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Level structure of the odd-A Rb isotopes

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The odd-A Rb isotopes ^{83}Rb and ^{85}Rb are studied in a semimicroscopic model which couples the proton quasiparticle motion in the $Z=28-50$ shell to the quadrupole vibrations of the neighboring even core. The agreement between the calculated and experimental level spectra, spectroscopic factors, and electromagnetic transition rates is found to be satisfactory. The scope of further theoretical study is discussed in the light of recent experimental findings.

[NUCLEAR STRUCTURE ^{83,85}Rb; calculated levels, J, π, $B(E2)$, $B(M1)$, τ, γ branching. Quasiparticle-phonon coupling.]

I. INTRODUCTION

The nuclear structure of the Rb ($Z=37$) nuclei is of general interest for the systematics of nuclei around mass $A=90$. Of the two naturally occurring Rb isotopes, ^{87}Rb with 50 neutrons has a closed neutron shell and one might expect that the low-lying levels of this isotope could be described as single-hole proton states. The level structure of the other stable isotope ^{85}Rb is interesting because it is in a region of transition away from the $N=50$ closed shell. Until recently the knowledge of the levels of ^{85}Rb has been meager.[1] Only two negative parity excited states below 1 MeV were known and no reactions leading to states in ^{85}Rb had been reported. However, in the last two years there has been a sudden spurt in the experimental investigations on the level structure of this isotope[2-5] and a large amount of experimental data has been made available. This has prompted us to undertake a detailed theoretical study of the level structure of this isotope. The Coulomb excitation[2] and single-particle transfer reaction data[3-5] indicate the presence of significant coupling between the single proton and collective motion of the core even in the low energy excitations of this isotope. Intermediate coupling in the unified model has previously been applied[6] to explain the properties of low-lying levels of several isotopes in this mass region. In the present work, we have calculated the level structure of ^{85}Rb in a quasiparticle-phonon coupling model[7,8] which couples the single-proton motion in the $1f_{5/2}$, $2p_{1/2}$, $2p_{3/2}$, and $1g_{9/2}$ orbitals to the vibrations of the neighboring even core. Paradellis and Hontzeas[9] have calculated the level structure of ^{83}Rb in a quasiparticle-phonon coupling model. However, more experimental data on the level properties of ^{83}Rb have been available since their calculation was made,[10] so we have repeated the calculation for this isotope with modified values of model parameters to compare the theoretical results with experimental data in more detail. As the model used in the present work has been discussed in detail by several workers,[7,8] only the important features of the model necessary for subsequent discussions are given below.

II. MODEL

The total Hamiltonian of the coupled system is given by

$$H=H_c+H_{sp}+H_{int},$$

where H_{sp} is the usual single-particle shell model Hamiltonian and H_c describes the core vibrations. The basis states used are of the type $|j;NR:IM\rangle$ in which H_c+H_{sp} is diagonal; j is the particle angular momentum, R is the core angular momentum for a state of N phonons, and $I=R+j$ with the Z component equal to M. The eigenfunction of H at an energy E^α can be expanded as

$$|E^\alpha;IM\rangle=\sum_{jNR}C_\alpha(j;NR;I)|j;NR;IM\rangle .$$

The core-particle interaction consists of a dipole-dipole[11] and a quadrupole-quadrupole term:

$$H_{int}=-\eta(J_c\cdot j_p)-\xi\hbar\omega_2(\pi/5)^{1/2}\sum_\mu Q_{2\mu}Y_{2\mu}(\theta,\phi).$$

The parameters η and ξ describe the strength of the dipole-dipole and quadrupole-quadrupole interactions, respectively. The pairing effects are introduced to the calculations through the interaction Hamiltonian whose matrix elements are

$$\langle j';N'R';IM|H_{\rm int}|j;NR;IM\rangle = -\eta[j(j+1)(2j+1)R(R+1)(2R+1)]^{1/2}(-)^{I+R+j}$$
$$\times\begin{Bmatrix} j & R & I \\ R & j & 1 \end{Bmatrix}\delta_{jj'}\delta_{NN'}\delta_{RR'} + (-)^{I+R+1/2}\xi\hbar\omega_2[\tfrac{1}{4}(2j+1)(2j'+1)]^{1/2}$$
$$\times\begin{pmatrix} j & 2 & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}\begin{Bmatrix} I & R & j \\ 2 & j' & R' \end{Bmatrix}\langle N'R'\|Q_2\|NR\rangle(U_jU_{j'}-V_jV_{j'})\,.$$

