exp[-i\vec{q}'_\perp\cdot(\vec{b}-\vec{s}_i)]\times\exp(iq'_\parallel z_i)f_i^{\gamma\pi}(\vec{q}'_\perp,q'_\parallel), \tag{5}$$

$$\Gamma_j^{\pi\pi}=(2\pi ip)^{-1}\int d^2q''_\perp\exp[-i\vec{q}''_\perp\cdot(\vec{b}-\vec{s}_j)]\times\exp(iq''_\parallel z_j)f_j^{\pi\pi}(\vec{q}''_\perp,q''_\parallel), \tag{6}$$

with $\vec{b}$ being the impact parameter and $\vec{s}_i$ the projection of the spatial coordinate $\vec{r}_i$ of the ith nucleon in the nucleus onto the impact parameter plane, and $\vec{q}_\perp$ is the transverse momentum transfer. Further $|\gamma\rangle$ describes the isospin properties of the electromagnetic current, and $|\pi\rangle$ is the isospinor of the produced pion. In Eq. (4) we have assumed that after it has been produced the pion is scattered only once before it leaves the nucleus; and while production or scattering takes place on a nucleon, the other nucleons in the nucleus sit as spectators. Furthermore, $f_i^{\gamma\pi}(\vec{q}'_\perp,q'_\parallel)$ is the amplitude for pion photoproduction on the ith nucleon with transverse momentum transfer $\vec{q}'_\perp$ and longitudinal momentum transfer $q'_\parallel$ and $f_j^{\pi\pi}(\vec{q}''_\perp,q''_\parallel)$ is the amplitude for πN scattering with momentum transfers $\vec{q}''_\perp$ and $q''_\parallel$. The θ function ensures the scattering of the produced pion by the nucleons sitting in front of where the production takes place.

Using Eqs. (2) to (6) we get

$$T_{ii}=\langle\gamma|\langle i|\left(\frac{ik}{2\pi}\right)\int d^2b\exp(i\vec{q}_\perp\cdot\vec{b})\Bigg((2\pi ik)^{-1}\int d^2q'_\perp\sum_{i=1}^{A}\exp(-i\vec{q}'_\perp\cdot\vec{b})\exp(i\vec{q}'\cdot\vec{r}_i)f_i^{\gamma\pi}(\vec{q}')$$
$$-\frac{1}{kp}(2\pi i)^{-2}\sum_{i=1}^{A}\sum_{j\neq i}^{A}\int d^2q'_\perp d^2q''_\perp\exp[-i(\vec{q}'_\perp+\vec{q}''_\perp)\cdot\vec{b}]\exp(i\vec{q}'\cdot\vec{r}_i)f_i^{\gamma\pi}(\vec{q}')$$
$$\times\exp(i\vec{q}''\cdot\vec{r}_j)f_j^{\pi\pi}(\vec{q}'')\theta(z_j-z_i)\Bigg)|i\rangle|\pi\rangle, \tag{7}$$

where $\vec{q}'=\vec{q}'_\perp+\hat{k}q'_\parallel$ and $\vec{q}''=\vec{q}''_\perp+\hat{k}q''_\parallel$.

We shall further assume that all the energy goes to the pion, $k^2=\mu^2+p'^2=\mu^2+p^2$, where $\vec{p}'$ is the momentum of the intermediary pion and μ is the pion mass. The longitudinal momentum transfers are then given by

$$q_{\parallel}=k-p\cos\theta,\quad q'_{\parallel}=k-(p'^2-q_\perp'^2)^{1/2},$$
$$q''_{\parallel}=q_{\parallel}-q'_{\parallel}, \tag{8}$$

where θ is the production angle.

The first term in Eq. (7) is the impulse-approximation term $T_{ii}^{(I)}$ and the second one is the correction term $T_{ii}^{(C)}$. The impulse-approximation term can also be written in the following way:

$$T_{ii}^{(I)}=\langle\gamma|\langle i|\sum_{i=1}^{A}\exp(i\vec{q}\cdot\vec{r}_i)f_i^{\gamma\pi}(\vec{q})|i\rangle|\pi\rangle \tag{9}$$

and the correction term as

$$T_{ii}^{(C)}=-\frac{ik}{2\pi}\frac{1}{kp}(2\pi i)^{-2}\times\int d^2b\exp(i\vec{q}_\perp\cdot\vec{b})\times\int d^2q'_\perp d^2q''_\perp\exp[-i(\vec{q}'_\perp+\vec{q}''_\perp)\cdot\vec{b}]\,\mathfrak{F}, \tag{10}$$

where

$$\mathfrak{F}=\langle\gamma|\langle i|\sum_{i=1}^{A}\sum_{j\neq i}^{A}\exp(i\vec{q}'\cdot\vec{r}_i)f_i^{\gamma\pi}(\vec{q}')\exp(i\vec{q}''\cdot\vec{r}_j)\times f_j^{\pi\pi}(\vec{q}'')\,\theta(z_j-z_i)|i\rangle|\pi\rangle. \tag{11}$$

Writing

$$f^{\gamma\pi}=L^{\gamma\pi}+i\vec{\sigma}\cdot\vec{K}^{\gamma\pi} \tag{12}$$

with $L^{\gamma\pi}$ and $\vec{K}^{\gamma\pi}$ being the spin-independent and spin-dependent parts of the photoproduction amplitude, respectively, and $\vec{\sigma}$ the Pauli spin matrix, Eq. (9) becomes

$$T_{ii}^{(I)}=\langle\gamma|\langle i|\sum_{i=1}^{A}\exp(i\vec{q}\cdot\vec{r}_i)(L_i^{\gamma\pi}+i\vec{\sigma}\cdot\vec{K}_i^{\gamma\pi})|i\rangle|\pi\rangle. \tag{13}$$

This is evaluated as in Ref. 2. Similarly, we write

$$f^{\pi\pi}=L^{\pi\pi}+i\vec{\sigma}\cdot\vec{K}^{\pi\pi}, \tag{14}$$

where $L^{\pi\pi}$ and $\vec{K}^{\pi\pi}$ are the spin-independent and spin-dependent parts of the πN amplitude.

We take the representation

$$\theta(z_j-z_i)=\lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{da}{a-i\epsilon}\exp[ia(z_j-z_i)] \tag{15}$$

and define

$$\vec{q}_1=\vec{q}'-a\hat{k},\quad \vec{q}_2=\vec{q}''+a\hat{k}. \tag{16}$$

Further, we expand

$$\exp(i\vec{q}\cdot\vec{r})=4\pi\sum_{l\mu}i^l j_l(qr)\,Y_{l\mu}(\hat{q})\,Y_{l\mu}^*(\hat{r}), \tag{17}$$

where j_l is a spherical Bessel function of order l and $Y_{l\mu}$ is a spherical tensor as in Ref. 4. Writing the nuclear wave function $|i\rangle$ as an antisymmetrized product of single-nucleon states, and doing some angular momentum algebra, we can write $\mathfrak{F}$ as follows:

$$\mathfrak{F}=\lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{da}{a-i\epsilon}(\mathfrak{F}^{(D)}+\mathfrak{F}^{(E)}), \tag{18}$$

$$\mathfrak{F}^{(D)}=\langle\gamma|\sum_{\alpha=1}^{A}\sum_{\beta=1}^{A}J_{\alpha\alpha}(1)\,J_{\beta\beta}(2)|\pi\rangle, \tag{19}$$

$$\mathfrak{F}^{(E)}=-\langle\gamma|\sum_{\alpha=1}^{A}\sum_{\beta=1}^{A}J_{\alpha\beta}(1)\,J_{\beta\alpha}(2)|\pi\rangle. \tag{20}$$

