Final-state interactions in muon capture by deuterons

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Prompted in part by the disparate theoretical estimates in the literature, we present a systematic study of the influence of the $^1S_0$ n-n final-state interaction on rates for the reaction $\mu^- + d \rightarrow n + n + \nu_\mu$, treated in impulse approximation with the standard Primakoff effective weak-interaction Hamiltonian. Instead of beginning with a particular n-n potential, we use as our basic input a set of four-parameter Bargmann models of the $^1S_0$ n-n phase shift. By solving a "restricted inverse problem" we generate the necessary radial functions from these phase shifts. This procedure enables us to examine in a consistent way the sensitivity of the total capture rates and neutron energy spectrum to uncertainties in the phase shifts. We find that changes in the phase shifts at either high or low energies do not affect the rates noticeably. Hence, we conclude that this reaction is not a good tool for determining the n-n scattering length. In addition, we investigate the effects of varying the off-shell behavior of the n-n interaction by means of the method of short-range unitary transforms of Coester *et al.* We then find that the rates change dramatically. The doublet capture rate, for example, can be shifted from the 375 sec^{-1} typical of the local potentials of the restricted inverse problem down to 236 sec^{-1}. More accurate measurements of this rate could thus serve as constraints on the off-shell extrapolation of the n-n interaction. Our results are in general agreement with those of Sotona and Truhlík.

[NUCLEAR REACTIONS $\mu^- + d \rightarrow n + n + \nu_\mu$; effect of $^1S_0$ n-n final-state interactions on rates, neutron spectra. Varied n-n phase shifts and off-shell behavior.]

I. INTRODUCTION

Because the final state of the muon capture reaction

$$\mu^- + d \rightarrow n + n + \nu_\mu \tag{1}$$

contains two neutrons which, for all practical purposes, interact only with one another, hope has been expressed that careful analysis of this process will yield empirical information about the neutron-neutron interaction which is otherwise relatively inaccessible.[1-3] In particular, one would like to use reaction (1) to determine the n-n scattering length and effective range as well as to probe the off-shell properties of the n-n interaction. However, calculations to date have given varying estimates of the contribution of n-n final-state interactions (FSI) to the capture rates, making it difficult to assess the likelihood that the desired information can be gleaned from experiment. For example, using dispersion theory, Cremmer[4] finds that the $^1S_0$ n-n FSI decreases the capture rate from the doublet μ-d hyperfine state by about 4%. By contrast, Truhlík[5] and Dogotar, Salganic, and Eramzhyan[6] calculate a 10% or more enhancement of this rate employing a simple potential model. More recently, Sotona and Truhlík[7] have shown that phase-shift-preserving unitary transforms of the $^1S_0$ n-n potential give FSI contributions which cover a fairly wide range of both enhancements and suppressions relative to the rate calculated without FSI. With these apparently conflicting—and in any case ambiguous—theoretical predictions in mind, and in anticipation of new data on reaction (1) from stopped-muon facilities at meson factories,[8] we have undertaken a systematic study of the effects of the $^1S_0$ FSI on the transition rates for muon capture by deuterons.[9] In the spirit of the Gel'fand-Levitan-unitary-transform method,[10] we allow both for variation of the n-n phase shifts at all energies and for the purely off-shell changes induced by phase-shift-preserving rank-one unitary transforms. One result of our work is an independent confirmation of the conclusions of Sotona and Truhlík[7] regarding the dependence of total capture rates on the FSI. In addition, we present a systematic study of the sensitivity of the neutron energy spectrum to changes (both on shell and off shell) in the FSI. While Sotona and Truhlík take a local n-n potential as basic input, we start with parametrizations of the n-n phase shift so that we are able to study the effects of phase shift uncertainties on the predicted capture rates and spectra.

For the description of the weak interaction responsible for muon capture, we found it convenient to adopt the notation, conventions, and choice of parameters of Pascual, Tarrach, and Vidal.[11] Following these authors, we represent the deuteron with the wave function of Gourdin *et al.*[12] The pertinent information from Refs. 11 and 12 is summarized in Sec. II.

