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Phase-shift analysis of the 20–30-MeV *pp* and *np* scattering data, including new high-precision *np* $P(\theta)$ measurements*

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The available *pp* and *np* scattering data in the energy range 20–30 MeV are analyzed and reduced in order to obtain a minimum set without loss of statistically significant information. The new Jones-Brooks and Morris-O'Malley-May-Thornton high-precision *np* $P(\theta)$ data are included. The resulting set of 43 *pp* and 65 *np* independent data are augmented by Coulomb splittings of the *pp* and *np* isospin-one phase shifts and by high and intermediate angular momentum phase shifts computed from πn, $\pi\pi$, and *en* data via analyticity and unitarity. Each of these three additions has a significant effect on the low angular momentum phase shifts. The resulting *D*-wave phase parameters are in much better agreement with the values produced by potential models than were those produced by the earlier Livermore continuous-energy analysis.

[NUCLEAR REACTIONS Phase parameter analysis, 20–30-MeV $pp+np$ data, theoretical input; low-*L* phases, other analyses, model values.]

I. INTRODUCTION

In recent papers Jones and Brooks[1] presented a new set of very high-precision neutron-proton polarization measurements at 16.2 and 21.6 MeV, and Morris, O'Malley, May, and Thornton[2] a set at 21.1 MeV. The region from 20 to 30 MeV has become rich in both *pp* and *np* scattering data, so we decided to use this occasion to thoroughly revise the data set in that energy range as well as to determine the effect of the new data.

II. DATA SELECTION

In Tables I and III we list all differences between our newly revised data set and those sets previously published by Wilson,[3] Nisimura,[4] and MacGregor, Arndt, and Wright.[5] In each instance we specify the difference between the sets and the reason for it. We are finally left with a *pp* data set consisting of 43 pieces plus 4 measured normalizations. The normalizations correspond to degrees of freedom which are not independent of the rest of the data, so do not contribute to the effective number of data. In the *np* set there are 69 measurements. However, in four of the angular distributions the normalizations were left unmeasured, and hence there are effectively 65 independent pieces of *np* data.

A case demanding special treatment is the new high-precision *np* $P(\theta)$ Cape Town data of Jones and Brooks,[1] which has an *additive* constant error.[6] By this we mean that separate constants, whose experimentally determined[1,6] values are

$$s(\exp'1) = 0.000 \pm 0.005,$$

are to be added to each of the three sets of 50°–170° data. These data then contribute to the least-squares sum in this way:

$$\chi^2(\text{Cape Town}) = \sum \left[\frac{p_n - (d_n + s)}{\epsilon_n}\right]^2 + \left[\frac{s - 0.000}{0.005}\right]^2,$$

where d_n is a measured datum, ϵ_n is the corresponding experimentally determined standard deviation (not including the uncertainty in s), and p_n is the corresponding predicted value determined from our least-squares-fitted parameters. These fitted parameters included phase shifts, normalization parameters, and s. In addition, each of the three Cape Town data sets has its own multiplicative normalization parameter.

The *np* total cross section value at 25.0 MeV, listed in Table III as Bowen, 1961, Harwell, was assigned the value 370 ± 12 mb by us. This was obtained from the published data by fitting, folding-in the beam profile, and interpolating.

Some data listed in Tables I and III are seen to have been dropped by us because of "insignificant influence on the analysis due to the presence of higher-precision data." In each such case, and cumulatively for all of such dropped data, the shifts in the deduced phase parameter values, produced when the data in question were dropped, were small compared to the corresponding phase

TABLE I. The 20–30 MeV pp data set corrections, additions, and deletions. NA denotes not available, D denotes drop, K denotes keep, and P denotes preliminary values.

No., type	Energy (MeV)	First author, publ. year, laboratory	Recommendations Wilson	Nisimura	MAW-X	Present work	Footnotes	Data Ref.
1, $\sigma(\theta)$	21.95	Batty, 1964, Rutherford	NA	D	D	D	a, b, c	pp1
1, $\sigma(\theta)$	25.62	Batty, 1964, Rutherford	NA	K	D	K	a	pp1
1, $\sigma(\theta)$	30.33	Batty, 1964, Rutherford	NA	D	D	K	a, c	pp1
8, $P(\theta)$	20.2	Catillon, 1968, Saclay	NA	NA	K	K		pp2
1, $P(\theta)$	27.4	Christmas, 1963, Harwell	K	K	D	D	d	pp3
3, $R(\theta)$	27.6	Ashmore, 1965, Rutherford	NA	K	K	K	e	pp4
3, $A(\theta)$	27.6	Ashmore, 1965, Rutherford	NA	K	K	K		pp4
1, $C_{nn}(\theta)$	27.05	Jarmie, 1967, Los Alamos	NA	NA	K	K		pp5
1, A_{yy}/A_{xx}	23.45	Catillon, 1967, Saclay	NA	P	K	K	f	pp6
1, A_{yy}/A_{xx}	26.50	Catillon, 1967, Saclay	NA	P	K	K	f	pp6

[a] MAW drop the entire 21.95–30.33 MeV data set because of large χ^2.
[b] Large χ^2.
[c] Nisimura gives no explanations for dropping data.
[d] Insignificant influence on analysis due to presence of higher precision data.
[e] The 39° datum was dropped by us and MAW due to large χ^2.
[f] MAW-X use absolute A_{yy} and A_{xx} values.

parameter standard deviations.

