Study of proposed ^{27}Al models by an analysis of inelastic scattering data of 60 to 90 MeV deuterons

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Angular distributions for elastic and inelastic deuteron scattering on ^{27}Al were measured at $E_d = 60$, 77, and 90 MeV. The data are analyzed by coupled channels calculations and discussed in terms of the weak coupling, strong coupling, and rotational-vibrational models for ^{27}Al. The analysis supports the concepts of rotational-vibrational interaction in ^{27}Al and suggests the level observed at $E_x = 5.5$ MeV to be the third $9/2^+$ member of the $K = 5/2$ ground state band.

NUCLEAR REACTIONS ^{27}Al(d,d), (d,d'), E=60, 77, and 90 MeV; measured $\sigma(E,\Theta)$, ^{27}Al deduced levels; optical model and coupled channels analysis; deduced L, β_l; weak coupling model, rotational model, rotational-vibrational model.

I. INTRODUCTION

Since ^{27}Al is a nucleus in the transition region between prolate (^{25}Mg, ^{25}Al) and oblate (^{28}Si) deformation, it is not clear from the beginning which model should describe its properties best. Indeed, experiments investigating the low lying levels of ^{27}Al have been explained in the past by rather different approaches.[1-11]

Recently, great success has been achieved in the microscopic description of ^{27}Al by the shell-model calculations of Wildenthal and McGrory.[1] They succeed in reproducing the level energies and spin sequences for the low excitation region of the ^{27}Al level scheme as well as the characteristics revealed by single nucleon transfer and electromagnetic transition rates. On the other hand, additional physical insight into the band structure of ^{27}Al can be gained from various macroscopic models.[2,7-9] This aspect is somewhat hidden in a high-dimensional shell-model calculation. A natural way to test the macroscopic models, which should in fact reproduce also additional ^{27}Al data, e.g., corresponding to higher excitation energy, is provided by the analysis of experimental inelastic scattering results.

The weak coupling model[2] has been known for a long time to describe successfully inelastic scattering of light particles on ^{27}Al.[3-5] Also, the weak coupling model reproduces many of the electromagnetic transition probabilities. It fails, however, in predicting the correct spectroscopic factors for one nucleon transfer into or out of ^{27}Al.[6] Moreover, the existence of a low lying $\frac{11}{2}^+$ state at $E_x = 4.51$ MeV and its γ decay are not compatible with this model.[7]

These deficiencies are partly resolved by the strong coupling model of ^{27}Al,[7,8] which assumes a static prolate deformation of the nucleus. This model reproduces the positions of the low lying energy levels as well as many electromagnetic transitions rates and the lowest $\frac{11}{2}^+$ state in ^{27}Al. Assuming an oblate instead of a prolate deformation, Dehnhard[9] has shown that the strong coupling model including Coriolis band mixing is also able to account for the enhanced $E2$ transitions between ^{27}Al states and for spectroscopic factors of nucleon pickup from ^{28}Si. Nevertheless, inelastic scattering data and the properties of the $\frac{9}{2}^+$ state at 3.0 MeV, which is proposed to be the third member of the $K = \frac{5}{2}$ ground state band,[10] have not yet been shown to be compatible with this model.

A theoretical approach proposed by Röpke, Glattes, and Hammel[11] combines the complementary aspects of these two rather opposite models. This rotational-vibrational coupling scheme accounts well for much of the experimental data concerning ^{27}Al: inelastic scattering to states up to an excitation energy of 3 MeV has been well described as well as $E2$-transition rates and the level spacings. Some difficulties still remain in the calculation for $M1$ transition strengths and spectroscopic factors. This is not necessarily connected with a principal deficiency of the rotational-vibrational model for ^{27}Al, since both quantities are sensitive to an interplay of collective and single particle motion. Such effects are not yet included in the quoted[11] calculations.

