

Decays of ^{88}Kr and $^{88}\text{Rb}^\dagger$

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The γ rays following the β decays of ^{88}Kr and ^{88}Rb have been studied, using both large volume Ge(Li) detectors for singles and coincidence measurements and anti-Compton spectrometry for singles measurements. Level schemes for ^{88}Rb and ^{88}Sr were constructed from the data and 74 of 81 γ rays observed in the decay of ^{88}Kr were placed in the ^{88}Rb level scheme with 23 excited states. For the decay of ^{88}Rb , all of the observed 27 γ rays have been placed in the (well known) level structure of ^{88}Sr . Spin and parity assignments have been deduced from β -decay $\log ft$ values and γ -ray transition patterns. Possible shell-model interpretations are presented for the level schemes.

<p>RADIOACTIVITY ^{88}Kr, ^{88}Rb [from $^{235}\text{U}(n, f)$]; measured E_γ, I_γ, γ-γ coin. Ge(Li) detectors, Compton suppression. ^{88}Rb, ^{88}Sr deduced levels, J, π, $\log ft$. Mass-separated ^{88}Kr activity.</p>

I. INTRODUCTION

Much importance has been placed on the experimental data from decay scheme and reaction studies for nuclei near $Z=38$ and $N=50$ because of their proximity to these subshell and shell closures. For the decays of ^{88}Kr and ^{88}Rb several ambiguities and inconsistencies have been noted recently.¹ In this study, two different source production and isolation techniques were employed, and the results combined in order to obtain detailed information on the decays of 2.84-h ^{88}Kr and 17.8-min ^{88}Rb . The study of these decays is part of a continuing systematic investigation of radioactive gaseous fission products at the Ames Laboratory and of fission product activities at the Lawrence Livermore Laboratory.

The only effective ways to study the levels of ^{88}Rb are by the β decay of ^{88}Kr or by the $^{87}\text{Rb}(d, p)^{88}\text{Rb}$ reaction. Besides our initial reports of data on the decay of the fission product ^{88}Kr ,^{2,3} only two previous β -decay studies using Ge(Li) spectroscopy techniques have appeared in the literature.^{4,5} Of these, the second updates the first through improved chemical separation techniques as applied to the ^{88}Kr fraction obtained in ^{235}U fission. The γ -ray intensities and energies in the present work are notably different from those of Lycklama and Kennett,⁵ and the identification of an additional 53 γ -ray transitions has changed the deduced β branching significantly. The recent γ -ray data by Gehrke, McIsaac, and Heath⁶ and

Heath⁷ are in general agreement with those of this work. Several of the levels populated in β decay that are observed for the first time in this work correspond to levels reported in the $^{87}\text{Rb}(d, p)^{88}\text{Rb}$ reactions.⁸⁻¹⁰ The β -branching ratios deduced in this work are compatible with direct measurements of β -group intensities and Q_β determinations in companion studies.^{11, 12}

The level structure of ^{88}Sr has been the subject of extensive experimental and theoretical investigation. Of primary concern has been the existence and quality of the shell closure at $Z=38$. Theoretical treatments¹³⁻¹⁸ of the alternative "core" nuclei of ^{88}Sr or ^{90}Zr in the literature have yielded differing descriptions for the levels of ^{88}Sr .

The experimental status for the levels in ^{88}Sr has been summarized in the Nuclear Data Sheets.^{1, 19} The low-lying excited states of ^{88}Sr have been determined accurately from the study of ^{88}Y electron-capture decay, providing an excellent internal calibration for the present study. The β decay following thermal neutron capture of ^{87}Rb has been studied extensively to deduce the excited states of ^{88}Sr .²⁰⁻²³ These works include γ - γ angular correlation measurements as well as γ - γ coincidence determinations. Other β -decay experiments using gaseous fission products have been reported.^{4, 24} The levels of ^{88}Sr have also been probed using the $^{87}\text{Sr}(n, \gamma)$ reaction.²⁵ Our coincidence data corroborate the level structure suggested by Ragaini and Meyer²⁴ and resolves the discrepancies between the results reported by Kawase²¹ and by Ragaini

and Meyer. Some low-intensity transitions reported in the latter work were not observed, however, in the current set of experiments.

II. EXPERIMENTAL TECHNIQUES

Two different techniques were used to obtain the ^{88}Kr activity. One method was to perform an on-line mass separation of gaseous fission products formed in fission. This method was used to provide sources used for singles and coincidence γ -ray spectroscopy measurements using standard detector arrangements. The other method was to isolate Kr activity by gas chromatographic methods from the gross gas produced in a nuclear detonation. This technique provided high-intensity sources, which were then used for Compton-suppressed spectroscopy, singles, and energy calibration measurements.

A. Source preparation and isobaric enhancement for mass-separated activities

The mass-separated fission product activities of 2.84-h ^{88}Kr and its β -decay product, 17.8-min ^{88}Rb , were obtained using the TRISTAN on-line isotope separator facility at the Ames Laboratory Research Reactor. A comprehensive description of the facility can be found in the literature²⁶ but a brief discussion of the essential components is presented here.

The fission products were generated in a 1-g sample of ^{235}U in the form of uranyl stearate which was placed in an external thermal neutron beam having a flux of $3 \times 10^9 n_{\text{th}}/\text{cm}^2 \text{ sec}$. The fission product gases diffuse out of the stearate sample and are pumped 2 m to the ion source of the mass separator. The ion beam is selectively focused onto an aluminum-bonded-to-mylar tape in a moving tape collector (MTC) or onto a Cu foil which can be inserted into the beam line through an air lock a few cm upstream from the MTC. A second collection point allowed the deposited radioactivity to be removed quickly from the ion beam line for off-line chemical separation and analysis.

Because of the transient equilibrium nature of the ^{88}Kr β decay chain, several procedures were used to obtain a reliable isobaric identification of the two activities. The two most effective separation methods are described below.

The first procedure involved the chemical separation of the ^{88}Rb activity from an "equilibrium" ^{88}Kr - ^{88}Rb source. The initial source was collected on 0.05-mm Cu foil chosen for its solubility in concentrated acids and excellent ion beam retention. The ^{88}Kr activity was flushed from a 6N HNO_3 target solution into a liquid nitrogen trap. The initial ^{88}Rb activity was recovered from the

HNO_3 solution by precipitating with a Rb carrier in a solution containing Bio AMP-1 ammonium molybdophosphate cation crystals. At some subsequent time another Rb separation was performed on the activity buildup in the LN_2 trap. A total of five targets were processed in this manner over a period of 12 h. Since the chemical separation was not completely effective in removing all the ^{88}Kr activity, individual γ -ray spectra were taken for each source. Only those spectra which exhibited good ^{88}Rb separation were later combined through a point-by-point summation for final analysis. This procedure led to a 70-fold enhancement of the ^{88}Rb activity (compared to the equilibrium singles data described later).

The second enhancement method utilized the MTC in a sequential stepping mode to sweep away the bulk of the deposited activity before a significant amount of the ^{88}Rb activity could accumulate. In this mode of MTC operation, the detection system was gated on and the ion beam collected in front of the detector for a period of 2 min. At the end of this time, the analyzer was gated off, the beam was deflected from the deposit point, the tape advanced a predetermined distance to a well-shielded area, and then a new sample collected and counted. After several hours, the resulting spectrum showed enhancement for the ^{88}Kr activity by a factor of about 12 over the equilibrium data.

