Proton transfer reactions on Sr isotopes

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We have investigated the proton transfer reactions ^{86}Sr(d,n)^{87}Y, ^{87}Sr(d,n)^{88}Y, and ^{88}Sr(d,n)^{89}Y at 12 MeV incident deuteron energy. Spectroscopic factors are extracted from a distorted wave Born approximation analysis and compared to the spectroscopic factors extracted from the analogous (^{3}He,d) reactions. A comparison is also made of the (d,n) and (d,^{3}He) reactions to determine fractional emptiness and occupation probability of shell model orbitals. Additionally, ground state proton configurations of ^{86}Sr and ^{88}Sr are determined and the $T_<$ states of ^{89}Y are studied.

[NUCLEAR REACTIONS ^{86,87,88}Sr(d,n), E_d=12 MeV; measured angular distributions. Deduced spectroscopic factors, ground state configurations. Enriched targets.]

I. INTRODUCTION

The shell model prediction of the closure of the neutron shell at N=50 has produced extensive investigation of nuclei near the mass 90 region. Both ^{88}Sr and ^{90}Zr have been considered candidates for a doubly closed shell nucleus. Calculations based upon an inert core of ^{88}Sr have met with reasonable success.[1-3] However, Hughes[4] predicted a more configuration-mixed ground state for ^{88}Sr of the form

$$\Psi=[a(p_{1/2})^{-2}+b(p_{3/2})^{-2}+c(f_{5/2})^{-2}+d(f_{7/2})^{-2}]\,|0\rangle\,,$$

where $|0\rangle$ represents a core with all proton orbitals filled through $p_{1/2}$. Experiments with the proton-transfer reactions (^{3}He,d)[5-7] and (d,^{3}He)[5,8,9] support the model of Hughes. However, the coefficients extracted from these experiments suffer a problem in the overall normalization that produces a sum of the squares of the coefficients up to 50% greater than unity. To overcome this problem previous authors[9] have compared spectroscopic information extracted from (^{3}He,d) data[7] to complementary information extracted from (d,^{3}He) data.[9] The results of this comparison were renormalized to satisfy sum-rule constraints; however, the significantly different analyses used in the two experiments bring the validity of this procedure into question. In the present paper, when such comparisons are made, the previous data have been reanalyzed in a manner consistent with the present data.

It was felt that an investigation of the ^{88}Sr(d,n)-^{89}Y reaction, for which the absolute normalization of spectroscopic factors is better understood, could resolve some of the ambiguities in the existing data. Additionally, the behavior of the proton distribution in the presence of neutron holes was of interest. This effect may be studied by comparing the ^{86}Sr(d,n), ^{87}Sr(d,n), and ^{88}Sr(d,n) reactions.

A proton stripping reaction can also yield information on the $T_<$ states in the residual nucleus. These $T_<$ states result from an antianalog state sharing its strength with "core polarization" states which have the same spin and parity.[10] The spreading of these states and the position of their energy centroid can provide information on the magnitude of the isospin potential responsible for the splitting of the analog and antianalog states.

II. EXPERIMENTAL ARRANGEMENT AND RESULTS

The experiment was performed in the low background neutron room at the U. S. Army Ballistic Research Laboratories (BRL) FN tandem Van de Graaff facility. The BRL pulsed source consisted of a direct-extraction-diode source with a 2-MHz rf pulser and a 4-MHz klystron buncher. Average current on target was 750 nA and typical time resolution of the system was 1.5 nsec full width at half maximum.

The low background neutron room was a large 25-m×25-m×18-m room with the beam line at the 5.5-m level and the neutron detectors out of the

TABLE I. A tabulation of target thicknesses and enrichments.

Target	Thickness (μg/cm^2)	Percent Enrichment
^{86}Sr	2000	97.6
^{87}Sr	2200	93.29
^{88}Sr	2300	99.84

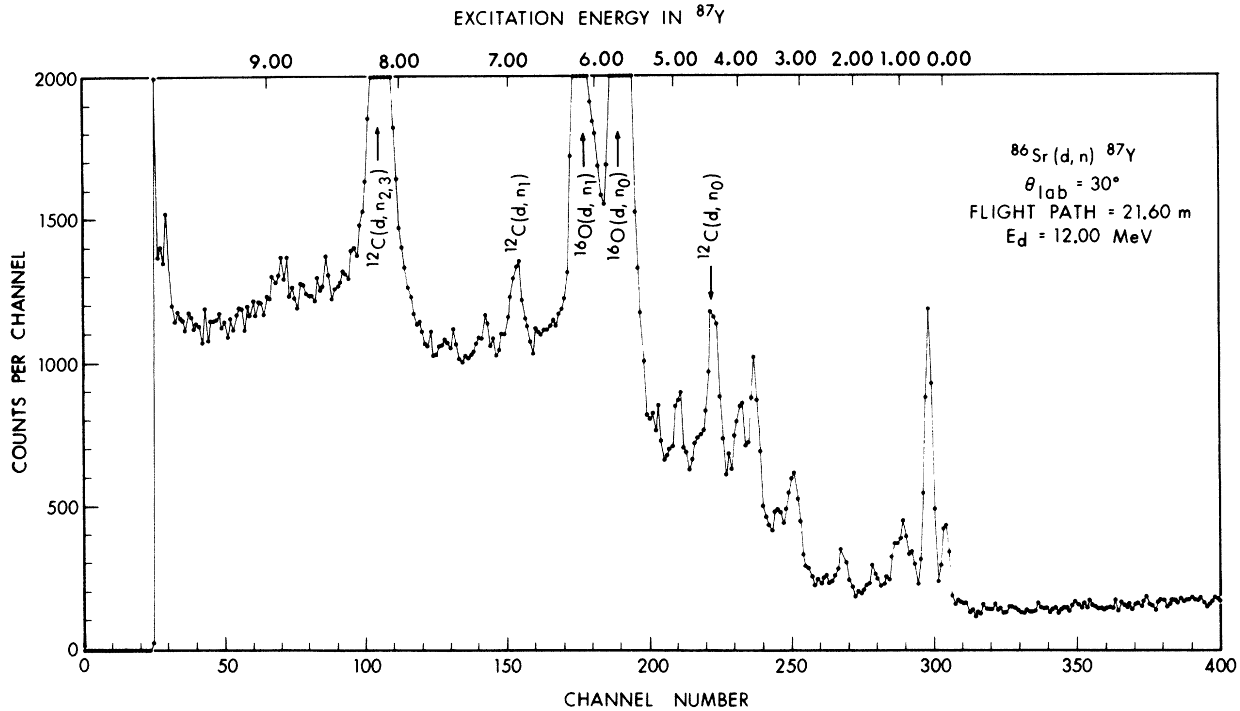

FIG. 1. A typical ^{86}Sr$(d,n)^{87}$Y neutron time-of-flight spectrum.