In view of these rather different approaches, additional experimental information seems desirable in order to test the various macroscopic models more rigorously. The present work has been prompted by the observation that inelastic deuteron scattering at higher incident energies has proven to be an effective spectroscopic tool for the investigation of the collective structure of light nuclei.[12,13] Therefore, inelastic deuteron scattering on ^{27}Al has been studied[14] at different energies between 60 and 90 MeV and has been analyzed on the basis of the three models mentioned above. Consistent results at all incident energies would indicate that the scattering mechanism is well accounted for.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The measurements were performed using the beam of the 3.30 m variable energy Jülich isochronous cyclotron, JULIC. The deuteron beam extracted from the cyclotron impinged upon a 3.0 mg/cm^2 natural Al foil located at the center of a 20 cm diameter scattering chamber.[15] The reaction products were analyzed by means of ΔE-E telescope counters mounted in cryostates around the scattering chamber. The telescopes consisted of a 0.5 mm thick surface barrier ΔE counter[16] and of a 20 mm Ge(Li) diode as E counter.[17] The resolution obtained in the deuteron spectra was about 200–250 keV full width at half-maximum (FWHM) corresponding to the 0.3% energy spread of the unanalyzed beam of JULIC.

A typical deuteron spectrum measured at E_d = 77 MeV and $\theta_{lab} = 50°$ is shown in Fig. 1. The three lowest groups can be attributed to the excitation of known[18] levels at 0.0 MeV $\frac{5}{2}^+$, 0.84 MeV $\frac{1}{2}^+$, and 1.01 MeV $\frac{3}{2}^+$ (unresolved doublet), and 2.21 MeV $\frac{7}{2}^+$. The fourth group very likely arises mainly from the $\frac{9}{2}^+$ level at 3.00 MeV, though it is not resolved from the neighboring $\frac{3}{2}^+$ state at 2.98 MeV. Only negligible contributions are expected from this $\frac{3}{2}^+$ level, since a high resolution experiment has demonstrated that inelastic p scattering[19] excites this $\frac{3}{2}^+$ level by almost one order of magnitude less than the $\frac{9}{2}^+$ state. A few further peaks at 4.6, 5.5, 6.7, and 7.4 MeV excitation energy are clearly visible (Fig. 1). In Sec. III C some evidence is presented that the first two peaks correspond essentially to the $\frac{11}{2}^+$ and $\frac{9}{2}^+$ levels at 4.51 and 5.43 MeV, respectively.[18]

The deuteron spectra were unfolded by an automatic computer program[20] fitting the peaks on top of a smooth background by a Gaussian distribution with an exponential low energy tail (Fig. 1). At three incident deuteron energies, E_d = 60, 77, and 90 MeV, angular distributions in the range from 12° to about 90° have been determined for ^{27}Al

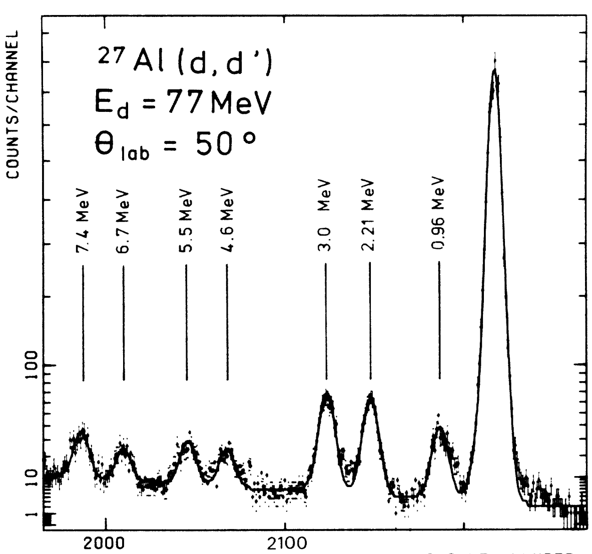

FIG. 1. Spectrum of scattered deuterons on ^{27}Al at 77.3 MeV incident energy and $\Theta_{lab} = 50°$. The solid line represents an over-all fit to the spectrum.