The last columns of Tables II and IV list the ratios of the least-squares-error sums to their expected values for the various data subsets. One sees that some subsets with relatively large numbers of data have strikingly low χ^2 values; this implies that the corresponding experimental errors may have been substantially overestimated.

III. PHASE SHIFT PARAMETER SELECTION

The phase shifts and coupling parameters were all treated in the "nuclear bar" representation and were divided into three groups. For $L \geq 4$ the phases were represented by lower-wave-subtracted one-pion-exchange amplitudes. For the $L = 2$ and $L = 3$ phases, shown in Table VII, we used the new

TABLE II. The final 43-piece pp data set in the energy range 20–30 MeV. The ratio of χ^2 to its expected value, χ^2_{exp}, corresponds to the "present work" line of Table VIII.

No., type	Energy (MeV)	Angular range (deg)	Data error	Norm error (%)	First author, publ. year, laboratory	Data Ref.	χ^2/χ^2_{exp}
1, $\sigma(\theta)$	25.62	90	0.6%		Batty, 1964, Rutherford	pp1	0.07
23, $\sigma(\theta)$	25.63	10–90	0.73%–2.6%	0.93	Jeong, 1960, Minnesota	pp7	0.43
1, $\sigma(\theta)$	28.16	90	1.9%		Johnston, 1959, Minnesota	pp8	0.19
1, $\sigma(\theta)$	30.33	90	0.6%		Batty, 1964, Rutherford	pp1	0.02
8, $P(\theta)$	20.2	17–45	0.0006–0.003	12	Catillon, 1968, Saclay	pp2	0.18
1, $P(\theta)$	30	45	0.0033		Batty, 1963, Rutherford	pp9	3.51
2, $R(\theta)$	27.6	23–55	0.026–0.063	3	Ashmore, 1965, Rutherford	pp4	0.52
3, $A(\theta)$	27.6	23–55	0.012–0.090	3	Ashmore, 1965, Rutherford	pp4	1.53
1, $C_{nn}(\theta)$	27.05	90	0.07		Jarmie, 1967, Los Alamos	pp5	0.59
1, A_{yy}/A_{xx}	23.45	90	0.014		Catillon, 1967, Saclay	pp6	0.47
1, A_{yy}/A_{xx}	26.5	90	0.015		Catillon, 1967, Saclay	pp6	0.07

TABLE III. The 20–30 MeV np data set corrections, additions, and deletions. Na denotes not available, D denotes drop, K denotes keep, P denotes preliminary values, and E denotes erroneous values.

No., type	Energy (MeV)	First author, publ. year, laboratory	Recommendations Wilson	Nisimura	MAW-X	Present work	Footnotes	Data Ref.
3, σ_{tot}	19.565, 23.951, 27.950	Groce, 1966, Canberra	NA	NA	K	K		$np1$
3, σ_{tot}	20.6, 25.3, 28.3	Peterson, 1960, Livermore	D	D	K	D	a	$np2$
2, σ_{tot}	24.63, 29.25	Brady, 1970, Davis	NA	NA	NA	K		$np3$
1, σ_{tot}	25.0	Bowen, 1961, Harwell	K	K	K	D	a	$np4$
6, $\sigma(\theta)$	22.5	Scanlon, 1963, Harwell	K	D	K	D	a	$np5$
4, $\sigma(\theta)$	24.0	Rothenberg, 1970, Wisconsin	NA	NA	P	K		$np6$
2, $\sigma(\theta)$	24	Masterson, 1972, Wisconsin	NA	NA	NA	K		$np7$
4, $\sigma(\theta)$	24.0	Burrows, 1973, Wisconsin	NA	NA	NA	K		$np8$
5, $\sigma(\theta)$	27.2	Burrows, 1973, Wisconsin	NA	NA	NA	K		$np8$
3, $\sigma(\theta)$	27.5	Scanlon, 1963, Harwell	K	K	K	D	b	$np5$
8, $\sigma(\theta)$	27.5	Scanlon, 1963, Harwell	K	K	K	D	a	$np5$
9, $P(\theta)$	20.5	Langsford, 1965, Harwell	NA	D	K	D	a, c	$np9$
6, $P(\theta)$	21.1	Morris, 1974, Charlottesville	NA	NA	NA	K		$np10$
6, $P(\theta)$	21.6	Jones, 1974, Cape Town	NA	NA	NA	K	d, e	$np11$(D)
4, $P(\theta)$	21.6	Jones, 1974, Cape Town	NA	NA	NA	D	a, d	$np11$(E)
6, $P(\theta)$	21.6	Jones, 1974, Cape Town	NA	NA	NA	K	d	$np11$(F)
4, $P(\theta)$	21.6	Jones, 1974, Cape Town	NA	NA	NA	K	d	$np11$(G)
6, $P(\theta)$	22.5	Bowen, 1961, Harwell	E	E	D	D	a	$np4$
6, $P(\theta)$	23.1	Mutchler, 1971, Los Alamos	K	K	K	K	f, g	$np12$
2, $P(\theta)$	23.1	Mutchler, 1971, Los Alamos	NA	D	K	K	c, f, h	$np12$
4, $P(\theta)$	23.7	Benenson, 1962, Wisconsin	K	K	K	D	a	$np13$
3, $P(\theta)$	29.6	Mutchler, 1971, Los Alamos	NA	NA	NA	K		$np12$
6, $P(\theta)$	30	Bowen, 1961, Harwell	E	E	D	D	a	$np4$
12, $P(\theta)$	30	Langsford, 1965, Harwell	NA	D	K	K	c, i	$np9$
3, $D(\theta)$	23.1	Perkins, 1963, Los Alamos	D	D	D	D	a	$np14$
2, $C_{nn}(\theta)$	23.1	Malanify, 1966, Los Alamos	NA	D	K	K	c	$np15$
2, $C_{nn}(\theta)$	23.1	Simmons, 1967, Los Alamos	NA	NA	K	K	j	$np16$