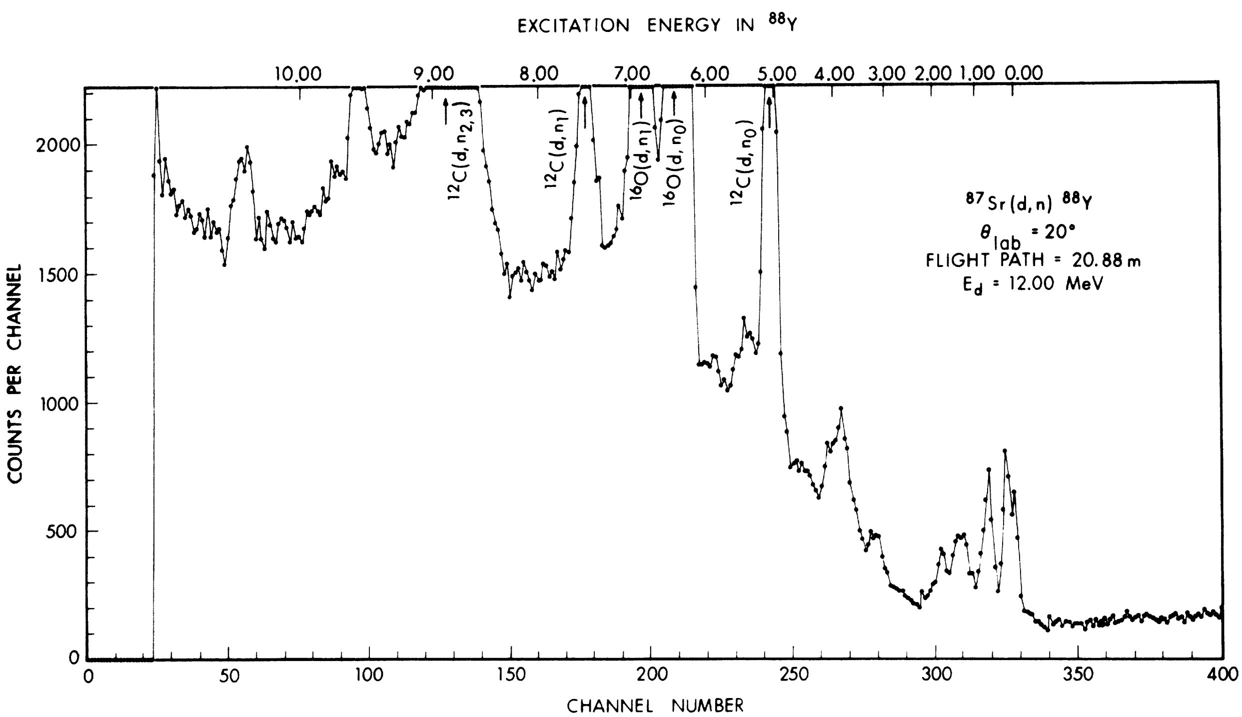

FIG. 2. A typical ^{87}Sr$(d,n)^{88}$Y neutron time-of-flight spectrum.

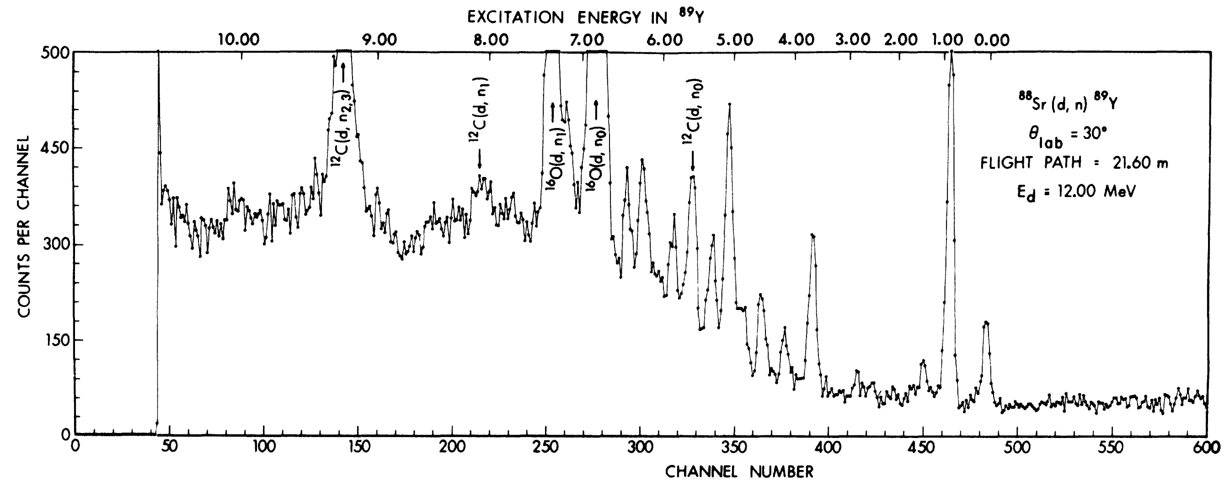

FIG. 3. A typical ^{88}Sr$(d,n)^{89}$Y neutron time-of-flight spectrum.

reaction plane at the 8-m level. The experiment was performed in an open geometry. The only shielding necessary was a shadow bar placed between the beam stop and the detectors.

The target chamber was a stainless steel cylinder 10 cm in diameter and 30 cm in height, with a wall thickness of 1 mm. The beam was tuned such that less than 1% of it intersected three slits, two upstream and one downstream from the target. Beam current was measured in a Faraday cup with a Pb beam stop 3 m downstream from the target. The targets were enriched self-supporting foils supplied by Oak Ridge National Laboratory. Their thicknesses, given in Table I, were determined by weighing.

Neutron detection was accomplished with two NE-213 liquid scintillators, 2.5 cm in thickness and 17.8 cm in diameter, Lucite coupled to XP-1041 photomultiplier tubes. The detectors were positioned at a 20-m flight path. Neutron energies were determined with standard time-of-flight circuitry employing n-γ discrimination.

Detector efficiency was determined experimentally by measuring the neutron yield from the ^{2}H$(d,n)^3$He reaction. The target was deuterium gas contained in a 2.5-cm long stainless steel gas cell. The neutron yields were then compared to the cross sections measured by Brolley, Putnam, and Rosen[11] to determine detector efficiency. Cross section data were corrected also for neutron absorption along the neutron flight path and in the walls of the chamber.

Typical time-of-flight spectra are displayed in Figs. 1–3. Excitation energies were determined by a least squares fitting procedure to states of known energy. The excitation energy of a state was calculated for each angle and the result at each angle was checked for consistency with results from other angles. Values quoted are believed to be accurate to within ±20 keV.

