Delbrück scattering of 2.754-MeV photons by Pb for angles from 15° to 150°

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Differential cross sections for the elastic scattering of 2.754-MeV photons by lead have been measured for angles ranging from 15° to 150°. A comparison with theoretical data obtained from the Rayleigh scattering amplitudes of Cornille and Chapdelaine and the Delbrück scattering amplitudes of Papatzacos and Mork shows good agreement at 15°, 120°, and 150° and a discrepancy of up to a factor 1.7 at intermediate angles. Definite evidence for the real part of the Delbrück scattering amplitude has been obtained.

[NUCLEAR REACTIONS Pb(γ, γ) E=2.754 MeV; measured Delbrück scattering, $\sigma(\theta)$, θ=15°–150°.]

I. INTRODUCTION

In recent time considerable progress has been made in the investigation of Delbrück scattering. At photon energies between 1 and 7 GeV differential cross sections have been measured by Jarlskog *et al.*[1] The results are in agreement with the predictions of quantum electrodynamics, if the exchange of a very large number of photons with the nucleus is taken into account. At lower energies differential cross sections measured by Jackson and co-workers[2,3] at 10.83 MeV and Moreh and Kahane[4,5] at 7.9 MeV have been reanalyzed by Papatzacos and Mork,[6,7] who calculated Delbrück scattering amplitudes in lowest-order Born approximation. The results of Jackson *et al.*[2,3] show fairly good agreement with theory. The results of Moreh and Kahane[4,5] show perfect agreement in the case of Ta and deviations of up to twice the experimental error in the case of Th and U.

From these investigations quantitative information is obtained only for the imaginary part of the Delbrück scattering amplitude but not for the real part. This is due to the fact that at energies around 10 MeV the contribution of the real part of the Delbrück scattering amplitude to the differential cross section is only of the order of the experimental error. Furthermore, at energies of several GeV the Delbrück scattering amplitude is purely imaginary.

For an investigation of the real part of the Delbrück scattering amplitude, lower energies are much more favorable. In a previous paper[8] it was shown that the scattering of 2.754-MeV photons by lead through an angle of 120° is dominated by the real part of the Delbrück scattering amplitude and nuclear Thomson scattering.[9] Since Thomson scattering can be calculated accurately, the good agreement between theory and experiment was interpreted as definite evidence for the real part of the Delbrück scattering amplitude.

The present paper describes an extension of our previous work[8] to the angular range from 15° to 150°.

II. EXPERIMENT

Sources of ^{24}Na were prepared by bombarding aluminum with deuterons in the internal beam of the Göttingen synchrocyclotron. With a deuteron current of 9 μA, a bombarding time of 5 h, and an average deuteron energy of 21 MeV, a source strength of about 140 mCi was obtained. The detector was a 76-cm^3 Ge(Li) crystal with a resolution of 2.1 keV [full width at half-maximum (FWHM)] at 1.33 MeV.

The setup shown in Fig. 1 is typical for scattering angles between 75° and 150° where great effort had to be made in order to achieve a sufficiently large counting rate of elastically scattered photons. The scatterer had a width of 10 cm, a height of 12 cm, and a thickness of 0.75 cm. The distance between source and scatterer was between 40 and 60 cm; the distance between scatterer and detector between 10 and 20 cm. The shielding between source and detector consisted of tungsten and lead.

For scattering angles between 15° and 60° a strong dependence of the differential cross section on the scattering angle was expected. Therefore, great care was taken to minimize the range of scattering angles admitted by the setup. For this purpose, equal distances between the scatterer and source and detector, respectively, were chosen, so that the scatterer became tangential to the Thales circle through the effective centers of source, scatterer, and detector. The distances were between 35 and 50 cm.

In order to obtain a sufficient statistical accuracy, between two and five sources were prepared for each scattering angle. The spectrum in Fig. 2 contains the data obtained from one source at $\theta = 120°$. In addition to the inelastically scattered photons there is a contribution from pair annihilation and two lines at 1368 and 2754 keV, which are due to elastic scattering of photons emitted by the ^{24}Na source. As demonstrated by spectrum (b), the elastically scattered line at 2754 keV is well separated from the background of inelastically scattered photons. No attempt has been made to obtain information from the peak at 1368 keV. The background depicted by circles in spectrum (b) was measured after removing the scatterer. The data have been corrected by subtracting this background, which amounted to about 10% at all scattering angles. In doing this, one has to consider that the scatterer may act as an absorber for photons elastically scattered by the air behind the scatterer. An estimate showed that no correction was necessary for this effect.