Since numerical solution of the complete inverse scattering problem for fixed angular momentum poses severe computational difficulties,[13] we specify the ${}^1S_0$ n-n final-state wave function in terms of the phase shifts by means of a "restricted inverse problem" (RIP) described in Sec. III. It enables us to examine the influence of uncertainties in the phase shifts at both low and high energies. This RIP generates n-n wave functions which correspond to local potentials. As there is no reason to believe that the n-n interaction is local at short distances, it is necessary to investigate the sensitivity of the calculated capture rates and spectra to possible nonlocality of the short-range interaction. This is most easily accomplished by applying the rank-one unitary transforms of Haftel and Tabakin[14] to the wave functions obtained by means of the RIP, as indicated in Sec. III.

Section IV summarizes our results. We present total rates for capture from the doublet and quartet μ-d hyperfine states, the statistical average of these rates, and also the neutron energy spectrum for doublet capture. Our conclusions and proposals for further work are given in Sec. V. An appendix records the explicit form of the n-n radial functions obtained from the RIP.

II. CAPTURE-RATE FORMULAS

A. Weak interaction

We use the standard Primakoff effective weak-interaction Hamiltonian[15]

$$H_{\mathrm{eff}}^{(\mu)}=\tau^{(+)}\tfrac{1}{2}(G\cos\theta)(1-\vec{\sigma}\cdot\hat{\nu})\sum_{i=1}^{2}\tau_i^{(-)}\delta(\vec{r}-\vec{r}_i)\Big(G_V^{(\mu)}+G_A^{(\mu)}\vec{\sigma}\cdot\vec{\sigma}_i-G_P^{(\mu)}(\vec{\sigma}\cdot\hat{\nu}(\vec{\sigma}_i\cdot\hat{\nu}) - g_V^{(\mu)}\frac{(\vec{\sigma}\cdot\hat{\nu})(\vec{\sigma}\cdot\vec{p}_i)}{M}\,g_A^{(\mu)}\frac{(\vec{\sigma}\cdot\hat{\nu})(\vec{\sigma}_i\cdot\vec{p}_i)}{M}\Big) \tag{2}$$

to describe the interaction responsible for muon capture. Here unsubscripted operators and variables refer to the emitted neutrino (ν is the neutrino momentum); those with subscripts refer to the nucleons (p is the nucleon momentum and M is the nucleon mass). The effective coupling constants G_V, G_A, and G_P are given by

$$G_V^{(\mu)}=g_V^{(\mu)}(1+\nu/2M)\,,$$
$$G_A^{(\mu)}=g_A^{(\mu)}-(g_V^{(\mu)}+g_M^{(\mu)})\nu/2M\,, \tag{3}$$

and

$$G_P^{(\mu)}=(g_P^{(\mu)}-g_A^{(\mu)}-g_V^{(\mu)}-g_M^{(\mu)})\nu/2M\,,$$

where

$$g_M^{(\mu)}=g_V^{(\mu)}(\mu_p-\mu_n)\,.$$

Following Pascual *et al.*,[11] we take

$$G\cos\theta=1.1484\times10^{-11}\ \mathrm{MeV}^{-2}\,,$$

$$g_V=0.968,\quad g_A=-1.226,\quad g_M=3.6\,,$$

and

$$g_P=\begin{cases}-8.54\\-10.27\end{cases}. \tag{3a}$$

Here $\mu_p=1.793$ and $\mu_n=-1.913$.

Since the induced pseudoscalar coupling constant g_P is not very well determined, we calculate all rates with both of the above values. They span the generally accepted range for this parameter. Explicit formulas for the doublet capture rate $\Gamma_{1/2}$, the quartet rate $\Gamma_{3/2}$, and the statistical average $\Gamma=\frac{1}{3}(\Gamma_{1/2}+2\Gamma_{3/2})$ are given in Eqs. (11) through (17) of Ref. 11.

B. Deuteron wave function

Again following Ref. 11, we use the convenient analytical deuteron wave function of Gourdin *et al.*[12]:

$$u(r)=N\sum_{\lambda=1}^{4}c_\lambda\exp(-\tilde{\beta}_\lambda r)\,,$$
$$w(r)=N\rho\sum_{\lambda=1}^{4}c_\lambda\frac{\tilde{\beta}_\lambda^{2}}{\tilde{\beta}_1^{2}}\beta_\lambda r h_2(i\tilde{\beta}_\lambda r) \tag{4}$$

