"Moon crescent effect" in transfer form factors*

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It is shown that, when a nucleus is subdivided into a core and a cloud for the calculation of transfer form factors, one should allow for the possible displacement of the core with respect to the center of mass of the entire nucleus. This leads to corrections, which are estimated for a soluble model.

[NUCLEAR REACTIONS Generator coordinates prove core is not center of composite nucleus for calculation of DWBA form factor.]

I. INTRODUCTION

The physical picture associated with the transfer of a few nucleons ν from a nucleus B is usually as shown in Fig. 1: the residual nucleus A, considered as the core of B, is located at its center, while the transferred nucleons, which come from the outer shells of B, form a cloud around A. There is, *a priori*, no reason to disregard the situation illustrated in Fig. 2, where the transferred cloud is located on a side of B. If the process described schematically in Fig. 1 is indeed dominant, this should come out of the form factor calculated without such an assumption.

It must be stressed that the "moon crescent" mechanism for transfer[1] shown in Fig. 2 is not meant to imply an excitation of the core (although it may entail very interesting excitations of the cloud). Furthermore, it should be emphasized that while such a mechanism is a specific recoil effect, it has not been considered so far in various calculations devoted to the study of recoil corrections.[2] The latter have been concerned with the replacement of the traditional threefold integrals by sixfold ones, and do not question the dynamical definition of the transfer form factor. This paper presents a dynamical study of the influence of that new mechanism on the form factor itself. There is only one (major) physical assumption in our approach: we assume that the distorted wave Born approximation (DWBA) is valid and use its definition of the form factor. We work in the generator coordinate formalism,[3] which is very convenient and introduces no additional physical assumption.

Section II briefly recalls the definition of the DWBA form factor and of the spectroscopic factors. Section III compares the correct DWBA form factor with those obtained under additional approximations. Section IV presents an illustrative example of the considerations of Secs. II and III in the framework of an analytical model (harmonic oscillator orbitals). Our results are briefly discussed in Sec. V.

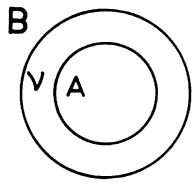

FIG. 1. Picture usually associated with the transfer of ν nucleons from a nucleus B, A being the residual nucleus.

II. REDUCTION OF THE DWBA FORM FACTOR

The form factor of a reaction $A+a \to B+b$, where, in an obvious notation, $B=A+\nu$ and $a=b+\nu$, can be defined as

$$W(\vec{S}',\vec{S}) = \int d\vec{r}_A d\vec{r}_b d\vec{r}_\nu \, \Psi_B^*(\vec{r}_A-\vec{s}_B, \vec{r}_\nu-\vec{s}_B)\Psi_b^*(\vec{r}_b-\vec{s}_b) \times V(\vec{r}_\nu-\vec{r}_A)\Psi_a(\vec{r}_b-\vec{s}_a, \vec{r}_\nu-\vec{s}_a)\Psi_A(\vec{r}_A-\vec{s}_A) . \quad (1)$$

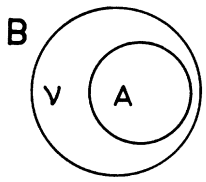

FIG. 2. The "moon crescent effect" for the reaction considered in Fig. 1.

In this expression, $\vec{r}_A$, $\vec{r}_b$, and $\vec{r}_\nu$ are the N_A, N_b, and N_ν coordinates of A, b, and ν, respectively, while the shell-model wave functions Ψ_B, Ψ_b, Ψ_a, and Ψ_A describe the nuclei B, b, a, and A shifted, respectively, by the amounts $\vec{s}_B = N_b\vec{S}'/N$, $\vec{s}_b = -N_B\vec{S}'/N$, $\vec{s}_a = -N_A\vec{S}/N$, and $\vec{s}_A = N_a\vec{S}/N$.