[a] Insignificant influence on analysis due to presence of higher precision data.

[b] Proton-detection backward-angle data contains probable multiple-scattering errors (private communication from M. Saltmarsh).

[c] No reason given by Nisimura for dropping these data.

[d] The four data sets published in Ref. 1 ($np11$ of Table VI) had uncorrelated values for their additive constants "s" (mentioned in our Sec. II). Therefore, strictly speaking, the simple combination into one set shown in Ref. 1 was not justified (private communication from D. T. L. Jones and F. D. Brooks). The four sets were treated as independent in the present analysis.

[e] The 110° datum was dropped due to large χ^2.

[f] From a data summary in this reference.

[g] Wilson, Nisimura, and MAW-X use originally published values, Phys. Rev. 130, 272 (1963).

[h] MAW-X use the originally published 140° datum, Phys. Rev. Lett. 9, 481 (1966).

[i] Incorrectly separated into two parts in MAW-X.

[j] Published only in graphical form in review article. Actual values obtained by private communication: $C_{nn}(130°) = 0.130 \pm 0.042$, $C_{nn}(150°) = 0.050 \pm 0.018$.

TABLE IV. The final 65-piece (69 minus 4 floating norms) *np* data set in the energy range 20–30 MeV. The ratio of χ^2 to its expected value, χ^2_{exp}, corresponds to the "present work" line of Table VIII.

No. type	Energy (MeV)	Angular range (deg)	Data error	Norm error	First author, publ. year, laboratory	Data ref.	χ^2/χ^2_{exp}
3, σ_{tot}	19.565, 23.951, 27.950		0.4%–0.6%		Groce, 1966, Canberra	*np*1	0.66
2, σ_{tot}	24.63, 29.25		0.60%, 0.82%		Brady, 1970, Davis	*np*3	0.36
12, $\sigma(\theta)$	22.5	65–175	3.6%–12.0%	∞	Flynn, 1962, Los Alamos	*np*17	0.56
4, $\sigma(\theta)$	24.0	89–164	0.9%–1.5%	∞	Rothenberg, 1970, Wisconsin	*np*6	0.32
2, $\sigma(\theta)$	24	39, 50	1.6%, 2.0%		Masterson, 1972, Wisconsin	*np*7	0.15
4, $\sigma(\theta)$	24.0	71–158	1.2%–2.9%	∞	Burrows, 1973, Wisconsin	*np*8	0.53
5, $\sigma(\theta)$	27.2	71–158	1.3%–4.9%	∞	Burrows, 1973, Wisconsin	*np*8	0.44
6, $P(\theta)$	21.1	40–140	0.0027–0.011	3%	Morris, 1974, Charlottesville	*np*10	1.96
6, $P(\theta)$	21.6	50–170	0.002–0.018	1%, 0.005 [a]	Jones, 1974, Cape Town	*np*11(D)	1.73
6, $P(\theta)$	21.6	70–170	0.002–0.011	1%, 0.005 [a]	Jones, 1974, Cape Town	*np*11(F)	0.95
4, $P(\theta)$	21.6	90–150	0.002–0.009	2%, 0.005 [a]	Jones, 1974, Cape Town	*np*11(G)	2.34
6, $P(\theta)$	23.1	50–150	0.0035–0.015	4%	Mutchler, 1971, Los Alamos	*np*12	0.75
2, $P(\theta)$	23.1	140, 150	0.0060, 0.0095	20%	Mutchler, 1971, Los Alamos	*np*12	0.38
3, $P(\theta)$	29.6	60–120	0.0066–0.010	10%	Mutchler, 1971, Los Alamos	*np*12	0.65
2, $C_{nn}(\theta)$	23.1	140, 174	0.011, 0.024		Malanify, 1966, Los Alamos	*np*15	0.04
2, $C_{nn}(\theta)$	23.1	130, 150	0.018, 0.042		Simmons, 1967, Los Alamos	*np*16	0.17