with

$$xh_2(ix) = (1 + 3/x + 3/x^2)e^{-x}\,,$$
$$\tilde{\beta}_1 = 45.683 \text{ MeV}, \quad \tilde{\beta}_2 = 8.55\tilde{\beta}_1\,, \quad \tilde{\beta}_3 = 11.13\tilde{\beta}_1\,,$$
$$\tilde{\beta}_4 = (\tilde{\beta}_2{}^2 + \tilde{\beta}_3{}^2 - \tilde{\beta}_1{}^2)^{(1/2)}\,,$$
$$c_1 = -c_4 = 1\,, \quad c_2 = -c_3 = -\left(\frac{\tilde{\beta}_1{}^2 - \tilde{\beta}_4{}^2}{\tilde{\beta}_2{}^2 - \tilde{\beta}_3{}^2}\right), \tag{4a}$$
$$\rho = 0.0265\,,$$

and

$$N^{-2} = \sum_{\lambda,\mu=1}^{4} \frac{c_\lambda c_\mu}{\tilde{\beta}_\lambda + \tilde{\beta}_\mu} + \frac{\rho^2}{\tilde{\beta}_1{}^4} \sum_{\lambda,\mu=1}^{4} \frac{\tilde{\beta}_\lambda{}^2 \tilde{\beta}_\mu{}^2}{\tilde{\beta}_\lambda + \tilde{\beta}_\mu} c_\lambda c_\mu\,.$$

This wave function has a D-state probability of 6.5%.

III. MODELS OF THE $^1S_0$ n-n FINAL-STATE WAVE FUNCTION

To complete the ingredients needed for evaluation of the capture matrix elements, we must specify the n-n final-state wave function. Treating the interaction only in the s wave, we write this as

$$\psi_{nn}^{(+)}(\vec{k}, \vec{r}) = \frac{e^{i\delta(k)}}{kr} w(k,r) + e^{i\vec{k}\cdot\vec{r}} - \frac{\sin kr}{kr}\,,$$
$$w(k,r) \underset{r\to\infty}{\sim} \sin[kr + \delta(k)]\,,$$

thereby defining the n-n s-wave radial function $w(k,r)$. Here k is the usual center-of-mass wave number and $\delta(k)$ is the s-wave phase shift.

A. Restricted inverse problem

Since we are interested in determining the sensitivity of the capture rates to changes in the n-n scattering length and effective range, as well as to off-shell variations, we cannot simply adopt one of the standard two-nucleon potential models to generate the $^1S_0$ n-n wave function. Commitment to such a potential implies commitment to a fixed set of phase shifts. An orderly procedure for generating n-n wave functions compatible with an arbitrarily prescribed set of phase shifts is provided by the Gel'fand-Levitan equation for the inverse scattering problem.[16] We have found that numerical solution of this equation presents severe computational difficulties.[13] However, the problem becomes analytically and numerically tractable if the energy dependence of the $^1S_0$ phase shift is restricted to the form

$$\delta(k) = \operatorname{Im}\ln\left[\frac{(2k + i\alpha_1)(2k + i\alpha_2)}{(2k + i\beta_1)(2k + i\beta_2)}\right]. \tag{5}$$

This form provides four free parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ and is thus flexible enough to reproduce the major features of the empirical $\delta(k)$. In fact, Srivastava and Sprung[17] have obtained a very precise fit to the experimentally determined $^1S_0$ n-p and p-p phase shifts using a parametrization of the type (5). (Unlike these authors, however, we require that α_1, α_2, β_1, and β_2 be purely real. In this way we insure that the corresponding Bargmann potentials—see below—do not contain the long-range oscillations which arise in the potentials of Ref. 17.) As is well known,[16,17] this functional form of $\delta(k)$ gives rise to an analytically soluble inverse problem corresponding to a class of two-term Bargmann potentials for which the scattering radial functions may be expressed in closed form. The explicit formulas which accomplish the translation of phase shifts into radial functions in thise case were derived by Blažek[18]; we have relegated the cumbersome expressions for the radial functions to an appendix.