In order to determine the differential cross section from the number of elastically scattered photons, a computer program was used which calculated an average of the contributions to the counting rate arising from a sufficiently large number of volume elements of the scatterer. This calculation had to take into account the absorption of photons in the scatterer. The corresponding attenuation coefficient has been measured and was found to be $\mu = 0.480\ \mathrm{cm}^{-1}$. In addition, double elastic scattering inside the scatterer had to be considered. A detailed estimate showed, however, that even at $\theta = 15°$ this contribution amounted to less than 1%.

Because of the finite sizes of scatterer and detector, the differential cross sections obtained in the way described above are averages over angular intervals, which amounted to $\Delta\theta = 6°$ at $\theta = 15°$ and $\Delta\theta = 15°$ at $\theta = 150°$. In order to correct for this averaging, the distribution of scattering angles admitted by each setup has been accurately calculated by means of a computer program. Then the correction has been carried out, making use of this distribution of scattering angles and of a smooth curve drawn through the uncorrected experimental differential cross sections.

The final results listed in Table I have been obtained with a total of 25 sources. The errors take into account the statistical error, the error due to the geometry, and an error due to the correction for the finite angular interval.

III. THEORY

The elastic scattering process is a coherent superposition of nuclear Thomson scattering, Rayleigh scattering, nuclear resonance scattering, and Delbrück scattering. The scattering process may be described either in terms of linearly polarized waves or in terms of circularly polarized waves. The corresponding scattering amplitudes

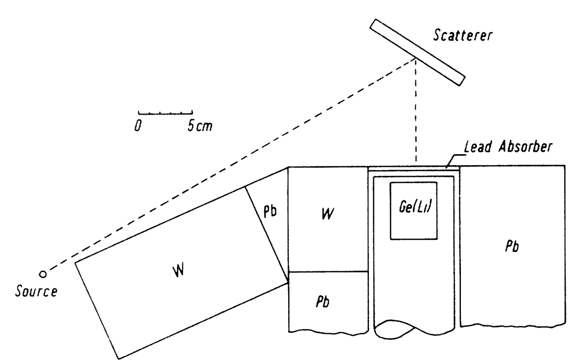

FIG. 1. Experimental setup used at the scattering angle $\theta = 120°$.

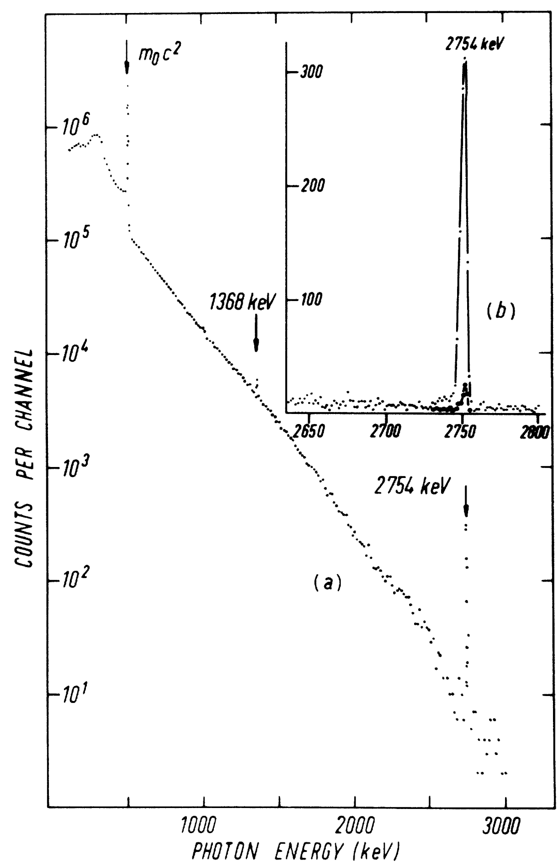

FIG. 2. (a) Spectrum of photons scattered by lead through an angle of $\theta = 120°$. Outside the lines only one out of ten channels is plotted. (b) Part of spectrum (a): —·— spectrum obtained with scatterer; —∘— spectrum obtained without scatterer.