[a] Norm error, additive constant error (see text).

two-pion-exchange values[7] computed from πn, $\pi\pi$, and *en* data via analyticity, crossing, and unitarity, with additional contributions from one-pion exchange and one-ω exchange. The two phases in Table VII shown with uncertainties were added to the data set as "data from theory." The standard deviations shown are 15% of the two-pion-exchange contributions and result from uncertainties in that calculation. No uncertainty was attached to the other phase parameters in Table VII because of the smallness of their 2π-exchange parts. The lowest-L partial wave phases, those shown in Table VIII, were searched upon in order to minimize the least-squares sum. The method used for minimizing χ^2 was that used previously.[8,9]

TABLE V. Burrows, 1973, Wisconsin *np* relative $\sigma(\theta)$ data, Ref. *np*8 in Table VI, as used in the present analysis. These values are from a private communication from T. W. Burrows; they contain one more digit of accuracy than those published. We wish to thank Dr. Burrows for permission to publish these values.

E_{lab}	$\theta_{c.m.}$ (deg)	$\sigma(\theta)/\sigma(158°)$
24.0 MeV	157.9	1.000 ± 0.012
	103.0	0.898 ± 0.026
	88.7	0.912 ± 0.017
	71.3	0.884 ± 0.015
27.2 MeV	157.8	1.000 ± 0.013
	117.0	0.924 ± 0.030
	111.4	0.885 ± 0.043
	88.6	0.864 ± 0.013
	71.3	0.846 ± 0.020

The dividing line between the two lowest angular momentum regions was determined by comparing the calculated "theoretical" values for the various phases at various energies to the corresponding values produced by single energy phase-shift analyses at the same energies.[10] The idea was to follow a particular theoretical phase shift up in energy until its value began to pull away from the experimental one and then consider the calculated value as valid up to that energy.

Since the data used in the analysis were at a number of energies between 20 and 30 MeV, the phase parameters had to be given an energy dependence. The highest-L group of parameters had values calculated at each energy separately, while the middle group had central-energy values, slopes, and curvatures calculated from theory. The lowest-L group, whose 25 MeV values were searched upon, had fixed slopes and curvatures calculated by us from the Hamada-Johnston potential.[11] The curvatures were found to have an insignificant effect on the analysis, however, so were discarded for all results reported here. The same

TABLE VI. Data references for Tables I–V.

