

Resonance neutron capture γ rays from $^{168}\text{Er}^\dagger$

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γ rays following neutron capture in seven major s -wave resonances in ^{168}Er up to 1 keV have been measured. The results serve to confirm and extend the knowledge of levels in ^{169}Er obtained from previous (d,p) , (d,t) , and thermal (n,γ) experiments. Absolute width determinations are made and a photon strength function is obtained.

[NUCLEAR REACTIONS ^{169}Er energy levels, spins, parities, resonance parameter, partial radiative widths, photon strength functions.]

I. INTRODUCTION

A great deal of nuclear structure information can be gleaned by combining radioactivity studies and single and multiple nuclear transfer reactions with neutron-capture γ -ray data. The specific and selective character of the former technique can be used to probe the nature of the wave functions, while the nonspecific (n,γ) reaction can be used to ensure that all final states in a given spin-parity range are populated. This feature of the (n,γ) reaction is operative only, however, when a sufficient number of capture states are included in the experiment. Averaging over a number of such resonances ensures that the fluctuations of the Porter-Thomas¹ distribution are reduced. In the past, many (d,p) and (d,t) experiments have been combined with thermal neutron (n,γ) data to study energy levels; in this case the Porter-Thomas distribution severely limits the usefulness of the (n,γ) data. For such comparisons it is useful to have the results for epithermal capture over several discrete resonances. The present experiment is an illustration of this point.

The $^{168}\text{Er}(n,\gamma)^{169}\text{Er}$ reaction has been studied for the major s -wave resonances of ^{168}Er up to a neutron energy of ~ 1 keV. This nuclide is a particularly difficult one to study via the (n,γ) reaction because of the high absorption cross section of the neighboring ^{167}Er isotope, especially at neutron energies near thermal. Previous (n,γ) work at thermal was reported by Mulligan, Sheline, Bunker, and Journey,² who used a special high purity sample of 99.987% ^{168}Er with only 0.13% of ^{167}Er . In the present experiment, a lower sample purity of 95.47% was available, but the use of resonance capture avoided the problem of competing isotopes. We could not, however, investigate off-resonance capture, and particularly thermal capture, be-

cause of the 2.44% ^{167}Er in the sample. On the other hand, the spectra from seven s -wave resonances provides additional information missed in thermal capture because of Porter-Thomas fluctuations of partial radiative widths.

Previous (n,γ) data from Mulligan *et al.*² were combined with (d,p) and (d,t) information from the same authors to establish the nuclear structure of ^{169}Er . Furthermore, (d,p) and (d,t) data have been published by Tjøm and Elbek³ as part of their study of energy levels in the erbium isotopes.

II. DESCRIPTION OF EXPERIMENT AND RESULTS

The fast chopper facility of the high flux beam reactor of Brookhaven National Laboratory was used for these measurements. A sample of 110.2 g of Er_2O_3 wrapped in a 7.68 cm \times 7.68 cm \times 0.79 cm polyethylene packet was used as the capturing target. The isotopic components of the sample used are the following: 162 (<0.02%), 164 (0.04%), 166 (1.44%), 167 (2.44%), 168 (95.47%), and 170 (0.61%).

To cover the neutron energy region from thermal to 1 keV, flight paths of 48 and 22 m were used with rotor speeds ranging from 15 000 to 1500 rpm. The γ -ray spectra were recorded with a Ge(Li) detector of nominal 10% efficiency relative to a standard 7.6 cm NaI(Tl) crystal (Ortec VIP-10 series). This detector has a resolution of 6 keV at 7.6 MeV. The total duration of the runs was about two weeks.

