Deuteron stripping reaction to unbound states*

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Two features of the (d, p) stripping reaction to the continuum, the parallelism between the excitation functions of energy-differential stripping cross section $d^2\sigma/d\Omega dE$ and the total neutron-target elastic scattering cross section $\sigma_{tot}(n, n)$, and the dependence of the ratio $(d^2\sigma/d\Omega dE)/\sigma_{tot}(n, n)$ on transferred angular momentum (l), are explained satisfactorily by a plane wave Butler approximation for a ^{32}S target nucleus. It appears that the l dependence of the above ratio is so dominant that even the crude Butler approximation suffices for identification of l values. The (d, p) and (d, n) angular distributions are also calculated for the targets ^{12}C, ^{14}C, and ^{16}O, leading to unbound states by this method. The results are compared with experiment and with these obtained by distorted wave Born approximation method employing an extrapolation technique to treat the neutron wave in the continuum.

[NUCLEAR REACTIONS Calculated $(d^2\sigma/d\Omega dE)/\sigma_{tot}(n,n)$, $d\sigma(d,p)/d\Omega$, $d\sigma(d,n)/d\Omega$; unbound states; $E_D = 6-15$ MeV.]

I. INTRODUCTION

Deuteron stripping reactions to the bound levels of the residual nucleus have been the subject of investigation for a long time.[1] However, within the last few years an increased amount of effort has been made to study the deuteron stripping reaction leading to the unbound states. As a result, some data on the differential cross section for the stripping reaction of the type $A(d,p)B^*$ into the continuum are available now.

It was first observed experimentally by Fuchs *et al.*,[2] that the formation of a target-neutron resonance in the (d,p) reaction is reflected in the spectrum of the emitted spectator proton in a way very parallel to the total cross section of the free neutrons. It is also interesting to notice that the transitions to resonances of high l values are strongly enhanced in (d,p) reaction with respect to the neutron scattering. Hence the ratio of the (d,p) energy differential cross section at some fixed angle to the total elastic neutron-target cross section is highly dependent on the orbital angular momentum l of the transferred particle.[3] This is a very important observation, as it serves to determine the orbital angular momentum of the resonant state without necessarily measuring the cross sections for all angles.

The purpose of the present paper is to make a theoretical analysis of some of the recently available experimental data on (d,p) and (d,n) reactions leading to the unbound states, for target nuclei $^{12,14}C$, ^{16}O, and ^{32}S and for incident deuteron energies in the range 6–15 MeV. The above bombarding deuteron energies enabled levels up to excitation energy $E_x = 9-12$ MeV to be reached in the reaction. The main aim of the present work is to study the resonance structure of the unbound system as well as the relationship between the deuteron stripping and the elastic scattering with respect to the transferred angular momentum.

There are several theories for the deuteron stripping reaction to unbound levels. One of them, developed by Vincent and Fortune[4] and by Huby and his collaborators,[5] employs the distorted wave Born approximation (DWBA) in its form familiar from stripping to bound levels, except that the wave function used for the captured neutron is proportional to a scattering wave function in a potential well adjusted to produce an exact resonance.

Another prescription[6] is to replace the resonant state by a weekly bound state of the order of a few keV binding energy without altering the energy of the outgoing particle and perform the usual DWBA calculation. In a more refined version[7] of the above prescription, the usual DWBA calculations are carried out for a particular lj state in the negative energy region, varying it towards positive energy region. The obtained cross sections are extrapolated into the positive energy region and read off for the corresponding lj state at various angles.

In the present calculation (d,p) continuum stripping cross section is expressed in terms of the phase shifts of the neutron-nucleus elastic scattering and a well-defined "distortion matrix element." The "parallelism" and the "l effect" find a complete explanation within the framework of this model. This treatment is similar to that of Baur and Trautmann[8] which is based on the earlier works of Butler[9] and Friedman and Tobocman.[9] The contributions from the nuclear interior and the breakup process are neglected throughout. A brief summary of the theory is given below.