The phonon matrix elements have been calculated following the method of Ford and Levinson.[12] The phonon energies have been taken from the excitation spectra of the neighboring even core.[10,13]

The static electric and magnetic moments of different levels and $E2$, $M1$ rates for several transitions are also calculated. The corresponding expressions are almost identical to those given by Heyde and Brussaard[14] except for a multiplicative factor in the particle part involving U_j and V_j. This factor is taken to be $(U_jU_{j'} \pm V_jV_{j'})$, where the upper (lower) sign refers to the magnetic (electric) operator. The spectroscopic factor for a stripping (pickup) reaction leading from the core nucleus to a state of spin $I=j$ is given by the absolute square of that coefficient in its wave function which corresponds to a pure particle (hole) state multiplied by U_j^2 (V_j^2):

$$S_\alpha(l,j) = U_j^2|C_\alpha(j;00;j)|^2\,, \quad \text{particle}$$
$$= V_j^2|C_\alpha(j;00;j)|^2\,, \quad \text{hole.}$$

III. RESULTS AND DISCUSSIONS

There are several parameters in this calculation, viz., quasiparticle energies, U_j factors, ξ, and η. In the case of ^{85}Rb, we have used two different cores, ^{84}Kr and ^{86}Sr, and the U_j factors are taken from the work of Medsker *et al.*[5] and that of Comfort, Duray, and Braithwaite,[4] respectively, in the two cases. In the case of ^{83}Rb, U_j factors are taken to be the same as used by Paradellis and Hontzeas[9] in their calculation and the phonon energies are taken from the excitation spectrum of ^{82}Kr. To fix up the parameter value η, the positive parity spectrum of ^{83}Rb is first calculated. Since this calculation involves only one single-particle orbital $1g_{9/2}$, there are two adjustable parameters ξ and η. It is found that a good agreement with experimental data can be achieved with $\xi = 2.0$ and $\eta = -0.03$ MeV. Negative parity levels of ^{83}Rb are then calculated with these ξ and η values. The $2p_{3/2}$ and $2p_{1/2}$ quasiparticle energies are adjusted to get a good fit to the experimental spectrum. To calculate the energy levels of ^{85}Rb, η is taken to be the same as in the case of ^{83}Rb and ξ and $2p_{3/2}$, $2p_{1/2}$ quasiparticle energies are adjusted so as to obtain a good agreement with the experimental level spectrum. Although there are several model parameters in the present work, a majority of them are fixed from the available data and effectively ξ and the two quasiparticle energies ($2p_{3/2}$, $2p_{1/2}$) are treated as free parameters. The parameter values used in the present work are listed in Table I. The calculated and experimental energy spectra are shown in Figs. 1 and 2. In the case of ^{85}Rb, the calculated energy spectra corresponding to two different cores used are shown separately. Static electromagnetic moments for some of the levels and $E2$, $M1$ transition rates for several transitions are also calculated. The magnetic moments and transition rates are calculated with $g_l = 1$, $(g_s)_{\rm eff} = 4.0$, and $g_R = 0.2$. The electric quadrupole moments and $E2$ transition rates are calculated with $(e_p)_{\rm eff} = e_p$, $eZ(\hbar\omega_2/2C_2)^{1/2} = 3e_p$, and $R = 1.2A^{1/3}$ fm. The average value of the core effective charge $eZ(\hbar\omega_2/2C_2)^{1/2}$ is determined from the $\hbar\omega_2$ and C_2 values of the core nuclei. The ra-

TABLE I. Parameter values used in this calculation.

		U_j [b]				Quasiparticle energies [a] (MeV)			Quadrupole coupling strength
Nucleus	Core	$1f_{5/2}$	$2p_{3/2}$	$2p_{1/2}$	$1g_{9/2}$	$2p_{3/2}$	$2p_{1/2}$	$1g_{9/2}$	(ξ)
^{83}Rb	^{82}Kr	0.447	0.447	1.0	1.0	0.02	0.27	0.14	2.0
^{85}Rb	^{84}Kr	0.547	0.547	0.910	0.927	0.19	0.45	0.89	3.5
^{85}Rb	^{86}Sr	0.489	0.436	0.794	0.954	0.25	0.30	0.75	3.3

[a] These energies are relative to the $1f_{5/2}$ orbital.

[b] These values are taken from Refs. 4, 5, and 9.