We fix the values of the four parameters by specifying the scattering length, the effective range, the energy at which the phase shift passes through zero, and the rate at which the phase shift approaches zero at high energy. Explicitly, the scattering length a is

$$a = \frac{\beta_1\beta_2(\alpha_1 + \alpha_2) - \alpha_1\alpha_2(\beta_1 + \beta_2)}{\alpha_1\alpha_2\beta_1\beta_2}\,,$$

the effective range r_0 is

$$r_0 = \frac{4}{\alpha_1\alpha_2(\beta_1 + \beta_2) - \beta_1\beta_2(\alpha_1 + \alpha_2)} \times \Bigg[-\alpha_1\alpha_2 - \beta_1\beta_2 + \alpha_1\beta_1 + \alpha_1\beta_2 + \alpha_2\beta_1 + \alpha_2\beta_2 + \frac{\alpha_1\alpha_2\beta_1\beta_2(\beta_1 + \beta_2 - \alpha_1 - \alpha_2)}{\alpha_1\alpha_2(\beta_1 + \beta_2) - \beta_1\beta_2(\alpha_1 + \alpha_2)}\Bigg]\,,$$

the laboratory energy $\tilde{E}_{\text{lab}}$ at which δ goes through zero is

$$\tilde{E}_{\text{lab}} = 82.94\, \frac{\beta_1\beta_2(\alpha_1 + \alpha_2) - \alpha_1\alpha_2(\beta_1 + \beta_2)}{4(\alpha_1 + \alpha_2 - \beta_1 - \beta_2)} \;\text{MeV}\,,$$

and the coefficient A in the asymptotic form $\delta(k) \underset{k\to\infty}{\sim} A/k$ [16] is

$$A = \tfrac{1}{2}(\alpha_1 + \alpha_2 - \beta_1 - \beta_2)\,.$$

The associated potentials are positive and finite at $r = 0$, change sign at about 0.7 fm, and then remain attractive as $r \to \infty$.

B. Unitary transforms

The n-n radial functions obtained through the RIP correspond to potentials which are local in the radial coordinate r. In this respect, they are somewhat unrealistic, since it is generally believed that the two-nucleon interaction is intrinsically nonlocal at short distances. Lacking any reliable theory of the short-range n-n interaction, we cannot specify the nonlocality. Instead, we must investigate the extent to which the capture rates depend upon the significant parameters of the nonlocal part of the interaction when these parameters are varied subject to the constraint that the fit to the empirical phase shifts is not altered. This is most easily accomplished by the method of short-range unitary transforms.[19] Starting with $w(k,r)$ from the RIP, we generate a new set of radial functions

$$\tilde{w}(k,r)=w(k,r)+\int_0^\infty \Lambda(r,r')w(k,r')dr' .$$

The transform kernel $\Lambda(r,r')$ is designed so that the $\tilde{w}(k,r)$ form a complete orthonormal set, just as the $w(k,r)$ do, and

$$\tilde{w}(k,r)\underset{r\to\infty}{\longrightarrow} w(k,r)\underset{r\to\infty}{\longrightarrow}\sin[kr+\delta(k)] .$$

The necessary and sufficient conditions for $\Lambda(r,r')$ to fulfill these requirements are given, e.g., in Ref. 10. For simplicity, we have restricted our investigation to a two-parameter family of rank-one transforms characterized by

$$\Lambda(r,r')=-2g(r)g(r') , \tag{6a}$$

$$\int_0^\infty g^2(r)dr=1 , \tag{6b}$$

in which we take

$$g(r)=C\exp(-\gamma r)(r+\beta r^2) . \tag{6c}$$

These transforms, which were also used by Sotona and Truhlík,[7] were introduced by Haftel and Tabakin.[14] The parameter γ controls the range of the nonlocality and β allows us to vary its form somewhat. Note that the normalization condition (6b) fixes the "strength" C of the nonlocality, which we are not free to change independently of γ and β.

IV. RESULTS

A. Rates without FSI: Estimates of contributions from deuteron D-state and relativistic corrections

The numerical evaluation of capture rates was time-consuming. Therefore, to obtain estimates of the dominant contributions and of the most important correction terms, we first computed rates without any FSI (i.e., using plane waves to describe the n-n final state). The numerical integrations were sensitive to the number and distribution of the sample points. The energy integrations required a large number of sample points at low energy before a stable result was obtained. We used 120 sample points for the energy integration with the larger range. The r integration was done accurately with a 15-point Laguerre formula. Our results are summarized in Table I. They show that explicit inclusion of the deuteron D wave with or without relativistic corrections, results in negligibly small changes in the doublet and spin-averaged rates. The quartet rate remains small in all cases. Hence, in order to simplify subsequent calculations, we omit both the deuteron D-wave and the relativistic correction terms. Note, however, that our deuteron S wave is normalized to $1-P_D$, where P_D is the deuteron D-wave probability.