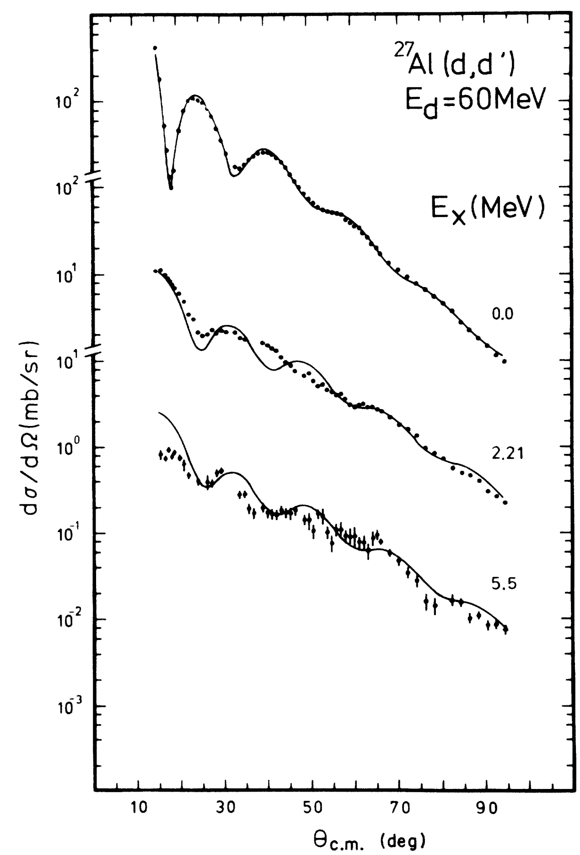

FIG. 2. Angular distributions of deuterons scattered elastically and inelastically from ^{27}Al at an incident energy of 60 MeV. The full curves result from coupled channels calculations with parameters given in Table I.

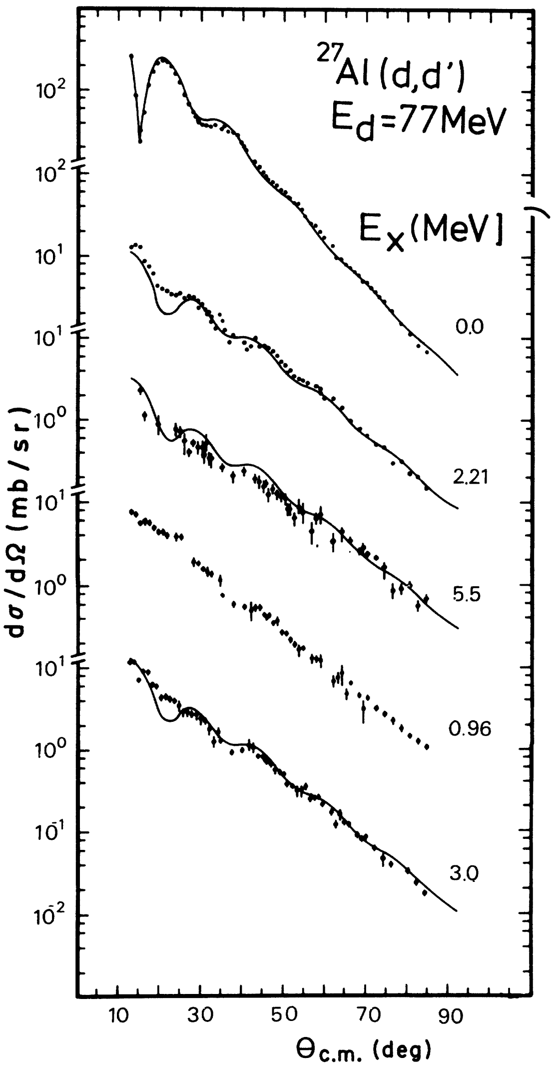

FIG. 3. Angular distributions of deuterons scattered elastically and inelastically from ^{27}Al at an incident energy of 77 MeV. The full curves result from coupled channels calculations with parameters given in Table I.