Arguments 1 and 2 denote production and scattering, respectively. The above sums can be written as

$$\sum_{\alpha=1}^{A}J_{\alpha\alpha}(1)=\sum_{L_\alpha J_\alpha M_\alpha m_\alpha}\sum_{\lambda n\Lambda}O(\lambda n\Lambda;\;L_\alpha J_\alpha)\,F_\lambda^{(L_\alpha L_\alpha)}(q_1^2)[Y_\lambda(\hat{q}_1)\times K_n^{\gamma\pi(\alpha\alpha)}(\vec{q}')]_\Lambda^0, \tag{21}$$

$$\sum_{\beta=1}^{A}J_{\beta\beta}(2)=\sum_{L_\beta J_\beta M_\beta m_\beta}\sum_{\lambda n\Lambda}O(\lambda n\Lambda;\;L_\beta J_\beta)\,F_\lambda^{(L_\beta L_\beta)}(q_2^2)[Y_\lambda(\hat{q}_2)\times K_n^{\pi\pi(\beta\beta)}(\vec{q}'')]_\Lambda^0, \tag{22}$$

$$\sum_{\alpha=1}^{A}\sum_{\beta=1}^{A}J_{\alpha\beta}(1)\,J_{\beta\alpha}(2)=\sum_{L_\alpha J_\alpha L_\beta J_\beta m_{\tau_\alpha} m_{\tau_\beta}}\;\sum_{\lambda n\lambda' n'\Lambda N}O'(\lambda n\Lambda;L_\alpha J_\alpha L_\beta J_\beta)\,O'(\lambda' n'\Lambda;L_\alpha J_\alpha L_\beta J_\beta)$$
$$\times(-)^{\lambda'+n'+1}\hat{\Lambda}^3\hat{N}^2\begin{Bmatrix}\lambda & n & \Lambda\\ \lambda' & n' & \Lambda\\ N & N & 0\end{Bmatrix}F_\lambda^{(L_\alpha L_\beta)}(q_1^2)\,F_{\lambda'}^{(L_\beta L_\alpha)}(q_2^2)$$
$$\times\{[Y_\lambda(\hat{q}_1)\times Y_{\lambda'}(\hat{q}_2)]_N\times[K_n^{\gamma\pi(\alpha\beta)}(\vec{q}')\times K_{n'}^{\pi\pi(\beta\alpha)}(\vec{q}'')]_N\}_0^0. \tag{23}$$

In the above expression L_α, J_α, M_α, and m_α are the orbital angular momentum, total angular momentum, total angular momentum projection, and isospin projection quantum numbers of the αth nucleon. The quantities O and O' are given by

$$O(\lambda n\Lambda;L_\alpha J_\alpha)=\sqrt{8\pi}\,i^{\lambda+n}(-)^{\lambda+n-\Lambda}\langle J_\alpha M_\alpha \Lambda 0|J_\alpha M_\alpha\rangle \hat{J}_\alpha \hat{L}_\alpha \hat{\lambda}\hat{n}\hat{\Lambda}\langle L_\alpha 0\lambda 0|L_\alpha 0\rangle \begin{Bmatrix} L_\alpha & \lambda & L_\alpha \\ \frac{1}{2} & n & \frac{1}{2} \\ J_\alpha & \Lambda & J_\alpha \end{Bmatrix}, \tag{24}$$

$$O'(\lambda n\Lambda;L_\alpha J_\alpha L_\beta J_\beta)=(-)^{\lambda+n+\Lambda}O'(\lambda n\Lambda;L_\beta J_\beta L_\alpha J_\alpha)=\sqrt{8\pi}\,i^{\lambda+n}(-)^{\lambda+n-\Lambda+J_\alpha}\hat{J}_\alpha \hat{J}_\beta \hat{L}_\beta \hat{\lambda}\hat{n}\langle L_\beta 0\lambda 0|L_\alpha 0\rangle \begin{Bmatrix} L_\beta & \lambda & L_\alpha \\ \frac{1}{2} & n & \frac{1}{2} \\ J_\beta & \Lambda & J_\alpha \end{Bmatrix}, \tag{25}$$

We use $\hat{A}=(2A+1)^{1/2}$ and Clebsch-Gordan coefficients, $9j$ symbols, and angular momentum couplings as defined in Ref. 4. The form factors are defined as below,

$$F_\lambda^{(L_\alpha L_\beta)}(q^2)=\int_0^\infty R^*_{n_\alpha L_\alpha}(r)\,j_\lambda(qr)\,R_{n_\beta L_\beta}(r)\,r^2dr, \tag{26}$$

where R_{nl} is the radial wave function, for which we take the harmonic oscillator one.

For both the photoproduction ($\gamma\pi$) and the scattering ($\pi\pi$) amplitudes, the following notation is used:

$$K_n^{(\alpha\beta)}=\langle m_\alpha|K_n|m_\beta\rangle, \tag{27}$$

where $|m\rangle$ is the isospin state of a nucleon with projection m, and K_n is defined by

$$L+i\vec{\sigma}\cdot\vec{K}=\sum_{n=0,1} i^n\sigma_n K_n, \tag{28}$$

where

$$\sigma_0=1,\quad \sigma_1=\vec{\sigma} \tag{29}$$

and

$$K_0=L,\quad K_1=\vec{K}. \tag{30}$$

Using the expressions for

$$\{[Y_\lambda(\hat{q}_1)\times Y_{\lambda'}(\hat{q}_2)]_N\times[K_n^{\gamma\pi(\alpha\beta)}(\vec{q}')\times K_{n'}^{\pi\pi(\beta\alpha)}(\vec{q}'')]_N\}_0^0$$

as given in Appendix B, we get:

for ^{4}He:

$$\mathfrak{F}^{(D)}=\sum_{m_\alpha m_\beta}\langle\gamma|4L^{\gamma\pi(\alpha\alpha)}(\vec{q}')L^{\pi\pi(\beta\beta)}(\vec{q}'')|\pi\rangle F_0^{(00)}(q_1^2)F_0^{(00)}(q_2^2), \tag{31}$$

$$\mathfrak{F}^{(E)}=-\sum_{m_\alpha m_\beta}\langle\gamma|2[L^{\gamma\pi(\alpha\beta)}(\vec{q}')L^{\pi\pi(\beta\alpha)}(\vec{q}'')-\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')]|\pi\rangle F_0^{(00)}(q_1^2)F_0^{(00)}(q_2^2); \tag{32}$$

for ^{12}C:

$$\mathfrak{F}^{(D)}=\sum_{m_\alpha m_\beta}\langle\gamma|L^{\gamma\pi(\alpha\alpha)}(\vec{q}')L^{\pi\pi(\beta\beta)}(\vec{q}'')|\pi\rangle[2F_0^{(00)}(q_1^2)+4F_0^{(11)}(q_1^2)][2F_0^{(00)}(q_2^2)+4F_0^{(11)}(q_2^2)], \tag{33}$$