Excitation functions of ^{86}Sr(d,n_0) and ^{88}Sr(d,n_0) were taken from 11.5 MeV to 12.5 MeV in steps of 100 keV. In this energy range the excitation functions showed a nearly monotonic increase in magnitude of approximately 15% with increasing energy and exhibited no evidence of compound nuclear processes.

Angular distributions were taken in 5° intervals from 5° to 55°. Peak areas were extracted by hand. For closely spaced doublets the hand calculations were checked with the program AUTOFIT.[12] In general, agreement between the two methods was better than 10%. The distorted wave Born approximation (DWBA) calculations are compared to the measured cross sections in Figs. 4–6.

The major sources of uncertainty in the absolute cross sections were background subtraction, target thickness, and the efficiency of the neutron detectors. The uncertainty in background subtraction was less than 10% for most states, although for some of the weakly populated states this uncertainty was higher. The uncertainties of the target thicknesses and the efficiency curve were 10% and 15%, respectively. Consequently, the overall uncertainty of the absolute cross sections was 20%.

III. DWBA ANALYSIS AND OPTICAL MODEL PARAMETERS

Several sets of optical model parameters were considered for the entrance and exit channels in the DWBA calculations performed with code DWUCK.[13] The form of the potential was

$$U(r) = V_c - f(x) + \left(\frac{\hbar}{m_\pi c}\right)^2 V_{so}(\vec{\sigma}\cdot\vec{1})\frac{1}{r}\frac{d}{dr}f(x_{so}) - i\left[wf(x_w) - 4w_D\frac{d}{dx_D}f(x_D)\right],$$

$$V_c = \begin{cases} \dfrac{Z_1Z_2e^2}{r} & \text{for } r \geq R_c \\ \dfrac{Z_1Z_2e^2}{2R_c}\left(3 - \dfrac{r^2}{R_c^2}\right) & \text{for } r \leq R_c \text{ with } R_c = r_c A^{1/3} f(x) = \dfrac{1}{1+e^x} \text{ with } x = r - r_0 A^{1/3}/a\,. \end{cases}$$

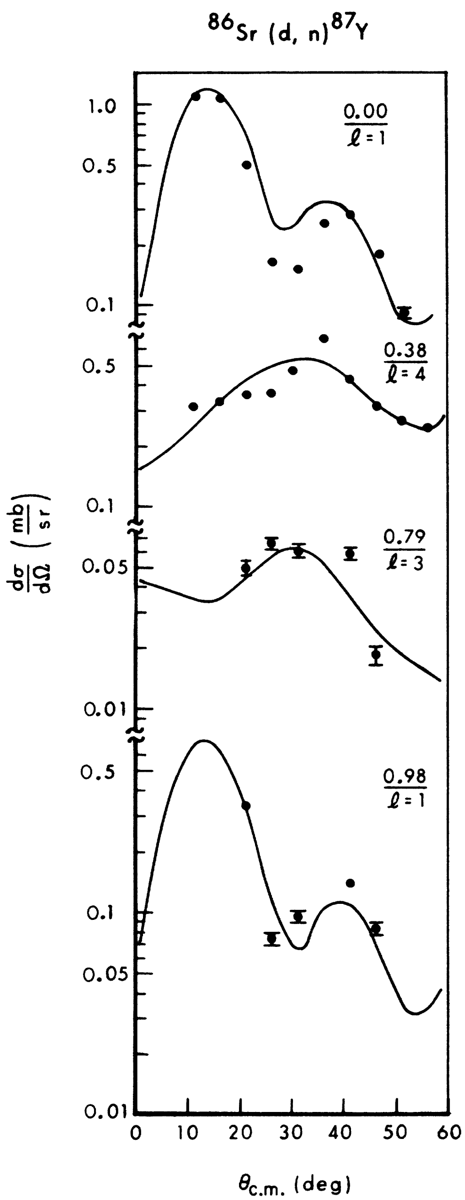

FIG. 4. DWBA fits to ^{86}Sr$(d,n)^{87}$Y angular distribution data. The l values and excitation energies are indicated.

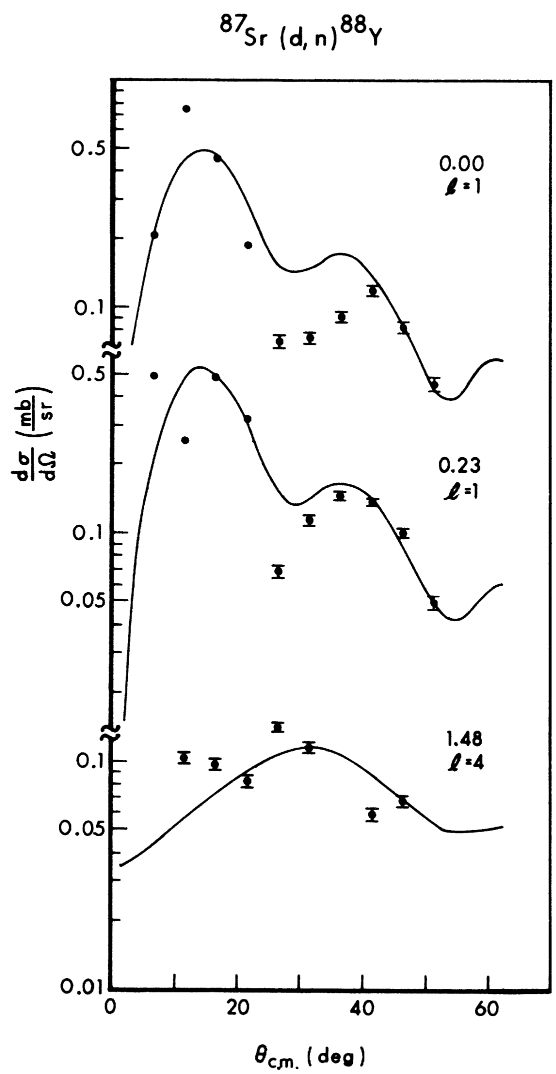

FIG. 5. DWBA fits to ^{87}Sr$(d,n)^{88}$Y angular distribution data. The l values and excitation energies are indicated.