II. FORMALISM

We consider a reaction of the type

$$d+A=(A+n)_{\rm res}+p\,. \tag{1}$$

The T matrix for this process is given in the post-interaction form of DWBA as[10]

$$T=\langle\Phi^{(-)}_{\vec{k}_n\nu_f}(r_n)\chi^{(-)}_{\vec{k}_f}(r_p)\,|\,V_{np}(r_{np})\,|\,\chi^{(+)}_{\vec{k}_i}\varphi_d(r_{np})\rangle\,. \tag{2}$$

The in- and outgoing scattering wave functions of deuteron (d) and proton (p) with wave vectors $\vec{k}_i$ and $\vec{k}_f$ are denoted by $\chi^{(+)}_{\vec{k}_i}$ and $\chi^{(-)}_{\vec{k}_f}$, respectively. $\Phi^{(-)}_{\vec{k}_n\nu_f}(r_n)$ is the neutron wave function in the continuum. $\vec{k}_n$ is the wave vector corresponding to the energy of the neutron in the continuum, ν_f is the neutron spin projection. The differential cross section for the stripping reaction to a three body final state in which the neutron is not observed is given by[11]

$$\frac{d^2\sigma_{lj}}{d\Omega_p dE_p}=\frac{m_i m_f}{\hbar^6}\frac{1}{(2S+1)}\frac{k_n m_n}{(2\pi)^5}\frac{k_f}{k_i}\sum_{\nu_f}\int d\Omega_{\vec{k}_n}\,|T|^2\,, \tag{3}$$

where m_i and m_f are the reduced masses in the initial and final channels, m_n is the reduced mass of the neutron-target system, and S is deuteron spin.

Employing the partial wave expansion of $\Phi^{(-)}_{\vec{k}_n\nu_f}(r_n)$ in Eq. (3) and carrying out the spin algebra, we obtain for a particular resonance (fixed l and j)

$$\frac{d^2\sigma_{lj}}{d\Omega_p dE_p}=\frac{m_i m_f}{2(\pi\hbar^2)^3}\frac{k_n m_n}{(2S+1)}\frac{k_f}{k_i}\frac{2J_B+1}{2J_A+1}\frac{1}{2l+1}\sum_{\mu}|\langle Y_{l\mu}(r_n)f_{lj}(r_n)\chi^{(-)}_{\vec{k}_f}(r_p)\,|\,V_{np}(r_{np})\,|\,\chi^{(+)}_{\vec{k}_i}(r_p)\varphi_d(r_{np})\rangle|^2\,. \tag{4}$$

Here J_A and J_B are the total spins of target nucleus and the resonant state. The symbols $f_{lj}(r_n)$ denote the radial wave function for the neutron, μ is the projection of l, and

$$d^2\sigma/d\Omega_p dE_p=\sum_{lj} d^2\sigma_{lj}/d\Omega_p dE_p\,.$$

The dominant contribution to the DWBA matrix element will come from the values of radial distances larger than the nuclear radius. In this region we can express the neutron wave function in terms of phase shifts δ_{lj} in the following way

$$f_{lj}(r_n)=j_l(k_n r_n)+ie^{i\delta_{lj}}\sin\delta_{lj}h_l^{(+)}(k_n r_n)\,, \tag{5}$$

where the spherical Bessel and Hankel functions are denoted by j_l and $h_l^{(+)}$, respectively. The spherical Bessel function term in Eq. (5) arises from the unscattered part of the neutron wave function. This term will be important when we consider nonresonant deuteron breakup. The three body model of deuteron breakup and stripping has been nicely described by Farrell, Vincent, and Austern.[12] However, in many applications, this term is small compared to the scattered part when the phase shift shows a resonance. This is specially the case for high l values and low energies.

Hence we can write

$$f_{lj}(r_n)=t_{lj}h_l^{(+)}(k_n r_n)\,, \tag{6}$$

where

$$t_{lj}=ie^{i\delta_{lj}}\sin\delta_{lj}\,.$$

In the interior region, the Hankel function should be replaced by the correct scattering solution. The quantity t_{lj} is directly related to the total neutron elastic scattering cross section in the channel lj as[13]

$$\sigma_{lj}(n,n)=\frac{2J_B+1}{(2J_A+1)(2S_n+1)}\frac{4\pi}{k_n^2}|t_{lj}|^2\,, \tag{7}$$

so that $\sigma_{\rm tot}(n,n)=\sum_{lj}\sigma_{lj}$.