The equilibrium γ -ray singles data were collected and counted by viewing the point of deposit on the MTC with the tape remaining stationary. The effectiveness of the separation techniques described above is illustrated in the spectra shown in Figs. 1 and 2. The isobaric identification of the γ rays was made by comparing the ratios of the γ -ray intensities in the enhanced and equilibrium spectra.

1. Detection methods

Two 60-cm³ true coaxial Ge(Li) detectors were used in the γ -ray singles and coincidence experiments. The γ -ray singles resolution was 2.5 keV full width at half-maximum (FWHM) and the detector efficiencies were 9 and 11% compared to a 7.6-cm by 7.6-cm NaI(Tl) crystal at 1332 keV. Low-energy γ -ray transitions were observed with a 1-cm³ planar Ge(Li) low-energy photon spectrometer. For high-energy data, a 0.476-cm lead absorber was used to attenuate the low-energy γ rays and low-energy data from the MTC were obtained using a 0.008-cm Mylar window in place of the standard 0.16-cm aluminum window.

The amplifiers and analog-to-digital converters (ADC's) were temperature stabilized and counting rates were controlled to prevent gain shifts during the long counting periods. Pole-zero cancellation

and base-line restoration were used in both detector channels.

2. γ - γ coincidence measurements

The γ - γ coincidence system utilized two 60-cm³ Ge(Li) detectors. The data were accumulated on a buffer tape system in the form of address pairs. Constant fraction timing circuitry was used and the timing window was typically 40 nsec. Each magnetic tape contained about 3.5×10^6 address pairs (coincidence events) which were processed by two 4096-channel ADC's. The sources for these experiments were deposited in equilibrium form on the MTC and a total of four tapes were accumulated. The data were analyzed at a later time by playing the tapes back through a format selector. The selector routes all events in coincidence with a preselected digital gate into an appropriate memory subsection of a 16 384-channel analyzer.

By accumulating a 4096×4096 channel coinci-

dence matrix it was generally possible to set the gating regions such that only one transition was contained in each gate. For each such gate a background region just above the peak was also processed. This background was a measure of the accidental and Compton coincidence events. A total of 26 transitions plus associated background coincidence gates were processed for the ^{88}Kr decay and eight such pairs for the ^{88}Rb decay. The coincidence data for the 391-keV gate were grouped into two sets consistent with double placement suggesting that the 391-keV transition was an unresolved doublet. The intensity of the 391-keV transition was divided between the two placements from the observed intensity ratios of the 391- and 1179-keV transitions in the 1185-keV gate.

B. Compton suppression measurements

The Kr activity for the Compton suppression spectroscopy part of the experiment was obtained

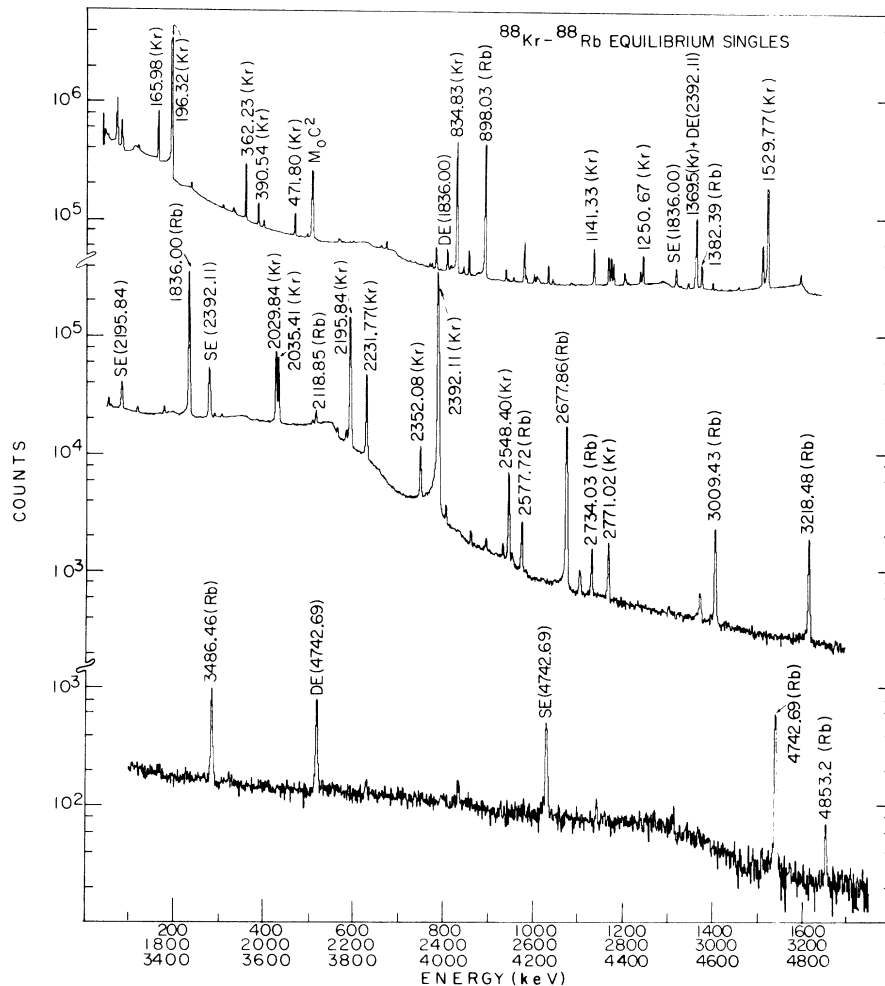


FIG. 1. ^{88}Kr - ^{88}Rb equilibrium γ -ray spectrum from mass-separated source.

from the prompt gross gas collected from an underground nuclear detonation.²⁷ Upon delivery to Lawrence Livermore Laboratory (LLL), the fission product gases were subjected to gas chromatographic separation and frozen onto charcoal in a quartz ampule, or a standard aluminum gas ampule, and sealed.²⁸ Krypton gas was isolated from the gross fission product gas of several underground nuclear detonations over the period of this investigation. Some of these ampules were opened, releasing the Kr gas and leaving the daughter ⁸⁸Rb activity. These sources were measured using a Ge(Li) Compton suppression spectrometer of the design by Camp.²⁹ Subsequent measurements on these sources are given in the next section.

C. γ -ray singles measurements

For precision γ -ray energy and intensity determinations, independent measurements were made at both laboratories. At Ames Laboratory each experiment required a minimum of four measurements. The mass-separated activity measurements resulted in 8192-channel spectra representing the calibration, unknown, calibration plus unknown, and background activities. Each calibration data set included the ^{56,57}Co, ¹⁸²Ta, ¹⁹²Ir, ²²⁸Ra, and ²⁴¹Am γ -ray standards.^{30,31} A similar

procedure was used for the fission product sources at LLL. These data were used to obtain the integral nonlinearity of the electronics and relative efficiency corrections for the counters. The calibration-plus-unknown spectra were used to calibrate the intense peaks from the unknown contribution for later use as internal calibration points. The quality of the calibration is reflected in the values returned for the ⁸⁸Sr transitions at 898.03 ± 0.04 , 1836.00 ± 0.05 , and 2734.03 ± 0.07 keV which have been calibrated in the ⁸⁸Y electron capture decay³² as 898.022, 1836.016, and 2734.018 keV (1836 + 898 keV cascade corrected for nuclear recoil).