$$\begin{aligned}\mathfrak{F}^{(E)}=-\sum_{m_\alpha m_\beta}\{&\langle\gamma|L^{\gamma\pi(\alpha\beta)}(\vec{q}')L^{\pi\pi(\beta\alpha)}(\vec{q}'')|\pi\rangle[2F_0^{(00)}(q_1^2)F_0^{(00)}(q_2^2)-8P_1(\hat{q}_1\cdot\hat{q}_2)F_1^{(10)}(q_1^2)F_1^{(10)}(q_2^2)\\&+4F_0^{(11)}(q_1^2)F_0^{(11)}(q_2^2)+4P_2(\hat{q}_1\cdot\hat{q}_2)F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)]\\&+\langle\gamma|\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')|\pi\rangle[-2F_0^{(00)}(q_1^2)F_0^{(00)}(q_2^2)+8P_1(\hat{q}_1\cdot\hat{q}_2)F_1^{(10)}(q_1^2)F_1^{(10)}(q_2^2)\\&-\tfrac{20}{9}F_0^{(11)}(q_1^2)F_0^{(11)}(q_2^2)-\tfrac{52}{9}P_2(\hat{q}_1\cdot\hat{q}_2)F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)\\&+\tfrac{4}{9}F_2^{(11)}(q_1^2)F_0^{(11)}(q_2^2)+\tfrac{4}{9}F_0^{(11)}(q_1^2)F_2^{(11)}(q_2^2)\\&+\tfrac{4}{3}(Q^2+\tfrac{2}{3})F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)]\\&+\vec{Q}\cdot\langle\gamma|\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')\times\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')|\pi\rangle\\&\times[8P_1(\hat{q}_1\cdot\hat{q}_2)F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)-4F_1^{(10)}(q_1^2)F_1^{(10)}(q_2^2)]\\&-4\langle\gamma|[\vec{Q}\cdot\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')][\vec{Q}\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')]|\pi\rangle F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)\\&-\tfrac{4}{3}\langle\gamma|[\hat{q}_1\cdot\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')][\hat{q}_1\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')]|\pi\rangle[F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)+F_2^{(11)}(q_1^2)F_0^{(11)}(q_2^2)]\\&-\tfrac{4}{3}\langle\gamma|[\hat{q}_2\cdot\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')][\hat{q}_2\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')]|\pi\rangle[F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)+F_0^{(11)}(q_1^2)F_2^{(11)}(q_2^2)]\};\end{aligned} \tag{34}$$

for ^{16}O:

$$\mathcal{F}^{(D)}=\sum_{m_\alpha m_\beta}\langle\gamma|L^{\gamma\pi(\alpha\beta)}(\vec{q}')L^{\pi\pi(\beta\alpha)}(\vec{q}'')|\pi\rangle[2F_0^{(00)}(q_1^2)+6F_0^{(11)}(q_1^2)][2F_0^{(00)}(q_2^2)+6F_0^{(11)}(q_2^2)], \quad (35)$$

$$\mathcal{F}^{(E)}=-\sum_{m_\alpha m_\beta}\langle\gamma|[L^{\gamma\pi(\alpha\beta)}(\vec{q}')L^{\pi\pi(\beta\alpha)}(\vec{q}'')-\vec{K}^{\gamma\pi(\alpha\beta)}(\vec{q}')\cdot\vec{K}^{\pi\pi(\beta\alpha)}(\vec{q}'')]|\pi\rangle$$
$$\times[2F_0^{(00)}(q_1^2)F_0^{(00)}(q_2^2)+6F_0^{(11)}(q_1^2)F_0^{(11)}(q_2^2)-12P_1(\hat{q}_1\cdot\hat{q}_2)F_1^{(10)}(q_1^2)F_1^{(10)}(q_2^2)$$
$$+12P_2(\hat{q}_1\cdot\hat{q}_2)F_2^{(11)}(q_1^2)F_2^{(11)}(q_2^2)]. \quad (36)$$

In the above expressions $\vec{Q}=\hat{q}_1\times\hat{q}_2$ and $P_l(\hat{q}_1\cdot\hat{q}_2)$ is the Legendre polynomial.

For π^0 production the isospin decompositions are given as below:

$$\sum_{m_\alpha}\sum_{m_\beta}\langle\gamma|K_n^{\gamma\pi(\alpha\alpha)}\cdot K_n^{\pi\pi(\beta\beta)}|\pi^0\rangle=4K_n^{\gamma\pi(+)}\cdot K_n^{\pi\pi(+)}, \quad (37)$$

$$\sum_{m_\alpha}\sum_{m_\beta}\langle\gamma|K_n^{\gamma\pi(\alpha\beta)}\cdot K_n^{\pi\pi(\beta\alpha)}|\pi^0\rangle$$
$$=2K_n^{\gamma\pi(+)}\cdot K_n^{\pi\pi(+)}+4K_n^{\gamma\pi(-)}\cdot K_n^{\pi\pi(-)}, \quad (38)$$

where

$$K_n^{(+)}=\tfrac{1}{3}(K_n^{(1/2)}+2K_n^{(3/2)}),$$
$$K_n^{(-)}=\tfrac{1}{3}(K_n^{(1/2)}-K_n^{(3/2)}). \quad (39)$$

Upper indices $\frac{1}{2}$ and $\frac{3}{2}$ refer to the total isospin in the production or scattering channel.

The photoproduction amplitudes $L^{\gamma\pi}$ and $\vec{K}^{\gamma\pi}$ are in the nonrelativistic case given in terms of electric and magnetic multipoles as[4]

$$L^{*\gamma\pi}=\hat{p}^*\cdot(\hat{k}^*\times\hat{\epsilon})(2M_{1+}+M_{1-}), \quad (40)$$

$$\vec{K}^{*\gamma\pi}=\hat{\epsilon}[E_{0+}+\cos\theta^*(M_{1+}+3E_{1+}-M_{1-})]$$
$$+\hat{k}^*(\hat{\epsilon}\cdot\hat{p}^*)(3E_{1+}+M_{1-}-M_{1+}). \quad (41)$$

Starred quantities refer to the πN c.m. system, and $\hat{\epsilon}$ is the photon polarization vector. The normalization is as follows

$$\left(\frac{d\sigma}{d\Omega}\right)^*_{\text{unpol}}(\gamma N\to N\pi)=\frac{p^*}{k^*}\overline{\sum_\epsilon}\{|L^{*\gamma\pi}|^2+|\vec{K}^{*\gamma\pi}|^2\}. \quad (42)$$

We need the amplitudes in the lab system [see Eq. (1)]. By invariance of $d\sigma$ we get

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}(\gamma N\to N\pi)=\left(\frac{d\sigma}{d\Omega}\right)^*_{\text{unpol}}(\gamma N\to N\pi)\frac{d(\cos\theta^*)}{d(\cos\theta)}, \quad (43)$$

and normalizing L and $\vec{K}$ with respect to the differential cross section in the lab system by

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}(\gamma N\to N\pi)=\frac{p}{k}\overline{\sum_\epsilon}\{|L^{\gamma\pi}|^2+|\vec{K}^{\gamma\pi}|^2\}, \quad (44)$$

it follows that

$$L^{\gamma\pi}=\eta L^{*\gamma\pi},$$
$$\vec{K}^{\gamma\pi}=\eta\vec{K}^{*\gamma\pi}, \quad (45)$$

where

$$\eta^2=\frac{p^*}{p}\frac{k}{k^*}\frac{d(\cos\theta^*)}{d(\cos\theta)}. \quad (46)$$