To carry out the integration in Eq. (4), we employ the addition theorem for spherical Hankel functions.[14] Thus the transition matrix is expressed in terms of a single coordinate r and the form factor D_0. Now we can split Eq. (4) into two parts as

$$\frac{d^2\sigma_{lj}}{d\Omega_p dE_p}=\sigma_{\rm tot}(n,n)F_l\,, \tag{8}$$

where $\sigma_{\rm tot}(n,n)$ is the total neutron elastic scattering cross section as defined in Eq. (7) and

$$F_l=\frac{m_i m_f}{4(\pi\hbar^2)^3}\frac{k_n^3 m_n}{(2S+1)}\frac{1}{\pi}\frac{k_f}{k_i}\frac{D_0^2}{2l+1}\sum_{\mu}\left|\int\chi^{(-)*}_{\vec{k}_f}(r)h_l^{(+)}(k_n r)Y^*_{l\mu}(\hat{r})\chi^{(+)}_{\vec{k}_i}(r)d^3r\right|^2 \tag{9}$$

with

$$D_0=-\frac{\hbar^2}{M}(8\pi\alpha)^{1/2}\frac{(\beta+2\alpha)^{3/2}(\beta+\alpha)^{1/2}}{(\beta+\alpha)^2+k_n^2}$$

and

$$\alpha=0.2317\ {\rm fm}^{-1}\ \text{ and }\ \beta=5.39\alpha\,.$$

The quantity $F_l=(d^2\sigma_{lj}/d\Omega_p dE_p)/\sigma_{\rm tot}(n,n)$ is called "stripping enhancement factor," which expresses the "parallelism" between neutron elastic scattering and deuteron stripping to continuum.

Employing plane waves for $\chi_{\vec{k}_f}^{(-)}$ and $\chi_{\vec{k}_i}^{(+)}$ and taking the transferred momentum $\vec{k}=(\vec{k}_i-\vec{k}_f)$ along the Z direction, we obtain

$$F_l=\frac{1}{(2S+1)}\frac{m_i m_f}{(\pi\hbar^2)^3}\frac{k_f}{k_i}k_n{}^3 m_n D_0{}^2 \times\left|\int j_l(kr)h_l^{(+)}(k_n r)r^2dr\right|^2 . \tag{10}$$

The radial integral, evaluated by introducing a Butler cutoff radius R_0, is given by[15]

$$\int_{R_0}^{\infty} j_l(kr)h_l^{(+)}(k_n r)r^2dr =\frac{R_0{}^2}{k^2-k_n{}^2}\left[kh_l^{(+)}(k_nR_0)j_{l-1}(kR_0) - k_n j_l(kR_0)\,h_{l-1}^{(+)}(k_nR_0)\right], \tag{11}$$

where the integral vanishes at the upper limit, because of the physical condition $k\neq k_n$.

The (d,p) angular distribution is given by

$$\frac{d\sigma_{lj}}{d\Omega_p}=\int dE_p\frac{d^2\sigma_{lj}}{d\Omega_p dE_p} . \tag{12}$$

The energy-differential cross section in the integral contains scattering phase shifts δ_{lj} which can be written near a resonance as

$$\delta_{lj}(E_n)=\delta_{lj}^{(0)}+\tan^{-1}\left[\frac{\tau_{lj}}{2(E_n-E_0)}\right] \tag{13}$$

under the assumption that only one channel is open. Here τ_{lj} and E_0 denote the width and the position of the resonance, respectively. Since $\delta_{lj}(E_n)$ is the only rapidly varying function in the energy interval of the resonance, we can easily carry out the integration over the energy in Eq. (12). In the case of vanishing background phase shift $\delta_{lj}^{(0)}=0$, this leads to the cross section for stripping to an unbound state. As already mentioned, we replace the scattering wave function of deuteron and proton by plane waves and take the effect of distortion and absorption into account by introducing Butler cutoff radius R_0. The (d,p) angular distribution is then given by

$$\frac{d\sigma_{lj}}{d\Omega_p}=\frac{4\pi}{(2S+1)}\frac{m_i m_f}{(2\pi\hbar^2)^2}\frac{k_f}{k_i}\frac{2J_B+1}{2J_A+1}\frac{D_0{}^2k_n m_n}{\hbar^2} \times\tau_{lj}\left|\int j_l(kr)h_l^{(+)}(k_n r)r^2dr\right|^2 , \tag{14}$$

where the explicit value of the integral is given by Eq. (11).