The data were recorded on magnetic tape, event by event, and the magnetic tapes thus obtained were subsequently scanned to assemble the spectra for each resonance or energy region desired. The individual γ -ray peaks were analyzed by assuming a Gaussian resolution function on a slowly varying continuum term, and employing least-

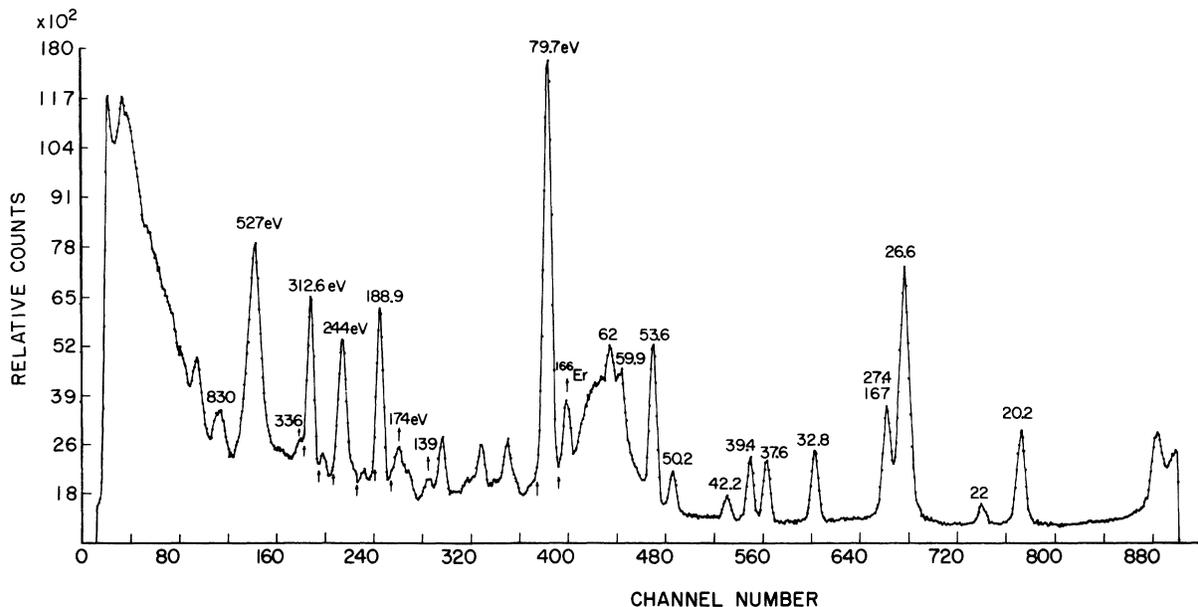


FIG. 1. The time-of-flight spectrum for neutron capture γ rays from the sample enriched in ^{168}Er at 48 m for a chopper speed of 15 000 rpm. Although the sample contains 95% ^{168}Er , many ^{167}Er impurity resonances are visible in the spectrum. Seven resonances from ^{168}Er were analyzed, ranging from the 79.7 eV resonance to the (unlabeled) resonance at 1005 eV.

squares fitting procedures. In view of the precise Q value reported by Mulligan *et al.*² ($Q = 6003.1 \pm 0.3$) keV no attempt was made at a precise determination of γ -ray energies in this experiment; however, by calibration against known γ -ray background lines, we find our energies consistent with those of Mulligan *et al.*² within an error of ± 0.5 keV.

Figure 1 shows the neutron time-of-flight spectra taken at 48 m. The first seven strong s -wave resonances of ^{168}Er are clearly visible. The arrows show some of the scan limits used for obtaining the resonance γ -ray spectra. The high resolution total neutron cross section of Er isotopes has been investigated by the Columbia group⁴ and these are compiled in the BNL neutron cross section compilation, BNL-325.⁵ Looking at these tables it is evident that there are many s -wave neutron resonances in ^{168}Er . The resonances with large neutron widths are observed at 79.7 eV ($\Gamma_n = 41$ meV), 189 ($\Gamma_n = 76$), 244 ($\Gamma_n = 550$), 312 ($\Gamma_n = 160$), 527 ($\Gamma_n = 845$), 830 ($\Gamma_n = 1020$), and 1005 ($\Gamma_n = 400$). There are also many weak resonances but since the capture cross section depends on the product of $\Gamma_n \Gamma_\gamma$, only the strong resonances have been clearly observed in our studies. The resolution above 1 keV is too poor to resolve resonances. At energies below 79.7 eV, a large number of resonances due to ^{167}Er are also observed in spite of its low abundance (2.44%). The

thermal capture cross section for ^{167}Er is 670 b as compared to the 1.95 b of thermal capture cross section in ^{168}Er . This made it impractical to measure intensities for neutron capture γ rays at thermal and between ^{168}Er resonances.