B. Rates and spectra with RIP model of FSI: Sensitivity to n-n phase shifts

To investigate the sensitivity of the capture rates and the neutron energy spectrum to changes in the n-n phase shifts, we calculated these quantities for a number of different RIP models of the n-n $^1S_0$ FSI. Each model is specified by the set of four parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2)$. Figure 1 shows the $^1S_0$ phase shift as a function of laboratory energy for a representative sample of the various RIP models. The resulting total capture rates are presented in Table II, which lists the values of the scattering length a, the effective range r_0, the laboratory energy $\tilde{E}_{\rm lab}$ at which the phase shift passes through zero, and the value of the potential at the origin, $V(0)$, for each model, along with the Bargmann parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2)$. The effect of RIP $^1S_0$ FSI on the neutron spectrum is illustrated in Figs. 2 and 3. The spectrum for only one of the models is presented; all other parameter sets yielded nearly identical curves. Figure 2 shows the spectra for $g_p=-10.27$ and Fig. 3 shows them for $g_p=-8.54$. For reference, the spectra computed without FSI are also displayed.

It is clear that, as one might expect, the rates

TABLE I. Capture rates with no final-state interactions.

g_p	$\Gamma_{1/2}$ (sec^{-1})	$\Gamma_{3/2}$ (sec^{-1})	Γ (sec^{-1})
	S-wave deuteron		
−8.54	331.07	5.77	114.21
−10.27	319.75	7.29	111.44
	$S+D$-wave deuteron		
−8.54	329.46	7.93	115.10
−10.27	317.96	9.26	112.16
	$S+D$-wave deuteron with relativistic corrections		
−8.54	334.49	7.71	116.64
−10.27	322.91	9.06	113.68

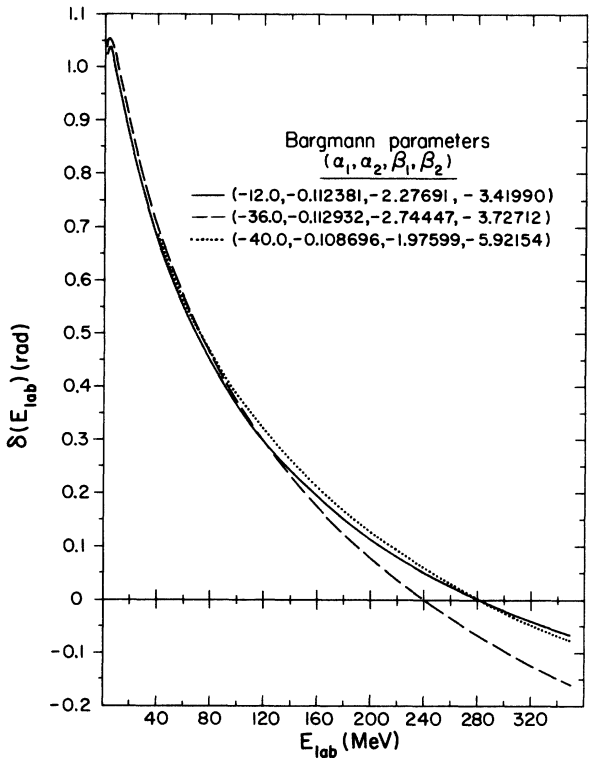

FIG. 1. The $^1S_0$ n-n phase shift for three typical RIP Bargmann parametrizations.

and spectra are completely insensitive to the high-energy phase shifts. More significant is the observation that these quantities are also unaffected by reasonable (i.e., small) changes in the n-n scattering length. This confirms the finding of Truhlík,[5] who used a simple separable potential model of the FSI. For these local potential models of the $^1S_0$ n-n interaction, the FSI increase the doublet rate by about 12%. Our typical value of $\Gamma_{1/2}$ of 374 sec^{-1} for $g_p = -8.54$ is very close to Truhlík's best estimate of 376 sec^{-1} for $g_p = -8.5$[5] and also to the more recent value of 377.1 sec^{-1} (again for $g_p = -8.5$) quoted by Sotona and Truhlík,[7] even though the potentials used in Refs. 5 and 7 differ from ours as well as from one another. Our rates also resemble those of Mintz,[20] who uses an entirely different method of calculation.