Although the coordinates $\vec{r}_A$ and $\vec{r}_\nu$ have been distinguished inside Ψ_B, the wave function Ψ_B is antisymmetric with respect to the whole set $\vec{r}_A$ and $\vec{r}_\nu$. The same property holds for $\vec{r}_b$ and $\vec{r}_\nu$ in Ψ_a. Finally, the interaction between the cloud ν and the core A, which is the driving potential responsible for the transfer, reads, explicitly,

$$V(\vec{r}_\nu - \vec{r}_A) = \sum_{j=1}^{N_A}\sum_{i=1}^{N_\nu} v(\vec{r}_{\nu i} - \vec{r}_{Aj}).$$

Equivalently, one may use the potential $V(\vec{r}_\nu - \vec{r}_b)$. There is nothing more in Eq. (1) than the standard DWBA expressed in the generator coordinate formalism. The form factor to be integrated with usual optical waves is actually a defolded one,[3] $w = \Gamma_{Bb}^{-1} W \Gamma_{Aa}^{-1}$, but this subtlety is irrelevant for the present discussion.

It is interesting to express Eq. (1) in the formalism of second quantization. Let $A^\dagger(\vec{s}_A)$, $a^\dagger(\vec{s}_a)$, $b^\dagger(\vec{s}_b)$, and $B^\dagger(\vec{s}_B)$ be the creation operators for nuclei A, a, b, and B at positions $\vec{s}_A$, $\vec{s}_a$, $\vec{s}_b$, and $\vec{s}_B$, respectively. For instance, when there is configuration mixing, $A^\dagger(\vec{s}_A)$ is a sum of products of N_A creation operators $\eta^\dagger(\vec{s}_A)$ for shell-model single particle states centered at $\vec{s}_A$. The equivalent of Eq. (1) is[3,4]

$$W(\vec{S}', \vec{S}) = \tfrac{1}{2}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|v|\gamma\delta\rangle \langle 0|B^{123}(\vec{s}_B)^4 b(\vec{s}_b)^1 \eta_\alpha^{\dagger\,2}\eta_\beta^\dagger \times {}^5\eta_\delta{}^6\eta_\gamma a^{46\dagger}(\vec{s}_a)^3 A^{5\dagger}(\vec{s}_A)|0\rangle, \quad (2)$$

where $\langle\alpha\beta|V|\gamma\delta\rangle$ is the usual antisymmetrized matrix element of the two-particle interaction on any orthonormal basis. The superscripts show how the set of allowed contractions must be limited and pairs of identical superscripts must be understood as "arrows". For instance, the superscript "arrow 5" from the left side of η_δ to the right side of $A^\dagger$ means that η_δ can be contracted only with a creation operator $\eta^\dagger$ contained in $A^\dagger$. Similarly, the superscript arrow 3 from the left side of $A^\dagger$ to the right side of B means that the creation operators left in $A^\dagger$ after the contraction with η_δ can be contracted only with annihilation operators available in B, and so on.

Although these restrictions pick out the same two-body matrix elements that appear in (1), the normalization of the wave functions is not the same in both expressions: in (2) it corresponds to complete antisymmetrization between all the nucleons, in (1) to antisymmetrization between the nucleons of each fragment only. If both expressions are to be equal, one of them should contain an inessential (and, here, irrelevant) constant that we have dropped so that the expressions do not become unduly cumbersome.

The physical interpretation of the rules expressed by the arrows is the following: (i) η_γ and η_δ are contracted separately within $a^\dagger$ and $A^\dagger$, respectively, since we have chosen, e.g., the "prior" representation in which the interaction between the fragments of the incoming channel is switched off at infinity; (ii) the rest of $A^\dagger$ is contracted within B, since nucleus A is understood as the core of nucleus B; accordingly, one among the two creation operators $\eta_\alpha^\dagger$ and $\eta_\beta^\dagger$ must be contracted within B, since that contraction corresponds to the core side of the core-cloud driving potential V; (iii) the remaining creation operator $\eta_\alpha^\dagger$ or $\eta_\beta^\dagger$ is contracted within B also and corresponds to the cloud side of the core-cloud driving potential V; accordingly, the only possibility for b is to be contracted within $a^\dagger$.

Another way of grasping the physical interpretation of these rules is to take notice of the fact that, in Eq. (1) the coordinates $\vec{r}_A$ and $\vec{r}_\nu$ have been put into Ψ_B, and the coordinates $\vec{r}_b$ and $\vec{r}_\nu$ into Ψ_a. Of course, antisymmetrization between A and a is broken. It is broken also between B and b. It is conserved only within A, a, B, and b separately.