In a separate experiment, the absolute values of the partial radiative widths for $^{168}\text{Er}(n, \gamma)^{169}\text{Er}$ were obtained by comparison with the reaction $^{197}\text{Au}(n, \gamma)^{198}\text{Au}$, which we have adopted for our intensity standards. The widths for the 4.9 eV resonance for gold have been measured by Kane,⁶ and the thermal intensities by Kane⁶ and Loper, Thomas, and Bollinger.⁷ A composite sample of ^{168}Er with a 0.010 cm Au foil was used. The absolute capture rate in the 4.9 eV resonance was calculated from the known resonance parameters, and likewise for the 79 eV resonance of ^{168}Er . The relative flux ratio between 4.9 and 79 eV was derived from the signal from the $^{10}\text{B}(n, \alpha \gamma)$ reaction, the ^{10}B being contained in a borosilicate glass used as a beam monitor. From a knowledge of the relative capture rates (corrected for multiple scattering), the known Au partial widths, and detector efficiency variations with photon energy, the absolute widths are obtained to an estimated accuracy of 10%.

To determine the relative capture rates in the various ^{168}Er resonances, we used the strong low energy γ -ray lines at 159, 430, 498, and 714 keV. The weighted sum of these transitions was as-

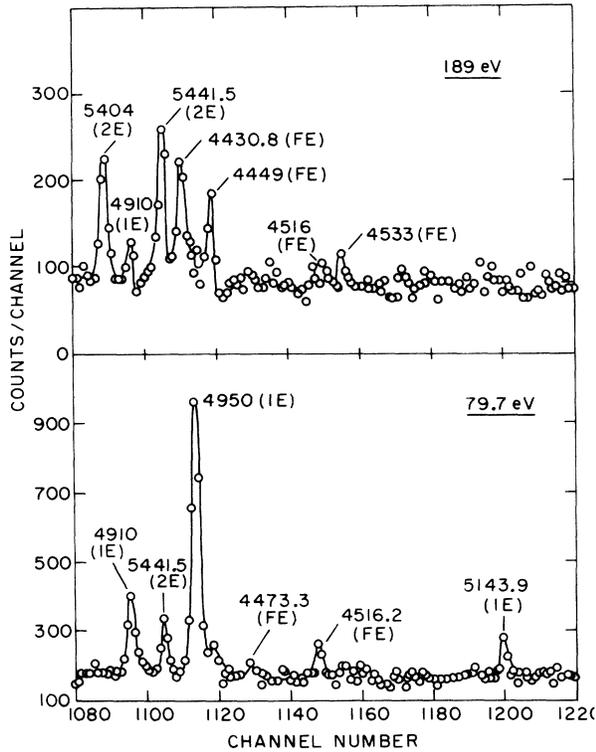


FIG. 2. Portions of the 79.7 and 189 eV spectra. The former shows the one-escape peak (1E) of 4950 keV γ ray, while the latter shows the two-escape peak (2E) of the 5404 keV γ ray. Neither of these was seen in the thermal capture spectrum of Ref. 2.

sumed proportional to the total capture rate, and was used to intercalibrate the resonances against the 79 eV standard.

Portions of the high energy γ -ray spectra of the 79 and 189 eV resonances observed in the present experiment are shown in Fig. 2. We have assigned 29 primary γ -ray transitions populating levels in ^{169}Er up to an excitation energy of 3781 keV. The absolute partial widths (in meV) for these transitions are given in Table I. We estimate the uncertainty in energy to be about ± 0.5 keV. In addition to the 29 transitions listed we have observed several more in the 79 eV resonance, where our spectrum is of relatively high statistical quality. These γ rays have energies 4222, 4209, 4107, 4036.3, 4005, 3905, and 3738.7 keV.

III. DISCUSSION OF RESULTS

A. Average transition strengths

The $J = \frac{1}{2}^+$ compound states in the present experiment populate $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states by electric dipole emission. From the systematics of neutron-capture reactions, we can be confident that average

$M1$ widths to $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states are an order of magnitude smaller. Our experimental sensitivity is such that we would observe only the strongest $M1$'s. Hence, most of the transitions listed in Table I are surely $E1$ transitions.

