Communications

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Sensitivity of π^+ and π^- inelastic scattering to differences in the neutron and proton amplitudes*

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We derive an expression relating the ratio of the cross sections for π^+ and π^- inelastic scattering to the ratio of the deformation parameters for the neutron and proton distributions. Calculations are described which determine the parameters in this relation, and the sensitivity of these parameters to the potential model used.

[NUCLEAR REACTIONS Pion inelastic scattering. DWBA. Separate neutron and proton amplitudes.]

Pion-nucleus scattering in the region of 100–300 MeV is dominated by the 3-3 resonance term in the interaction potential. It has been shown that because of the strong isospin dependence of the interaction in this energy region, both the reaction cross section[1] and the elastic scattering angular distribution[2] for $\pi^{\pm}$ scattering are sensitive to differences in the proton and neutron ground state nuclear distributions. Several experiments[3,4] have used comparison of π^+ and π^- reaction cross sections to look for such differences.

In the present work we examine the influence of the $T=\frac{3}{2}$ term in the π^+ and π^- inelastic scattering. One would expect comparisons of π^+ and π^- inelastic scattering to be sensitive to differences in the neutron and proton contributions to the final nuclear state. If this sensitivity, which has been shown in elastic scattering, persists in inelastic scattering, many facets of nuclear structure calculations can be tested. Specific examples include: (1) measuring the purity of single particle states in the ^{208}Pb region[5]; (2) studying Pauli blocking effects on the relative contributions of neutrons and protons to collective states in the f-p shell[6]; (3) looking for possible differences[7] in the ground state neutron and proton deformations in deformed nuclei.[8,9]

We know in the case of free π-p elastic scattering on the 3-3 resonance, the ratio of the cross sections for $\pi^+ + p$ and $\pi^- + p$ scattering is 9 : 1. If Coulomb, off-mass shell, and distortion effects can be neglected, then one would expect the same ratio for $\pi^{\pm}$-nucleus inelastic scattering to a single particle proton state. We have used a modified version distorted wave Born approximation (DWBA) code DWPI[10] to estimate the influences of distortion effects and model uncertainties. In all of these calculations, we have used a simple collective form factor for the excitations,[11,12] but we have allowed the transition strengths (β_N and β_Z) to be different for neutrons and protons. Studies which have compared the collective model with microscopic models have shown the differences to be small.[11]

If one assumes that in an inelastic transition from the ground state of a nucleus to an excited state that (1) the reaction is direct, i.e., there are no multiple step processes; (2) the neutron and proton form factors $F_N(r,\theta)$ and $F_Z(r,\theta)$ have the same radial shape and can be expanded as $F(r,\theta)=F_L(r)[1+\beta_L Y_L(\theta,\phi)+\cdots]$ where different β_L's are allowed for the proton and neutron parts of the form factors; (3) and that the interaction potential (H') depends only linearly on F_N and F_Z (that tensor isospin terms and correlation effects not linear in F can be neglected) then H' in the collective mac-

TABLE I. Variations in a, b, and K^+/K^- for different potential parameters. All calculations used a modified Kisslinger potential for ^{152}Sm at 200 MeV.

Parameter change[a]	a^+	a^-	b^+	b^-	K^+/K^-
No change	0.495	0.486	0.076	0.100	0.969
$R_0 \rightarrow R_0 + 0.5$ fm	0.496	0.487	0.077	0.096	0.978
$t \rightarrow t + 0.5$ fm	0.499	0.491	0.075	0.091	1.000
$\mathrm{Re}(b_0) \rightarrow 1.2\ \mathrm{Re}(b_0)$	0.499	0.489	0.092	0.108	0.968
$\mathrm{Im}(b_0) \rightarrow 1.2\ \mathrm{Im}(b_0)$	0.492	0.483	0.076	0.099	0.972
$\mathrm{Re}(b_1) \rightarrow 1.2\ \mathrm{Re}(b_1)$	0.497	0.487	0.076	0.099	0.968
$\mathrm{Im}(b_1) \rightarrow 1.2\ \mathrm{Im}(b_1)$	0.492	0.483	0.063	0.098	0.965

[a] R_0 is the nuclear radius; t is the 10–90 % skin thickness; and b_0 and b_1 are the usual combinations of the phase parameters used in defining the potential.

roscopic model can be written as

$$H'_L = \sum_i V_i(\beta_S - \epsilon_\pi k_i \beta_D) F_L(r, \theta)\,. \tag{1}$$

Here V_i is some potential operator (this expression can accommodate any of the commonly used coordinate space potentials), L indicates the multipolarity of the transition being considered, β_S and β_D are, respectively, the sum and the difference of $N\beta_N$ and $Z\beta_Z$, ϵ_π is the charge of the pion, and k_i is a constant which expresses the magnitude of the isospin dependence of the ith part of the potential.