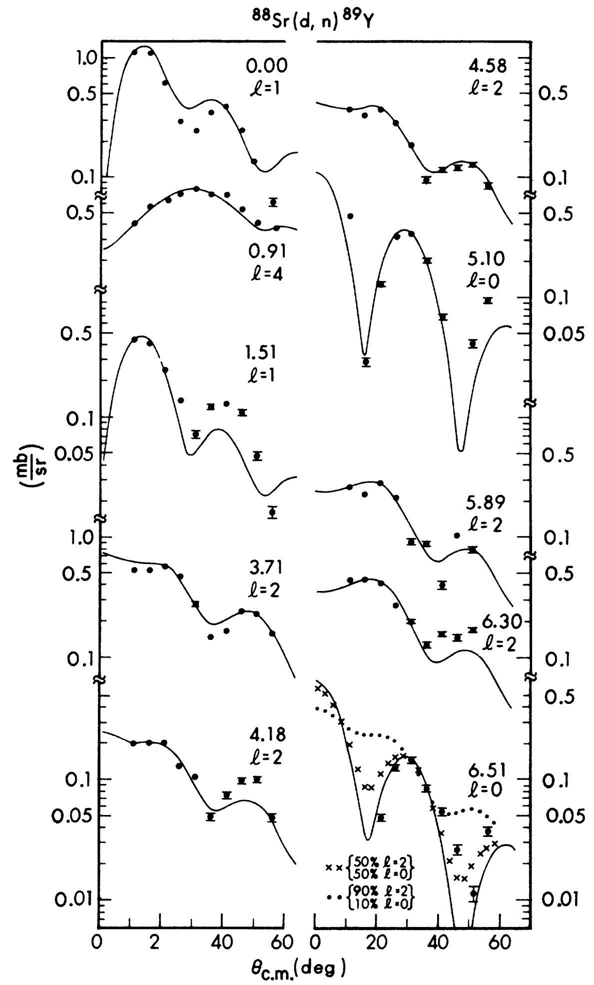

FIG. 6. DWBA fits to ^{88}Sr$(d,n)^{89}$Y angular distribution data. The l values and excitation energies are indicated.

A standard Woods-Saxon geometry with $a = 0.65$, $r_0 = 1.25$ and a Thomas-type spin-orbit coupling with $\lambda = 25$ was used to obtain the bound-state wave functions. The well depth was varied to bind the state at the correct separation energy.

Deuteron parameters considered were from a study of charge-exchange effects in the ^{92}Mo-$(d, n\bar{p})$ reaction by Coker and Tamura[14] and previously employed in an investigation of the (d, n) reaction at 12 MeV on Mo isotopes[15]; a global analysis of Perey and Perey,[16] which has been used extensively in stripping calculations; and a survey of polarized-deuteron scattering from several elements at 11.8 MeV by Griffith *et al.*[17] Three different parameter sets from Griffith *et al.* were considered. No appreciable differences in quality of the fits were observed among the various sets from Griffith. Neutron parameters were from global analyses by Wilmore and Hodgson[18] and by Becchetti and Greenlees.[19] These parameters and the finite-range and nonlocal corrections are given in Table II.

Calculations with these parameters were compared to the measured angular distributions of four states in ^{89}Y. The ground state and states with excitation energies of 0.91, 3.71, and 5.10 MeV were chosen as representative of $l = 1$, 4, 2, and 0 transfers, respectively. Finite-range nonlocal (FRNL) calculations are displayed in Fig. 7.

The best fits to the measured angular distributions of the four states were predicted by the Coker-Tamura deuteron parameters combined with the Becchetti-Greenlees neutron parameters. The Perey and Perey deuteron parameters combined with either set of neutron parameters failed to reproduce the measured angular distribution of the $g_{9/2}$ transfer to the 0.91-MeV state and the Griffith *et al.* deuteron parameters predicted deeper minima than measured for the $p_{1/2}$ transfer. The spectroscopic factors predicted by the different parameters varied by approximately 30%. Local zero-range (LZR) calculations were performed, but no appreciable improvement in the fits was found.

Spectroscopic factors were extracted from the DWBA predictions with the following relation

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{exp}} = NC^2S\frac{2J_f+1}{2J_i+1}\,\frac{2s+1}{2(2j+1)}\left(\frac{d\sigma}{d\Omega}\right)^{\text{DWUCK}},$$

where J_i and J_f were the initial and final state spins and s and j were the spin and total angular momentum of the transferred nucleon. The square of the Clebsch-Gordan isospin coupling coefficient was equal to $2T_0/(2T_0+1)$ for the states considered in this analysis. The normalization constant N was equal to 1.65×10^4 MeV^2 fm^3.

IV. DISCUSSION

A. ^{88}Sr$(d,n)^{89}$Y

1. Comparison of ^{88}Sr(d,n) *to* ^{88}Sr$(^3$He$,d)$

A reanalysis of previous proton-transfer reactions on ^{88}Sr was performed to achieve consistency with the present analysis. Three previous $(^3$He$, d)$[5-7] experiments were analyzed in the LZR approximation. Harrison and Hiebert[5] and Vourvopoulos *et al.*[6] used the standard normalization factor for $(^3$He$, d)$ of 4.42. Picard and Bassani[7] used a different geometry in obtaining the bound-state wave functions. Sum-rule limits were used to normalize the spectroscopic factors. This procedure produced a normalization factor of 7.5. However, even if Picard and Bassani had employed

TABLE II. A summary of optical model parameters investigated in the present analyses.

	V (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	r_w (fm)	a_w (fm)	W_d (MeV)	r_d (fm)	a_d (fm)	V_{so} (MeV)	r_{so} (fm)	a_{so} (fm)	r_c (fm)
							Deuteron parameters						
Coker-Tamura	88.0	1.28	0.72				14.0	1.41	0.694	6.0	1.28	0.72	1.3
Perey-Perey	V^2	1.15	0.81				W_d^2	1.34	0.68				1.15
Griffith *et al.*	V^3	1.05	0.85				12.9	r_d^3	a_d^3	13.05	0.84	0.46	1.3
							Neutron parameters						
Wilmore-Hodgson	V^4	r_0^4	0.66				W_d^4	r_d^4	0.48				
Becchetti-Greenlees	V^5	1.17	0.75	W^5	1.26	0.58	W_d^5	1.26	0.58	6.2	1.01	0.75	

Finite range correction = 0.621; nonlocality correction: $\beta^d = 0.54$, $\beta^n = 0.85$

$V^2 = 81.0 - 0.22E + \frac{2Z}{A^{1/3}}$; $W_d^2 = 14.4 + 0.24E$

$V^3 = 90 + 2.36\frac{Z}{A^{1/3}}$; $r_d^3 = 1.19 + \frac{0.9}{A^{1/3}}$; $a_d^3 = 0.398 + 0.082A^{1/3}$

$V^4 = 47.01 - 0.267E$; $r_0^4 = 1.322 - 7.6A \times 10^{-4}$; $W_d^4 = 9.52 - 0.53E$; $r_d^4 = 1.266 - 3.7A \times 10^{-4}$

$V^5 = 56.3 - 0.32E - 24\frac{(N-Z)}{A}$; $W^5 = 0.22E - 1.56$; $W_d^5 = 13.0 - 0.25E - 12\frac{(N-Z)}{A}$

the same bound-state geometry as Harrison and Hiebert and Vourvopoulos *et al.*, the sum-rule limits still would have yielded a normalization factor of 5.5.