If the captured particle is a proton then this will be subject to the Coulomb field even outside the radius R_0. The wave function of the transferred proton is then replaced by a Coulomb modified Hankel function[16] $h_l^{(+)}(k_p^* r)$ with

$$k_p^{*2}=k_p{}^2-\frac{2m_p}{\hbar^2}\frac{Ze^2}{R} . \tag{15}$$

Here we have assumed that the Coulomb barrier has a constant height corresponding to radius R. Z represents the charge number of the target nucleus and k_p is the wave vector of proton in the continuum.

III. RESULTS AND DISCUSSION

The l dependence of the "stripping enhancement" factor (F_l), i.e., the ratio of a neutron resonance in (d,p) and the total neutron cross section, was first experimentally observed by Bommer *et al.* for ^{14}N and ^{24}Mg target nuclei. However, the recent experimental data of Bommer *et al.*[17] for ^{32}S(d,p) reactions to the unbound states of ^{33}S reveal a similar l dependence. Figure 1 shows the results of our calculation for F_l for the nucleus

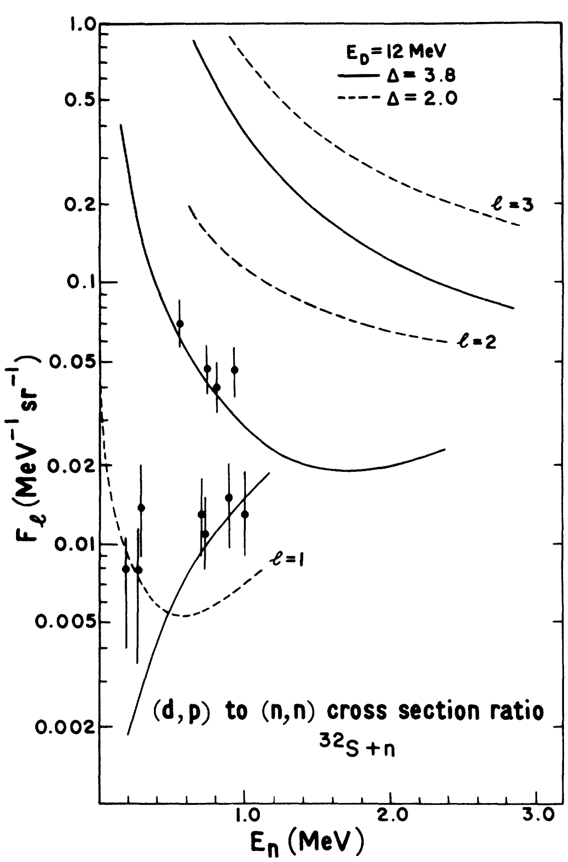

FIG. 1. Variation of stripping enhancement factor (F_l) for ^{32}S target with the neutron resonance energy and the orbital angular momentum. The energy-differential cross section is measured at 10° for 12 MeV incident deuteron energy. The experimental points are taken from Ref. 17.

TABLE I. Kinematics of (d,p) and (d,n) reactions to unbound states.

Nucleus	Type of reaction	Deuteron energy (MeV)	Excitation energy (MeV)	Level width (keV)	Resonance energy (MeV)
^{12}C	(d,p)	15.0	9.899	30	5.368
^{14}C	(d,n)	6.5	12.097	1.7	2.025
^{16}O	(d,p)	9.3	4.554	45	0.442
	(d,p)	13.3	5.083	94	1.00

^{32}S and their comparison with experimental values. The only free parameter in our calculation is the cutoff radius R_0. It is taken as

$$R_0=(1.4A^{1/3}+\Delta)\text{ fm}.$$

To vary R_0 we change Δ in our calculation. For D_0 we have used the actual form factor dependent upon k_n instead of a zero range constant as was done by Baur and Trautmann.