Since the transversal components of $\vec{p}$ and $\vec{p}^*$ are the same, the vectorial structures of $L^{\gamma\pi}$ and $\vec{K}^{\gamma\pi}$ in the lab system are given by

$$L^{\gamma\pi}=\eta\frac{p}{p^*}\hat{p}\cdot(\hat{k}\times\hat{\epsilon})(2M_{1+}+M_{1-}), \quad (47)$$

$$\vec{K}^{\gamma\pi}=\eta\left\{\hat{\epsilon}[E_{0+}+\cos\theta^*(M_{1+}+3E_{1+}-M_{1-})]\right.$$
$$\left.+\frac{p}{p^*}\hat{k}(\hat{\epsilon}\cdot\hat{p})(3E_{1+}+M_{1-}-M_{1+})\right\}. \quad (48)$$

For the spin-independent pion-nucleon scattering amplitudes we use

$$L^{\pi\pi}=f_{\pi N}(0)\exp(-\tfrac{1}{2}\beta^2 q''^2), \quad (49)$$

where $f_{\pi N}(0)$ is the forward amplitude and β^2 is the slope parameter. The spin-dependent amplitude is given by an expansion in terms of the derivatives of the Legendre polynomials, so s-wave phase shifts will not contribute. Further, since it is a good approximation to consider only the (3, 3) resonance part, we write

$$\vec{K}^{\pi\pi}=(\vec{p}\times\vec{q}'')B^{\pi\pi}, \quad (50)$$

where $B^{\pi\pi}$ is a constant.

To evaluate $T_{fi}^{(C)}$, we first perform the trivial integrations over $\vec{b}$ and one of the transverse momentum transfers in Eq. (10). Using also Eq. (18), we get

$$T_{fi}^{(C)}=\lim_{\epsilon\to 0}\frac{1}{2\pi i}\times\int_{-\infty}^{\infty}\frac{da}{a-i\epsilon}\frac{i}{2\pi p}\int d^2q'_\perp(\mathcal{F}^{(D)}+\mathcal{F}^{(E)})\bigg|_{\vec{q}''_\perp=\vec{q}_\perp-\vec{q}'_\perp} \quad (51)$$

Since the expressions for ^{12}C are more involved, we shall only discuss the numerical results for ^{4}He and ^{16}O.

The terms $\mathfrak{F}^{(D)}$ and $\mathfrak{F}^{(E)}$ have the following structure [compare Eqs. (31), (32), (35), and (36)]:

$$\mathfrak{F}^{(D,E)}=\sum \text{const}\, P_l(\hat{q}_1\cdot\hat{q}_2)\, F_X^{(\gamma\delta)}(q_1^{\,2})\, F_{X'}^{(\epsilon\eta)}(q_2^{\,2}) \times K_n^{\gamma\pi}(\vec{q}\,')\cdot K_n^{\pi\pi}(\vec{q}\,''), \quad (52)$$

where the form factors are Gaussians multiplied with polynomials in the momentum transfers. This permits us to do the a integration in Eq. (51), writing out the expressions Eq. (51), and using Eq. (16). We get integrals of the following type:

$$A^{(m)}(s)=\lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{da}{a-i\epsilon}\,a^m \exp[-(as+a^2)\tfrac{1}{2}R^2], \quad (53)$$

where

$$s=q_\parallel''-q_\parallel' \quad (54)$$

and R is the oscillator constant. These are evaluated in Appendix C. In the limit of no longitudinal momentum transfer $q_\parallel'=q_\parallel''=0$ we get

$$A^{(m)}(0)=\tfrac{1}{2}\delta_{m0}. \quad (55)$$

Before proceeding with the angular integration in Eq. (51), we discuss polarizations. Since we consider coherent production on $J=0$ nuclei, only the term $L^{\gamma\pi}$ will contribute to the impulse approximation. It is clear from Eq. (47) that $L^{\gamma\pi}$ vanishes for $\hat{\epsilon}\,||\,\vec{p}_\perp$, i.e., only the photons polarized perpendicular to the production plane will contribute. This will also be the case for higher-order terms. For the second-order term, which we consider here, it follows from the angular integration in Eq. (51) that the expression becomes odd in $\phi_{q_\perp'}$ for $\hat{\epsilon}\,||\,\vec{p}_\perp$. With photon polarization perpendicular to the ultimate (not intermediary) production plane, the angular integrals in Eq. (51) are of the type

$$\int_0^{2\pi} d\phi_{q_\perp'}\cos^\nu\phi_{q_\perp'}\exp(A\cos\phi_{q_\perp'}),$$

where A is real. They give modified Bessel functions, or linear combinations thereof, according to the relation

$$I_\nu(A)=\frac{1}{\pi}\int_0^\pi d\phi\cos(\nu\phi)\exp(A\cos\phi). \quad (56)$$

The remaining integration over $q_\perp'$ is performed numerically, since the expression will contain the partial longitudinal momentum transfers $q_\parallel'$ and $q_\parallel''$, which are not trivially expressed by $q_\perp'$ and q. In Appendix D we give the expressions for the contributions to $T_{fi}^{(C)}$ arising from the "direct" terms $\mathfrak{F}^{(D)}$ for ^{4}He and ^{16}O. The expressions for the "exchange" terms become much more complicated in appearance, so we do not write them out. However, we take them into account in the numerical calculations, and discuss their relative importance in the next section.

III. RESULTS AND CONCLUSIONS

The expressions for the direct terms given in Appendix D and similar (though considerably more complicated) expressions for the exchange terms are evaluated numerically. For $L^{\gamma\pi}$ and $\vec{K}^{\gamma\pi}$ we use the multipole amplitudes of Berends, Donnachie, and Weaver.[10] The forward scattering amplitudes $f_{\pi N}(0)$ in Eq. (49) we take from the Karlsruhe table.[11] The slope parameter β^2 and numerical values of the constant $B^{\pi\pi}$ in Eq. (50) are taken from Ref. 12. The harmonic oscillator parameter R is taken to be 1.58 and 1.71 fm for ^{4}He and ^{16}O, respectively.

In Figs. 1–4, the differential cross sections for ^{4}He and ^{16}O for photon energies 260 and 380 MeV are shown together with the impulse-approximation results. In Fig. 5 the total cross section is shown for ^{4}He in the energy region ranging from 260 to 410 MeV. It is clear that calculations taking only direct terms into account give too low values for all the cases. The inclusion of the exchange terms

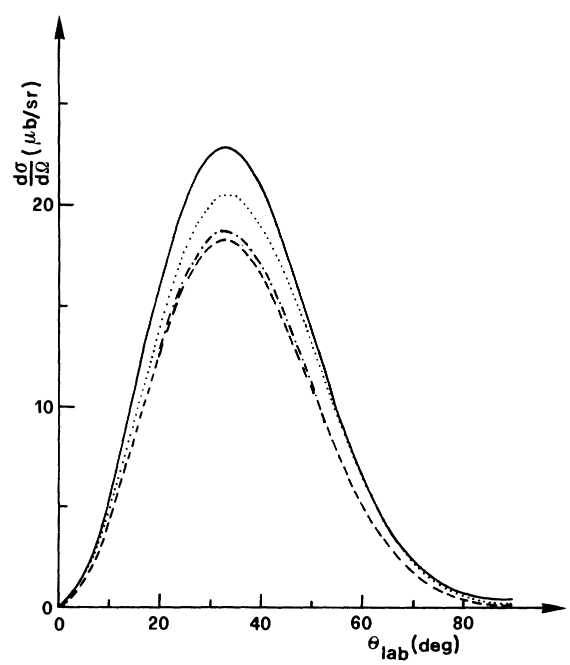

FIG. 1. Differential cross section for the coherent π^0 photoproduction on ^{4}He at $E_\gamma=260$ MeV. The dotted curve is the impulse approximation, the dashed curve includes second-order direct terms, the dot-dashed curve includes also the second-order exchange terms except for double spin flip, the solid curve includes all second-order corrections.