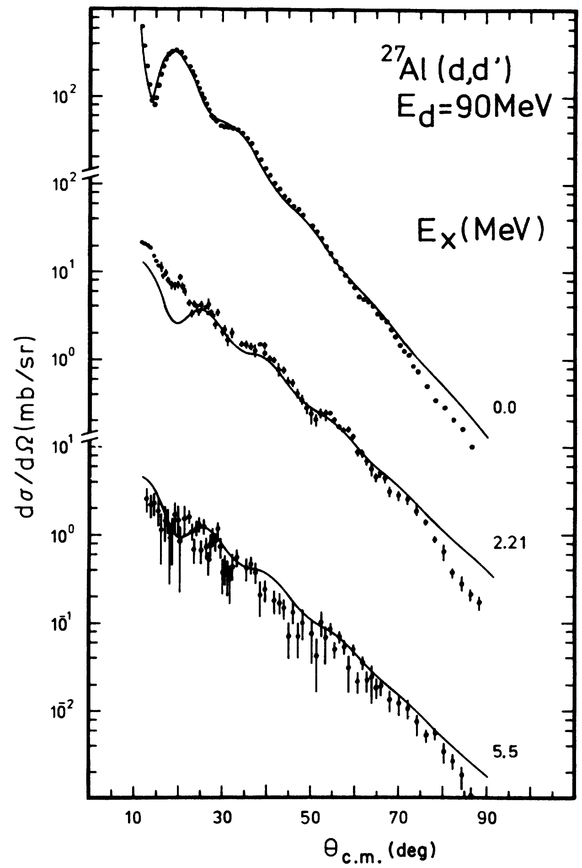

FIG. 4. Angular distributions of deuterons scattered elastically and inelastically from ^{27}Al at an incident energy of 90 MeV. The full curves result from coupled channels calculations with parameters given in Table I.

states up to excitation energies of about 8 MeV. In Figs. 2–4 the angular distributions of deuterons ($E_d = 60$, 77, and 90 MeV) scattered from the ground state, and some excited states, which are relevant to further discussion, are shown. The absolute error of the cross sections is estimated to be about 15%, while the errors indicated in Figs. 2–4 are due only to statistics.

The elastic angular distributions show a clear diffraction pattern at forward angles. All the other angular distributions do not have such a pronounced structure. Therefore, they are characterized mainly by two parameters: the magnitude of the absolute cross section and the slope. For all states the slopes are about the same for a fixed incident deuteron energy and increase gradually with increasing energy.

III. DISCUSSION

A. Weak coupling model

A simple but quite appropriate approach to the analysis of the data consists in the comparison of the integrated experimental cross sections with the intensity predictions of the weak coupling model.

In this model,[2] a $d_{5/2}$ proton hole is coupled to the 0^+ ground and first excited 2^+ state in ^{28}Si, generating in this way the $\frac{5}{2}^+$ ground state and five low lying excited states in ^{27}Al with spins $\frac{1}{2}^+$ to $\frac{9}{2}^+$ (Fig. 5). According to this model, the scattering cross sections for the excited states with spins J_f should be proportional to $(2J_f+1)$. If the $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{7}{2}^+$, $\frac{5}{2}^+$, and $\frac{9}{2}^+$ states in ^{27}Al at $E_x = 0.84$, 1.01, 2.21, 2.73, and 3.00 MeV are considered as weak

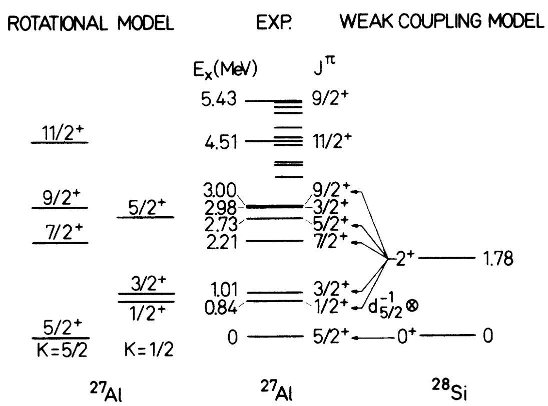

FIG. 5. The weak coupling model and the strong coupling model for ^{27}Al in comparison with the experimental level scheme. For convenience the theoretical levels are shown at the energy of the corresponding experimental states.

coupling states with the same core then their cross sections should relate as 2 : 4 : 8 : 6 : 10.