We now turn to the task of breaking the calculations described by Eqs. (1) and (2) into simpler calculations, involving products of terms which can receive, as much as possible, a physically useful interpretation. In particular, we would like to break the highly restricted set of contractions of Eq. (2) into a product of unrestricted sets of contractions, thus easier to handle through Wick's theorem.

A trivial way of rewriting Eq. (1) is

$$W(\vec{S}', \vec{S}) = \int d\vec{r}_\nu \Big[\int d\vec{r}_A \Psi_B^*(\vec{r}_A - \vec{s}_B, \vec{r}_\nu - \vec{s}_B) V(\vec{r}_\nu - \vec{r}_A) \Psi_A(\vec{r}_A - \vec{s}_A) \Big] \chi^{a/b}(\vec{r}_\nu, \vec{s}_a, \vec{s}_b), \quad (3)$$

where

$$\chi^{a/b}(\vec{r}_\nu, \vec{s}_a, \vec{s}_b) = \int d\vec{r}_b \Psi_b^*(\vec{r}_b - \vec{s}_b) \Psi_a(\vec{r}_b - \vec{s}_a, \vec{r}_\nu - \vec{s}_a) \quad (4)$$

is the cloud wave function of nucleus a having nucleus b as its core. There is of course a typical moon-crescent effect in Eq. (4) since a and b are not centered at the same origin.

In the formalism of second quantization, the analog of Eq. (3) is

$$W(\vec{S}', \vec{S}) = \tfrac{1}{2} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | V | \gamma\delta \rangle \langle 0 | B(\vec{s}_B) \eta_\alpha^\dagger \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger}(\vec{s}_A) \eta_\gamma b(\vec{s}_b) a^\dagger(\vec{s}_a) | 0 \rangle , \tag{5}$$

which is obtained from Eq. (2) by commuting $A^\dagger$ with $a^\dagger$, η_γ with $A^\dagger$ (since η_γ can only be contracted with $a^\dagger$), and, finally, b with $\eta_\alpha^\dagger \eta_\beta^\dagger \eta_\delta A^\dagger \eta_\gamma$ (since b cannot be contracted with either $\eta_\alpha^\dagger$, $\eta_\beta^\dagger$, or $A^\dagger$). Each of these commutations introduces a phase. The resulting over-all (and, here, irrelevant) phase has been omitted in Eq. (5) and so will be all new over-all phases in the following equations. It is gratifying to notice that all the restrictions that appeared in Eq. (2) are now useless, except that going from η_δ to $A^\dagger$.

Let now $\{\varphi_m(\vec{r}_\nu)\}$ be any complete set of antisymmetric wave functions for the N_ν coordinates $\vec{r}_\nu$, and let $\{D_m^\dagger\}$ be the corresponding set of creation operators. For instance, $D_m^\dagger$ may be a product $\eta_{\alpha_1}^\dagger \eta_{\alpha_2}^\dagger \cdots \eta_{\alpha_{N_\nu}}^\dagger$, where $m \equiv (\alpha_1, \ldots, \alpha_{N_\nu})$ is ordered, or it may be any suitable configuration mixing of such products. Inserting $\sum_m D_m^\dagger D_m$ between η_γ and b in the last factor of Eq. (5), one obtains

$$\langle 0 | B \eta_\alpha^\dagger \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} \eta_\gamma b a^\dagger | 0 \rangle = \sum_m \langle 0 | B \eta_\alpha^\dagger \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} \eta_\gamma D_m^\dagger | 0 \rangle \langle 0 | D_m b a^\dagger | 0 \rangle . \tag{6}$$

The quantity $\langle 0 | D_m b a^\dagger | 0 \rangle$ is nothing but a spectroscopic amplitude and

$$\chi^{a/b}(\vec{r}_\nu, \vec{s}_a, \vec{s}_b) = \sum_m \varphi_m(\vec{r}_\nu) \langle 0 | D_m b a^\dagger | 0 \rangle . \tag{7}$$

It will be noticed again that the spectroscopic amplitude depends on $\vec{s}_a$ and $\vec{s}_b$.