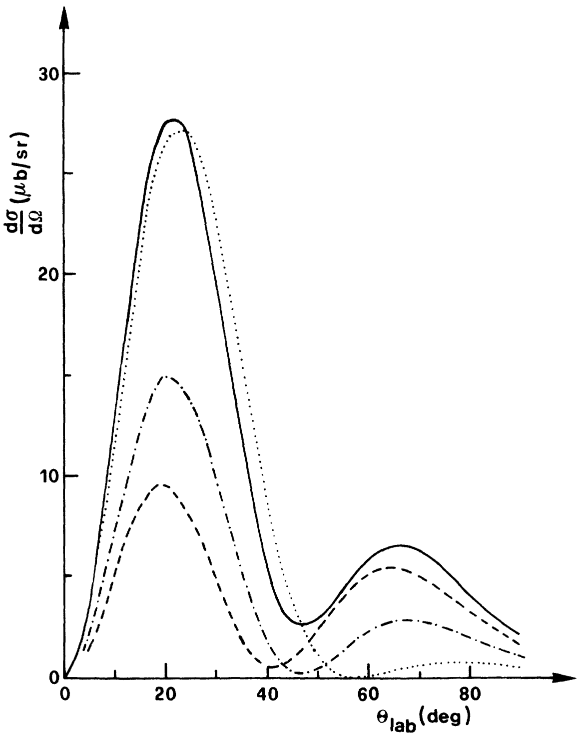

FIG. 2. Same as Fig. 1 but for $E_\gamma = 380$ MeV.

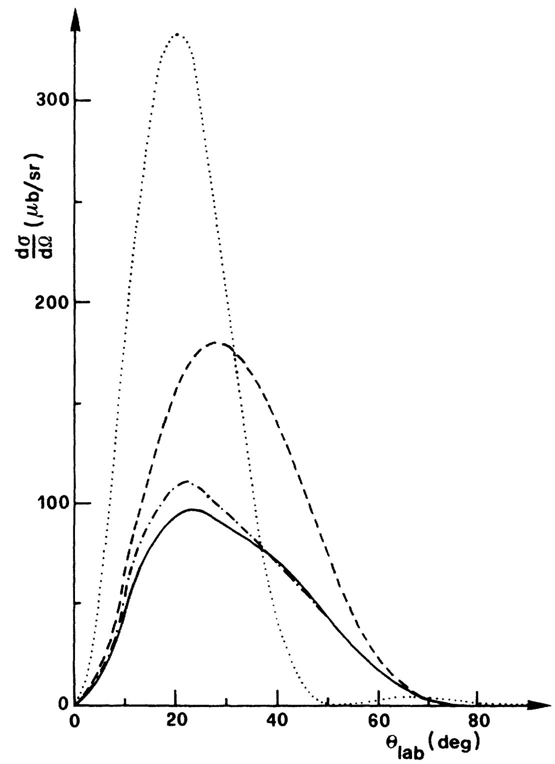

FIG. 4. Same as Fig. 3 but for $E_\gamma = 380$ MeV.

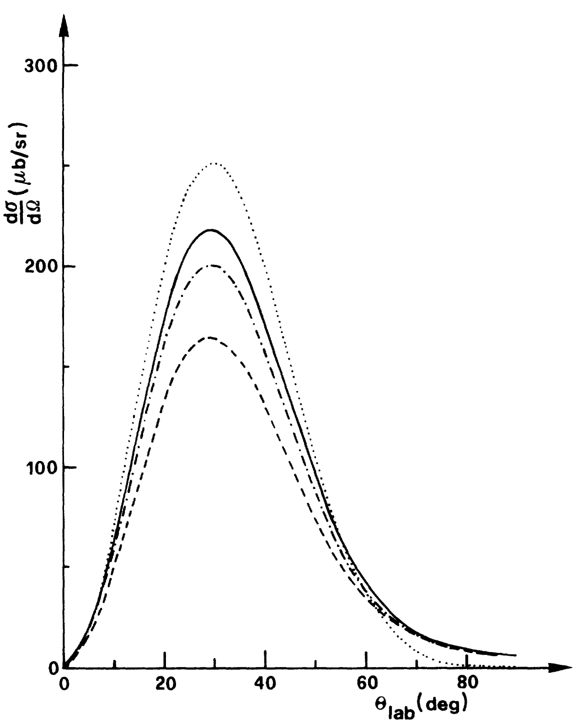

FIG. 3. Differential cross section for the coherent π^0 photoproduction on ^{16}O at $E_\gamma = 260$ MeV. The interpretation of the separate curves is as in Fig. 1.

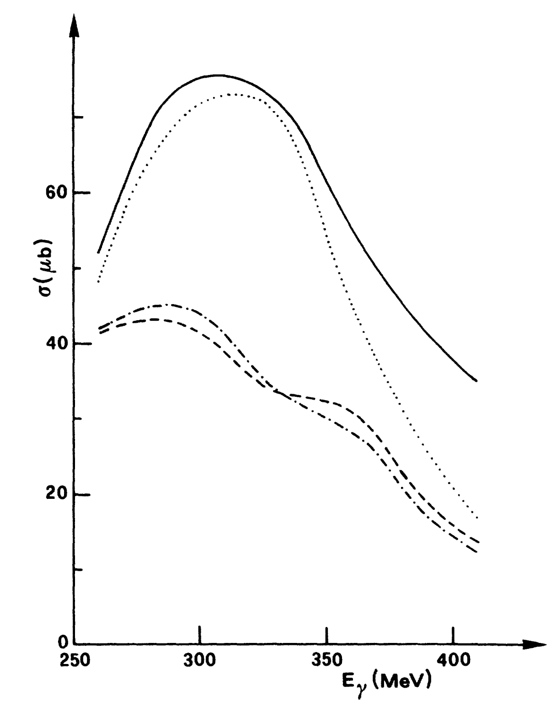

FIG. 5. Total cross sections for the coherent π^0 photoproduction on ^{4}He from 260 to 410 MeV. The interpretation of the separate curves is as in Fig. 1.

is very important. Particularly, the double spin-flip term increases the cross section for ^{4}He by a factor of 2 at higher energies. The exchange terms are responsible for the double spin-flip and since these are smaller than the direct terms by roughly a factor $1/A$, this contribution is most important for ^{4}He among the nuclei considered. Including all the exchange effects, the ultimate results with second-order corrections appear not much different from the impulse-approximation results for ^{4}He. However, for ^{16}O it turns out to be very different.