TABLE I. Differential cross sections for elastic scattering of 2.754-MeV photons by Pb.

θ (deg)	$d\sigma/d\Omega$ (exp.) (cm^2/sr)	$d\sigma/d\Omega$ (theor.) (cm^2/sr)
15	$(4.70\pm 0.7)\ -27$	$(4.18\pm 0.9)\ -27$
30	$(2.16\pm 0.17)\ -28$	$(1.25\pm 0.1)\ -28$
45	$(8.39\pm 0.4)\ -29$	$(5.95\pm 0.4)\ -29$
60	$(4.09\pm 0.2)\ -29$	$(2.85\pm 0.4)\ -29$
75	$(3.34\pm 0.2)\ -29$	$(2.52\pm 0.13)\ -29$
90	$(3.10\pm 0.16)\ -29$	$(2.56\pm 0.1)\ -29$
120	$(3.47\pm 0.16)\ -29$	$(3.18\pm 0.13)\ -29$
150	$(4.01\pm 0.19)\ -29$	$(3.88\pm 0.13)\ -29$

$A_{\parallel}$ and $A_{\perp}$ for the case of linear polarization parallel and perpendicular to the scattering plane, respectively, are related to the scattering amplitudes A (no spin flip) and A' (spin flip) for the case of circular polarization by the equations

$$A_{\parallel}=A+A', \qquad (1)$$

$$A_{\perp}=A-A'. \qquad (2)$$

The differential cross section is given by

$$d\sigma/d\Omega=r_0^2\,\tfrac{1}{2}(|A_{\parallel}|^2+|A_{\perp}|^2), \qquad (3)$$

with r_0 the classical electron radius. Different sign conventions have been used in the literature, depending on the sign adopted in the optical theorem:

$$\sigma=\pm 4\pi\lambda\!\!\!^{-} r_0\,\mathrm{Im}A(\theta=0). \qquad (4)$$

Differing from our previous work,[8,10-12] and in accordance with recent theoretical work on elastic scattering,[6,7,13] we adopt the plus sign in the following discussion.

Table II contains the scattering amplitudes used to calculate the theoretical differential cross sections of Table I. The Delbrück amplitudes have been calculated by Papatzacos[14] for the case covered by this experiment. The Thomson amplitudes are given by

$$A_{\perp}^{T}=-\frac{Z^2m}{M};\quad A_{\parallel}^{T}=A_{\perp}^{T}\cos\theta. \qquad (5)$$

Retardation corrections to the Thomson amplitudes[15] are negligible in the present case. Nuclear resonance scattering from the lead isotopes has been discussed by Jackson *et al.*[3] Following their arguments we used the giant-dipole-resonance parameters given by Veyssiere *et al.*[16] to calculate the scattering amplitudes for nuclear resonance scattering.

Rayleigh scattering amplitudes have been calculated by Cornille and Chapdelaine[17] for the K electrons of mercury and $E=2616$ keV, using second-order perturbation theory. We have used these amplitudes without any modification though, in principle, corrections should be applied for contributions of the L shells and the differences

TABLE II. Scattering amplitudes in units of $10^{-3}r_0$ (r_0: classical electron radius) calculated for Pb and E=2.754 MeV: D—Delbrück, T—Thomson, R—Rayleigh, N—nuclear resonance fluorescence. Errors of Delbrück scattering amplitudes given by Papatzacos (Ref. 14) are denoted by footnotes. The imaginary parts have errors of 1% or less.