Because of the similar method of analysis between the present data and that of Vourvopoulos *et al.*, as to bound-state geometry and standard normalization procedure, the Vourvopoulos *et al.* data were reanalyzed to include nonlocal and finite-range corrections. The spectroscopic factors were lowered 20 to 25% in magnitude. This led to better agreement among the present data, Vourvopoulos *et al.*, and Harrison and Hiebert. Although Harrison and Hiebert performed an LZR calculation, r_{so} was set equal to $0.9r_0$. They commented that with this prescription there was little difference between LZR and FRNL calculations. This is borne out by the agreement of the spectroscopic factors given in Table III. It is noted that, except for the first excited state, the present results are somewhat lower than previous results.

The present results cannot distinguish between spins of the final state except, of course, for the $l=0$ transfer. Tentative spin assignments are based on expected shell model ordering and previous experiments. All $l=2$ transfers have been assigned a spin and parity of $\frac{5}{2}^+$.

There is disagreement between the present results and those of Vourvopoulos *et al.* on the spin of the 6.51-MeV state. The earlier results indicated an $s_{1/2}$-$d_{5/2}$ doublet in the ratio of 10% $s_{1/2}$ to 90% $d_{5/2}$. The angular distribution of the 6.51-MeV state measured in the present experiment is compared in Fig. 6 with calculated angular dis-

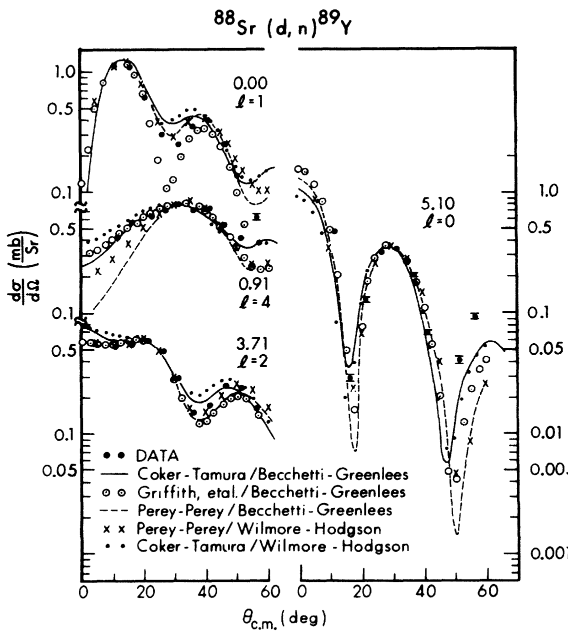

FIG. 7. Comparison of fits to angular distribution data predicted with different optical parameters.

TABLE III. A comparison of spectroscopic factors extracted from the present $^{88}Sr(d,n)^{89}Y$ analysis and previous $^{88}Sr(^{3}He,d)^{89}Y$ analyses. The data of Ref. 6 have also been reanalyzed to include FRNL corrections to those states observed in both (d,n) and $(^{3}He,d)$.

	Present		Reanalyzed Vourvopoulos *et al.*		Vourvopoulos *et al.*		Picard-Bassani			Harrison-Hiebert			
E_x	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	J^π	Coefficient
0.00	1.31	$\frac{1}{2}^-$	0.00	1.47	1.85	$\frac{1}{2}^-$	0.00	1.8	$\frac{1}{2}^-$	0.00	1.6	$\frac{1}{2}^-$	$2a^2$
0.91	6.45	$\frac{9}{2}^+$	0.91	5.87	7.40	$\frac{9}{2}^+$	0.896	8.8	$\frac{9}{2}^+$	0.91	6.9	$\frac{9}{2}^+$	$10a^2$
1.51	0.28	$\frac{3}{2}^-$	1.50	0.30	0.38	$\frac{3}{2}^-$	1.49	0.44	$\frac{3}{2}^-$	1.49	0.5	$\frac{3}{2}^-$	$2b^2$
1.74	(0.37)	$\frac{5}{2}^-$	1.74	0.63	0.79	$\frac{5}{2}^-$	1.735	0.55	$\frac{5}{2}^-$	1.74	0.4	$\frac{5}{2}^-$	$2c^2$
2.26			2.22			$(l=2)$							
2.61	(0.41)	$\frac{9}{2}^+$	2.61		0.25	$\frac{9}{2}^+$							
2.87			2.88			$(l=1,4)$							
3.09			3.06		0.02	$\frac{3}{2}^-$							
					0.02	$\frac{1}{2}^-$							
3.51			3.51		0.06	$\frac{3}{2}^-$	3.49	0.07	$\frac{3}{2}^-$				
					0.07	$\frac{1}{2}^-$		0.08	$\frac{1}{2}^-$				
3.71	0.40	$\frac{5}{2}^+$	3.72	0.67	0.84	$\frac{5}{2}^+$	3.69	0.95	$\frac{5}{2}^+$				
4.00	(0.01)	$\frac{1}{2}^+$	4.00	0.03	0.036	$\frac{1}{2}^+$							
4.18	0.15	$\frac{5}{2}^+$	4.18	0.22	0.27	$\frac{5}{2}^+$	4.16	0.3	$\frac{5}{2}^+$				
4.46			4.46		0.018	30% $\frac{1}{2}^+$							
					0.027	70% $\frac{5}{2}^+$							
4.58	0.22	$\frac{5}{2}^+$	4.58	0.32	0.40	$\frac{5}{2}^+$	4.57	0.5	$\frac{5}{2}^+$				
4.86			4.84		0.32	$\frac{5}{2}^+$	4.86	0.4	$\frac{5}{2}^+$				
			4.94										
5.10	0.065	$\frac{1}{2}^+$	5.08	0.19	0.25	$\frac{1}{2}^+$	5.06	0.6	$\frac{5}{2}^+$				
			5.19		0.04	$\frac{5}{2}^+$							
			5.27		0.05	$\frac{1}{2}^+$							
5.35			5.34		0.08	$\frac{1}{2}^+$	5.32	0.4	$\frac{5}{2}^+$				
			5.43		0.09	$\frac{5}{2}^+$							
5.63			5.62			$(l=2)$	5.61						
			5.72			$(l=2)$							

TABLE III (*Continued*)