We immediately notice that the stripping enhancement factor calculated in plane wave approximation is strongly dependent upon resonance angular momentum l. The ratio of the resonance cross section in (d,p) to that in (n,n) increases by up to 1 order of magnitude if the l value is augmented by one unit. It is important to note that the l-value determination of the nucleon resonances from their formation in deuteron stripping can be made from such a simple theory. The agreement between these crude theoretical estimates and the experimental values also appears satisfactory. One can understand the l effect qualitatively as well. The effect is actually due to the strong l dependence of the penetrability that governs the magnitude of the form factor in the barrier region. This remark has been made both by Lipperheide and Mohring[18] and by Vincent and Fortune.

In order to establish the practicability of the method, we have calculated in Butler approximation the stripping angular distribution for several

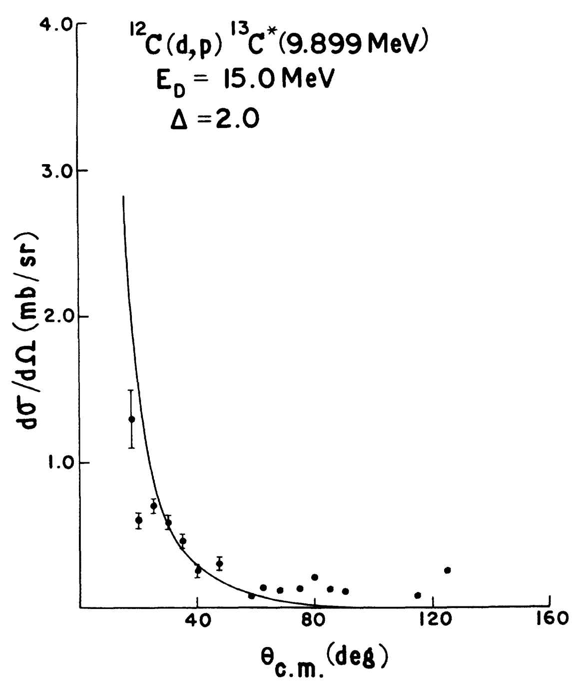

FIG. 2. Angular distribution for $^{12}C(d,p)^{13}C^*$ (9.899 MeV) reaction obtained by Butler cutoff method. The measurements are taken from Ref. 6.

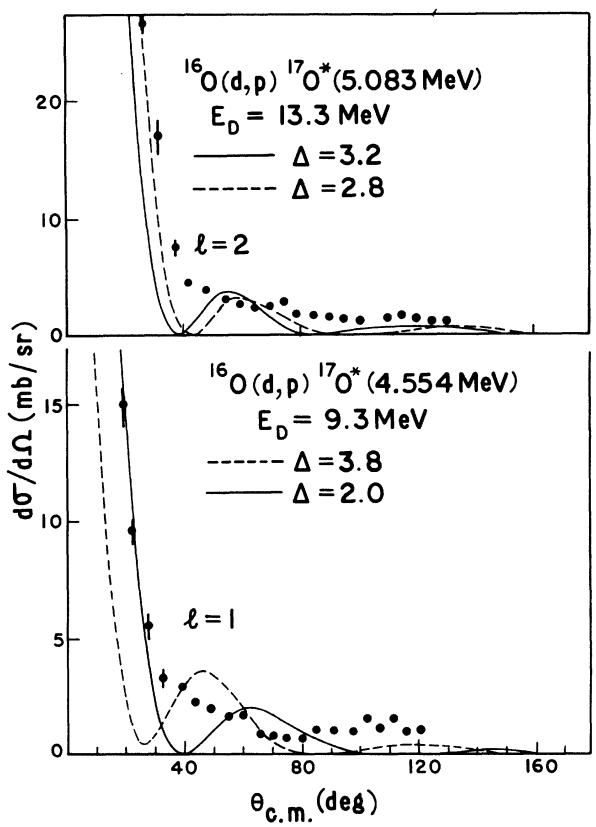

FIG. 3. Stripping differential cross section for ^{16}O-$(d,p)^{17}O^*$ reaction for two levels having $l=1$ and $l=2$. Plane wave calculations are shown for two values of cutoff radii. The measurements are taken from Ref. 6.