θ (deg)	$A_{\parallel}^{D}$	$A_{\parallel}^{T}$	$A_{\parallel}^{R}$	$A_{\parallel}^{N}$
15	136.1[a] $+i$ 69.8	−17.2	−311.4 $+i$ 6.3	1.56 $+i$ 0.1
30	62.7[a] $+i$ 35.5	−15.4	−19.8 $+i$ 2.8	1.40 $+i$ 0.09
45	35.8[a] $+i$ 20.4	−12.6	3.8 $+i$ 2.5	1.14 $+i$ 0.07
60	23.6[b] $+i$ 10.7	−8.9	4.5 $+i$ 1.6	0.81 $+i$ 0.05
75	16.5[a] $+i$ 9.4	−4.6	3.2 $+i$ 0.7	0.42 $+i$ 0.03
90	12.5[a] $+i$ 7.16	0.0	2.3 $+i$ 0.1	0.0 $+i$ 0.0
120	7.9[b] $+i$ 4.90	8.9	1.4 $-i$ 0.6	−0.81 $-i$ 0.05
150	6.1[b] $+i$ 4.01	15.4	1.1 $-i$ 1.0	−1.40 $-i$ 0.09
	$A_{\perp}^{D}$	$A_{\perp}^{T}$	$A_{\perp}^{R}$	$A_{\perp}^{N}$
15	103.8[a] $+i$ 53.7	−17.8	−329.6 $+i$ 12.2	1.62 $+i$ 0.1
30	33.6[a] $+i$ 16.1	−17.8	−32.4 $+i$ 8.8	1.62 $+i$ 0.1
45	12.9[a] $+i$ 3.79	−17.8	−4.3 $+i$ 7.1	1.62 $+i$ 0.1
60	4.6[b] $-i$ 5.66	−17.8	−1.3 $+i$ 5.2	1.62 $+i$ 0.1
75	0.4[c] $-i$ 2.33	−17.8	−1.2 $+i$ 3.9	1.62 $+i$ 0.1
90	−1.8[d] $-i$ 3.11	−17.8	−1.3 $+i$ 2.9	1.62 $+i$ 0.1
120	−4.3[b] $-i$ 3.65	−17.8	−1.4 $+i$ 1.8	1.62 $+i$ 0.1
150	−5.4[b] $-i$ 3.76	−17.8	−1.1 $+i$ 1.2	1.62 $+i$ 0.1

[a] $\leq 5\%$.
[b] From 5 to 10%.
[c] 100%.
[d] 20%.

in energy and charge number. The arguments leading to this procedure are contained in Table III where the real parts of the Rayleigh scattering amplitudes $A_\perp$ of Cornille and Chapdelaine[17] are compared with the corrected form factor approximation[18-20] and with the high-energy matrix element of Florescu and Gavrila.[21] The corrected form factor approximation is given by

$$A_\perp^{\mathrm{CFF}} = -\tfrac{1}{2}(1+\cos\theta)g - \tfrac{1}{2}(1-\cos\theta)f\,, \qquad (6)$$

$$A_\parallel^{\mathrm{CFF}} = -\tfrac{1}{2}(1+\cos\theta)g + \tfrac{1}{2}(1-\cos\theta)f\,, \qquad (7)$$

where f stands for the relativistic form factor and g for the "corrected" form factor.[18-20] The high-energy matrix element of Florescu and Gavrila[21] has been shown to be a good approximation to second-order perturbation theory for high energies and small scattering angles.

In the angular range from 15° to 45° both approximations indicate that for a given angle the K-shell Rayleigh scattering amplitudes at $E=2754$ keV, $Z=82$ are about 10% smaller than at $E=2616$ keV, $Z=80$. This difference is of about the same magnitude as the contribution expected from the L shells. Therefore, we expect that the use of the K-shell scattering amplitudes of Cornille and Chapdelaine[17] approximately compensates for the neglect of the contribution of the L shells.

In the angular range from 75° to 150° both approximations show large deviations from the real parts of the Rayleigh scattering amplitudes. However, at these large angles advantage may be taken of the observation[8,11,12] that the ratio $A_\perp^{\mathrm{CFF}}/\mathrm{Re}A_\perp$ is an almost linear function of energy and charge number. Since for the K shell the corrected form factor approximations are in very close agreement with each other at $E=2616$ keV, $Z=80$ and at $E=2754$ keV, $Z=82$, respectively, the same is expected to be true for the corresponding real parts of the scattering amplitudes. In estimating the L-shell Rayleigh scattering amplitudes for the angular range from 75° to 150° use can be made of recent investigations of Johnson and Cheng,[13] who calculated the Rayleigh scattering amplitudes for the K shell and higher atomic shells for photon energies up to 889 keV from the second-order S matrix. At 889 keV and large angles the Johnson-Cheng[22] calculation shows, that in case of the L shell, the ratio $A_\perp^{\mathrm{CFF}}/\mathrm{Re}A_\perp$ has the tendency to be even larger than in case of the K shell. Taking this into account an estimate of $(1\pm 1)\times 10^{-4}$ is obtained for the L-shell scattering amplitudes from the corrected form factors of Table III. Since this value is of the order of the uncertainty of the extrapolation of the K-shell scattering amplitudes from $E=2616$ keV, $Z=80$ to $E=2754$, $Z=82$, we decided not to correct for the L-shell scattering amplitudes.