*pp*1.	C. J. Batty, G. H. Stafford, and R. S. Gilmore, Nucl. Phys. 51, 225 (1964).
*pp*2.	P. Catillon, J. Sura, and A. Tarrats, Phys. Rev. Lett. 20, 602 (1968).
*pp*3.	P. Christmas and A. E. Taylor, Nucl. Phys. 41, 388 (1963).
*pp*4.	A. Ashmore, B. W. Davies, M. Devine, S. J. Hoey, J. Litt, and M. E. Shepherd, Nucl. Phys. 73, 256 (1965).
*pp*5.	N. Jarmie, J. E. Brolley, H. Kruse, H. C. Bryant, and R. Smythe, Phys. Rev. 155, 1438 (1967).
*pp*6.	P. Catillon, M. Chapellier, and D. Garreta, Nucl. Phys. B2, 93 (1967).
*pp*7.	T. H. Jeong, L. H. Johnston, D. E. Young, and C. N. Waddell, Phys. Rev. 118, 1080 (1960).
*pp*8.	L. H. Johnston and Y. S. Tsai, Phys. Rev. 115, 1293 (1959).
*pp*9.	C. J. Batty, R. S. Gilmore, and G. H. Stafford, Nucl. Phys. 45, 481 (1963).
*np*1.	D. E. Groce and B. D. Sowerby, Nucl. Phys. 83, 199 (1966).
*np*2.	J. M. Peterson, A. Bratenahl, and J. P. Stoering, Phys. Rev. 120, 521 (1960).
*np*3.	F. P. Brady, W. J. Knox, J. A. Jungerman, M. R. McGie, and R. L. Walraven, Phys. Rev. Lett. 25, 1628 (1970).
*np*4.	P. H. Bowen, J. P. Scanlon, G. H. Stafford, J. J. Thresher, and P. E. Hodgson, Nucl. Phys. 22, 640 (1961).
*np*5.	J. P. Scanlon, G. H. Stafford, J. J. Thresher, P. H. Bowen, and A. Langsford, Nucl. Phys. 41, 401 (1963).
*np*6.	L. N. Rothenberg, Phys. Rev. C 1, 1226 (1970).
*np*7.	T. G. Masterson, Phys. Rev. C 6, 690 (1972).
*np*8.	T. W. Burrows, Phys. Rev. C 7, 1306 (1973) and Table IX.
*np*9.	A. Langsford, P. H. Bowen, G. C. Cox, G. B. Huxtable, and R. A. J. Riddle, Nucl. Phys. 74, 241 (1965).
*np*10.	C. L. Morris, T. K. O'Malley, J. W. May, Jr., and S. T. Thornton, Phys. Rev. C 9, 924 (1974).
*np*11.	D. T. L. Jones and F. D. Brooks, Nucl. Phys. A222, 79 (1974); and private communications from D. T. L. Jones and F. D. Brooks. The letters (D), (E), (F), (G) refer to data set labels in this reference (see footnote d, Table III).
*np*12.	G. S. Mutchler and J. E. Simmons, Phys. Rev. C 4, 67 (1971).
*np*13.	W. Benenson, R. L. Walter, and T. H. May, Phys. Rev. Lett. 8, 66 (1962).
*np*14.	R. B. Perkins and J. E. Simmons, in *Comptes Rendus du Congres International De Physique Nucleaire II*, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, 1964), p. 164.
*np*15.	J. J. Malanify, P. J. Bendt, T. R. Roberts, and J. E. Simmons, Phys. Rev. Lett. 17, 481 (1966).
*np*16.	J. E. Simmons, Rev. Mod. Phys. 39, 542 (1967); and private communication.
*np*17.	E. R. Flynn and P. J. Bendt, Phys. Rev. 128, 1268 (1962).

was true of curvatures for the middle-L group.

Charge splitting of the isospin-one phases was introduced in two ways. First, the *pp* and *np* phases were computed via the Coulomb and Hamada-Johnston potentials, assuming the latter as a charge-independent strong interaction. The resulting nuclear bar phase-shift differences $\Delta_C \equiv \bar{\delta}(np) - \bar{\delta}(pp)$ are shown in Tables VII and VIII. These differences were maintained for all phases used in this analysis, excepting those labeled $^1S_0$ and $^3P_{LS}$. The splitting value for the $^1S_0$, $\Delta(^1S_0)$, was included in the set of parameters adjusted for a least-squares fit on the grounds that the *threshold* splitting of this phase is well known to be more than that produced by simple removal of the Coulomb potential.[12] The splitting value for the "spin-orbit" combination of P waves,[13] $\Delta(^3P_{LS})$, was included because of a suggestion by Morris *et al.*[2] that its value might be obtained from the new *np* $P(\theta)$ data. Finally, the splitting of the one-pion-exchange phases due to the pion mass splittings[12] was introduced. This latter splitting was found to have no significant effect on the analysis and so was discarded for all results reported here.

IV. ANALYSIS AND GENERAL RESULTS

The low-L phase shifts, corresponding to the minimized value of χ^2 for our final data set, are shown in Table VIII. The number of independent data was 110; 108 coming from the *np* and *pp* scattering experiments of Tables II and IV, and 2 from the $^3D_2$ and $^3D_3$ calculated particle-physics values of Table VII. The number of freely searched upon phase parameters was 13, so the number of degrees of freedom was 97. This, then, was the expected value of the least-squares sum, denoted χ^2_{exp}. The actual value of χ^2 was 72.5 so the ratio of χ^2 to its expected value was $\chi^2/\chi^2_{exp} = 0.75$, a very reasonable value.

Also shown in Table VIII are the values published by MacGregor, Arndt, and Wright (MAW-X).[5] The significant changes from the MAW-X phase parameters would appear to be these: (a) the $^1S_0$, $^3P_0$, $^3P_1$, and $\Delta(^1S_0)$ *central values* changed by about 1 standard deviation, while those for $^1P_1$, $^3S_1$, ϵ_1, and $^3D_1$ changed by more than 2 new standard deviations; (b) the phase parameter *standard deviation* for the $^3S_1$ dropped to 60% of its former value, while that for $^1P_1$ decreased to 70% of its former value; and (c) the standard deviation on the $^3P_0$ phase shift doubled. The shifts in central values are presumably due to the inclusion of new data and to the inclusion of the Coulomb splittings, while the lowering of phase shift uncertainties

TABLE VII. The intermediate-L phase parameters used in the analyses. The values are from our most recent particle-physics calculation and include the Coulomb splittings (Δ) which were computed via the Hamada-Johnston potential (see text). All values are nuclear bar, in degrees. The isospin-one values shown are pp; the np values may be obtained by adding the corresponding values of Δ.