The remaining quantity of interest is now

$$[\chi' U]^{B/A}(\vec{r}_\nu, \vec{s}_B, \vec{s}_A) = \int d\vec{r}_A \Psi_B^*(\vec{r}_A - \vec{s}_B, \vec{r}_\nu - \vec{s}_B) V(\vec{r}_\nu - \vec{r}_A) \Psi_A(\vec{r}_A - \vec{s}_A) , \tag{8}$$

which is the product of the cloud wave function of B having A as its core, and an average nuclear field (analogous to a Hartree-Fock potential) generated by the folding of $V(\vec{r}_\nu - \vec{r}_A)$ with the density of core A. In terms of contractions, it is interesting to consider the remaining factor of Eq. (6):

$$\langle 0 | B \eta_\alpha^\dagger \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} \eta_\gamma D_m^\dagger | 0 \rangle = \langle 0 | B \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle , \tag{9}$$

where the commutation of $\eta_\alpha^\dagger$ with η_δ and $A^\dagger$ is possible since η_δ is blocked with $A^\dagger$. The insertion of a complete set $\sum_{m'} D_{m'}^\dagger D_{m'}$ between $A^\dagger$ and $\eta_\alpha^\dagger$ in Eq. (9) yields

$$\langle 0 | B \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle = \sum_{m'} \langle 0 | B \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} D_{m'}^\dagger | 0 \rangle \langle 0 | D_{m'} \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle . \tag{10}$$

Finally, the restriction arrow becomes unnecessary if one writes

$$\langle 0 | B \eta_\beta^{\dagger 5} \eta_\delta A^{5\dagger} D_{m'}^\dagger | 0 \rangle = \langle 0 | B D_{m'}^\dagger \eta_\beta^\dagger \eta_\delta A^\dagger | 0 \rangle . \tag{11}$$

As a consequence of Eq. (10) one finds, except for an over-all multiplicative constant, the alternate definition

$$[\chi' U]^{B/A}(\vec{r}_\nu, \vec{s}_B, \vec{s}_A) = \sum_{mm'} \sum_{\alpha\beta\gamma\delta} \langle \alpha\beta | v | \gamma\delta \rangle \langle 0 | B D_{m'}^\dagger \eta_\beta^\dagger \eta_\delta A^\dagger | 0 \rangle \times \langle 0 | D_{m'} \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle \varphi_m^*(\vec{r}_\nu) . \tag{12}$$

Thus, when the form factors are calculated in the formalism of second quantization, Wick's theorem can be used without any restriction in $\langle 0 | D_m b a^\dagger | 0 \rangle$, $\langle 0 | D_m \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle$, and $\langle 0 | B D_{m'}^\dagger \eta_\beta^\dagger \eta_\delta A^\dagger | 0 \rangle$. As pointed out earlier, the first of these factors contains the physical information that results from integrating over $\vec{r}_b$ in Eqs. (1) and (4). It is equally obvious that the quantities $\langle 0 | B D_{m'}^\dagger \eta_\beta^\dagger \eta_\delta A^\dagger | 0 \rangle$ determine the average potential U due to core A through integration over $\vec{r}_A$. Finally, the factors $\langle 0 | D_{m'} \eta_\alpha^\dagger \eta_\gamma D_m^\dagger | 0 \rangle$ correspond to the matrix element of U between the clouds thus extracted from B and a, or, in other words, to the integration over $\vec{r}_\nu$.

III. APPROXIMATIONS

From now on, we concentrate on the calculation of the function $[\chi' U]^{B/A}$ and the corresponding matrix element, Eqs. (8) or (11). These quantities are more complicated than $\chi^{a/b}$ and $\langle 0 | D_m b a^\dagger | 0 \rangle$, and are more important to analyze. An exact calculation of $[\chi' U]^{B/A}$ is possible in special cases such as the one considered in Sec. IV, but most often approximations are necessary.