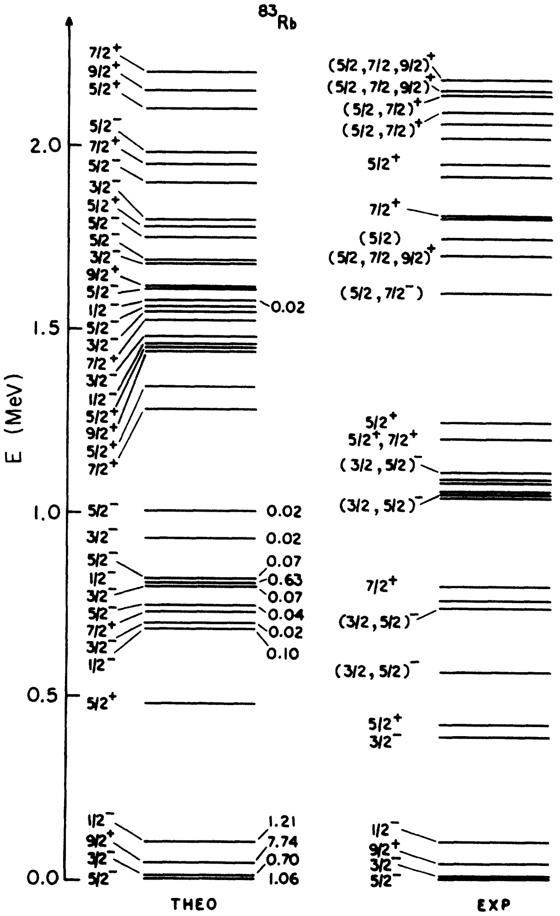

FIG. 1. Calculated and experimental level schemes of ^{83}Rb. The excitation energies and spin parities of the levels are shown. The spectroscopic factors are given at the right hand side of the levels provided $(2J+1)S \geq 0.01$. The experimental level scheme is taken from Ref. 10.

dial matrix elements are calculated in the harmonic oscillator basis. The calculated results are shown in Tables II and III. Branching ratios for several transitions are also calculated using the computed $E2$, $M1$ transition rates. Since the branching ratios are very sensitive to the transition energies involved, experimental transition energies are used to calculate them. The results are listed in Table IV. The comparison between the calculated and observed values in each individual isotope is made below.

^{83}Rb

A detailed investigation of the level scheme of this isotope was first done by Etherton *et al.*[15] from the decay of 32 h ^{83}Sr. Recently Broda *et al.*[10] have made a number of unambiguous spin parity assignments from electron conversion and angular correlation measurements. The level scheme proposed by them is shown in Fig. 1 along with the calculated one. Low-lying negative and positive parity levels excepting the 389 keV $\frac{1}{2}^-$ state are more or less correctly reproduced. In the higher energy part of the spectrum, the calculated level density is also in agreement with that observed experimentally. In the absence of sufficient data, the correspondence between levels in the higher energy part of the calculated and experimental spectra cannot be established. However, the calculated decay modes of the 697 ($\frac{3}{2}^-$), 743 ($\frac{5}{2}^-$), 1006 ($\frac{5}{2}^-$), 1280 ($\frac{7}{2}^+$), and 1520 ($\frac{7}{2}^+$) keV levels (Table IV) suggest that they may correspond to the observed 737 ($\frac{3}{2}^-, \frac{5}{2}^-$), 565 ($\frac{3}{2}^-, \frac{5}{2}^-$), 1044 ($\frac{5}{2}^-$), 1202 ($\frac{5}{2}^+, \frac{7}{2}^+$), and 1696 ($\frac{5}{2}^+, \frac{7}{2}^+, \frac{9}{2}^+$) keV levels.