Phase parameter	$^3D_2$	$^3D_3$	ϵ_3	$^1F_3$	ϵ_2	$^3F_2$	$^3F_3$	$^3F_4$
25 MeV value	3.89 ± 0.10	0.038 ± 0.023	0.590	−0.451	−0.833	0.114	−0.241	0.025
$\Delta \equiv \delta(np) - \delta(pp)$					−0.027	0.005	−0.009	0.001

should be due solely to the new data. The increase in the $^3P_0$ standard deviation is certainly due to the variable splitting introduced here.

V. SPIN-ORBIT COMBINATION RESULTS

One of the purposes of the new np $P(\theta)$ measurements was to determine the spin-orbit combination of P and D phases, independently of any theoretical assumptions about the individual P and D phases. The point here is that most models are in disagreement with the MAW-X values for the "central" combinations[13] of D-wave phases,

$$^3D_C \equiv (1/15)(3\,^3D_1 + 5\,^3D_2 + 7\,^3D_3)\,,$$

are in agreement for the "tensor" combination,[13]

$$^3D_T = (-7/120)(3\,^3D_1 - 5\,^3D_2 + 2\,^3D_3)\,,$$

and are in strong disagreement with the MAW-X values for the "spin-orbit" combination,[13]

$$^3D_{LS} = (-1/60)(9\,^3D_1 + 5\,^3D_2 - 14\,^3D_3)\,,$$

as shown in lines 1-6 and 9 and 10 of Table IX. The numerical values for the various models and for the two energy-dependent representations of MAW-X are shown in Table IX. Our "present work" combination values, corresponding to the values in Table VIII, are shown in line 7 of Table IX. The new "experimental" values for ^3D_C and $^3D_{LS}$ are seen to be in agreement with the models, rather than with those of MAW-X(Exp'l) or MAW-X (Constr). In fact, the MAW-X central values of $^3D_{LS}$ differ from the new "experimental" ones of line 7 by five new-analysis standard deviations.

Jones and Brooks[1] suggested that the value of $^3D_{LS}$ should be very well pinned down by the data, regardless of whether the other D-wave parameters are fixed or not. In line 8 of Table IX we show the result of removing the $^3D_2$ and $^3D_3$ theoretical phase shift values of Table VII from the data set, allowing them to be free parameters. Line 8 shows that $^3D_{LS}$ is indeed still very well defined at 25 MeV. Even in this case, the probability of the MAW-X $^3D_{LS}$ being correct is quite small.

Finally, in Table X we examine the effect of various combinations of the np $P(\theta)$ data on the uncer-

TABLE VIII. The 25 MeV low-L phase parameters corresponding to a least-squares fit to the 110 data of Tables II, IV, and VII. The fixed Coulomb splittings Δ are defined in Table VII. All phase parameter values shown are nuclear bar, in degrees. The isospin-one values shown are pp; the np values may be obtained by adding the corresponding values of Δ. Values for $^3D_2$ and $^3D_3$ are not shown because they turned out to be identical to the "theoretical data" values used (see Table VII).

Phase parameter	$^1S_0$	$^3P_0$	$^3P_1$	$^3P_2$	$^1D_2$
$\Delta \equiv \delta(np) - \delta(pp)$	(See below)	0.28	−0.22	0.16	0.031
Present work	48.80 ± 0.24	7.67 ± 0.68	−4.84 ± 0.16	2.48 ± 0.12	0.720 ± 0.029
MAW-X	48.60 ± 0.26	8.52 ± 0.31	−5.04 ± 0.15	2.45 ± 0.08	0.74 ± 0.03

Phase parameter	$\Delta(^1S_0)$	$\Delta(^3P_{LS})$	$^3S_1$	$^1P_1$	ϵ_1	$^3D_1$
Present work	2.2 ± 1.5	−0.3 ± 0.15	81.0 ± 1.6	−5.18 ± 0.47	1.03 ± 0.58	−2.91 ± 0.09
MAW-X	0.2 ± 1.8	···	84.5 ± 2.7	−4.00 ± 0.69	−0.34 ± 0.73	−3.21 ± 0.18

TABLE IX. Values of the central, tensor, and spin-orbit combinations of the nuclear bar $^3D_1$, $^3D_2$, and $^3D_3$ phase shifts, in degrees. Line 7 corresponds to the "present work" line of Table VIII and uses the theory data of Table VII. PW denotes present work.