For the mass-separated sources, background data were used to identify long-lived and other background contaminants in the spectra. In low specific activity experiments with the reactor in operation, a few neutron-capture γ rays from Ar, Ge, Cd, Al, H, and Fe were noted. Also, the long-lived activities ¹⁴⁰La and ¹⁴⁰Ba were observed. Furthermore, activities from 56-sec ⁸⁷BrH⁺ and 16-sec ⁸⁸Br were recorded during the ⁸⁸Kr-enhancement experiment with the MTC in the stepping mode. Transitions from the Br activities were identified from unpublished data of recent experiments.³³ The escape-peak-to-photo-peak intensity ratios were determined for the specific detectors from a large body of experimental data and appro-

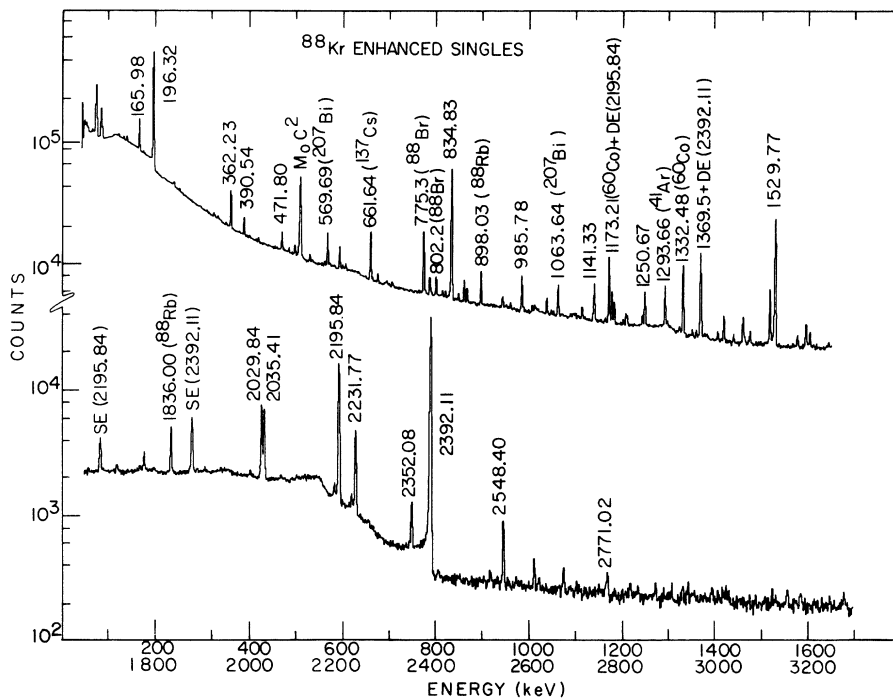


FIG. 2. ⁸⁸Kr-enhanced γ -ray spectrum from mass-separated source.

priate intensity corrections were made wherever necessary.

As mentioned in the previous section, the intense fission product sources of krypton obtained at LLL were measured using a Ge(Li) Compton suppression spectrometer. Where possible, several spectra were taken in order to check the half-life of the parent decay from individual γ rays. Less intense fission product sources were used to measure the γ rays above 1 MeV by placing up to 1.2 cm of lead between the source and a large volume Ge(Li) detector connected to an 8192-channel analyzer. The weakest sources were used to measure the γ rays from 0 to 2 MeV with Ge(Li) spectrometers of various efficiencies. All of the sources were used in the energy calibration experiments. The energies and relative intensities were determined from computer analysis of the raw data at both laboratories using a skewed Gaussian function with low-energy exponential tail as the analytical peak shape in the fitting codes.^{34, 35}

III. RESULTS

The transition energies and intensities resulting from the experiments performed at TRISTAN and

at Livermore were combined into a weighted average list. In Figs. 1 and 2, we present the ^{88}Kr - ^{88}Rb equilibrium and ^{88}Kr enhanced spectra taken with mass-separated sources, respectively, and in Fig. 3 we present a Compton suppression spectrum of ^{88}Kr isolated from the prompt gas of an underground nuclear detonation. The level schemes for ^{88}Rb and ^{88}Sr were constructed using the transitions resulting from the combination of the results of the two laboratories and from the coincidence data. For the determination of β feeding of excited levels, γ -ray intensities were corrected for internal conversion, and the transition imbalances were equated to the β feeding. $\text{Log}ft$ values were determined for decay branches in both decays using decay energies of 2930 ± 30 keV for ^{88}Kr decay¹² and 5338 ± 4 keV for ^{88}Rb decay.³⁶ Ground-state β branches used are preliminary results from absolute decay rate determinations using a 4π counter with the on-line facility.³⁷ The value of $(13 \pm 5)\%$ for the ground-state β branch in the decay of ^{88}Kr is in excellent agreement with that predicted from systematics.³⁸ The ground-state branch for ^{88}Rb decay is $(77.4 \pm 1.4)\%$. These results, and those presented below, have been incorporated in the latest compilation of $A = 88$ nuclei.¹

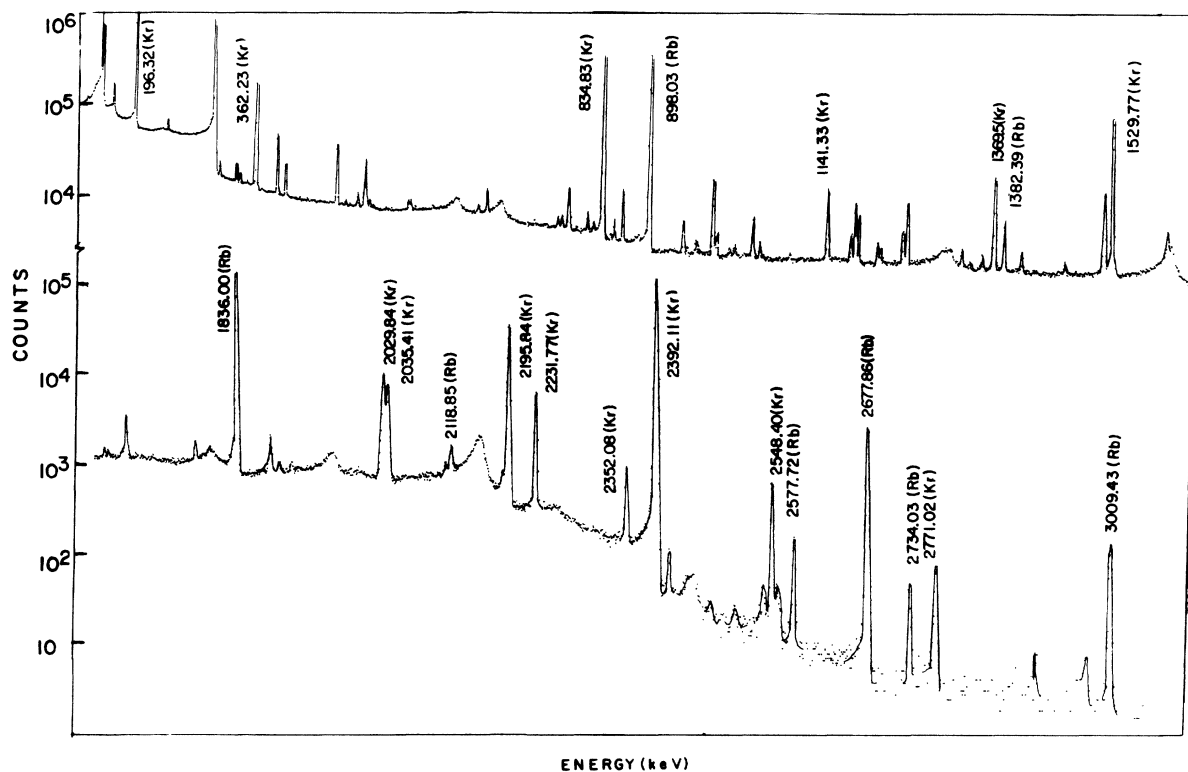


FIG. 3. ^{88}Kr - ^{88}Rb equilibrium γ -ray spectrum taken with Compton suppression spectrometer.