^{85}Rb

This isotope has been studied from the stripping,[5] pickup,[3,4] and Coulomb excitation[2] experiments as well as from the decay of ^{85}Kr and ^{85}Sr isomers.[16,17] The calculated and experimental energy spectra are in good agreement (Fig. 2). Within the experimental uncertainties, the agreement between the calculated and experimental spectroscopic factors for most of the levels obtained from both stripping and pickup reactions can be considered to be satisfactory. From spectroscopic factor consideration, the levels at 735 and 1294 keV observed in the stripping reaction[5] should have spin $\frac{3}{2}^-$. Nonobservation of any excited $\frac{5}{2}^-$ state in the stripping reaction may partly be explained by rather small values of the calculated spectroscopic factors for the $\frac{5}{2}_2^-$ and $\frac{5}{2}_3^-$ states using ^{84}Kr as a core. Several positive parity levels corresponding to the $l=2$ transition have been observed in the stripping reaction. To reproduce these levels, the positive parity spectrum is calculated including the $2d_{5/2}$ orbital. The $2d_{5/2}$-$1g_{9/2}$ energy spacing is estimated from the experimental work of Picard and Bassani.[18] The ground state moments[19] are correctly reproduced (Table II). The $B(E2\uparrow)$ values calculated for the $\frac{3}{2}_1^-$ and $\frac{5}{2}_2^-$ states using ^{84}Kr as the core are close to the values obtained in the Coulomb excitation experiment,[2] whereas the calculated $B(E2\uparrow)$ values for the $\frac{1}{2}_1^-$ and $\frac{3}{2}_2^-$ states are larger than their experimental values (Table III). Using ^{86}Sr as the core, the overall situation remains more or less the same. The present calculation predicts a large Coulomb excitation probability for two levels near 1 MeV excitation having spin values $\frac{7}{2}^-$ and $\frac{9}{2}^-$. It would be interesting to verify these predictions experimentally. The calculated decay modes of the 732 ($\frac{3}{2}^-$) and 868 ($\frac{5}{2}^-$) keV levels show that these levels should decay predominantly to the ground state and the first excited $\frac{3}{2}^-$ state (Table IV). However, experimentally no transition to the first excited state from these lev-

TABLE II. Calculated and experimental electric quadrupole and magnetic dipole moments.

			Quadrupole moment (e b) Theo.			Magnetic moment (μ_N) Theo.		
Isotope	E (keV)	J^π	Kr core	Sr core	Exp. [a]	Kr core	Sr core	Exp. [a]
^{83}Rb	0	$\frac{5}{2}^-$	0.30			1.53		1.4
	5	$\frac{3}{2}^-$	0.20			2.48		
	42	$\frac{9}{2}^+$	−0.64			5.66		
	99	$\frac{1}{2}^-$				0.105		
	423	$\frac{5}{2}^+$	−0.37			4.72		
^{85}Rb	0	$\frac{5}{2}^-$	0.30	0.35	0.26	1.41	1.35	1.35
	151	$\frac{3}{2}^-$	0.19	0.25		2.50	2.54	
	281	$\frac{1}{2}^-$				0.098	0.029	
	514	$\frac{9}{2}^+$	−0.64	−0.70		5.80	5.77	6.16
	735	$\frac{3}{2}^-$	−0.015			0.89		

[a] Reference 19.

els has been observed.[2] The calculated lifetimes of these levels are close to the corresponding measured values.

IV. CONCLUSION

The present work shows that a majority of the available experimental data on the low-lying state of the odd-mass rubidium isotopes can be successfully explained in a model which couples the proton-quasiparticle motion to the vibrations of the neighboring even core by a judicious choice of the model parameters. Recent ^{85,87}Rb(^{3}He,d) experiments by Schneider, Anderson, and Brabeck[20] and ^{86,88}Sr(d, ^{3}He) experiments by Comfort, Duray, and Braithwaite[4] indicate that the single proton-hole description is poorer for ^{85}Rb than for ^{87}Rb and $Z = 38$ forms a better closed shell when $N = 50$ than when $N = 48$. The two $1g_{9/2}$ neutron holes in ^{85}Rb produce a substantial redistribution of proton orbital populations and it is suggested[20] that the ground state wave function of ^{85}Rb has a sizable $(1f_{5/2})^{-3}$ component. In that case, it is expected that in the low energy excitations of ^{83,85}Rb there should be several states arising from the $(1f_{5/2})^{-3}$ proton configuration. It would be interesting to see whether calculations of the type performed by

TABLE III. Calculated and experimental $B(E2\uparrow)$ values in ^{85}Rb. $B(E2\uparrow)$ values calculated using ^{84}Kr and ^{86}Sr as core are shown separately.