Line	Model	Ref.	^3D_C	^3D_T	$^3D_{LS}$	$^3P_{LS}(pp)$	Δ^3P_{LS}
1	Hamada-Johnston	a	0.69	1.60	0.13	0.70	0.08
2	Yale	b	0.80	1.57	0.12	0.92	0.09
3	Lomon-Feshbach	c	0.75	1.70	0.12	0.59	−0.06
4	Bryan-Scott III	d	0.63	1.46	0.11	0.80	0.07
5	Bryan-Gersten ("D")	e	0.81	1.70	0.12	0.80	0.08
6	de Tourreil-Sprung	f	0.75	1.58	0.13	0.75	0.08
7	Exp'l, $^3D_2$ and $^3D_3$ as theory data	PW	0.73	1.64	0.121	0.97	−0.03
			±0.03	±0.04	±0.008	±0.15	±0.15
8	Exp'l, $^3D_1$, $^3D_2$, $^3D_3$ free	PW	0.8	1.7	0.121	0.90	0.03
			±1.0	±2.8	±0.018	±0.15	±0.15
9	MAW-X ("Exp'l")	g	0.95	1.65	0.085	0.98	···
10	MAW-X ("Constr")	g	0.95	1.59	0.077	···	···

[a] See Ref. 10.
[b] K. E. Lassila, M. J. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. 126, 881 (1962).
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[g] See Ref. 4.

tainties in the spin-orbit and $\bar{\epsilon}_1$ phase parameters. The new (1974) data are seen to have caused a dramatic narrowing of the uncertainty in $^3D_{LS}$ to 33% of its former value, but have had little impact on $^3P_{LS}$ and $\bar{\epsilon}_1$. Morris *et al.*[14] noted a normalization discrepancy between their data and those of Jones and Brooks. This showed up in our analysis as: (1) substantially negative analysis values for the Jones-Brooks normalization constants "s" (−0.010, −0.007, and −0.010; all ±0.005, for the three sets D, F, and G); and (2) a drop of 14.1 in the x^2 of the analysis when the three Jones-Brooks s values were made infinitely uncertain (when the data were "floated"). Only the $^3P_{LS}$ splitting seems to have been affected by this normalization discrepancy (see Table X, line 6). Note, however, that all of the $^3P_{LS}$ splitting values shown in Table X are consistent with zero splitting.

TABLE X. The effect of the new np $P(\theta)$ data on the $^3P_{LS}$, $^3D_{LS}$, and $\bar{\epsilon}_1$ phase parameters. All values shown are nuclear bar, in degrees. Line 5 corresponds to both line 7 of Table IX and the "present work" line of Table VIII. The $^3P_{LS}$ combination is defined (Ref. 13) as $^3P_{LS} = (-1/12)(2^3P_0 - 3^3P_1 + 5^3P_2)$. Line 6 shows the effect of setting the errors on the Jones-Books normalization constants "s" equal to $\pm\infty$ ("floating" these data).

np $P(\theta)$ data used	$^3D_{LS}$	$^3P_{LS}$	$\Delta(^3P_{LS})$	$\bar{\epsilon}_1$
1 None	−0.12 ±0.85	0.97±0.15	0.3 ±4.8	1.01±0.60
2 Mutchler (1971)	0.114±0.024	0.97±0.15	−0.06±0.16	1.01±0.57
3 Mutchler and Morris (1974)	0.120±0.012	0.96±0.15	−0.10±0.15	1.02±0.57
4 Mutchler and Jones (1974)	0.120±0.010	0.98±0.15	0.05±0.15	1.00±0.57
5 Mutchler, Morris, and Jones	0.121±0.008	0.97±0.15	−0.03±0.15	1.03±0.57
6 Mutchler, Morris, and Jones (Jones floated)	0.120±0.008	0.97±0.15	−0.09±0.15	1.03±0.57

VI. RESULTS FOR $\bar{\epsilon}_1$

The parameter ϵ_1, which couples the $^3S_1$ and $^3D_1$ states,[15] has long been a problem at low energies. A recent manifestation of this problem was the negative "experimental" value for $d\epsilon_1/dE$ near threshold found by MAW-*X*: this violated expectations from the deuteron data, analyticity, and models.[16] In fact, comparison to a number of models[13] shows only a very narrow range of *positive* values at 25 MeV for all models which satisfy the deuteron data; the Hamada-Johnston value of $\bar{\epsilon}_1$(25 MeV) = 1.93° is typical. It is therefore gratifying that the somewhat negative "experimental" value found by MAW-*X*, and quoted in our Table VIII, has now become a fairly definite positive value (Table VIII). The importance of the parameter $\bar{\epsilon}_1$ for nuclear calculations has been discussed elsewhere.[15]

VII. SECOND DERIVATIVE MATRIX

It has been noted by Arndt and MacGregor[17] that a considerable saving of time and effort can be effected by matching models to the second derivative matrices $s_{ij} = \partial^2\chi^2/\partial\delta_i\partial\delta_j$ rather than to the experimental data. In Table XI we display the matrix $\alpha_{ij} \equiv \frac{1}{2}s_{ij}$ in the format developed by MacGregor, Arndt, and Wright.[18] Since modelists are unlikely to introduce splittings other than that produced by the Coulomb potential, only the latter was retained in the calculation of that matrix. Thus, for example, one should not expect the standard deviations quoted in Table VIII to be given exactly by the square roots of the diagonal elements of the error matrix in Table XI.