TABLE I. γ -ray transitions observed in the decay of ^{88}Kr .

Energy (keV)	Intensity ^a	Placement	Energy (keV)	Intensity ^a	Placement
27.513±0.014	56 ± 4	28- 0	1039.59 ± 0.03	14.0±0.5	2392-1352
28.26 ± 0.11	0.8±0.3	391- 362	1049.48 ± 0.12	4.1±0.3	2232-1182
122.27 ± 0.06	5.7±0.2	391- 268	1054.54 ± 0.20	0.9±0.3	
165.98 ± 0.04	89.7±0.4	362- 196	1090.53 ± 0.12	1.8±0.4	2232-1141
168.5 ± 0.2	≤0.2	196- 28	1141.33 ± 0.06	37.1±0.9	1141- 0
176.71 ± 0.17	0.7±0.2		1179.51 ± 0.03	28.8±0.6	2392-1213
196.320±0.015	751 ± 5	196- 0	1184.95 ± 0.04	19.9±0.8	1213- 28
240.71 ± 0.04	7.3±0.2	268- 28	1209.84 ± 0.08	4.1±0.7	2392-1182
311.69 ± 0.03	3.1±0.2	1915-1604	1212.73 ± 0.17	4.0±1.4	1213- 0
334.71 ± 0.03	4.2±0.2	362- 28	1245.22 ± 0.04	10.5±0.5	1441- 196
350.04 ± 0.19	0.5±0.2	1213- 862	1250.67 ± 0.04	32.4±0.6	2392-1141
362.226±0.013	65.0±1.6	362- 0	1298.78 ± 0.15	2.7±0.6	1661- 362
363.5 ± 0.5	1.4±0.9	391- 28	1303.09 ± 0.24	1.9±0.7	2548-1245
390.543±0.011	18.6±1.2	391- 0	1324.98 ± 0.04	4.6±1.0	1352- 28
391.20 ± 0.10	2.3±1.2	1604-1213	1335.81 ± 0.14	1.9±0.3	2548-1213
421.70 ± 0.18	0.3±0.1	1604-1182	1352.32 ± 0.11	4.6±0.6	1352- 0
471.80 ± 0.03	21.0±0.4	862- 391	1369.5 ± 0.2	42.7±1.7	2232- 862
500.02 ± 0.06	2.8±0.2	862- 362	1406.94 ± 0.10	6.3±0.5	2548-1141
517.00 ± 0.08	1.0±0.3	2232-1715	1464.84 ± 0.09	3.3±0.4	1661- 196
570.57 ± 0.07	1.8±0.2	2232-1661	1518.39 ± 0.03	62.2±1.6	1715- 196
573.27 ± 0.06	2.1±0.2	1715-1141	1529.77 ± 0.03	316 ± 5	2392- 862
579.04 ± 0.14	0.7±0.3	1441- 862	1603.79 ± 0.05	13.2±0.8	1604- 0
603.21 ± 0.13	1.2±0.3		1608.01 ± 0.20	2.0±0.5	
665.94 ± 0.06	2.5±0.4	862- 196	1661.31 ± 0.3	2.6±0.6	1661- 0
677.34 ± 0.05	6.8±0.4	2392-1715	1685.6 ± 0.4	19.2±2.1	2548- 862
731.01 ± 0.09	1.0±0.3	2392-1661	1789.14 ± 0.22	1.3±0.5	
741.34 ± 0.18	1.0±0.3	2456-1715	1793.3 ± 0.3	1.0±0.4	1793- 0
774.14 ± 0.06	2.8±0.4	1915-1141	1801.3 ± 0.3	1.1±0.4	
779.12 ± 0.08	2.8±0.6	1141- 362	1892.76 ± 0.13	4.0±0.7	2089- 196
788.28 ± 0.04	15.4±0.4	2392-1604	1908.7 ± 0.4	2.9±0.4	2771- 862
790.32 ± 0.07	3.6±0.3	2232-1441	2029.84 ± 0.03	130.9±2.5	2392- 362
798.65 ± 0.21	0.8±0.3	1661- 862	2035.41±0.018	108 ± 3	2232- 196
822.01 ± 0.12	2.6±0.3	1213- 391	2186.5 ± 0.3	8.3±1.7	2548- 362
834.830±0.003	375 ± 4	862- 28	2195.842±0.007	381 ± 3	2392- 196
850.34 ± 0.05	5.0±0.3	1213- 362	2231.772±0.021	98.0±1.9	2232- 0
862.327±0.019	19.4±0.5	862- 0	2259.5 ± 0.3	0.9±0.4	2456- 196
879.51 ± 0.19	0.7±0.2	2232-1352	2352.08 ± 0.04	21.1±0.6	2548- 196
883.06 ± 0.14	1.2±0.2	1245- 362	2364.7 ± 0.3	0.9±0.4	2392- 28
944.92 ± 0.04	8.5±0.4	1141- 196	2392.11 ± 0.04	1000 ± 3	2392- 0
950.49 ± 0.12	1.1±0.3	2392-1441	2408.91 ± 0.07	3.0±0.3	2771- 362
961.83 ± 0.06	2.4±0.3	1352- 391	2535.52 ± 0.11	1.2±0.1	
985.780±0.016	38.0±0.7	1182- 196	2548.40 ± 0.03	18.0±0.3	2548- 0
990.09 ± 0.09	4.1±0.5	1352- 362	2771.02 ± 0.05	4.3±0.2	2771- 0

^a Relative to I_{2392} . Can be converted to I_γ per 100 decays by multiplication by the factor 0.0349, assuming a ground-state β branch of 13%.