The average values of partial widths in a population of seven resonances will follow a χ^2 distribution with $\nu=7$. This distribution is much narrower than the Porter-Thomas distribution ($\nu=1$) characteristic of single transitions. We are able to draw conclusions on final state spin-parities within the fluctuations allowed about the mean. The 5 and 95 percentiles for this distribution lie at 0.3 and 2.0 times the mean value, respectively.⁸

We can predict the expected average $E1$ strength from either a single particle estimate⁹ or from an extrapolation of the giant dipole resonance.¹⁰ Using the resonance parameters of BNL-325 (3rd ed.) and an expected value of $k_{E1} \approx 3$, from the relation

$$k_{E1} = [\langle \Gamma_{\gamma\lambda f} \rangle / D(J)] A^{-2/3} E_{\gamma}^{-3},$$

(where $\Gamma_{\gamma\lambda f}$ denotes the partial radiative width from the initial state λ to the final state f , and the average is over initial states of a given spin J) we expect $\langle \Gamma_{\gamma\lambda f} \rangle = 1.86$ meV at 6 MeV. The giant dipole resonance (GDR) extrapolation for Er is quoted by Bartholomew *et al.*¹⁰ They give for the expression

$$f = (\langle \Gamma_{\gamma\lambda f} \rangle / D) E^{-3} = 1.5 \times 10^{-7} \text{ MeV}^{-3}$$

or $\langle \Gamma_{\gamma\lambda f} \rangle = 3.04$ meV at 6 MeV.

From our discussion above we can expect our $E1$ transitions averaged over seven resonances to range from $\frac{1}{2}$ to twice the estimate. If we make the almost certain assumption (we will discuss this below) that the five lowest energy final states populated in the present experiment are negative parity states, we obtain:

$$\langle \Gamma_{\gamma\lambda f} \rangle (6/E_{\gamma})^3 = 1.56 \text{ meV};$$

(averaged over 35 transitions)

which is consistent with our hypothesis of $E1$ transitions. From the 35 transitions represented by our 7 resonances and 5 final states, we thus obtain

$$k_{E1} = (2.5 \pm 0.6) \times 10^{-9} \text{ MeV}^{-3}$$

and

$$f = (0.76 \pm 0.18) \times 10^{-7} \text{ MeV}^{-3}.$$

The $E1$ strength function is close to the value calculated from the Weisskopf estimate, but about a factor of 2 under the GDR extrapolation. This observation is supported by the 167 Er data of Bollinger and Thomas.^{10, 11}

TABLE I. The partial widths for radiative transitions from seven resonances in ^{168}Er are shown in this table. A number of entries show upper limits only, indicating nonobservance for that transition. In the last column the quantity $\langle \Gamma_{\gamma\lambda f} \rangle (B_n/E_\gamma)^3$ is shown. This represents the partial width evaluated at the neutron binding energy (B_n) by correcting the E_γ^3 phase-space factor for photon emission.