In Fig. 6, these relations are displayed by the left-hand dark columns (number 1) together with the corresponding experimental values (columns number 2, 3, and 4 for $E_d = 60$, 77, and 90 MeV), obtained by integrating the measured angular distributions between 15° and 85°. The cross section for the $\frac{5}{2}^+$ state at 2.73 MeV could only be estimated. The integrals are normalized to that of the $\frac{7}{2}^+$ state at $E_x = 2.21$ MeV. For the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states at $E_x = 0.84$ and 1.01 MeV the sum of their intensities is given, because they are not resolved in this experiment. As can be seen in Fig. 6, the normalized values of the integrated cross sections for an individual level are rather independent of the incident deuteron energy. The largest differences of about 10% appear for the unresolved $\frac{1}{2}^+$ and $\frac{3}{2}^+$ doublet. Furthermore, except for the $\frac{5}{2}^+$ level, there is a fair agreement with the weak coupling prediction, although the intensities corresponding to the $\frac{9}{2}^+$ and to the unresolved $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states seem to deviate systematically from their theoretical values by about 15%. The estimated scattering cross section to the $\frac{5}{2}^+$ level, however, is lower by a factor of about 2 to 3 than that given by the $(2J_f + 1)$ weak coupling intensity rule.

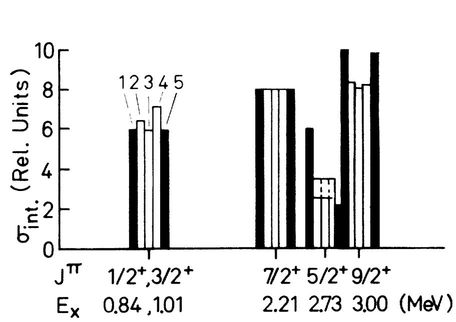

FIG. 6. Integrated cross sections (σ_{int}) of deuterons scattered to excited states in ^{27}Al at $E_d = 60$, 77, and 90 MeV (columns 2, 3, and 4, respectively) compared to the $(2J_f + 1)$ intensities of the weak coupling rule (column 1) and to the prediction of the weak coupling model taking into account mixing between the ^{27}Al ground state and the first excited $\frac{5}{2}^+$ level (column 5).

This discrepancy may be removed assuming a mixing between that level and the ^{27}Al ground state,[3,4] which has the same spin and parity. With a mixing parameter $A^2 = 0.23$, extracted in a previous analysis of proton scattering data,[4] the predicted cross section for the excited $\frac{5}{2}^+$ level is substantially reduced by a factor of about 3, while all other cross sections are subject to only a slight reduction. These new relationships are given in Fig. 6 by the dark columns on the right (No. 5), which are normalized again to the $\frac{7}{2}^+$ state.

B. Strong coupling model

In the strong coupling model,[7,8] the low lying ^{27}Al states are explained in terms of a $K^\pi = \frac{5}{2}^+$ ground state rotational band and a $K^\pi = \frac{1}{2}^+$ band (Fig. 5). The $\frac{5}{2}^+$ ground state, the 2.21 MeV $\frac{7}{2}^+$ and the 3.00 MeV $\frac{9}{2}^+$ levels are considered to be members of the $K^\pi = \frac{5}{2}^+$ ground state band. To check this classification, a coupled channels calculation with the code JUPITOR-1[21] has been carried out. In search runs, the coupled channels potential parameter sets (Table I) were deduced from the optical model parameter sets obtained from an analysis of the measured ground state angular distributions. The sets of both types are similar to each other. The quadrupole deformation parameter β_2 is determined by the cross section to the $\frac{7}{2}^+$ level, which in first order is proportional to $\beta_2{}^2$. Therefore, no free parameter is left in the coupled channels calculation for the $\frac{9}{2}^+$ level at $E_x = 3.00$ MeV. At all incident deuteron energies, the angular distributions of the ground state and the $\frac{7}{2}^+$ excited level can be reproduced quite well with the parameter sets given in Table I and discussed below, while no fit for the $\frac{9}{2}^+$ level can be achieved at the same time. The calculated cross sections are at least a factor of 3 lower