A. Decay of ^{88}Kr

The transitions observed in the decay of ^{88}Kr are listed in Table I. The placements for the transitions are listed in accordance with the decay scheme shown in Fig. 4. Coincidence information is given in Table II and is discussed in detail in

Ref. 3. The β branching to the levels in ^{88}Rb and the associated $\log ft$ values are indicated in Table III. For purposes of β -decay branching determinations, corrections to the transition intensities for internal conversion were made on the following basis: The 165.98-keV transition was assigned an $M1/E2$ multipolarity on the basis of its experim^e

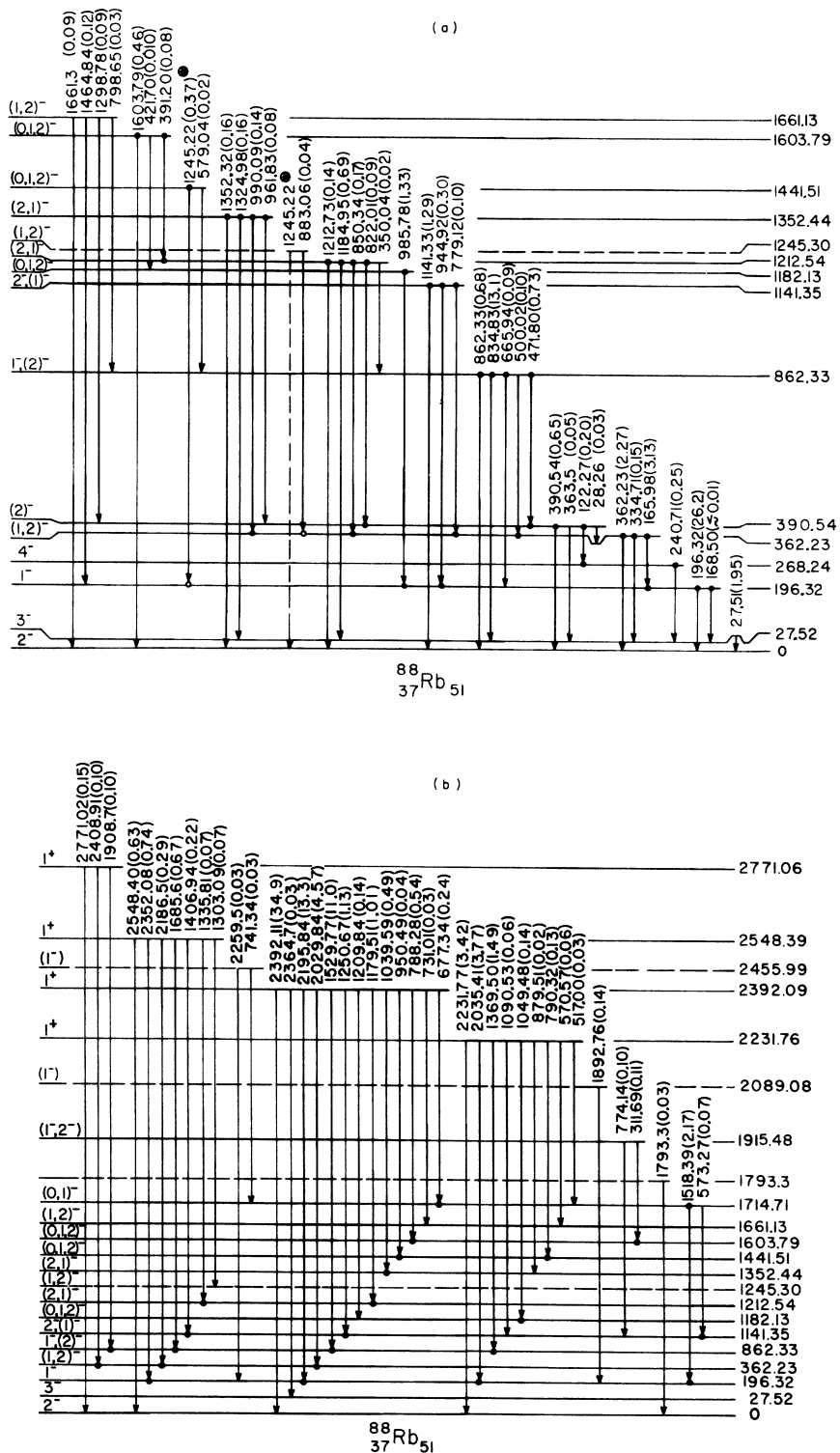


FIG. 4. Level scheme of ^{88}Rb populated in the decay of ^{88}Kr . Coincidences are indicated by filled circles at the transition start and termination points: (a) levels up to 1700 keV, (b) levels above 1700 keV.

TABLE II. Coincidence information for the decay of ^{88}Kr .

Gating transition (keV)	Observed coincidences ^a (keV)
122	240, 471, 1529
165	196, 779, 850, 883, 2029, 2195, 2352
196	165, 944, 985, 1518, 2035, 2195, 2352
240	122
362	(500), (779), 850, 990, 2029, 2186, 2408
390	471, 822, 1529
391	788, 822, 1184, 1212
788	(196), 1603
834	1369, 1529, 1685, 1908
985	196, (1039), 1049
1039	961, 990, 1324, 1352
1141	573, 1250, 1406
1179	822, 850, 1184, 1212
1184	1179, 1335
1250	779, 944, 1141
1324	(1039)
1369	471, 834
1518	196, 677
1529	471, 665, 834, 862
1603	788, (944)
1685	196, 834
2029	165, 334, 362
2035	196
2186	196, 362
2195	196
2352	196

^a Energies enclosed in parentheses indicate probable coincidences.

tal K -shell internal conversion coefficient (ICC) of 0.067, as was the 196.32-keV transition (with K -shell ICC = 0.044).^{11, 36} The 27.51- and 28.26-keV transitions were assigned as $M1$ transitions, and the 168.5-keV transition was assigned as an $E2$ transition, these assignments being made principally on the basis of the level structure indicated from other considerations. All the rest of the low-energy transitions were treated as equal mixtures of $M1/E2$ multipolarity. (The $Z \approx 38$ nuclei have conversion coefficients nearly equal for $M1$ and $E2$ transitions of 400 keV or greater.³⁹)

Spin-parity assignments were developed for the levels of ^{88}Rb , starting with the knowledge that the ^{88}Kr ground state has a spin-parity of 0^+ , and $J^\pi = 2^-$ for the ^{88}Rb ground state.⁴⁰ The β branch $\log ft$ values were used for spin-parity assignments according to the rules suggested by Raman and Gove.⁴¹ Further restrictions in the resulting assignments were often possible from consideration of the γ -ray relative branching and the results of $^{87}\text{Rb}(d, p)^{88}\text{Rb}$ studies.⁸⁻¹⁰ The spin-parity values thus deduced are indicated on the level scheme of Fig. 4, and are summarized as follows:

The spin sequence for the levels at 27.52, 196.32,

TABLE III. β branching and $\log ft$ values for ^{88}Kr decay.

Level energy (keV)	Percent β branching	Log ft
0.0	13 ± 5 ^a	9.37 ± 0.17 ^b
27.514 ± 0.026	~ 0	...
196.32 ± 0.04	1.8 ± 0.3	8.72 ± 0.07
268.24 ± 0.04	~ 0	...
362.23 ± 0.03	~ 0	...
390.542 ± 0.014	0.23 ± 0.09	9.48 ± 0.17
862.33 ± 0.03	1.3 ± 0.3	8.35 ± 0.09 ^c
1141.35 ± 0.08	0.11 ± 0.05	9.2 ± 0.2
1182.13 ± 0.07	1.03 ± 0.07	8.16 ± 0.03 ^c
1212.54 ± 0.07	~ 0	...
1245.30 ± 0.12	~ 0	...
1352.44 ± 0.08	~ 0	...
1441.51 ± 0.06	0.23 ± 0.03	8.54 ± 0.06
1603.79 ± 0.03	~ 0	...
1661.13 ± 0.07	0.23 ± 0.04	8.26 ± 0.08 ^c
1714.71 ± 0.04	1.94 ± 0.13	7.26 ± 0.04
1793.3 ± 0.3	0.035 ± 0.014	8.89 ± 0.18 ^c
1915.48 ± 0.04	0.207 ± 0.020	7.93 ± 0.05 ^c
2089.08 ± 0.14	0.14 ± 0.03	7.80 ± 0.09
2231.76 ± 0.04	9.1 ± 0.6	5.69 ± 0.04
2392.09 ± 0.05	67.5 ± 4.0	4.43 ± 0.05
2455.99 ± 0.16	0.066 ± 0.018	7.24 ± 0.13
2548.39 ± 0.05	2.68 ± 0.19	5.32 ± 0.07
2771.06 ± 0.06	0.36 ± 0.03	4.96 ± 0.14

^a Value adopted from Ref. 37.