Let $\mathcal{O}(\vec{r}_A, \vec{r}_A')$ be an arbitrary projector in the space of antisymmetric functions of the coordinate $\vec{r}_A$. An approximation of $[\chi' U]^{B/A}$ is

$$[\chi' U]_{\mathcal{O}}^{B/A} = \int d\vec{r}_A d\vec{r}'_A \Psi_B(\vec{r}_A - \vec{s}_B, \vec{r}_\nu - \vec{s}_B) \times \mathcal{O}(\vec{r}_A, \vec{r}'_A) V(\vec{r}_\nu - \vec{r}'_A) \Psi_A(\vec{r}'_A - \vec{s}_A) . \tag{13}$$

As a special case for $\mathcal{O}$ it is interesting to consider the projector upon all positions of the unexcited core A,

$$\mathcal{O}_A(\vec{r}_A, \vec{r}'_A) = \int d\vec{s}\, d\vec{s}' \Psi_A(\vec{r}_A - \vec{s}) \mathfrak{N}_A^{-1}(\vec{s}, \vec{s}') \Psi_A^*(\vec{r}'_A - \vec{s}') , \tag{14}$$

where $\mathfrak{N}_A^{-1}$ is the inverse of the overlap kernel

$$\mathfrak{N}_A(\vec{s}, \vec{s}') = \int d\vec{r}_A \Psi_A^*(\vec{r}_A - \vec{s}) \Psi_A(\vec{r}_A - \vec{s}') . \tag{15}$$

The presence of $\mathfrak{N}_A^{-1}$ in Eq. (14) is typical of the generator coordinate method. An insertion of Eq. (14) into Eq. (13) gives

$$[\chi' U]_{\mathcal{O}_A}^{B/A} = \int d\vec{s}\, d\vec{s}' \mathfrak{N}_A^{-1}(\vec{s}, \vec{s}') \left[\int d\vec{r}_A \Psi_B^*(\vec{r}_A - \vec{s}_B, \vec{r}_\nu - \vec{s}_B) \Psi_A(\vec{r}_A - \vec{s}) \right] \left[\int d\vec{r}'_A \Psi_A^*(\vec{r}'_A - \vec{s}') V(\vec{r}_\nu - \vec{r}'_A) \Psi_A(\vec{r}'_A - \vec{s}_A) \right] . \tag{16}$$

The first bracket on the right-hand side of Eq. (16) is a cloud wave function, quite analogous to $\chi^{a/b}$, Eq. (4), and the second bracket is clearly the direct term of an average potential calculated, however, with two wave functions at different positions. Moon crescent effects are again present in Eq. (16).

In the language of second quantization, the projector $\mathcal{O}_A$ reads

$$\mathcal{O}_A = \int d\vec{s}\, d\vec{s}' A^\dagger(\vec{s}) \mathfrak{N}_A^{-1}(\vec{s}, \vec{s}') A(\vec{s}') . \tag{17}$$

The analog of the insertion that leads to Eq. (16) is

$$\langle 0 | B D_{m'}^\dagger \mathcal{O}_A \eta_\beta^\dagger \eta_\delta A^\dagger | 0 \rangle = \int d\vec{s}\, d\vec{s}' \mathfrak{N}_A^{-1}(\vec{s}, \vec{s}') \langle 0 | B(\vec{s}_B) D_{m'}^\dagger A^\dagger(\vec{s}) | 0 \rangle \langle 0 | A(\vec{s}') \eta_\beta^\dagger \eta_\delta A^\dagger(\vec{s}_A) | 0 \rangle , \tag{18}$$

where one recognizes at once a spectroscopic factor and a typical contraction for an average field.

In the following we refer to the use of the projector $\mathcal{O}_A$ as the $\mathcal{O}_A$ approximation. Further approximations can be made in order to skip the integrals over $\vec{s}$ and $\vec{s}'$, namely, (1) set $\vec{s} = \vec{s}' = \vec{s}_A$, which retains only the situation shown in Fig. 3, characterized by the usual average field for A and an off-centered spectroscopic factor; (2) set $\vec{s} = \vec{s}' = \vec{s}_B$, as illustrated in Fig. 4, which yields the usual spectroscopic factor and an off-centered average field of A; (3) set $\vec{s} = \vec{s}_B$ and $\vec{s}' = \vec{s}_A$ (we are no longer dealing with a projector, then), which gives a usual spectroscopic factor and the usual average field (Fig. 5).

We will refer to these as approximations 1, 2, and 3, respectively. Actually, as will be shown in Sec. IV, there are cases where the integration over $\vec{s}$ and $\vec{s}'$ can be performed without difficulty, taking simultaneously into account these three approximations and even more.