TABLE I. Parameters of the (d,d') coupled channels calculations for ^{27}Al; $r_V = 1.25$ fm, $r_C = 1.3$ fm.

E_d (MeV)	V (MeV)	a_V (fm)	W_s (MeV)	r_W (fm)	a_W (fm)	β_2	$\beta_{vib}^{(9/2^+,3,0)}$
60	70.6	0.65	16.7	1.14	0.71	0.22	0.30
77	67.5	0.73	16.3	1.08	0.77	0.22	0.30
90	65.2	0.76	15.9	1.03	0.81	0.22	0.30

than the measured ones. Also, changing the sign of β_2 or including an extra hexadecapole component does not change the $\frac{9}{2}^+$ cross section noticeably. These calculations provide additional evidence for previously reported results[4] that the strong coupling model with a pure $K^\pi = \frac{5}{2}^+$ ground state rotational band is not appropriate for the description of inelastic processes on ^{27}Al. Nevertheless, the question remains whether any more extensive calculations, e.g., including other K^π components, would remove these discrepancies.

C. Rotational-vibrational model

Both weak coupling and strong coupling features are unified in the rotational-vibrational model of ^{27}Al.[11] Here, by coupling a $K^\pi = \frac{5}{2}^+$ single proton state to the second 2^+ level in ^{26}Mg, known to be a γ-vibrational state, two additional rotational bands result which are built on $K = \frac{5}{2} \pm 2$ γ-vibrational bandheads[11] (Fig. 7). Since ^{27}Al is quite a soft nucleus, the rotational-vibrational interaction could be of such an importance that a $K = \frac{5}{2} - 2$ character might also be admixed to the 3.68 MeV level, assumed to be the $K = \Omega = \frac{1}{2}$ single particle bandhead with one particle in Nilsson orbit 9 (Ref. 11) (Fig. 7). An additional $K = \frac{3}{2}$ band results from putting one particle into Nilsson orbit 8. The $\frac{9}{2}^+$ state at $E_x = 3.00$ MeV is interpreted[11] as head of a $K^\pi = \frac{9}{2}^+$ band, while the third member of the ground state rotational band is postulated to be located at an excitation energy of about 5.5 MeV. Indeed, a promising candidate for such a state has been reported[11,18,22] to be located at $E_x = 5.43$ MeV, and we also observe in our spectra a distinct deuteron group at 5.5 MeV. The fairly strong intensity of this observed group at rather high excitation energy makes a collective nature quite probable; the measured angular distribution can be reproduced

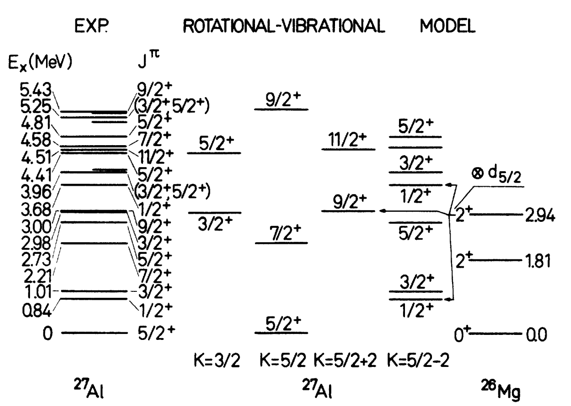

FIG. 7. Classification of ^{27}Al states according to the rotational-vibrational model. For convenience theoretical levels are shown at the energy of the corresponding experimental states.