If the cross section for such a transition is evaluated in the DWBA its dependence on β_S and β_D can be expressed as

$$\sigma^{\pm}_{\mathrm{inel}} = (K^{\pm})^2[(\beta_S - \epsilon_\pi a^{\pm}\beta_D)^2 + (b^{\pm}\beta_D)^2]\,. \tag{2}$$

Here $a^{\pm}$, $b^{\pm}$, and $K^{\pm}$ are constants which depend upon the charge of the pion and the nucleus being considered. The constants a and b have been evaluated, using the code DWPI, and are graphed in Figs. 1 and 2, for two different nuclei (^{152}Sm and ^{24}Mg). The effects of varying the surface radius and thickness and the potential parameters are presented in Table I. In Table II, we show how these parameters vary for several different potential models. From the figures we notice that both a and b are reasonably independent of the model assumption, and that they are close to equal for both π^+ and π^- scattering. Since b is a small number we can write

$$R = \left(\frac{\sigma^+_{\mathrm{inel}}}{\sigma^-_{\mathrm{inel}}}\right)^{1/2} \approx [K^+(\beta_S - a\beta_D)]/[K^-(\beta_S + a\beta_D)] \tag{3}$$

$$\approx [K^+(\alpha f + 1)]/[K^-(f + \alpha)]\,, \tag{4}$$

where we have defined the new quantities

$$\alpha = N\beta_N/Z\beta_Z \quad \text{and} \quad f = (1-a)/(1+a)\,.$$

We have already shown that a and b, and consequently f, are nearly potential model independent. The usefulness of the above formula rests on how well the quantity K^+/K^- can be determined. This is illustrated in Table II again, for the two nuclei and for three potential models considered above. For ^{24}Mg the uncertainty in this ratio is 5%, but for ^{152}Sm the uncertainty is 15%. Since this variation amounts to an uncertainty in the optical potential used in the elastic channel, elastic scattering data can be used to reduce it.

Inserting some numbers into Eq. (4) one finds

$$R = \frac{0.33\alpha + 1}{0.33 + \alpha} = \begin{cases} 3 & \text{for } \alpha = 0 \\ \frac{1}{3} & \text{for } \alpha \gg 1 \end{cases}$$

at 200 MeV. These limits correspond to single

TABLE II. Effects of different potential models on the parameters defined in the text. These calculations are for a pion energy of 200 MeV, and for $L = 2$ transitions.

	Potential	a^+	a^-	b^+	b^-	K^+/K^-
^{24}Mg	Kisslinger	0.504	0.503	0.076	0.071	0.973
	Local	0.492	0.492	0.067	0.067	0.997
	Modified Kisslinger	0.502	0.498	0.077	0.087	0.952
^{152}Sm	Kisslinger	0.499	0.498	0.079	0.064	1.045
	Local	0.486	0.486	0.057	0.053	1.104
	Modified Kisslinger	0.495	0.486	0.076	0.100	0.969

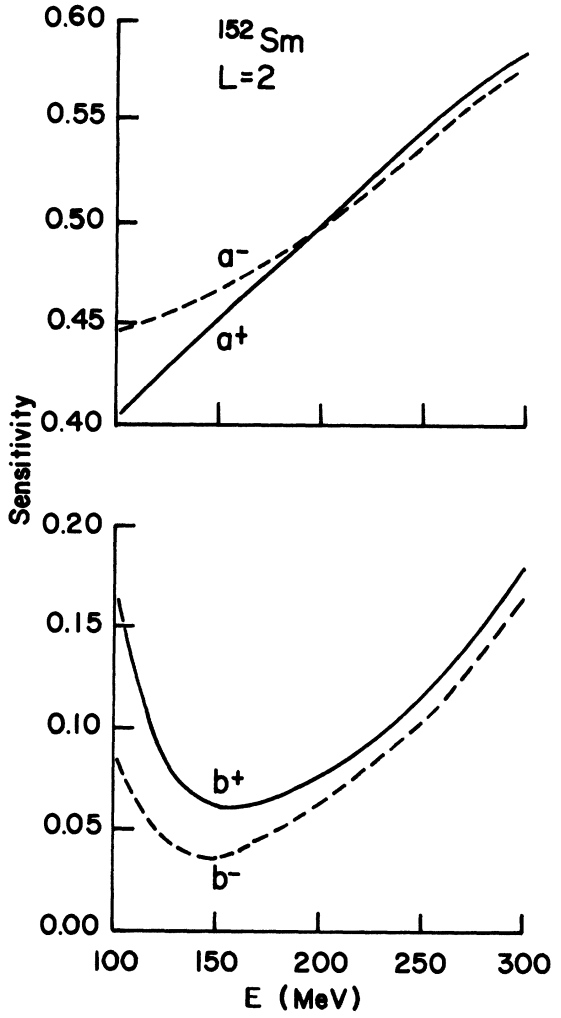

FIG. 1. Plots of a and b (as described in the text) for π^+ and π^- inelastic scattering to the first excited state of ^{152}Sm. We have used the Kisslinger potential for these calculations.