In the case of elastic pion scattering on ^{4}He at pion kinetic energies of 120 MeV (corresponding to $E_\gamma = 260$ MeV) and 280 MeV ($E_\gamma = 420$ MeV) it was noted previously[13] that the inclusion of the second-order terms was enough to assess the rescattering effects. At the top of the resonance the contribution from the third-order term is around 30%. In our case the contribution from the second order is rather small at $E_\gamma = 260$ MeV and one would expect the higher-order terms to give negligible contributions, especially for smaller angles. At this energy the correction to the differential cross sections is around 15%. Again this correction decreases with decreasing energies. For example, our calculations at $E_\gamma = 200$ MeV show that the correction would be less than 5%. Therefore, in the case of coherent photoproduction on ^{4}He near threshold one can safely use the impulse approximation.

From Fig. 2 one may be tempted to believe that for ^{4}He the impulse approximation is good enough at $E_\gamma = 380$ MeV too, for angles up to 40–50°. However, this is purely accidental; there are large cancellations between direct and exchange terms. Third-order terms might well be important at this energy. This is even more so at the top of the resonance. Further, the spin-exchange terms play a very important role, and have to be investigated to the third order.

For ^{16}O the situation is very different. As noted in the pion-scattering case,[12] the rescattering series requires more terms to be included. This will also be the case for photoproduction. However, we note that the effect of the double spin flip is only around 5% at higher energies ($E_\gamma > 350$ MeV), and it will not be necessary to include the spin terms in higher-order calculations. This will reduce the complications to a great extent.

For ^{16}O, Fig. 3 shows little difference between the impulse-approximation results and the corrected ones at $E_\gamma = 260$ MeV. But the second-order amplitudes are not small; the phase is such that they do not show up in the cross sections. Higher-order calculations might change the picture. However, our calculations at $E_\gamma = 200$ MeV show that at this energy the contribution from the correction term would be less than 10% for ^{16}O and we expect the impulse approximation to be good around threshold.

As regards the Fresnel correction, we note that with increasing transverse momentum transfer, the longitudinal momentum transfer will increase, causing the dominant form factors to decrease. In Table I we have shown what this would mean to the differential cross sections in the single-scattering approximation. The second-order term will also receive corrections due to the inclusion of the longitudinal momentum transfer. In Table II we have shown the second-order amplitudes where the double spin-flip terms are not included, with and without the Fresnel correction. We note that the inclusion of the longitudinal momentum transfer causes the amplitudes to decrease. This is true also at smaller angles. On the other hand, the double spin-flip terms increase when the Fresnel correction is introduced. In both cases, the Fresnel correction causes a considerable change of phase.

Unfortunately, the general importance of exchange terms makes the distorted-wave approach to higher-order terms inapplicable. This is so since in that approach one does not treat the nu-

TABLE I. Differential cross sections in the impulse approximation in units of μb/sr. The upper line includes the Fresnel correction, whereas the lower line does not.

θ_{lab}	10°	20°	30°	40°	50°	60°
^{4}He, 260 MeV	4.6	14.1	20.1	18.9	12.9	6.5
	4.7	14.8	23.0	25.3	22.8	18.5
^{4}He, 380 MeV	11.8	26.6	22.2	8.6	1.1	0.0
	12.0	28.4	27.4	15.5	5.8	1.4
^{16}O, 260 MeV	71	200	251	197	105	38
	72	213	301	293	230	162
^{16}O, 380 MeV	177	333	201	44	1	2
	180	363	269	104	22	2

TABLE II. Second-order amplitudes in units of fm. Double spin flip is not included. The upper line includes the Fresnel correction, whereas the lower line is pure Glauber theory.

θ_{lab}	10°	20°	30°	40°	50°	60°
^{4}He, 260 MeV	0.0021 −0.0016i	0.0038 −0.0029i	0.0051 −0.0037i	0.0058 −0.0038i	0.0058 −0.0034i	0.0053 −0.0027i
	0.0032 −0.0010i	0.0058 −0.0018i	0.0074 −0.0023i	0.0080 −0.0025i	0.0080 −0.0024i	0.0076 −0.0023i
^{4}He, 380 MeV	−0.0034 0.0044i	−0.0061 0.0076i	−0.0076 0.0088i	−0.0077 0.0081i	−0.0068 0.0063i	−0.0054 0.0043i
	−0.0059 0.0035i	−0.0103 0.0061i	−0.0124 0.0073i	−0.0125 0.0074i	−0.0115 0.0068i	−0.0101 0.0060i
^{16}O, 260 MeV	0.0335 −0.0189i	0.0605 −0.0331i	0.0768 −0.0398i	0.0808 −0.0388i	0.0743 −0.0325i	0.0610 −0.0237i
	0.0527 −0.0196i	0.0939 −0.0348i	0.1172 −0.0434i	0.1236 −0.0457i	0.1187 −0.0438i	0.1091 −0.0401i
^{16}O, 380 MeV	−0.0547 0.0326i	−0.0912 0.0456i	−0.1000 0.0358i	−0.0837 0.0156i	−0.0547 −0.0005i	−0.0268 −0.0067i
	−0.0657 0.0424i	−0.1064 0.0689i	−0.1148 0.0746i	−0.1008 0.0658i	−0.0792 0.0518i	−0.0598 0.0392i

cleons individually, and different spin and isospin states cannot be assigned to them. Furthermore, the importance of Fresnel corrections makes the summation over Glauber terms untrustworthy. The only way we see of including higher-order terms, is by explicit calculation, term by term. Of course, at particular energies and angles, the simpler methods will be useful.

Finally, the binding-energy corrections and the Fermi motion could also be important. The binding-energy correction has been found to be important for ^{2}D at large angles.[14] However, on larger nuclei the main production takes place at smaller momentum transfers where the free-nucleon approximation should be better. From studies done on elastic π ^{4}He scattering,[12] the effect of Fermi motion is to reduce the cross section by around 20–25% in the forward direction at the top of the resonance. The effect is less pronounced on larger nuclei, as one comes closer to the black-disk limit, where there is complete independence of the elementary amplitude. We expect similar effects in the photoproduction case.

We acknowledge useful discussions with Dr. Fäldt, Dr. Glauber, Dr. Pilkuhn, Dr. Reitan, and Dr. Wilkin.

APPENDIX A

We assume (i) the binding energy is negligible, (ii) the nucleons are "frozen" in the nucleus, and expand the scattering operator T in the Watson series, keeping only the single- and double-scattering terms as follows:

$$T=\frac{k}{2\pi}\langle\gamma|\langle\vec{k}\,|\,\tilde{T}\,|\,\vec{p}\rangle\,|\,\pi\rangle, \tag{A1}$$

$$\tilde{T}=\sum_{i=1}^{A} t_i^{\gamma\pi}+\sum_{i=1}^{A}\sum_{j\neq i}^{A} t_i^{\gamma\pi}G t_j^{\pi\pi}, \tag{A2}$$

where $|\gamma\rangle$ and $|\pi\rangle$ are the isospin states of the incoming photon and the outgoing pion, respectively; $t_i^{\gamma\pi}$'s are transition operators for pion photoproduction; and $t_j^{\pi\pi}$'s are for pion scattering. The matrix elements of the transition operators are

$$\langle\vec{k}\,|\,t_i^{\gamma\pi}\,|\,\vec{p}\,'\rangle=\frac{2\pi}{k}\,f_i^{\gamma\pi}(\vec{k}-\vec{p}\,')\,e^{i(\vec{k}-\vec{p}\,')\cdot\vec{r}_i},$$
$$\langle\vec{p}\,'\,|\,t_j^{\pi\pi}\,|\,\vec{p}\rangle=\frac{2\pi}{p}\,f_j^{\pi\pi}(\vec{p}\,'-\vec{p}\,)\,e^{i(\vec{p}\,'-\vec{p})\cdot\vec{r}_j}, \tag{A3}$$

where $f_i^{\gamma\pi}(\vec{q}\,')$ is the amplitude for pion photoproduction and $f_j^{\pi\pi}(\vec{q}\,'')$ the amplitude for pion scattering.