assuming $J^\pi = \frac{9}{2}^+$. Therefore, we identify the 5.5 MeV group with the known $\frac{9}{2}^+$ level at 5.43 MeV. The full curves in Figs. 2–4 show the results of our coupled channels calculations coupling the $\frac{5}{2}^+$, $\frac{7}{2}^+$, and $\frac{9}{2}^+$ states at $E_x = 0.0$, 2.21, and 5.43 MeV as members of the ground state rotational band using the potential parameters of Table I and a deformation parameter $\beta_2 = 0.22$. The parameters[23] show a smooth, almost linear dependence on the incident deuteron energy (Table I); the real potential depth has a marked linear decrease with increasing incident deuteron energy, which is expected because of the momentum dependence of the nuclear forces. The coupling parameters are independent of the incident deuteron energies, which proves the reliability of the collective model for ^{27}Al in this energy range. The absolute value of $\beta_2 = 0.22$, extracted from our analysis, is somewhat lower than the values reported earlier,[10] which fluctuate between 0.27 and 0.3. A change of the sign of β_2 does not significantly affect the calculated distributions.

The $\frac{9}{2}^+$ state at $E_x = 3.0$ MeV can be reasonably described in the rotational-vibrational model for ^{27}Al. Figure 3 shows a fit to this state at $E_d = 77$ MeV with a coupling parameter $\beta_{(5/2)+2}(\text{vib}) = 0.3$, assuming it to be the head of the $K = \frac{5}{2} + 2$ rotational band. The second member of this $K^\pi = \frac{9}{2}^+$ band is predicted[11] to be located at about 4.5 MeV and indeed, an $\frac{11}{2}^+$ level is experimentally known[18] at $E_x = 4.51$ MeV. In the present experiment a deuteron group at 4.6 MeV was observed which could be associated with this $\frac{11}{2}^+$ level. However, up to now a quantitative test has not been possible, since for an odd nucleus the higher members of such a band cannot be treated in the code JUPITOR-1. The same applies to the sum of the angular distributions (see Fig. 3) of the two unresolved $\frac{1}{2}^+$ and $\frac{3}{2}^+$ peaks at 0.84 and 1.01 MeV, respectively. In the rotational-vibrational model of ^{27}Al these states are the two lowest members of the $K = \frac{5}{2} - 2$ band.

IV. CONCLUSIONS

The weak coupling, the strong coupling, and the rotational-vibrational models have been applied to the analysis of inelastic deuteron scattering on ^{27}Al in the energy range $60 \leq E_d \leq 90$ MeV. While the weak coupling model, including mixing between ground state and first excited $\frac{5}{2}^+$ state, was able to account for the observed cross sections for inelastic scattering to low lying states in ^{27}Al, the strong coupling model, assuming a pure $K^\pi = \frac{5}{2}^+$ ground state rotational band, failed in that respect. The success of the weak coupling model, however, should not be stressed too much, because it is certainly too simple[18] and not adequate to describe further essential aspects[6,7] of the ^{27}Al nucleus (e.g., see Introduction).

On the other hand, the rotational-vibrational model accounts well for our measured data at higher excitation energy. At all incident deuteron energies three members of the $K^{\pi} = \frac{5}{2}^+$ ground state rotational band could consistently be described by coupled channels calculations assigning spin $J^{\pi} = \frac{9}{2}^+$ and $K = \frac{5}{2}$ to a state observed at $E_x = 5.5$ MeV and predicted by theory in this energy range. Also, the assumed $K = 2 + \frac{5}{2}$ bandhead at $E_x = 3.00$ MeV can be reproduced with a reasonable coupling parameter. For technical reasons, however, it was not possible to test the rotational-vibrational model more rigorously by calculating the scattering to the higher members of the $K = \frac{5}{2}^+ \pm 2$ bands. Also, better agreement with data on the basis of a modified strong coupling model, including Coriolis band mixing, cannot be ruled out completely. However, it may be concluded that our analysis of deuteron inelastic scattering data does favor the concept of rotational-vibrational interaction in ^{27}Al.

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