	Present work	$B(E2\uparrow)$ (e^2 b^2) Core nucleus			Experiment [a]	
E (keV)	J^π	^{84}Kr	^{86}Sr	E (keV)	J^π	$B(E2\uparrow)$ (e^2 b^2)
161	$\frac{3}{2}^-$	0.0029	0.0037	151	$\frac{3}{2}^-$	0.0035 ±0.0004
262	$\frac{1}{2}^-$	0.0072	0.0059	281	$\frac{1}{2}^-$	0.0016 ±0.0002
850	$\frac{3}{2}^-$	0.0194	0.0161	732	$\frac{3}{2}^-$	0.0101 ±0.0010
886	$\frac{1}{2}^-$	0.0082	0.0116			
906	$\frac{5}{2}^-$	0.0274	0.0174			
				868	($\frac{5}{2}^-$, $\frac{7}{2}^-$)	0.036 ±0.004
909	$\frac{7}{2}^-$	0.0624	0.0692			
1031	$\frac{9}{2}^-$	0.0731	0.0760			

[a] Reference 2.

TABLE IV. Calculated and experimental branching ratios for several transitions in ^{83}Rb and ^{85}Rb.

					Theo.				Exp.[a]		τ (psec)	
Isotope	E_i (keV)	J_i^π	E_f (keV)	J_f^π	$P(E2)$ (sec^{-1})	$P(M1)$ (sec^{-1})	P_{tot} (sec^{-1})	Branching ratio	E_i (keV)	Branching ratio	Calc.	Exp.[a]
^{83}Rb	697	$\frac{3}{2}^-$	0	$\frac{5}{2}^-$	0.56×10^{11}	1.21×10^{11}	1.77×10^{11}	100	737	100	2.3	
			12	$\frac{3}{2}^-$	0.04×10^{11}	1.36×10^{11}	1.40×10^{11}	78		40		
			99	$\frac{1}{2}^-$	0.02×10^{10}	1.22×10^{11}	1.22×10^{11}	66		25		
	743	$\frac{5}{2}^-$	0	$\frac{5}{2}^-$	0.15×10^{11}	0.22×10^{11}	0.37×10^{11}	38	565	62	5.5	
			12	$\frac{3}{2}^-$	0.10×10^{11}	0.13×10^{12}	0.14×10^{12}	100		100		
	1006	$\frac{5}{2}^-$	0	$\frac{5}{2}^-$	0.12×10^{9}	0.84×10^{11}	0.84×10^{11}	100	1044	100	10	
			12	$\frac{3}{2}^-$	0.02×10^{11}	0.10×10^{11}	0.12×10^{11}	14		17		
	1280	$\frac{7}{2}^+$	45	$\frac{9}{2}^+$	0.05×10^{10}	0.35×10^{12}	0.35×10^{12}	31	1202	78	0.7	
			479	$\frac{5}{2}^+$	0.07×10^{12}	0.10×10^{13}	0.11×10^{13}	100		100		
	1520	$\frac{7}{2}^+$	45	$\frac{9}{2}^+$	0.14×10^{10}	0.16×10^{12}	0.16×10^{12}	19	1696	48	0.8	
			479	$\frac{5}{2}^+$	0.04×10^{12}	0.17×10^{12}	0.21×10^{12}	25		48		
			730	$\frac{7}{2}^+$	0.14×10^{12}	0.71×10^{12}	0.85×10^{12}	100		100		
^{85}Rb	261	$\frac{1}{2}^-$	0	$\frac{5}{2}^-$	0.47×10^{9}		0.47×10^{9}	3.6	281	4.7	77	
			161	$\frac{3}{2}^-$	0.08×10^{8}	0.13×10^{11}	0.13×10^{11}	100		100		
	850	$\frac{3}{2}^-$	0	$\frac{5}{2}^-$	0.07×10^{12}	0.14×10^{12}	0.21×10^{12}	100	732	100	2.5	6.4
			161	$\frac{3}{2}^-$	0.02×10^{11}	0.16×10^{12}	0.16×10^{12}	78		...		
			261	$\frac{1}{2}^-$	0.03×10^{9}	0.20×10^{11}	0.20×10^{11}	10		66		
	906	$\frac{5}{2}^-$	0	$\frac{5}{2}^-$	0.20×10^{12}	0.10×10^{12}	0.30×10^{12}	100	868	100	2	4.2
			161	$\frac{3}{2}^-$	0.02×10^{12}	0.18×10^{12}	0.20×10^{12}	66		...		
			261	$\frac{1}{2}^-$	0.08×10^{9}		0.08×10^{9}	...		...		

[a] References 2 and 10.