^b $\log f_1 t$, since a known first-forbidden unique β transition.

^c $\log f_1 t > 8.5$, so cannot exclude a first-forbidden unique β transition.

and 268.24 keV was developed using spectroscopic factors (with $l_n = 2$) for the $^{87}\text{Rb}(d, p)^{88}\text{Rb}$ reaction, $M1$ multipolarity for the 196.32-keV transition from internal conversion coefficient measurements,^{11, 36} and presence of a β branch to the 196.32-keV level with a $\log ft$ value in the first-forbidden range (and absence of β branches to the other two levels).

The levels at 862.33 and 1141.35 keV have $\log ft$ values and γ deexcitations which indicate J^π ranges of $(1, 2)^-$. They are seen in (d, p) studies with $l_n = 0$ and the relative spectroscopic factors indicate a preference for 1^- and 2^- for the two levels, respectively.

The levels at 2231.76, 2392.09, 2548.39, and 2771.06 keV are all assigned $J^\pi = 1^+$ on the basis of the allowed β -decay $\log ft$ values.

The spin-parity assignments for the rest of the levels of ^{88}Rb involve the use of weaker arguments, including the absence of observed γ transitions to higher spin states and/or β feeding. The levels at 1245.30, 1793.3, 2089.08, and 2455.99 keV which have no definite coincidence basis are also observed in (d, p) reactions.

TABLE IV. γ -ray transitions observed in the decay of ^{88}Rb .

Energy (keV)	Relative intensity ^a	Placement
338.95±0.07	2.8 ± 0.3	4853-4513
416.2 ± 0.3	0.17± 0.06	3634-3218
439.2 ± 0.3	0.67± 0.16	4853-4413
484.53±0.16	1.3 ± 0.3	3218-2734
898.03±0.04	656 ± 4	2734-1836
1027.3 ± 0.3	0.5 ± 0.2	4513-3486
1217.97±0.18	2.4 ± 0.3	4853-3634
1366.26±0.12	4.8 ± 0.6	4853-3486
1382.39±0.05	34.7 ± 1.2	3218-1836
1679.6 ± 0.3	2.1 ± 0.4	4413-2734
1779.83±0.07	10.1 ± 0.6	4513-2734
1798.35±0.19	2.9 ± 0.6	3634-1836
1836.00±0.05	1000 ± 11	1836- 0
2111.22±0.12	5.5 ± 0.5	4845-2734
2118.85±0.07	19.7 ± 0.9	4853-2734
2388.0 ± 0.6	1.3 ± 0.4	4224-1836
2577.72±0.06	8.4 ± 0.4	4413-1836
2677.86±0.05	91.5 ± 1.3	4513-1836
2734.03±0.07	5.1 ± 0.3	2734- 0
3009.43±0.07	11.4 ± 0.4	4845-1836
3017.19±0.20	0.2 ± 0.1	4853-1836
3218.48±0.08	10.0 ± 0.3	3218- 0
3486.46±0.09	6.1 ± 0.2	3486- 0
3523.97±0.6	0.3 ± 0.2	3523- 0
4035.5 ± 0.4	0.5 ± 0.1	4035- 0
4742.69±0.11	6.7 ± 0.3	4742- 0
4853.2 ± 0.3	0.42± 0.06	4853- 0

^a Relative to I_{1836} . Can be converted to I_γ per 100 decays by multiplication by the factor 0.0220, assuming a ground state β branch of 77.4%.

The 1212.54-keV, 1352.44-keV, and 1661.13-keV levels were all observed in (d, p) studies with $l_n = 0$ which favors spin possibilities of 1 or 2. The absence of β branches to the 1212.54- and 1352.44-keV levels is a weak argument for preferring spin 2, whereas no preference can be stated for the 1661.73-keV level.

The level at 362.23 keV is assigned a spin-parity of $(1, 2)^-$ on the basis of γ deexcitations and population from the 1^+ 2392.09-keV state. The 390.54-keV level is limited to a $(2)^-$ assignment from the presence of β feeding and from the spin-parity assignments for levels fed by transitions from this level.

The levels at 1182.13, 1441.51, 1603.79, and 1714.71 keV are assigned negative parity on the basis of feeding transitions from the 1^+ states higher in energy and the spin ranges are determined on the basis of β branching and γ excitation patterns. The level at 1915.48 keV is assigned

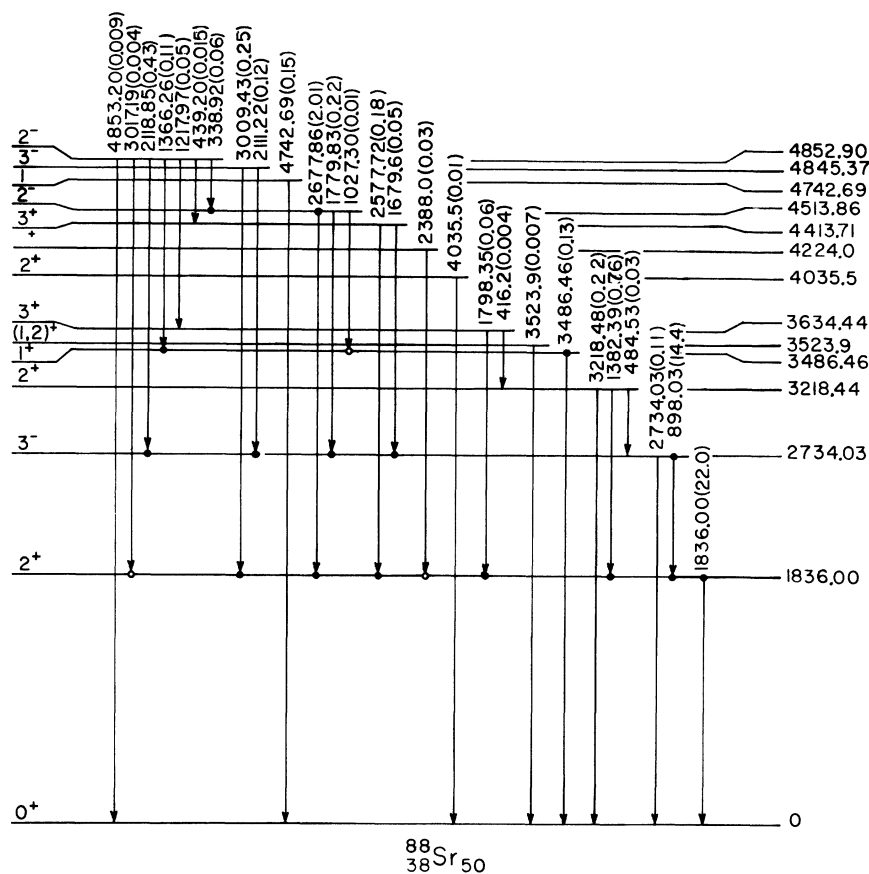
$(1, 2)^-$ on the basis of β and γ branchings, plus a reported $l_n = 2$ transfer in a (d, p) study.