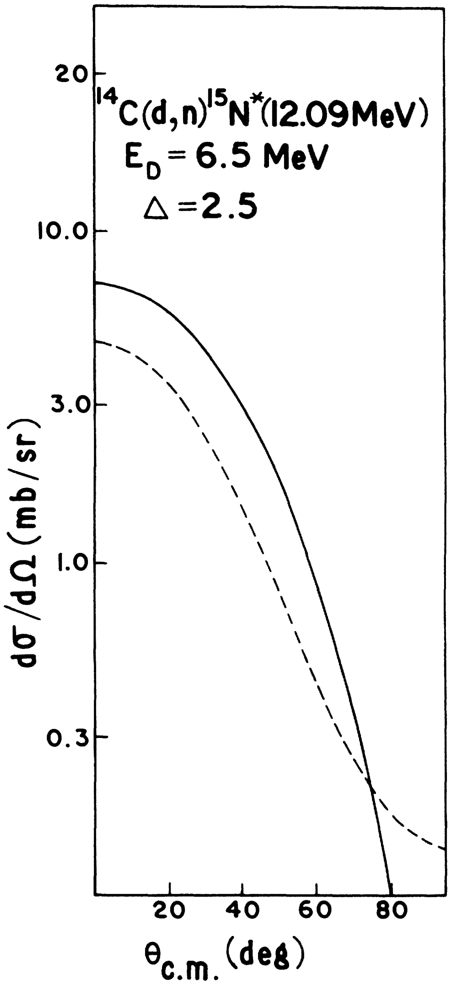

FIG. 4. Comparison of stripping differential cross section for $^{14}C(d,n)^{15}N^*$ (12.09 MeV) calculated with DWBA (dotted line) and plane wave (solid line). The DWBA results are taken from Ref. 7.

(d,p) and (d,n) reactions leading to unbound states. The kinematic parameters[19] of these reactions are listed in Table I. The results of these calculations for target nuclei ^{12}C and ^{16}O are shown in Figs. 2 and 3, respectively. The experimental points are from Ref. 6. We notice that results of calculations are in fair agreement with experiment. The main features of the angular distribution are unaltered by varying the cutoff radius R_0, as can be observed in Fig. 3. It is also clear that the angular distributions for stripping to the unbound states do not show any l dependence. So it is favorable to determine the l values of the resonances by calculating the stripping enhancement factor rather than the angular distributions, since the former is strongly l dependent. The same conclusion has been reached by Mohring and Lipperheide[20] in a more sophisticated calculation.

In Fig. 4 the results of calculation for (d,n) differential cross section employing Butler approximation are shown along with those obtained by DWBA calculations which use an extrapolation method for the evaluation of neutron wave function in the continuum. It is apparent that the plane wave calculations discussed here, though they require the adjustment of the cutoff radius, give results in a way very similar to those produced by DWBA.

In a more accurate treatment for the evaluation of F_l, one should employ the method suggested by Huby and Kelvin. According to them the energy-differential cross section is calculated by the usual DWBA formula except that the bound state wave function is replaced by a scattering wave function as follows:

A real Woods-Saxon potential well is adjusted in depth so that it produces an (lj) orbit resonating (i.e., having phase shift $\frac{1}{2}\pi$) at the neutron energy which corresponds to the formation of $(A+n)$ resonance. The problem of slow convergence of the radial integral in DWBA claculations and alternative methods to circumvent it and also the relevance of spectroscopic factors will be discussed in a future publication.[21]

IV. CONCLUSION

In summary we can say that the simple method of Butler cutoff discussed here accounts well for the observed parallelism between the energy differential (d,p) cross section and the total neutron-target elastic scattering cross section. The l dependence of the stripping enhancement factor is also accounted for. The angular distributions obtained for (d,n) reactions by this method are comparable with those obtained by DWBA.

Butler approximation, therefore, seems to be a good approximation to the real situation. The agreement between measured and calculated cross sections strongly supports this too. It should further be noted that the method described here is not only conceptually simple but also leads to a small amount of numerical work. One of the reasons for the success of this method for unbound stripping may be that the unbound states occur at relatively higher excitation energies in comparison with bound states, and hence the transferred particle is less sensitive to the nuclear interior. Thus the approximation of neglecting the contributions from the nuclear interior seems to be reasonable for such cases. The other reason may be due to the presence of the factor $(k^2 - k_n^{\,2})^{-1}$ in Eq. (11). As k tends to k_n, the plane wave part of the full DWBA matrix element will, perforce, dominate.

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