Present			Reanalyzed Vourvopoulos *et al.*		Vourvopoulos *et al.*		Picard-Bassani			Harrison-Hiebert			
E_x	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	J^π	E_x	$(2J_{f+1})C^2S$	J^π	Coefficient
5.89	0.13	$\frac{5}{2}^+$	5.87	0.25	0.31	$\frac{5}{2}^+$							
6.10			6.10		0.10	$\frac{5}{2}^+$							
6.20			6.20		0.82	$\frac{7}{2}^+$							
6.30	0.18	$\frac{5}{2}^+$	6.28	0.34	0.42	$\frac{5}{2}^+$							
6.51	0.03	$\frac{1}{2}^+$	6.48	0.014	0.018	$\frac{1}{2}^+$							
				0.110	0.136	$\frac{5}{2}^+$							
			6.59		0.06	$\frac{5}{2}^+$							
			6.68										
			6.78										
			7.72		0.155	$\frac{1}{2}^+$							

tributions of 100% $s_{1/2}$ (solid line), 90% $d_{5/2}$–10% $s_{1/2}$ (dotted line), and 50% $d_{5/2}$–50% $s_{1/2}$ (××× line). The present data are fitted only by the 100% $s_{1/2}$ calculations. Also, there was an earlier disagreement between Vourvopoulos *et al.* and Picard and Bassani on the spin of the 5.10-MeV state. Present results favor the $s_{1/2}$ assignment of Vourvopoulos *et al.*

The states between 1.74 and 3.71 MeV were too weakly populated in the present experiment to allow extraction of reliable angular distributions. However, tentative spectroscopic factors were assigned to the 1.74 and the 2.61-MeV states by choosing an angle at which their cross sections could be extracted and comparing to a DWUCK calculation based on previously assigned spins. Additionally, some states observed in previous experiments were seen as unresolved doublets or not fully resolved from more strongly populated states.

2. *Comparison of* ^{88}Sr(d,n) *to* ^{88}Sr$(d,{}^3$He$)$

Proton stripping and proton pickup reactions on the same target yield complementary spectroscopic information; however, previous $(d,{}^3\text{He})$ experiments have not been analyzed in a manner consistent with the present experiment. The data of Comfort, Duray, and Braithwaite[9] were analyzed with FRNL corrections but with a different bound-state geometry. Harrison and Hiebert[5] employed LZR approximations as discussed above.

The data of Comfort *et al.* were reanalyzed with the same bound-state geometry as the present analysis. The results are given in Table IV in terms of the quantity v_J^2, the occupational probability of the appropriate shell model orbital. This quantity is defined by

$$v_J^2 = \frac{1}{2J+1} \sum_f C^2 S_f(J)\,,$$

where the sum is taken over all final states of the appropriate J value. Also given in Table IV is u_J^2, the fractional emptiness of a shell model orbital. This is defined by

$$u_J^2 = \sum_f C^2 S_f(J)\,,$$

where the sum is again taken over all appropriate final states.

For a consistent analysis u_J^2 and v_J^2 should sum to unity. The sums for each orbital are close to unity except for the $g_{9/2}$. No $g_{9/2}$ strength was observed by either Harrison and Hiebert or by Comfort *et al.*

There was disagreement between the two exper-

TABLE IV. u_J^2, v_J^2 for ^{88}Sr. A comparison of occupation probability and fractional emptiness of ^{88}Sr shell model proton orbitals.

	Present	Harrison Hiebert		Comfort *et al.*		Reanalyzed Comfort *et al.*	
	$\sum u_J^2$	$\sum v_J^2$	$\sum u_J^2+v_J^2$	$\sum v_J^2$	$\sum u_J^2+v_J^2$	$\sum v_J^2$	$\sum u_J^2+v_J^2$
$g_{9/2}$	0.69	0.00	0.69	0.00	0.69	0.00	0.69
$p_{1/2}$	0.65	0.30	0.95	0.24	0.89	0.28	0.93
$p_{3/2}$	0.07	0.73	0.80	0.78	0.85	0.90	0.97
$f_{5/2}$	0.06	0.85	0.91	0.88	0.94	1.01	1.07

iments on the spin of the state at 1.58 MeV. Comfort *et al.* tentatively assigned a spin and parity of $\frac{7}{2}^-$ to this state and Harrison and Hiebert assigned $\frac{1}{2}^-$. Harrison and Hiebert suggested this state may be a doublet. Two previous studies of the γ decay following the β decay of ^{87}Kr[20,21] disagreed on the spin of this state. One assigned a value of $\frac{7}{2}^-$[21] and the other a value of $\frac{9}{2}^+$.[20] A recent study of ^{86}Kr(^{3}He, d)^{87}Rb,[22] however, furnished strong evidence that the 1.58-MeV state had a spin and parity of $\frac{9}{2}^+$. The question of a possible doublet was unresolved. However, if the 1.58-MeV state populated in the ^{88}Sr(d, ^{3}He) reaction was $\frac{9}{2}^+$, it would indicate an occupation probability of 0.1 for the $g_{9/2}$ orbital. The sum, $u_{9/2+} + v_{9/2+}$, would be 0.79 in closer agreement with unity.

B. ^{86}Sr(d,n)^{87}Y

1. Comparison of ^{86}Sr(d,n) *to* ^{86}Sr(^{3}He,d)

The most obvious difference between the ^{86}Sr(d,n)^{87}Y reaction and the ^{88}Sr(d,n)^{89}Y reaction is the much higher density of levels populated in ^{87}Y than in ^{89}Y due to the fractionation of the single particle strength by the two neutron holes. Due to this high level density it was not possible to extract spectroscopic factors for levels beyond the first four states. However, these four states are the most significant in determining the ground state configuration of ^{86}Sr. Other levels carry only a small fraction of the total $g_{9/2}$, $p_{1/2}$, $p_{3/2}$, and $f_{5/2}$ strength.

The spectroscopic factors for the first four states are given in Table V. Given for comparison are the spectroscopic factors extracted for these states from ^{86}Sr(^{3}He, d)^{87}Y.[23] It must be noted that Maher *et al.*[23] performed an FRNL calculation with the standard normalization of 4.42 using the parameters of Picard and Bassani.[7] As mentioned above, this is a significantly different approach than that used by Picard and Bassani and brings into question the absolute normalization of the values of Maher *et al.* This must be borne in mind in comparing the present results to those of Ref. 23.

2. Comparison of ^{86}Sr(d,n) *to* ^{86}Sr(d, ^{3}He)

The occupation probability and the fractional emptiness of orbitals in ^{86}Sr are given in Table VI. The v_J^2 values are taken from Ref. 9. As noted, a different bound-state geometry was employed in this analysis; therefore, Table VI lists both the original values of v_J^2 and those obtained in a reanalysis of the data with the same bound-state geometry as the present work.

All sums agree well with the expected value of unity except the sum for the $g_{9/2}$ orbital which is somewhat low. Maher *et al.*[23] observed some high lying $g_{9/2}$ states in the (^{3}He, d) reaction but these

TABLE V. Absolute spectroscopic factors extracted from the present ^{86}Sr(d,n)^{87}Y analysis compared to the values extracted from ^{86}Sr(^{3}He, d)^{87}Y previously.