B. Decay of ^{88}Rb

The transitions observed in the decay of ^{88}Rb are listed in Table IV. The decay scheme shown in Fig. 5 accommodates all of the observed transitions, with all but three of the placed transitions having assignments between levels confirmed by the coincidence information given in Table V. The levels at 3523.9, 4035.5, and 4742.69 keV are established on the basis that the transitions to the ^{88}Rb ground state are not allowed to feed even the first excited state at 1836.00 keV from decay energy limitations. The decay scheme shown in Fig. 5 is in good agreement with previous studies.^{4, 20-25, 42} The intensity for the 898.03-keV transition reported in Ref. 24 is incorrect, due to a typographical error. The β branchings and $\log ft$ values are given in Table VI.

The more detailed study in this work using both intense fission product and mass-separated sources has allowed us to appraise a number of low-intensity γ rays that were reported²⁴ as belonging to the decay of 18-min ^{88}Rb which was produced by neutron capture rather than by fission. Of 20 transitions, some of which were only tentatively proposed, the transitions at 416.2, 439.2, 1679.6, 2388.0, 3523.97, and 4035.5 keV were observed in this work in singles measurements. The 1027.3- and 3017.19-keV transitions were seen in the coincidence studies and in the equilibrium measurements at TRISTAN. The remaining transitions were not observed using the intense fission product and mass-separated sources. The coincidence measurements did not exhibit the 1217.97-keV transition to be in coincidence with the 898.03-keV transition as would be expected from the placement reported by Ragaini and Meyer,²⁴ and the 1366.25-keV transition was confirmed by coincidences to have the placement reported by Kawase.²¹ Singles spectra using more intense sources with the LLL Compton suppression spectrometer and the mass-separated singles spectra, give the following limits for the tentative transitions reported in Ref. 24: For unplaced transitions, the energies and relative intensity limits are: 625.27 keV, ≤ 0.2 ; 890.68 keV, ≤ 0.2 ; 916.90 keV, ≤ 0.06 ; 1555.7 keV, ≤ 0.2 ; and 1668.8 keV, ≤ 0.4 . For tentatively placed transitions; 1239.50, ≤ 0.05 ; 1257.20, ≤ 0.05 ; and 1297.00, ≤ 0.1 . The relative intensity limits on the four remaining γ rays in Ref. 24, used to assign new levels in ^{88}Sr , are: 2797.4, ≤ 0.08 ; 3611.5, ≤ 0.06 ; 3966.20 ≤ 0.06 ; and 4633.50, ≤ 0.03 .

These comments on low-intensity transitions are

FIG. 5. Level scheme of ^{88}Sr populated in the decay of ^{88}Rb .

supported in part by the recent report of Erten and Blachot.⁴³ While transitions at 626.2, 1257.7, 1556.0, and 1666.7 keV are reported by Erten and Blachot as confirming those of Ragaini and Meyer,²⁴ the intensity limits for those transitions from this work are considerably lower than the intensities assigned in Ref. 43. Furthermore, the transitions at 558.5 and 1599.5 keV reported by Bunting³ and also by Erten and Blachot (at significantly different energies) are not included in this work, since they were not observed at all in the Livermore studies. Finally, Erten and Blachot report two transitions of low intensity, not observed in this work, at 2007.7 and 2905.7 keV. We can place upper limits in intensity to these transitions, from spectra with several-fold better statistics, of less than half the intensity values reported in Ref. 43. Furthermore, the placements of transitions suggested by Erten and Blachot are incorrect in energy by two to three standard deviations, more than would be expected.

Spin-parity assignments shown in Fig. 5 for the levels of ^{88}Sr are derived mostly from combining reaction data,¹⁹ neutron capture data, and the decay data reported here. The levels up through 3634.44

keV, with the exception of the 3523.9-keV level, are documented rather thoroughly in reaction, neutron capture, and previous decay studies (including angular correlation).^{1, 19} The spin-parity assignments for these levels are well known enough to be considered firm. The 3523.9-keV level is populated also in neutron capture (but not by a direct

TABLE V. Coincidence information for the decay of ^{88}Rb .

Gating transition (keV)	Observed coincidences ^a (keV)
898	1836, 1679, 1779, 2111, 2118
1779	898, 1836
1836	(484), 898, 1382, 1798, (2388), 2577, 2677, 3009, (3017)
2111	898, 1836
2118	898, 1836
2677	338, 1836
3009	1836
3486	(1027), 1366

^a Energies enclosed in parentheses indicate probable coincidences.

TABLE VI. β branching and $\log ft$ values for ^{88}Rb decay.

Level energy (keV)	β branching %	$\log ft$
0.0	77.4 \pm 1.4 ^a	9.29 \pm 0.01 ^b
1836.00 \pm 0.05	4.3 \pm 0.4	7.88 \pm 0.04 ^c
2734.03 \pm 0.05	13.7 \pm 0.9	6.83 \pm 0.03
3218.44 \pm 0.06	1.01 \pm 0.07	7.59 \pm 0.03 ^c
3486.46 \pm 0.09	0.018 \pm 0.015	9.1 \pm 0.4
3523.9 \pm 0.6	0.007 \pm 0.004	9.5 \pm 0.3
3634.44 \pm 0.17	\sim 0	...
4035.5 \pm 0.4	0.011 \pm 0.002	8.73 \pm 0.09
4224.0 \pm 0.6	0.029 \pm 0.009	8.07 \pm 0.14 ^c
4413.71 \pm 0.08	0.216 \pm 0.019	6.89 \pm 0.04
4513.86 \pm 0.05	2.18 \pm 0.14	5.72 \pm 0.03
4742.69 \pm 0.11	0.147 \pm 0.011	6.41 \pm 0.03
4845.37 \pm 0.08	0.37 \pm 0.03	5.74 \pm 0.03
4852.88 \pm 0.17	0.68 \pm 0.05	5.46 \pm 0.03

^a Value adopted from Ref. 37.

^b $\log f_1 t$, since known first-forbidden unique transition.

^c $\log f_1 t > 8.5$, so cannot exclude first-forbidden unique β transitions.

transition from the capture state) and the single deexcitation γ transition to the ground state suggests a spin of 1 or 2. The levels at 4224.0, 4413.71, 4513.86, and 4852.9 keV have no reaction data, and so the spin-parity assignments for these levels are made on the basis of the β branching and γ -ray branching in this work, or from the angular correlation measurements of Kawase²¹ in combination with neutron capture data.²⁵

The levels at 4035.5 and 4742.68 keV are postulated on the basis of the observed ground-state γ -ray transitions, which cannot feed the first excited state from decay energy limitations. The 4035.5-keV level is also seen in reaction experiments, and its spin-parity assignment is based on the observed angular momentum transfers. The spin assignment for the 4742.68-keV level is based upon the resonant γ radiation angular distribution.⁴²

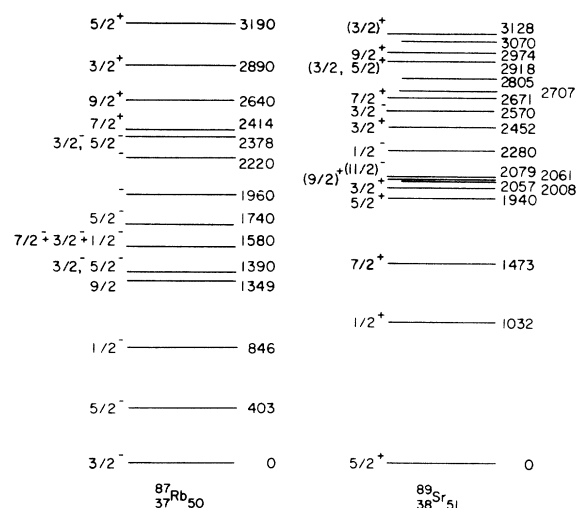
The level at 4413.71 keV is assigned a spin-parity of 3^+ on the basis of direct population from the neutron capture state and a $\log ft$ value indicative of a nonunique β transition. The 4513.86-keV level has a spin-parity of 2^- from the allowed $\log ft$ value for the populating β branch and the angular correlation results.