C. Rates and spectra with unitary transforms of the RIP FSI: Dependence on off-shell behavior of the n-n interaction

Having found that on-shell variations of the n-n interaction do not alter the capture rates or neutron spectrum very much, we turn to changes generated by the purely off-shell variations resulting from the unitary transforms defined in Sec. III B. When we applied the unitary transform of Eqs. (6a)–(6c) to the RIP radial function having Bargmann parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (-12.0, -0.112\,381, -2.276\,91, -3.4199)$ (fm^{-1}), we obtained the results displayed in Table III and in Fig. 4. In these calculations we used $g_p = -8.54$ and coarser grids for the energy integrations. We find, as did Sotona and Truhlík,[7] that the unitary transforms produce large variations in the total capture rates. It is also clear that the shape of the neutron spectrum, $d\Gamma_{1/2}/dE_n$, is sensitive to these changes, which alter both the shape and position of the low-energy peak. The source of these large variations be-

TABLE II. Capture rates for various Bargmann potential parametrizations of $^1S_0$ n-n FSI. In each pair of rows, the upper capture-rate entries are obtained with $g_p = -8.54$, the lower with $g_p = -10.27$.

a/α_1 (fm^{-1})	r_0/α_2 (fm^{-1})	$\bar{E}(\delta=0)/\beta_1$ (MeV/fm^{-1})	$V(0)/\beta_2$ (fm^{-2}/fm^{-1})	$\Gamma_{1/2}$ (sec^{-1})	$\Gamma_{3/2}$ (sec^{-1})	Γ (sec^{-1})
−16.5	2.8	280.0	63.56	374.59	8.44	130.49
−12.0	−0.112 381	−2.276 91	−3.419 90	361.45	10.02	127.16
−16.5	2.8	280.0	780.54	374.90	8.44	130.60
−40.0	−0.112 380	−1.982 88	−5.915 58	361.75	10.02	127.27
−16.5	2.6	240.0	637.29	375.55	8.46	130.82
−36.0	−0.112 932	−2.744 47	−3.727 12	362.38	10.04	127.49
−16.5	2.8	240.0	190.11	373.86	8.43	130.24
−20.0	−0.112 382	−2.294 79	−3.810 15	360.75	10.00	126.92
−17.1	2.8	240.0	190.10	374.07	8.43	130.31
−20.0	−0.108 698	−2.282 65	−3.820 71	360.96	10.00	126.99
−17.1	2.8	240.0	786.07	374.14	8.43	130.33
−40.0	−0.108 697	−2.138 35	−4.826 06	361.02	10.00	127.01
−17.1	2.8	280.0	63.56	374.80	8.44	130.56
−12.0	−0.108 697	−2.264 07	−3.430 49	361.66	10.02	127.23
−17.1	2.8	280.0	780.52	375.12	8.45	130.67
−40.0	−0.108 696	−1.975 99	−5.921 54	361.96	10.03	127.34

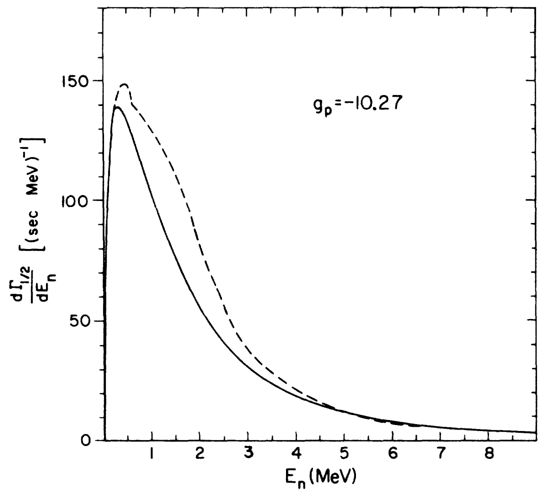

FIG. 2. Comparison of neutron energy spectrum, $d\Gamma_{1/2}/dE_n$ vs E_n, for RIP FSI (dashed curve) with the spectrum without FSI (solid curve). Here $g_p = -10.27$. We show the curve for Bargmann parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (-36.0, -0.112\,932, -2.744\,47, -3.727\,12)$ fm^{-1}; all other RIP FSI give spectra which are essentially indistinguishable on the scale of the figure.

comes apparent when we compare the transformed radial functions with the original ones. Such a comparison is presented in Fig. 5. The transforms induce drastic modifications in the probability that the two neutrons will be within 1 fm of one another; in addition, the radial functions are altered appreciably as far out as 2 fm.