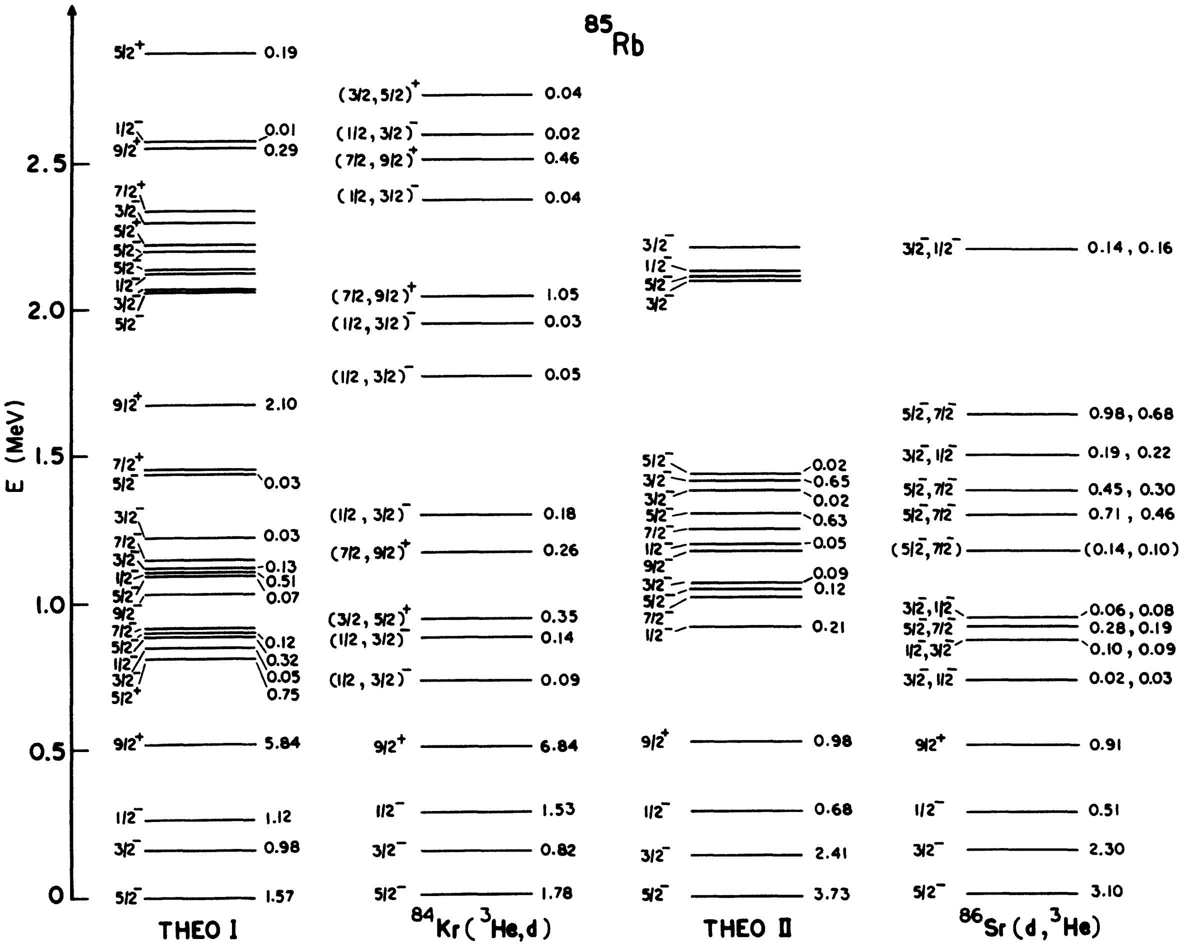

FIG. 2. Calculated and experimental level schemes of ^{85}Rb. The levels calculated with ^{84}Kr and ^{86}Sr cores are shown in THEO I and THEO II, respectively. The spectroscopic factors are given at the right hand side of the levels provided $(2J+1)S \geq 0.01$. The ^{84}Kr(^{3}He, d) and ^{86}Sr(d, ^{3}He) data are taken from Refs. 5 and 4, respectively.

Paar,[21] who considers the coupling of a few particle cluster to quadrupole vibrations, give a better agreement than the present work. Scholz and Malik[22] calculated the level spectra of the odd-A Rb nuclei with the inclusion of Coriolis coupling and a residual pairing interaction. They did not calculate the electromagnetic properties of the levels and as such their results cannot be compared in detail with the recent experimental findings. In conclusion, it may be said that in view of the availability of a large amount of experimental data in recent years, more theoretical investigations are necessary for understanding the nature of excitations of the nuclei in this mass region.

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