E_γ (keV)	E_x (keV)	$E_n = 1005$ eV		$E_n = 830$ eV		$E_n = 527$ eV		$E_n = 312$ eV		$E_n = 244$ eV		$E_n = 189$ eV		$E_n = 79$ eV		
		$\Gamma_{\gamma\lambda f}$ (meV)	$\Delta\Gamma_{\gamma\lambda f}$ (meV)	$\langle \Gamma_{\gamma f} \rangle$ (meV)												
6003.0	0	<0.3	<0.3	<0.3	<0.3	1.45	0.31	10.05	0.47	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	1.64
5939.0	64.0	4.40	0.51	<0.5	<0.5	0.98	0.24	1.09	0.31	<0.2	<0.2	1.20	0.16	<0.2	<0.2	1.13
5441.5	562.4	0.55	0.24	1.73	0.31	0.65	0.24	0.39	0.16	<0.2	<0.2	2.12	0.24	0.72	0.16	1.18
5404.0	599.0	1.19	0.47	<0.5	<0.5	6.22	0.94	0.92	0.31	0.42	0.16	1.87	0.31	<0.6	<0.6	2.08
5289.0	714.0	0.98	0.39	1.07	0.31	1.21	0.31	2.09	0.24	3.07	0.31	<0.2	<0.2	<0.1	<0.1	1.76
5143.9	859.1	<0.6	<0.6	0.77	0.24	<0.6	<0.6	<0.2	<0.2	<0.2	<0.2	0.32	0.12	0.65	0.16	0.32
4950.0	1053.0	0.68	0.24	<0.2	<0.2	1.42	0.16	<0.2	<0.2	<0.3	<0.3	<0.2	<0.2	5.82	0.31	2.22
4923.5	1079.5	0.8	0.3	1.0	0.4	0.5	0.2	0.5	0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	0.57
4910.0	1093.0	<0.4	<0.4	0.90	0.31	1.34	0.16	<0.4	<0.4	<0.3	<0.3	0.67	0.24	1.72	0.20	1.21
4617.0	1386.0	2.31	0.47	1.81	0.20	1.11	0.31	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	<0.2	1.64
4533.3	1469.7	<0.6	<0.6	<0.3	<0.3	0.57	0.16	<0.2	<0.2	0.29	0.08	0.65	0.16	0.26	0.08	0.58
4516.2	1486.8	<0.6	<0.6	<0.3	<0.3	2.47	0.47	<0.2	<0.2	<0.2	<0.2	0.36	0.12	0.57	0.12	1.15
4473.3	1529.7	<0.5	<0.5	<0.3	<0.3	1.69	0.47	0.42	0.16	<0.2	<0.2	0.57	0.12	0.31	0.08	1.03
4449.0	1554.0	<0.6	<0.6	<0.6	<0.6	0.73	0.31	<0.3	<0.3	0.39	0.12	1.16	0.24	0.61	0.16	1.02
4430.8	1572.2	<0.6	<0.6	<0.6	<0.6	0.53	0.31	<0.3	<0.3	0.45	0.15	1.30	0.25	<0.2	<0.2	0.82
4181.6	1821.4	<0.6	<0.6	0.65	0.19	1.00	0.39	<0.3	<0.3	0.37	0.12	<0.2	<0.2	<0.2	<0.2	0.85
4163.6	1839.4	<0.6	<0.6	<0.3	<0.3	1.04	0.31	<0.3	<0.3	0.25	0.08	0.31	0.12	0.52	0.16	0.91
4154.7	1848.3	0.91	0.24	<0.2	<0.2	1.13	0.31	<0.3	<0.3	0.83	0.16	0.35	0.12	<0.2	<0.2	1.38
4134.9	1868.1	0.61	0.24	<0.3	<0.3	<0.5	<0.5	<0.3	<0.3	0.48	0.16	0.42	0.12	<0.2	<0.2	0.66
4073.8	1929.2	<0.4	<0.4	1.29	0.31	0.91	0.24	<0.3	<0.3	0.28	0.08	<0.2	<0.2	1.12	0.12	1.64
4055.5	1947.5	<0.6	<0.6	1.53	0.39	<0.5	<0.5	<0.3	<0.3	<0.2	<0.2	<0.2	<0.2	0.82	0.16	1.09
4047.6	1955.4	<0.5	<0.5	0.55	0.16	1.21	0.47	0.57	0.16	<0.3	<0.3	<0.3	<0.3	<0.2	<0.2	1.09
3980.2	2012.8	0.82	0.24	0.69	0.20	2.50	0.63	0.98	0.24	<0.2	<0.2	<0.3	<0.3	3.53	0.39	4.19 ^a
3973.5	2029.5	1.09	0.31	0.57	0.16	1.57	0.47	<0.2	<0.2	<0.2	<0.2	<0.5	<0.5	1.26	0.24	2.21
3878.9	2124.1	0.72	0.31	<0.5	<0.5	<0.5	<0.5	<0.3	<0.3	0.81	0.24	0.50	0.12	<0.2	<0.2	1.07
3864.0	2139.0	<0.4	<0.4	<0.3	<0.3	<0.5	<0.5	0.69	0.20	<0.2	<0.2	0.68	0.12	0.20	0.08	0.84
3838.0	2165.0	<0.5	<0.5	<0.3	<0.3	0.75	0.28	0.50	0.20	1.28	0.24	<0.2	<0.2	0.22	0.08	1.50
3823.0	2180.0	0.56	0.24	<0.3	<0.3	0.97	0.31	<0.3	<0.3	1.81	0.31	<0.2	<0.2	<0.2	<0.2	1.84
3780.6	2222.4	0.61	0.24	<0.3	<0.3	0.92	0.31	<0.3	<0.3	0.27	0.08	0.61	0.24	0.73	0.16	1.80

^a Possibly a doublet.