The Green's function is given by

$$G=\frac{1}{E_k-E_{p'}+i\epsilon'}\simeq\frac{E_k+E_{p'}}{E_k{}^2-E_{p'}{}^2+i\epsilon}$$

$$\simeq\frac{2k}{k^2-p'^2-\mu^2+i\epsilon}, \qquad \text{(A4)}$$

where E_k and $E_{p'}$ are the energies of the photon and the intermediary pion, respectively, and where in the numerator we approximate $E_{p'}\simeq E_k=k$.

The single-scattering term is trivial, and we calculate it following Ref. 2.

The double-scattering term is

$$T^{(2)}=\frac{k}{2\pi}\langle\gamma|\sum_i\sum_{j\neq i}\int\frac{d^3p'}{(2\pi)^3}\frac{2\pi}{k}f_i^{\gamma\pi}(\vec{q}\,')\,G\frac{2\pi}{p'}f_j^{\pi\pi}(\vec{q}\,'')$$
$$\times e^{i\vec{q}\,'\cdot\vec{r}_i}e^{i\vec{q}\,''\cdot\vec{r}_j}|\pi\rangle, \qquad \text{(A5)}$$

where $\vec{p}\,'$ is the momentum of the intermediary pion,

$$\vec{q}\,'=\vec{k}-\vec{p}\,',$$
$$\vec{q}\,''=\vec{p}\,'-\vec{p}, \qquad \text{(A6)}$$
$$\vec{q}=\vec{q}\,'+\vec{q}\,''=\vec{k}-\vec{p},$$

and $\vec{r}_i$ and $\vec{r}_j$ are the coordinates of the ith and jth nucleons. Equation (A5) can be written as

$$T^{(2)}=-\frac{1}{(2\pi)^4}\langle\gamma|\int d^2b\,e^{i\vec{q}_\perp\cdot\vec{b}}\sum_i\sum_{j\neq i}\int d^2q'_\perp d^2q''_\perp dq'_\parallel\,dq''_\parallel\frac{1}{p'}f_i^{\gamma\pi}(\vec{q}\,')f_j^{\pi\pi}(\vec{q}\,'')\,G\,e^{-i\vec{q}\,'_\perp\cdot(\vec{b}-\vec{s}_i)}e^{-i\vec{q}\,''_\perp\cdot(\vec{b}-\vec{s}_j)}$$
$$\times e^{iq'_\parallel z_i}e^{iq''_\parallel z_j}\delta(q_\parallel-q'_\parallel-q''_\parallel)|\pi\rangle, \qquad \text{(A7)}$$

where we have written the momentum transfers in terms of transversal ($\perp\hat{k}$) and longitudinal components.

The propagator can be written as

$$G=\frac{1}{q'_\parallel-(\mu^2+\vec{q}\,'^2)/2k+i\epsilon'} \qquad \text{(A8)}$$

and we retain only the transverse components of $\vec{q}\,'$, which amounts to taking the Fresnel corrections into account (see Refs. 15 and 16). Defining

$$a=q'_\parallel-c,$$
$$c=\frac{\mu^2+\vec{q}\,'^2}{2k} \qquad \text{(A9)}$$

we write

$$G=\frac{1}{a+i\epsilon'}. \qquad \text{(A10)}$$

The integrations over the longitudinal components $q'_\parallel$ and $q''_\parallel$ are then performed as follows:

$$\int dq'_\parallel dq''_\parallel f_i^{\gamma\pi}(\vec{q}\,'_\perp;q'_\parallel)f_j^{\pi\pi}(\vec{q}\,''_\perp;q''_\parallel)\frac{1}{a+i\epsilon'}e^{iq'_\parallel z_i}e^{iq''_\parallel z_j}\frac{1}{p'}\delta(q_\parallel-q'_\parallel-q''_\parallel)$$
$$=e^{icz_i}e^{i(q_\parallel-c)z_j}\int da\frac{e^{ia(z_i-z_j)}}{a+i\epsilon'}f_i^{\gamma\pi}(\vec{q}\,'_\perp;q'_\parallel)f_j^{\pi\pi}(\vec{q}\,''_\perp;q_\parallel-q'_\parallel)\frac{1}{|\vec{k}-\vec{q}\,'|}.$$

We shall assume that all factors except the exponential and the propagator depend only weakly on $q'_\parallel$. This means that we take them outside the integral, evaluating them at the value of $q'_\parallel$ corresponding to $a=0$, where the dominant contribution comes. Using Eq. (15), the double-scattering term becomes

$$T^{(2)}=-\frac{ik}{2\pi}\langle\gamma|\int d^2b\,e^{i\vec{q}_\perp\cdot\vec{b}}$$
$$\times\sum_i\sum_{j\neq i}\Gamma_i^{\gamma\pi}\Gamma_j^{\pi\pi}\theta(z_j-z_i)|\pi\rangle, \qquad \text{(A11)}$$

where

$$\Gamma_i^{\gamma\pi}=\frac{1}{2\pi ik}\int d^2q'_\perp e^{-i\vec{q}\,'_\perp\cdot(\vec{b}-\vec{s}_i)}e^{iq'_\parallel z_i}f_i^{\gamma\pi}(\vec{q}\,'_\perp;q'_\parallel) \qquad \text{(A12)}$$

with $q'_\parallel=(\mu^2+\vec{q}\,'^2_\perp)/2k$ and

$$\Gamma_j^{\pi\pi}=\frac{1}{2\pi ip}\int d^2q''_\perp e^{-i\vec{q}\,''_\perp\cdot(\vec{b}-\vec{s}_j)}e^{iq''_\parallel z_j}f_j^{\pi\pi}(\vec{q}\,''_\perp;q'') \qquad \text{(A13)}$$

with $q''_\parallel=q_\parallel-(\vec{q}_\perp-\vec{q}\,''_\perp)^2/2k$.

APPENDIX B

We tabulate here the relevant values of the expression

$$X[(\lambda\lambda')N(nn')N]$$
$$=\{[Y_\lambda(\hat{q}_1)\times Y_{\lambda'}(\hat{q}_2)]_N[K_n(\vec{q}\,')\times K_{n'}(\vec{q}\,'')]_N\}_0^0:$$

$$X[(00)0(00)0]=\frac{1}{4\pi}L(\vec{q}\,')L(\vec{q}\,''),$$

$$X[(00)0(11)0]=-\frac{1}{4\sqrt{3}\pi}\vec{K}(\vec{q}\,')\cdot\vec{K}(\vec{q}\,''),$$

$$X[(11)\,0\,(00)\,0] = -\frac{\sqrt{3}}{4\pi} P_1(\hat{q}_1\cdot\hat{q}_2) L(\vec{q}\,') L(\vec{q}\,''),$$

$$X[(11)\,0\,(11)\,0] = \frac{1}{4\pi} P_1(\hat{q}_1\cdot\hat{q}_2)[\vec{K}(\vec{q}\,')\cdot\vec{K}(\vec{q}\,'')],$$