The foregoing discussion has been carried through in terms of $A_\perp$. However, all the conclusions are equally valid for $A_\parallel$. The errors attributed to the theoretical differential cross sections of Table I take into account the errors of the Delbrück scattering amplitudes as given in Table II and an error due to the adaptation of the Rayleigh scattering amplitudes. A reasonable estimate of the latter error seemed to us 10%.

TABLE III. Rayleigh scattering amplitudes for $E=2616$ keV, $Z=80$ and $E=2754$ keV, $Z=82$. Columns 2–6: K-shell scattering amplitudes: $A_\perp$ calculated by Cornille and Chapdelaine (Ref. 17); $A_\perp^{\mathrm{CFF}}$ corrected form factor approximation; $A_\perp^{\mathrm{FG}}$ high-energy matrix element of Florecu and Gavrila (Ref. 21). Column 7: corrected form factor approximation for the L shell of Pb and $E=2754$ keV.

θ (deg)	$-\mathrm{Re}A_\perp$ 2616 keV $Z=80$	$-A_\perp^{\mathrm{CFF}}$ 2616 keV $Z=80$	$-A_\perp^{\mathrm{CFF}}$ 2754 keV $Z=82$	$-A_\perp^{\mathrm{FG}}$ 2616 keV $Z=80$	$-A_\perp^{\mathrm{FG}}$ 2754 keV $Z=82$	$-A_\perp^{\mathrm{CFF}}$ L shell $Z=82$
0	1.7311	1.7313	1.7191	1.7228	1.7104	7.53
10	0.7092	0.7097	0.6990	0.7068	0.6762	0.0057
15	0.3296	0.3478	0.3278	0.3248	0.3034	0.0327
20	0.1473	0.1669	0.1552	0.1456	0.1333	0.0209
30	0.0324	0.0454	0.0417	0.0302	0.0264	0.0066
40	0.0080	0.0165	0.0153	0.0056	0.0045	0.0025
45	0.0043	0.0112	0.0104	0.0018	0.0013	0.0017
50	0.0025	0.0082	0.0078	0.0000	−0.0002	0.0013
60	0.0013	0.0054	0.0052	−0.0011	−0.0010	0.0008
75	0.0012	0.0040	0.0040	−0.0010	−0.0007	0.0006
90	0.0013	0.0035	0.0035			0.0005
120	0.0014	0.0031	0.0031			0.0005
150	0.0011	0.0029	0.0030			0.0004

IV. DISCUSSION

A comparison between theory and experiment is carried out in Table I and Fig. 3. The comparison reveals good agreement at 15°, 120°, and 150° and a discrepancy in the angular range from 30° to 90°. From the discussion given in Sec. III it appears to us unlikely that this discrepancy may be due to errors involved in the adaptation of the Rayleigh scattering amplitudes of Cornille and Chapdelaine[17] to the photon energy and charge number of the present experiment. However, a definite answer to this question requires a calculation of the Rayleigh scattering amplitudes for the present case.

Recently Johnson and Cheng[13] have calculated differential cross sections for Rayleigh scattering from the second-order S matrix for energies ranging from 145 to 889 keV. The calculations combined with Thomson scattering showed good agreement with accurate experimental data.[10-12] This investigation confirms that no principal difficulties are contained in the calculation of Rayleigh scattering amplitudes. The Delbrück scattering amplitudes[6,7] are calculated in lowest-order Born approximation, and the question has been considered[6,7] as to what extent Coulomb corrections might alter the results.

The differential cross sections represented by the dashed curve of Fig. 3 have been calculated by omitting the real parts of the Delbrück scattering amplitudes. In the angular range between 75° and 150° this dashed curve differs from the solid curve by about a factor of 2. This difference is obviously much larger than the discrepancy between theory, as represented by the solid curve, and experiment. Therefore, despite the discrepancy, we may conclude that a definite evidence for the real part of the Delbrück scattering amplitude has been obtained.