VIII. FURTHER EXPERIMENTS

The small discrepancy between the data sets of Jones *et al.* and Morris *et al.* could be fairly well resolved by, for example, measurements of *np* *P*(70°) and *P*(120°) to an accuracy of ±0.001. However, it is not at all clear that investiture of the considerable effort and funds necessary for such an experiment could be justified at this time.

It would be nice to have a smaller uncertainty on $\bar{\epsilon}_1$, but a few attempts to find a relatively inexpensive experiment which would do the job were unsuccessful. Further work with interested experimentalists might be more fruitful, however.

We would like to express our deep appreciation to D. T. L. Jones and F. D. Brooks for their patience and cooperation in assisting us with the proper interpretation of their experimental uncertainties.

TABLE XI. The second derivative (lower left) and error (upper right) matrices corresponding to "present work" of Table VIII but with fixed splittings given solely by the Coulomb effect in the Hamada-Johnston potential. Values of the second derivative matrix are in deg^{-2}, of the error matrix in deg^2.

	$^1S_0$	$^1D_2$	$^3P_0$	$^3P_1$	$^3P_2$	$^1P_1$	$^3S_1$	ϵ_1	$^3D_1$	$^3D_2$	$^3D_3$	
	4.9754 − 2	−2.7907 − 4	−6.2212 − 2	2.9674 − 2	−3.9870 − 3	1.2926 − 2	−7.0187 − 3	−4.1043 − 3	−4.2619 − 4	1.6590 − 5	7.0771 − 6	$^1S_0$
		8.1239 − 4	2.3623 − 3	−6.1636 − 4	5.3886 − 4	1.1388 − 3	−5.8757 − 4	2.0278 − 4	7.8527 − 6	−8.2732 − 6	−4.1382 − 7	$^1D_2$
$^1S_0$	7.8478 + 1		1.1756 − 1	−4.9305 − 2	6.6022 − 3	−2.2311 − 2	−2.7830 − 2	1.9634 − 2	6.1820 − 4	−1.9928 − 4	−2.2747 − 5	$^3P_0$
$^1D_2$	−9.4328 + 1	1.7027 + 3		2.6077 − 2	−2.8350 − 3	6.9843 − 3	5.2486 − 3	−2.6322 − 3	−3.7685 − 4	1.7505 − 5	1.6358 − 6	$^3P_1$
$^3P_0$	2.3878 + 1	−8.8351 + 1	5.4507 + 1		2.1896 − 3	4.9612 − 4	1.2236 − 3	−1.2116 − 3	1.1302 − 4	1.7049 − 5	2.8107 − 6	$^3P_2$
$^3P_1$	−4.2155 + 1	−5.9278 + 1	7.2475 + 1	2.2198 + 2		2.0945 − 1	8.7319 − 2	−1.6051 − 1	−8.0191 − 3	8.6800 − 3	1.6766 − 4	$^1P_1$
$^3P_2$	3.8098 + 1	−4.0203 + 2	−7.9186	3.7062	6.5832 + 2		6.3766 − 1	−1.1260 − 1	1.4916 − 3	−5.2362 − 3	4.5215 − 4	$^3S_1$
$^1P_1$	−2.4604	−1.7175 + 1	6.2471 − 3	6.5602 − 1	1.6671	9.9075		2.5451 − 1	7.9504 − 3	−7.4204 − 3	−3.2824 − 5	ϵ_1
$^3S_1$	2.2337	−1.3905	1.5668	8.6329 − 2	−8.6989 − 1	−3.9350 − 1	1.8218		7.2737 − 3	−5.5105 − 3	8.1482 − 4	$^3D_1$
ϵ_1	−1.2517	−9.8265	−2.4558	−3.7369	5.7793	5.8813	5.1714 − 1	8.1244		9.9708 − 3	−1.5608 − 6	$^3D_2$
$^3D_1$	6.3839 − 1	7.1810 − 1	7.2528	1.3950 + 1	−2.4459 + 1	3.2881	2.2764 − 1	−3.8193	3.4659 + 2		5.2880 − 4	$^3D_3$
$^3D_2$	3.0130	7.9012	3.8590	5.4734	−1.2807 + 1	−2.6526	1.8375	−9.7069 − 1	1.8604 + 2	2.0571 + 2		
$^3D_3$	−2.3514	6.0558	−1.0885 + 1	−1.9064 + 1	3.3548 + 1	−7.5053	−1.7055	3.9639	−5.3459 + 2	−2.8667 + 2	2.7175 + 3	

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