$$X[(11)\,1\,(11)\,1] = \frac{\sqrt{3}}{8\pi}\hat{Q}\cdot[\vec{K}(\vec{q}\,')\times\vec{K}(\vec{q}\,'')],$$

$$X[(22)\,0\,(00)\,0] = \frac{\sqrt{5}}{4\pi} P_2(\hat{q}_1\cdot\hat{q}_2) L(\vec{q}\,') L(\vec{q}\,''),$$

$$X[(22)\,0\,(11)\,0] = -\sqrt{\tfrac{5}{3}}\,\frac{1}{4\pi} P_2(\hat{q}_1\cdot\hat{q}_2)[\vec{K}(\vec{q}\,')\cdot\vec{K}(\vec{q}\,'')],$$

$$X[(22)\,1\,(11)\,1] = -\frac{\sqrt{15}}{8\pi} P_1(\hat{q}_1\cdot\hat{q}_2)\,\vec{Q}\cdot[\vec{K}(\vec{q}\,')\times\vec{K}(\vec{q}\,'')],$$

$$\begin{aligned} X[(22)\,2\,(11)\,2] = -\frac{3\sqrt{15}}{16\sqrt{7}\pi}\big(2[\vec{K}(\vec{q}\,')\cdot\vec{Q}][\vec{K}(\vec{q}\,'')\cdot\vec{Q}] &+ \tfrac{2}{3}\{[\vec{K}(\vec{q}\,')\cdot\hat{q}_1][\vec{K}(\vec{q}\,'')\cdot\hat{q}_1] \\ &+ [\vec{K}(\vec{q}\,')\cdot\hat{q}_2][\vec{K}(\vec{q}\,'')\cdot\hat{q}_2]\} \\ &- \tfrac{2}{3}(Q^2+\tfrac{2}{3})[\vec{K}(\vec{q}\,')\cdot\vec{K}(\vec{q}\,'')]\big), \end{aligned}$$

$$X[(20)\,2\,(11)\,2] = \frac{\sqrt{6}}{8\pi}\{[\hat{q}_1\cdot\vec{K}(\vec{q}\,')][\hat{q}_1\cdot\vec{K}(\vec{q}\,'')] - \tfrac{1}{3}\vec{K}(\vec{q}\,')\cdot\vec{K}(\vec{q}\,'')\}.$$

APPENDIX C

We give here the integrals defined by Eq. (53):

$$A^{(m)}(s) = \lim_{\epsilon\to 0}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{da}{a-i\epsilon}\,a^m \exp[-(as+a^2)\tfrac{1}{2}R^2].$$

Writing

$$\lim_{\epsilon\to 0}\frac{1}{a-i\epsilon} = P\frac{1}{a} + i\pi\delta(a)$$

we see that

$$\mathrm{Re}A^{(m)}(s) = \tfrac{1}{2}\delta_{m0},$$

$$\mathrm{Im}A^{(m)}(s) = -\frac{1}{2\pi}P\int_{-\infty}^{\infty} da\, a^{m-1}\exp[-(as+a^2)\tfrac{1}{2}R^2],$$

where P denotes the principal value part. For $m>0$ this integral becomes

$$\mathrm{Im}A^{(m>0)}(s) = -\frac{1}{\sqrt{\pi}}\left(\frac{1}{\sqrt{2}R}\right)^m i^{m-1} \times \exp\left(\frac{s^2R^2}{8}\right) H_{m-1}\left(i\frac{sR}{2\sqrt{2}}\right),$$

where H_{m-1} are Hermite polynomials. $\mathrm{Im}A^{(0)}(s)$ cannot be written in closed form, but the following expansion is rapidly convergent:

$$\mathrm{Im}A^{(0)}(s) = \frac{1}{\sqrt{2\pi}}\frac{sR}{4}\left\{\frac{8}{s^2R^2}\left[\exp\left(\frac{s^2R^2}{8}\right)-1\right] + \frac{1}{2}\sum_{l=0}^{\infty}\left(\frac{s^2R^2}{8}\right)^l \frac{1}{(l+\frac{1}{2})(l+1)l!}\right\}.$$

APPENDIX D

We give below the contribution to $T_{ii}^{(C)}$ from the "direct" terms, i.e., taking only $\mathcal{F}^{(D)}$ in Eq. (51):

for ^{4}He:

$$T_{ii}^{(C,D)} = \frac{16}{ip^2}(2M_{1+}^{(+)}+M_{1-}^{(+)})f_{\pi N}^{(+)}(0)\exp(-\tfrac{1}{4}R_\beta^2q^2)\int_0^{q'_{\perp\max}} dq'_\perp q'^2_\perp \exp[-\tfrac{1}{4}R^2q'^2 - \tfrac{1}{4}R_\beta^2(q'^2-2q_\parallel q'_\parallel)] \times A^{(0)}(s)\, I_1(\tfrac{1}{2}R_\beta^2 q_\perp q'_\perp);$$

for ^{16}O:

$$\begin{aligned} T_{ii}^{(C,D)} = {}& \frac{256}{ip^2}(2M_{1+}^{(+)}+M_{1-}^{(+)})f_{\pi N}^{(+)}(0)\exp(-\tfrac{1}{4}R_\beta^2q^2) \\ &\times\int_0^{q'_{\perp\max}} dq'_\perp q'^2_\perp\exp[-\tfrac{1}{4}R^2q'^2-\tfrac{1}{4}R_\beta^2(q'^2-2q_\parallel q'_\parallel)] \\ &\quad\times\big[\big([1-\tfrac{1}{8}R^2(q^2+2q'^2-2q_\parallel q'_\parallel)+\tfrac{1}{64}R^4q'^2(q^2+q'^2-2q_\parallel q'_\parallel)]A^{(0)}(s) \\ &\qquad+\{-\tfrac{1}{4}R^2s+\tfrac{1}{64}R^4[2q''_\parallel q'^2-2q'_\parallel(q^2+q'^2-2q_\parallel q'_\parallel)]\}A^{(1)}(s) \\ &\qquad+[-\tfrac{1}{4}R^2+\tfrac{1}{64}R^4(q^2+2q'^2-2q_\parallel q'_\parallel-4q'_\parallel q''_\parallel)]A^{(2)}(s)+\tfrac{1}{32}R^4sA^{(3)}(s)+\tfrac{1}{64}R^4A^{(4)}(s)\big)I_1(\tfrac{1}{2}R_\beta^2q_\perp q'_\perp) \\ &\qquad+[(\tfrac{1}{4}R^2q_\perp q'_\perp-\tfrac{1}{32}R^4q'^2q_\perp q'_\perp)A^{(0)}(s)+\tfrac{1}{16}R^4q'_\parallel q_\perp q'_\perp A^{(1)}(s)-\tfrac{1}{32}R^4q_\perp q'_\perp A^{(2)}(s)] \\ &\qquad\times\tfrac{1}{2}[I_0(\tfrac{1}{2}R_\beta^2q_\perp q'_\perp)+I_2(\tfrac{1}{2}R_\beta^2q_\perp q'_\perp)]\big], \end{aligned}$$

where

$$q'^2 = q'^2_\perp + q'^2_\parallel,\quad q^2 = q^2_\perp + q^2_\parallel,\quad s = q''_\parallel - q'_\parallel,\quad\text{and}\quad R_\beta^2 = R^2+2\beta^2.$$

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