E_x	Present $(2J_{f+1})C^2S$	J^π	E_x	Maher *et al* $(2J_{f+1})C^2S$	J^π	Coefficient
0.00	1.14	$\frac{1}{2}^-$	0.000	1.15	$\frac{1}{2}^-$	$2a^2$
0.38	5.48	$\frac{9}{2}^+$	0.380	7.19	$\frac{9}{2}^+$	$10a^2$
0.79	0.70	$\frac{5}{2}^-$	0.793	1.15	$\frac{5}{2}^-$	$2c^2$
0.98	0.43	$\frac{3}{2}^-$	0.982	0.54	$\frac{3}{2}^-$	$2b^2$

TABLE VI. u_j^2, v_j^2 for ^{86}Sr. A comparison of the ^{86}Sr(d,n) ^{87}Y and ^{86}Sr$(d,{}^3$He$)$ ^{85}Rb analyses. For case c Comfort *et al.* divided the $l=1$ transfers between the $p_{3/2}$ and $p_{1/2}$ orbitals and the $l=3$ transfers between the $f_{5/2}$ and $f_{7/2}$ orbitals with bias given to the $p_{3/2}$ and $f_{5/2}$ orbitals, respectively.

	Present	Comfort *et al.* case c		Reanalyzed Comfort *et al.* case c	
	$\sum u_j^2$	$\sum v_j^2$	$\sum u_j^2+v_j^2$	$\sum v_j^2$	$\sum u_j^2+v_j^2$
$g_{9/2}$	0.55	0.09	0.64	0.11	0.66
$p_{1/2}$	0.57	0.36	0.93	0.41	0.98
$p_{3/2}$	0.11	0.65	0.76	0.75	0.86
$f_{5/2}$	0.12	0.78	0.90	0.90	1.02

states were only weakly populated. Their contribution would not greatly affect the present results. There is qualitative agreement between the present analysis and previous analyses[9,23] that the $p_{1/2}$ and $g_{9/2}$ orbitals ^{86}Sr contain more proton particles than in ^{88}Sr. These particles seem to have been removed from the $f_{5/2}$ orbital.

C. ^{87}Sr$(d,n)^{88}$Y

An even higher level density exists in ^{88}Y than in ^{87}Y due to the nonzero spin of the target. Spectroscopic factors could be extracted for only the levels at 0.00, 0.23, and 1.48 MeV. Relative spectroscopic factors for these levels are compared to the relative values extracted from ^{87}Sr$(^3$He$,d)^{88}$Y [24] in Table VII. Spin values are taken from Ref. 24. Comfort and Schiffer[24] normalized their spectroscopic factors to the 1.478-MeV state because their target thickness was unknown. The results compare well. Absolute spectroscopic factors are presented in the last column of Table VII.

D. Ground state configuration of ^{86}Sr and ^{88}Sr

The ground state configuration of ^{88}Sr predicted by Hughes[4] is represented pictorially in Fig. 8. A similar configuration with two additional neutron holes would be expected for ^{86}Sr. Hughes calculated the coefficients for ^{88}Sr to be $a^2=0.694$, $b^2=0.208$, $c^2=0.083$, and $d^2=0.015$. These coefficients may be experimentally determined from the spectroscopic strengths extracted from proton-transfer reactions.

The proton stripping reactions indicate the ground state configuration of ^{89}Y is $(2p_{1/2})^{-1}$ and the 0.91-MeV state is $(lg_{9/2})(2p_{1/2})^{-2}$. The spectroscopic strength of the ground state then is essentially $2a^2$ and that of the 0.91-MeV state is $10a^2$. The agreement of a^2 extracted from these two states provides

TABLE VII. A tabulation of relative spectroscopic factors extracted from the present ^{87}Sr(d,n) ^{88}Y analysis to the similar quantities extracted from ^{87}Sr$(^3$He$,d)$ ^{88}Y analysis. Absolute spectroscopic factors extracted from the present analysis are also included.

Relative values						Absolute values		
	Present		Comfort and Schiffer			Present		
E_x	C^2S	J^π	E_x	C^2S	J^π	E_x	C^2S	J^π
0.00	1.00	4^-	0.00	0.82	4^-	0.00	0.58	4^-
0.23	0.85	5^-	0.234	0.83	5^-	0.23	0.49	5^-
1.48	1.00	9^+	1.478	1.00	9^+	1.48	0.58	9^+

a convenient check on the consistency of the analysis.

The configurations of the 1.51- and 1.74-MeV states are believed to be $(2p_{3/2})^{-1}$ and $(1f_{5/2})^{-1}$, respectively. This implies that the spectroscopic strength of the 1.51-MeV state is equal to $2b^2$ and that of the 1.74-MeV state is equal to $2c^2$. No $(1f_{7/2})^{-1}$ strength was observed, but this is not surprising in view of the small magnitude of d^2. This configuration could well be represented by one of the many weakly populated states between 1.74 and 3.71 MeV. These states carry such a small fraction of the spectroscopic strength as to have little effect on the values extracted for the other coefficients. The ^{88}Sr$(d,{}^3$He$)^{87}$Rb experiments yield the same information in a similar manner.[5]

The values of the coefficient extracted from the various experiments are given in Table VIII. The values of c^2 extracted from the ^{88}Sr$(d,{}^3$He$)$ experiment of Ref. 5 are dependent upon the 1.58-MeV state being $\frac{1}{2}^-$.

The present results indicate surprising agreement between the two values of a^2 extracted from the ground and first excited states. Also, the sum of a^2, b^2, c^2 is close to unity, indicating the accuracy of the overall normalization. The sums of the squares of the coefficients extracted from the $(^3$He$,d)$ experiments are significantly larger than unity. The $(d,{}^3$He$)$ results are in general agreement with the stripping results.

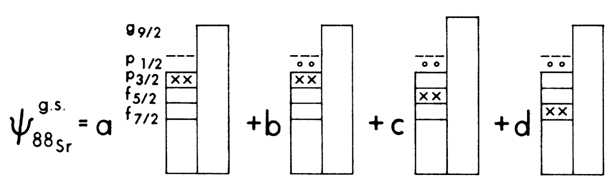

FIG. 8. Pictorial representation of ground state configuration of ^{88}Sr.

TABLE VIII. A summary of coefficients of the ^{88}Sr ground state proton configuration extracted from various proton transfer reactions. The first line for each reference contains the value for a^2 extracted from the $p_{1/2}$ ground state transfer in ^{88}Y. The second line contains the value for a^2 extracted from the $g_{9/2}$ transfer to the 0.91-MeV state in ^{89}Y.