IV. AN ANALYTICAL EXAMPLE

We consider here a fictitious nucleus B made of three identical fermions; say, neutrons with spin up. The wave function Ψ_B in the model is a Slater determinant containing a $0s$, a $0p_y$, and a $0p_z$ orbital,

$$\varphi_1(r) = \pi^{-3/4} \lambda^{-3/2} \exp(-r^2/2\lambda^2) ,$$

$$\varphi_2(r) = \pi^{-3/4} \lambda^{-5/2} 2^{1/2} y \exp(-r^2/2\lambda^2) ,$$

$$\varphi_3(r) = \pi^{-3/4} \lambda^{-5/2} 2^{1/2} z \exp(-r^2/2\lambda^2) .$$

(The choice is obviously arbitrary. One could select more traditional $0p_{\pm 1}$ orbitals.) The wave function Ψ_A is chosen to contain the $0s$ and the

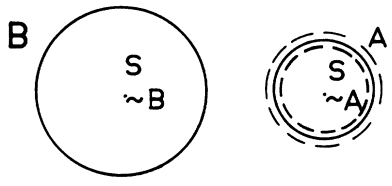

FIG. 3. Positions of A and B in approximation 1. The full lines correspond to the spectroscopic factor, the dashed lines to the average field.

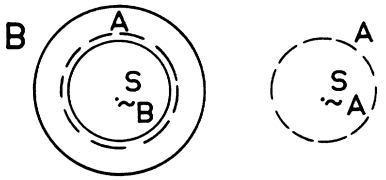

FIG. 4. Analog of Fig. 3 for approximation 2.

$0p_y$ orbitals. Depending on whether one considers Ψ_B or Ψ_A, the orbitals φ must be translated by $\vec{s}_B$ or $\vec{s}_A$, respectively. The coordinates $\vec{r}_1$ and $\vec{r}_2$ are attributed to Ψ_A, and thus $\vec{r}_\nu = \vec{r}_3$. Finally, the nuclear interaction $V(\vec{r}_3 - \vec{r}_1) + V(\vec{r}_3 - \vec{r}_2)$ is chosen as Gaussian, $V(\vec{r}_i - \vec{r}_j) = V_0 \exp[-(\vec{r}_i - \vec{r}_j)/\mu^2]$.

A straightforward calculation gives, as an illustration of Eq. (8),

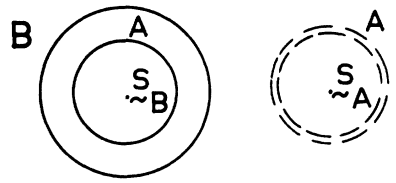

FIG. 5. Analog of Fig. 3 for approximation 3.

$$[\chi' U]^{B/A}(\vec{r}_3, \vec{s}_B, \vec{s}_A) = \frac{1}{\sqrt{3}} V_0 2^{3/2} \pi^{-3/4} \lambda^{-5/2} \left(1 + \frac{\lambda^2}{\mu^2}\right)^{-5/2} \left(z_3 - \frac{s_{Az} + s_{Bz}}{2}\right) \exp\left[-\frac{(\vec{r}_3 - \vec{s}_B)^2}{2\lambda^2}\right]$$

$$\times \exp\left\{-\frac{[\vec{r}_3 - \frac{1}{2}(\vec{s}_A + \vec{s}_B)]^2}{\lambda^2 + \mu^2}\right\} \exp\left[-\frac{(\vec{s}_A - \vec{s}_B)^2}{2\lambda^2}\right]. \tag{19}$$

The last exponential of this equation is related to the generator coordinate method and is not relevant to the present discussion. The next to last one demonstrates that the average potential acting as the transferred particle is centered halfway between A and B. Finally, one recognizes in Eq. (19) a $0p_z$-like wave function, the exponential and the polynomial parts of which, however, are not centered at the same point. This seems to indicate possibly significant moon crescent effects in the calculation of actual cross sections.