In our calculations, as in those of Ref. 7, the total rates computed with transformed radial functions are always smaller than those obtained with the untransformed RIP radial functions. An elegant argument sketched by Haftel[21] indicates that, for s

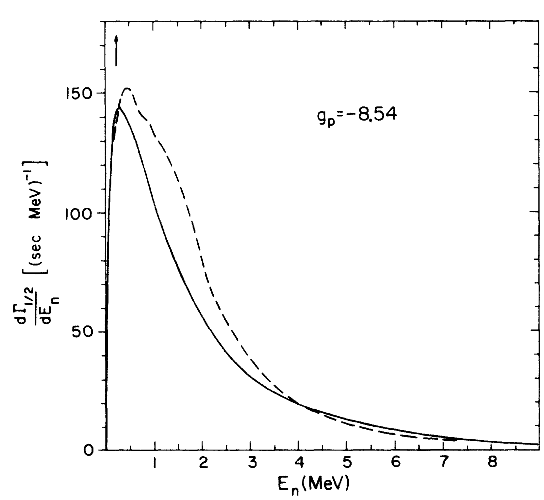

FIG. 3. Same as Fig. 2 for $g_P = -8.54$.

TABLE III. Variation of μ-capture rates with unitary transformation of ${}^1S_0$ n-n radial functions and $g_p = -8.54$. The parameters of the local Bargmann potential (untransformed) are $\alpha_1 = -12.0\ \text{fm}^{-1}$, $\alpha_2 = -0.112\,381\ \text{fm}^{-1}$, $\beta_1 = -2.276\,91\ \text{fm}^{-1}$, and $\beta_2 = -3.4199\ \text{fm}^{-1}$.

Transform parameters (γ, β)	$\Gamma_{1/2}$ (sec^{-1})	$\Gamma_{3/2}$ (sec^{-1})	Γ (sec^{-1})
(0, 0)(untransformed)	375.01	8.4	131
(3, 0)	293.23	6.9	102
(3, −0.5)	330.07	7.6	115
(3, −1.2)	373.69	8.3	130
(3, −2)	236.08	5.8	83
(3, +0.5)	272.43	6.5	95
(3, +1.2)	257.12	6.2	90
No FSI	332.75	7.60	116

waves, this suppression may be a general characteristic of all sufficiently localized unitary transforms.

V. CONCLUSIONS AND OUTLOOK

There are, to date, only two reported experimental determinations of the doublet capture rate. Wang *et al.*[22] give $\Gamma_{1/2} = 365 \pm 96\ \text{sec}^{-1}$, while the recent experiment of Bertin *et al.*[23] yielded $\Gamma_{1/2} = 445 \pm 60\ \text{sec}^{-1}$. The experiments are difficult

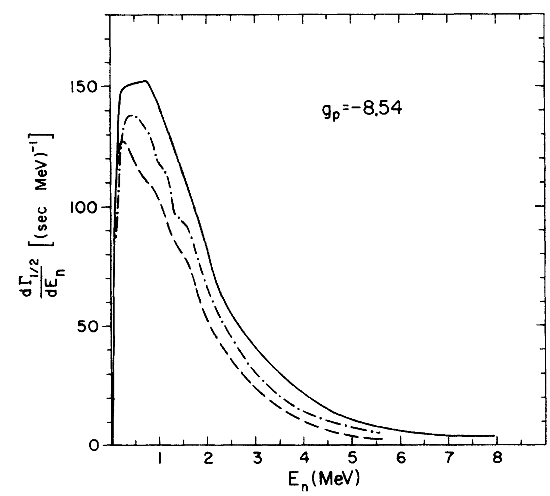

FIG. 4. Comparison of neutron spectra calculated by applying the unitary transforms with transform parameters $(\gamma, \beta) = (3.0, +1.2)$ (dashed curve) and $(\gamma, \beta) = 3.0, 0.0)$ (dashed-dotted curve) to the RIP radial function with Bargmann parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2) = (-12.0, -0.112\,381, -2.276\,91, -3.4199)\ \text{fm}^{-1}$ *with* the spectrum obtained using the untransformed radial function (solid curve). All curves are for $g_P = -8.54$. Other unitary transforms give spectra that roughly range from the solid curve to the dashed curve.