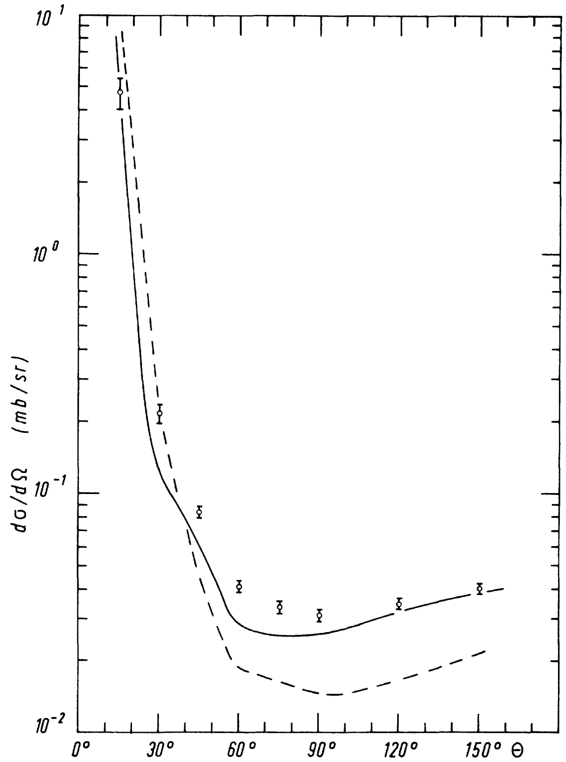

FIG. 3. Differential cross section for elastic scattering of 2.754-MeV photons by lead versus scattering angle. —— calculated cross section; – – – calculated cross section not including real parts of Delbrück scattering amplitudes.

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[1] G. Jarlskog, L. Jönsson, S. Prünster, H. D. Schulz, H. J. Willutzki, and G. G. Winter, Phys. Rev. D 8, 3813 (1973).

[2] H. E. Jackson and K. J. Wetzel, Phys. Rev. Lett. 22, 1008 (1969).

[3] H. E. Jackson, G. E. Thomas, and K. J. Wetzel, Phys. Rev. C 9, 1153 (1974).

[4] R. Moreh and S. Kahane, Phys. Lett. 47B, 351 (1973).

[5] S. Kahane and R. Moreh, Phys. Rev. C 9, 2384 (1974).

[6] P. Papatzacos and K. Mork, Phys. Rep. 21C, 81 (1975).

[7] P. Papatzacos and K. Mork, Phys. Rev. D 12, 206 (1975).

[8] M. Schumacher, I. Borchert, F. Smend, and P. Rullhusen, Phys. Lett. 59B, 134 (1975).

[9] Table I of Ref. 8 contains misprints. The correct numbers are contained in Table II of the present paper.

[10] M. Schumacher, Phys. Rev. 182, 7 (1969).

[11] M. Schumacher, F. Smend, and I. Borchert, Nucl. Phys. A206, 531 (1973).

[12] F. Smend, M. Schumacher, and I. Borchert, Nucl. Phys. A213, 309 (1973).

[13] W. R. Johnson and K.-t. Cheng, Phys. Rev. A 13, 692 (1976).

[14] P. Papatzacos (private communication).

[15] T. E. O. Ericson and J. Hüfner, Nucl. Phys. B57, 604 (1973).

[16]A. Veyssiere, H. Beil, R. Bergere, P. Carlos, and A. Lepretre, Nucl. Phys. A159, 561 (1970).
[17]H. Cornille and M. Chapdelaine, Nuovo Cimento 14, 1386 (1959).
[18]G. E. Brown and D. F. Mayers, Proc. Roy. Soc. (London) A242, 89 (1957).
[19]V. Florescu and M. Gavrila, Technical Report of the Lawrence-Livermore Laboratory, University of California, Livermore (unpublished).
[20]F. Smend and M. Schumacher, Nucl. Phys. A223, 423 (1974).
[21]V. Florescu and M. Gavrila (unpublished).
[22]W. R. Johnson and K.-t. Cheng (private communication).