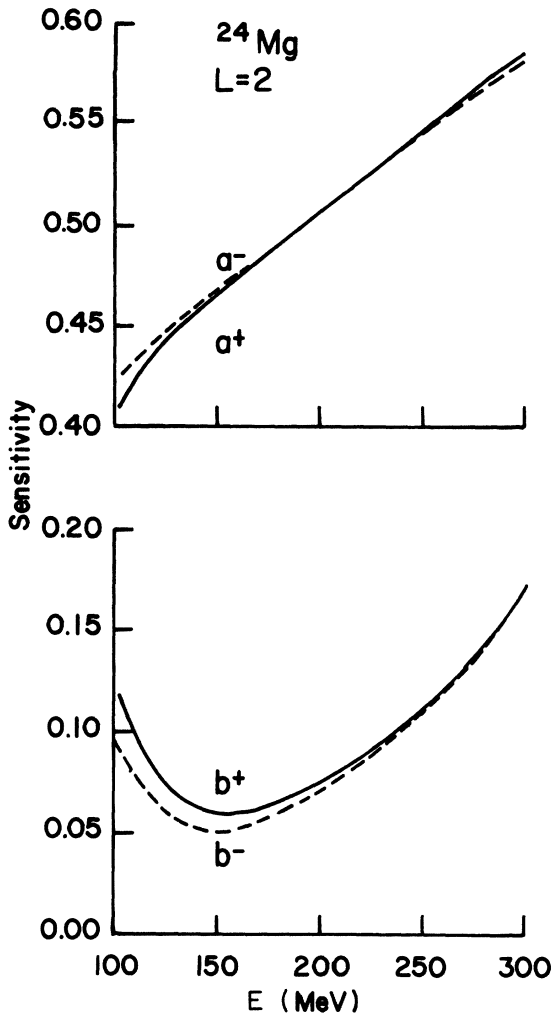

FIG. 2. Plots of a and b for π^+ and π^- inelastic scattering to the first excited state of ^{24}Mg. Again, these calculations have been made using the Kisslinger potential.

particle final states (pure neutron or proton excitations) and are seen to agree with free pion-nucleon scattering. It is also apparent from the figures that a increases above 0.5 at energies above resonance. This is a result of the interference between the $T=\frac{3}{2}$ and $T=\frac{1}{2}$ terms in the forward scattering amplitude for $T_z=\frac{1}{2}$ pion-nucleon scattering. These energies slightly above resonance may prove to be important because of the increased sensitivity and because of the decreased role of multiple step processes.

Although we have not studied the effects of many of the intricacies of pion-nucleus inelastic scattering the results of our simple model should be indicative of the results of more sophisticated models. In conclusion, since distortion effects and uncertainties in the optical model do not dominate calculations of the sensitivites, it appears that comparisons of π^+ and π^- inelastic scattering data will provide nuclear structure physicists with a valuable tool.

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[1]M. M. Sternheim and F. H. Auerbach, Phys. Rev. C 4, 1805 (1971).

[2]L. S. Kisslinger, R. L. Burman, J. H. Koch, and M. M. Sternheim, Phys. Rev. C 6, 469 (1972).

[3]A. Abashian, R. Cool, and J. W. Cronin, Phys. Rev. 104, 855 (1956).

[4]B. W. Allardyce, C. J. Batty, D. J. Baugh, E. Friedman, G. Heymann, M. E. Cage, C. J. Pyle, G. T. A. Squier, A. S. Clough, D. F. Jackson, S. Murugesu, and V. Rajaratnam, Nucl. Phys. A209, 1 (1973).

[5]W. J. Braithwaite, C. F. Moore, J. Bolger, K. Boyer, G. W. Hoffman, and C. L. Morris, LAMPF proposal, 1975 (unpublished).

[6]W. J. Braithwaite, C. F. Moore, J. Bolger, K. Boyer, G. W. Hoffman, and C. L. Morris, LAMPF proposal, 1975 (unpublished).

[7]D. L. Hendrie, Phys. Rev. Lett. 28, 1711 (1972).

[8]N. Ensslin, D. Madland, C. L. Morris, and H. A. Thiessen, LAMPF proposal, 1975 (unpublished).

[9]D. Madland, C. L. Morris, N. Ensslin, and H. A. Thiessen, LAMPF proposal, 1975 (unpublished).

[10]R. A. Eisenstein and G. A. Miller, private communication; and (unpublished).

[11]T.-S. H. Lee and F. Tabakin, Nucl. Phys. A226, 253 (1974).

[12]G. W. Edwards and E. Rost, Phys. Rev. Lett. 26, 758 (1971).