The β branches to the levels at 4845.37 and 4852.90 keV both have allowed $\log ft$ values, and angular correlation measurements suggest spins of 3 and 2 for the respective levels. The 3^- assignment to the 4845.37-keV level is also supported by the observation of direct population from the neutron capture state.

IV. DISCUSSION

The level structure of ^{88}Sr has been rather thoroughly investigated, at least for energies below 5 MeV, by β decay studies (this work) and reaction studies (see Ref. 1). The theoretical understanding of this structure has been less than satisfactory, either from consideration of one particle-one hole interactions for both neutrons and protons,¹³ or the coupling of the ^{90}Zr core with two proton holes.¹⁴ Very recently, Auerbach and Vary⁴⁴ have calculated the structure of ^{88}Sr using an expanded orbital space with both neutron and proton one particle-one hole excitations using surface restricted effective interactions. They obtain results which are in relatively good agreement with experiment, especially regarding relative transition strengths. Because of the current improvements to the interpretation of the level structure of ^{88}Sr , no further discussion of this structure will be made here, except to point out that the levels at 4224 and 4853 keV are not observed in reaction studies.

The interpretation of the level structure of ^{88}Rb is a different picture. There exist at present no theoretical calculations for the structures of odd-odd nuclei in this region. It is possible, however, to suggest the major configurations contributing to the levels of ^{88}Rb by using the known shell-model states and levels that have been observed experimentally in the neighboring odd nuclei ^{87}Rb and ^{89}Sr . These levels, shown in Fig. 6, are taken from the recent literature.⁴⁵⁻⁴⁸ The structures of ^{87}Rb ($Z=37$) and ^{89}Sr ($N=51$) up to 3 MeV are needed to predict that of ^{88}Rb ($Z=37$, $N=51$) up to 3 MeV, using a weak coupling scheme and neglect-

FIG. 6. Level structures of ^{87}Rb and ^{89}Sr up to 3 MeV.

ing the residual proton hole-neutron particle interaction. In this way, particle-hole interactions in the even "core" configurations are already approximately accounted for. In the discussion below, the odd-particle configurations will be referred to as $\pi(E_{\text{level}})$ for ^{87}Rb and $\nu(E_{\text{level}})$ for ^{89}Sr .

A. ^{88}Rb odd-parity levels

The lowest energy odd-parity multiplet expected in ^{88}Rb is the $\pi(0)\nu(0)$ multiplet, dominated by the $\pi(2p_{3/2})^{-1}\nu(2d_{5/2})$ shell-model configuration. The multiplet ordering of 2^- , 3^- , 1^- , and 4^- ascending in energy is not that expected from simple coupling rules.⁴⁹ However, the 2^- and 1^- levels have been reported¹⁰ to contain an admixture from the $\pi(2p_{3/2})^{-1}\nu(3s_{1/2})$ configuration, so the multiplet ordering may be distorted from configuration mixing.

The two levels at 362.23 and 390.54 keV show similar behavior in transitions to the ground-state and 27.52-keV level, and the 28.26-keV connecting transition suggests a similar nature for these levels. The $\pi(403)\nu(0)$ multiplet, which should be dominated by the $\pi(1f_{5/2})^{-1}\nu(2d_{5/2})$ configuration, may account for the 362.23- and 390.54-keV levels, with some mixing possible from the ground-state multiplet configuration. The higher spin members of the multiplet are not expected to be populated in β decay from even-even ^{88}Kr .

The levels at 862.33 and 1141.35 keV both exhibit strong $l_n=0$ transfer strength in the (d, p) reaction.¹⁰ Hence, they should have the $\pi(2p_{3/2})^{-1}\nu(3s_{1/2})^1$ configuration as the major part of their wave function, coming from the $\pi(0)\nu(1032)$ multiplet. The β -decay population of these levels suggests the presence of an admixture of the "ground-state multiplet" configuration.

The rest of the odd-parity levels are difficult to characterize on a simple basis. However, we note that there are 11 remaining levels (not counting the 1793.3-keV level) with spins ≤ 2 observed in β decay, while the remaining configuration combinations up to 2 MeV from Fig. 6 give rise to at least 15 additional levels of this low spin range.

B. ^{88}Rb even-parity levels

Four 1^+ levels from 2231 to 2771 keV are observed in ^{88}Rb , populated in the β decay of ^{88}Kr .

These levels are notable in that they dominate the β branching pattern, as evidenced from their $\log ft$ values. An interpretation of these levels can be made using the same weak coupling scheme suggested above for the odd-parity levels. With this scheme, four 1^+ levels with unperturbed energies below 2.9 MeV can be formed from the $\pi(0)\nu(2880)$, $\pi(0)\nu(2570)$, $\pi(2414)\nu(0)$, and the $\pi(2890)\nu(0)$ configurations.

The first two levels should contain as major components the orbitals $\pi(3p_{3/2})^{-1}\nu(3p_{1/2})^{-1}2d_{5/2}^2$ and $\pi(3p_{3/2})^{-1}\nu(3p_{3/2})^{-1}2d_{5/2}^2$, respectively. These configurations lead to a natural explanation of the large β branching to the 2231- and 2392-keV levels, resulting from the decay of a $p_{1/2}$ or $p_{3/2}$ neutron into a $p_{3/2}$ proton, respectively. In the latter case, the relatively strong γ branching to the ground-state configuration multiplet proceeds through the single-particle neutron transition $d_{5/2} \rightarrow p_{3/2}$. The similar branching from the 2231-keV level indicates the presence of mixing from the configuration dominant in the 2392-keV level.

The levels at 2548 and 2771 keV have different γ branching patterns and probably are associated with the other two possible weak coupling states. The β -branch and γ -deexcitation patterns for these levels are not so easy to interpret, due to a less well defined character of the ^{87}Rb levels attributed to these levels. Possibly, admixtures from the other configurations account for at least part of these transition strengths.

The above interpretations, though speculative, are consistent with the apparent absence of these levels in reaction studies^{9, 10} and the appearance of relatively low-lying two particle-one hole excitations of negative parity observed in odd Sb ($Z=51$) nuclei.^{50, 51} Vanden Berghe and co-workers^{52, 53} have performed calculations which show that such states will be low-lying at proton mid-shell for $N=51$ nuclei. The calculated properties for ^{119}Sb and ^{121}Sb agree well with those for experimentally observed levels in the region of 1.4 MeV.⁵¹ Similar calculations for odd-odd nuclei in the region of $A=88$ would be interesting to compare to the results of the present work.

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