	Un-normalized				Renormalized ($a^2+b^2+c^2=1$)		
	a^2	b^2	c^2	Total	a^2	b^2	c^2
(d,n)	0.65	0.14	0.18	0.97	0.67	0.14	0.19
	0.65			0.97	0.67		
Harrison and Hiebert	0.80	0.25	0.20	1.25	0.64	0.20	0.16
(^{3}He, d)	0.69			1.14	0.61	0.22	0.18
Vourvopoulos *et al.*	0.92	0.19	0.39	1.50	0.61	0.13	0.26
(^{3}He, d)	0.74			1.32	0.56	0.14	0.30
Picard and Bassini	0.90	0.22	0.27	1.39	0.65	0.16	0.19
(^{3}He, d)	0.88			1.37	0.64	0.16	0.20
Harrison and Hiebert	0.73	0.21	0.11	1.05	0.70	0.20	0.10
(d, ^{3}He)	0.85			1.17	0.73	0.18	0.09
Comfort *et al.*	0.78	0.24					
(d, ^{3}He)	0.88						
Theory					0.694	0.208	0.083

If the coefficients are renormalized so that the sum of their squares is unity, the agreement among the experiments and between the experiments and theory is very good. The major difference is that the experiments indicate more nearly equal values for b^2 and c^2 than does the theory.

Coefficients for the ^{86}Sr ground state proton configuration are given in Table IX. They were extracted in the same manner as those of ^{88}Sr. No state was observed in ^{86}Sr(d, ^{3}He) which may be definitely interpreted as arising from the $(f_{5/2})^{-1}$ $(p_{1/2})^{-1}$ proton configuration. The most striking effect of the neutron holes on the ground state configuration of ^{86}Sr is the number of proton particles from the $f_{5/2}$ orbital mixed into the $p_{1/2}$ and $g_{9/2}$ orbitals manifested by the smaller values of a^2 and the much larger value of c^2 for ^{86}Sr compared to ^{88}Sr.

E. $T_<$ states

All states observed in ^{89}Y above 3.71-MeV excitation energy were populated by either $l=2$ or $l=0$ transfers. These were the $T_<$ states resulting from the admixing of the antianalog states with the "core polarization" states.[10] Because of the inability of the present experiment to distinguish between $\frac{3}{2}^+$ and $\frac{5}{2}^+$ states any thorough analysis of these states was impossible. Under the assumption that all $l=2$ transfers were $d_{5/2}$, the centroids of the $d_{5/2}$ and $s_{1/2}$ orbitals are given in Table X. These values are compared to those extracted from Refs. 6 and 7. The energies of the corresponding analog states and the energy differences are also given. Although not all the spectroscopic strength for each orbital was observed, the cen-

TABLE IX. A summary of coefficients of the ground state of ^{86}Sr extracted from various proton transfer reactions. The two values of a^2 were extracted in a manner analogous to that explained in Table VIII.

	Un-normalized				Renormalized $a^2+b^2+c^2=1$		
	a^2	b^2	c^2	Total	a^2	b^2	c^2
Present	0.57	0.22	0.35	1.14	0.50	0.19	0.31
(d,n)	0.55			1.12	0.49	0.20	0.31
Maher *et al.*	0.58	0.27	0.58	1.45	0.41	0.18	0.41
(^{3}He, d)	0.72			1.57	0.46	0.17	0.37
Comfort *et al.*	0.51	0.26					
(d, ^{3}He)	0.58						

TABLE X. A comparison of the centroids of proton shell model orbitals determined in this analysis and previous (^{3}He, d) analyses. The excitation energies of the corresponding isobaric analog states (IAS) and the energy difference between the centroids extracted in the present analysis and the IAS are also included.

	Centroids (MeV) Present (d,n)	Vourvopoulos *et al.* (^{3}He, d)	Picard-Bassani (^{3}He, d)	$E_>$ (MeV)	$E_> - E_>(d,n)$ (MeV)
$d_{5/2}$	4.65	4.91	4.49	12.07	7.42
$s_{1/2}$	5.40	5.76		13.07	7.67

troids should not change much as the remaining strength is spread over many weakly populated states.

The high level density of the $T_<$ states in ^{87}Y and ^{88}Y made it impossible to extract the spectroscopic factors of these states in the present experiment. It is clear that the two neutron holes in ^{87}Y cause considerable fractionation of these states. The high level density in ^{88}Y is due to the one neutron hole and the odd-odd nature of the residual nucleus.

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[1] S. Cohen, R. D. Lawson, M. H. Macfarlane, and M. Soga, Phys. Lett. 10, 195 (1964).

[2] J. B. Ball, J. S. Larsen, and J. B. McGrory, Bull. Am. Phys. Soc. 17, 446 (1972).

[3] J. E. Kitching, W. G. Davies, W. J. Darcey, W. McLatchie, and J. Morton, Nucl. Phys. A177, 433 (1971).

[4] T. A. Hughes, Phys. Rev. 181, 1586 (1969).

[5] J. F. Harrison and J. C. Hiebert, Nucl. Phys. A185, 385 (1972).

[6] G. Vourvopoulos, R. Shoup, and R. A. Brown, Nucl. Phys. A174, 581 (1971).

[7] J. Picard and G. Bassani, Nucl. Phys. A131, 636 (1969).

[8] C. D. Kavaloski, J. S. Lilley, D. C. Shreve, and N. Stein, Phys. Rev. 161, 1107 (1967).

[9] J. R. Comfort, J. R. Duray, and W. J. Braithwaite, Phys. Rev. C 8, 1354 (1973).

[10] G. Vourvopoulos and J. D. Fox, Phys. Rev. 177, 1558 (1969).

[11] J. E. Brolley, Jr., T. M. Putnam, and L. Rosen, Phys. Rev. 107, 820 (1957).

[12] H. D. Jones, Ballistic Research Laboratories, Memorandum Report Number No. 2307, 1973 (unpublished).

[13] P. D. Kunz (private communication).

[14] R. Coker and T. Tamura, Phys. Rev. 182, 1277 (1969).

[15] P. J. Riley, J. L. Horton, C. L. Hollas, S. A. A. Zaidi, C. M. Jones, and J. L. C. Ford, Jr., Phys. Rev. C 4, 1864 (1973).

[16] C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).

[17] J. A. R. Griffith, M. Irshad, O. Karban, and S. Roman, Nucl. Phys. A146, 193 (1970).

[18] D. Wilmore and P. E. Hodgson, Nucl. Phys. 55, 673 (1964).

[19] F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[20] J. P. Bouquet, R. Brissot, J. Crancon, J. A. Pinston, F. Schussler, and A. Moussa, Nucl. Phys. A125, 613 (1969).

[21] A. Shihab-Eldin, S. G. Prussin, F. M. Bernthal, and J. O. Rasmussen, Nucl. Phys. A160, 33 (1971).

[22] L. R. Medsker, H. T. Fortune, S. C. Headley, and J. N. Bishop, Phys. Rev. C 11, 474 (1975).

[23] J. V. Maher, J. R. Comfort, and G. C. Morrison, Phys. Rev. C 3, 1162 (1971).

[24] J. R. Comfort and J. P. Schiffer, Phys. Rev. C 4, 803 (1971).