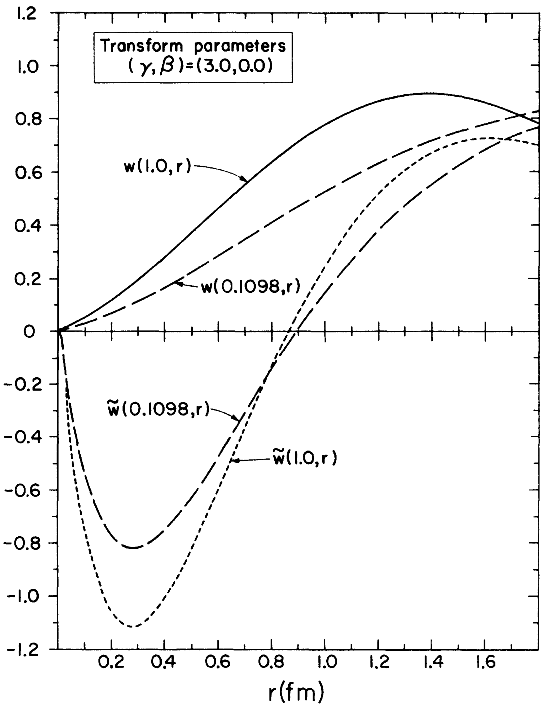

FIG. 5. Comparison of transformed radial functions with original RIP functions for energies of 82.94 and 1.00 MeV, which correspond to $k = 1.00$ and $0.1098\ \mathrm{fm}^{-1}$, respectively. The Bargmann parameters are those of Fig. 4; the transform parameters are $(\gamma,\beta) = (3.0, 0.0)$.

because it is necessary to account for the neutron background from reactions undergone by $d\mu d$ molecules.[23] It thus seems prudent to inquire whether our study suggests that measurements of rates for reaction (1) offer much prospect of additional insight into the n-n interaction. The answer is clearly negative for experiments at the current level of precision, for even our most drastic unitary-transform modification does not take us out of the range of capture rates spanned by the experimental results of Refs. 22 and 23. However, if we could discard the number from the earlier experiment, for example, we could rule out a large class of unitary transforms because they give doublet capture rates which are too low, as already noted in Ref. 7. Sauer[25] has shown that these transforms give rise to sizeable model-dependent charge asymmetries. Replication of the experiment and increased precision would thus be helpful in constraining the off-shell behavior of the n-n interaction, thereby providing independent empirical grounds for discarding those bothersome nonlocal modifications of the interaction for which the useful assumption of charge symmetry is invalid. On the other hand, our results of Sec. IV B show that there is not much hope of using reaction (1) to place significant restrictions on the on-shell behavior of the n-n interaction.

We have not addressed the important problem of estimating meson-exchange contributions. Dautry, Rho, and Riska find these to be appreciable when they are calculated using a standard local potential.[24] In the ultimate analysis, both meson-exchange effects and off-shell variations must be studied simultaneously.

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APPENDIX

We record here the explicit form of $w(k,r)$ found by solving the RIP according to Eqs. (14) and (16) of Ref. 18 for $n=2$. It is

$$w(k,r)=\frac{1}{|f(k)|}\left\{\sin kr\left[1-\sum_{i=1}^{2}\frac{2\beta_i}{4k^2+\beta_i^2}F_i(r)\cosh\tfrac{1}{2}(\beta_i r)\right]+k\cos kr\sum_{i=1}^{2}\frac{4}{4k^2+\beta_i^2}F_i(r)\sinh\tfrac{1}{2}(\beta_i r)\right\}, \tag{A1}$$

where

$$|f(k)|=\left[\frac{(4k^2+\alpha_1^2)(4k^2+\alpha_2^2)}{(4k^2+\beta_1^2)(4k^2+\beta_2^2)}\right]^{1/2} \tag{A2}$$

and

$$F_i(r)=N_i(r)/D(r). \tag{A3}$$

Here

$$D=x_{11}x_{22}-x_{12}x_{21}, \tag{A4a}$$

$$N_1=x_{22}-x_{12}, \tag{A4b}$$

and

$$N_2=x_{11}-x_{21}. \tag{A4c}$$

The x_{ij} are given by

$$x_{ij}=\frac{2}{\alpha_i^2-\beta_j^2}\left[\alpha_i \sinh\tfrac{1}{2}(\beta_j r)-\beta_j \cosh\tfrac{1}{2}(\beta_j r)\right], \quad i,j=1,2. \tag{A5}$$

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