Turning now to the $\mathcal{O}_A$ approximation, we obtain

$$\int d\vec{r}_1 d\vec{r}_2 \Psi_B^*(\vec{r}_1 - \vec{s}_B, \vec{r}_2 - \vec{s}_B, \vec{r}_3 - \vec{s}_B) \Psi_A(\vec{r}_1 - \vec{s}, \vec{r}_2 - \vec{s})$$

$$= 2^{1/2} 3^{-1/2} \pi^{-3/4} \lambda^{-5/2} \left(z_3 - \frac{s_z + s_{Bz}}{2}\right) \exp\left[-\frac{(\vec{r}_3 - \vec{s}_B)^2}{2\lambda^2}\right] \exp\left[-\frac{(\vec{s} - \vec{s}_B)^2}{2\lambda^2}\right], \tag{20}$$

which again exhibits a $0p_z$-like orbital with multiple centers, and

$$\int d\vec{r}_1 d\vec{r}_2 \Psi_A^*(\vec{r}_1 - \vec{s}', \vec{r}_2 - \vec{s}') [V(\vec{r}_3 - \vec{r}_1) + V(\vec{r}_3 - \vec{r}_2)] \Psi_A(\vec{r}_1 - \vec{s}_A, \vec{r}_2 - \vec{s}_A)$$

$$= V_0 \left(1 + \frac{\lambda^2}{\mu^2}\right)^{-3/2} \left[1 + \frac{\mu^2}{\lambda^2 + \mu^2} + \frac{2\lambda^2}{(\lambda^2 + \mu^2)^2} \left(y_3 - \frac{s'_y + s_{Ay}}{2}\right)^2\right] \exp\left\{-\frac{[\vec{r}_3 - \frac{1}{2}(\vec{s}' + \vec{s}_A)]^2}{\lambda^2 + \mu^2}\right\} \exp\left[-\frac{(\vec{s}' - \vec{s}_A)^2}{2\lambda^2}\right], \tag{21}$$

which corresponds to an average potential centered halfway between $\vec{s}'$ and $\vec{s}_A$.

A multiplication of Eqs. (20) and (21), where $\vec{s}$ and $\vec{s}'$ are given suitable values, gives at once the results of approximations 1, 2, and 3, namely

$$2^{1/2} 3^{-1/2} V_0 \pi^{-3/4} \lambda^{-5/2} \left(1 + \frac{\lambda^2}{\mu^2}\right)^{-3/2} \left(z_3 - \frac{s_{Az} + s_{Bz}}{2}\right) \exp\left[-\frac{(\vec{r}_3 - \vec{s}_B)^2}{2\lambda^2}\right]$$

$$\times \left[1 + \frac{\mu^2}{\lambda^2 + \mu^2} + \frac{2\lambda^2}{(\lambda^2 + \mu^2)^2} (y_3 - s_{Ay})^2\right] \exp\left[-\frac{(\vec{r}_3 - \vec{s}_A)^2}{\lambda^2 + \mu^2}\right] \exp\left[-\frac{(\vec{s}_B - \vec{s}_A)^2}{2\lambda^2}\right], \tag{22}$$

$$2^{1/2} 3^{-1/2} V_0 \pi^{-3/4} \lambda^{-5/2} \left(1 + \frac{\lambda^2}{\mu^2}\right)^{-3/2} (z_3 - s_{Bz}) \exp\left[-\frac{(\vec{r}_3 - \vec{s}_B)^2}{2\lambda^2}\right]$$

$$\times \left[1 + \frac{\mu^2}{\lambda^2 + \mu^2} + \frac{2\lambda^2}{(\lambda^2 + \mu^2)^2} \left(y_3 - \frac{s_{By} + s_{Ay}}{2}\right)^2\right] \exp\left\{-\frac{[\vec{r}_3 - \frac{1}{2}(\vec{s}_B + \vec{s}_A)]^2}{\lambda^2 + \mu^2}\right\} \exp\left[-\frac{(\vec{s}_B - \vec{s}_A)^2}{2\lambda^2}\right], \tag{23}$$

and

$$2^{1/2} 3^{-1/2} V_0 \pi^{-3/4} \lambda^{-5/2} \left(1 + \frac{\lambda^2}{\mu^2}\right)^{-3/2} (z_3 - s_{Bz}) \exp\left[-\frac{(\vec{r}_3 - \vec{s}_B)^2}{2\lambda^2}\right]$$

$$\times \left[1 + \frac{\mu^2}{\lambda^2 + \mu^2} + \frac{2\lambda^2}{(\lambda^2 + \mu^2)^2} (y_3 - s_{Ay})^2\right] \exp\left[-\frac{(\vec{r}_3 - \vec{s}_A)^2}{\lambda^2 + \mu^2}\right], \tag{24}$$

respectively. The differences which are present in these three results deserve little comment. They

clearly differ through the points with respect to which the transferred particle and the driving potential are centered.

Finally, a straightforward but slightly tedious calculation gives the exact result of the $\mathcal{O}_A$ approximation, Eq. (16),

$$[\chi' U]^{B/A}_{\mathcal{O}_A} = 2^{1/2}3^{-1/2}V_0\pi^{-3/4}\lambda^{-5/2}\left(1+\frac{\lambda^2}{\mu^2}\right)^{-5/2}\left(1+\frac{\lambda^2}{2\mu^2}\right)\left(z_3-\frac{s_{Az}+s_{Bz}}{2}\right)\exp\left[-\frac{(\vec{r}_3-\vec{s}_B)^2}{2\lambda^2}\right]$$
$$\times\left[1+\frac{\mu^2}{\lambda^2+\mu^2}+\frac{2\lambda^2}{(\lambda^2+\mu^2)^2}\left(y_3-\frac{s_{Ay}+s_{By}}{2}\right)^2\right]\exp\left\{-\frac{[\vec{r}_3-\frac{1}{2}(\vec{s}_A+\vec{s}_B)]^2}{\lambda^2+\mu^2}\right\}\exp\left[-\frac{(\vec{s}_A-\vec{s}_B)^2}{2\lambda^2}\right], \quad (25)$$

to be compared with the result of Eq. (19) which involves no approximation.

V. DISCUSSION AND CONCLUSION

This paper contains two kinds of results. Firstly, the calculation of the DWBA form factor has been split into three steps, namely the calculation of a spectroscopic amplitude $\langle 0|D_m ba^\dagger|0\rangle$, another amplitude $\langle 0|BD^\dagger_{m'}\eta^\dagger_\alpha\eta_\delta A^\dagger|0\rangle$, and a transition matrix element $\langle 0|D_{m'}\eta^\dagger_\beta\eta_\gamma D^\dagger_m|0\rangle$; these three quantities correspond to the integrations over $\vec{r}_b$, $\vec{r}_A$, and $\vec{r}_\nu$, respectively. The method followed in this paper illustrates clearly the nature of the "moon crescent effect" created by the different positions of B, A, b, and a. Secondly, approximation methods are given for breaking the calculation of $\langle 0|BD^\dagger_{m'}\eta^\dagger_\alpha\eta_\delta A^\dagger|0\rangle$ into that of a product $\langle 0|BD^\dagger_{m'}A^\dagger(s)|0\rangle\langle 0|A(s')\eta^\dagger_\alpha\eta_\delta A^\dagger|0\rangle$. The influence of shifted shell-model centers is again clearly displayed by the method that we have followed.

A question that has not been answered, however, is whether the "moon crescent effect" brings important corrections in practical calculations. Qualitatively, one may present the following argument. The illustrative example of Sec. IV displays a peaked function $\exp[-(\vec{s}_A-\vec{s}_B)^2/2\lambda^2]$, which prevents the core A from drifting very far away from the center of the complete nucleus B. Furthermore, it is familiar to practitioners of the generator coordinate formalism that such a function is very narrow when high masses are involved and becomes even narrower after the zero-point motion has been defolded. Roughly speaking, one may expect effects of the order of N_ν/N_A.

It can thus be concluded that the effect studied in this paper is not likely to be critical, but may change the spectroscopic factors calculated with the DWBA. For instance, it should be taken into account when one considers the transfer of four particles out of ^{20}Ne, the core ^{16}O being undisturbed. These four particles may be not only in the *sd* shell: for instance, they might dip a little into the *p* shell while pushing the ^{16}O core slightly aside without exciting it. Clearly, a more realistic calculation than that of Sec. IV is desirable in order to test whether the correction is significant or vanishingly small.

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