B. Nuclear structure of ^{169}Er

The most recent previous data relative to the nuclear structure of ^{169}Er are the (d, p) and (d, t) data of Tjøm and Elbek³ and the experiment of Mulligan *et al.*,² which uses thermal neutron capture data combined with (d, p) and (d, t) . In the latter experiment 45 high energy γ rays were observed, most prominent of which are those at 5938 (6.6%), 5441 (3%), 4908.8 (10.3%), and 4616.5 (6%) keV. All of these strong lines are seen in resonance capture.

The ground state band of ^{169}Er was well established by the early work of Funke *et al.*¹² and Harlan and Sheline¹³ as the Nilsson orbital $\frac{1}{2}^- [521]$. As expected the $\frac{1}{2}^-$ ground and $\frac{3}{2}^-$ 64 keV first excited states are populated strongly in the resonances; the average transition strengths confirm these negative parity assignments. In contrast to thermal capture, where the ground state transition is extremely weak, the resonance averaged ground state width is close to the average expected for $E1$ transitions.

The 5441.5 keV γ transition is in the $E1$ class, establishing a negative parity state of 562 keV, while a 5404 keV transition of comparable strength populates a negative parity state at 599 keV. The latter is not seen at all in the thermal capture spectrum, but it is strongly populated by (d, p) . It was supposed by Tjøm and Elbek³ that a $\frac{1}{2}^- [510]$ state, which is a component of a $K=2$ vibration built on the $\frac{5}{2}^- [512]$, can be placed at 562 keV. The present experimental results are consistent with $\frac{1}{2}^-$ 562 and $\frac{3}{2}^-$ 599 keV assignments for this sequence.

A 5289 keV γ ray is observed to populate a state at 714 keV in five resonances. The $E1$ assignment for this γ ray is definite and confirms the $\frac{3}{2}^-$ assignment of Mulligan *et al.*² as the bandhead of a $\frac{3}{2}^- [521]$ hole state. The average resonance width for this transition is in the $E1$ range and illustrates the fact that the (n, γ) reaction populates hole states and particle states with equal facility.

A relatively weak transition at 5143 keV is consistent with the $\frac{3}{2}^+$ assignment deduced by Mulligan *et al.*² for the 860 keV state, based on selection rule arguments involving the decay of the level. The average radiative width is inconsistent with the assumption that it belongs in the $E1$ class ($P \cong 0.02$). The conclusion that it is an $M1$ confirms the positive parity assignment. Mulligan *et al.* suppose that the $\frac{3}{2}^+$ 860 keV state is the $(K2)$ γ vibration coupled with the $\frac{7}{2}^+ [633]$ and admixed strongly with the $\frac{3}{2}^+ [651]$.

The 4950 keV γ ray is the strongest transition displayed by the 79 eV resonance, yet was unobserved in the thermal (n, γ) work, as is the case

for the 5441.5 keV γ ray. The latter, however, is weak also in the 79 eV resonance. The 79 eV resonance contributes to thermal capture (0.25b out of a total of 1.95b), but level-level interference effects can serve to minimize the strength of the 4950 at thermal. Another reason for the failure of Mulligan *et al.* to observe this transition may be the presence of the strong ^{13}C line near 4945 keV, caused by neutron capture in the graphite moderator of the Omega West reactor. The average strength in Table I allows us confidently to assign $\frac{1}{2}^-$ or $\frac{3}{2}^-$ spin-parity to the 1053 keV level populated by this transition. This level cannot be the $\frac{9}{2}^-$ member of the $\frac{5}{2}^- [523]$ hole state reported by both Refs. 2 and 3, nor the $\frac{11}{2}^-$ member of the $\frac{7}{2}^- [514]$ particle state detected in (d, p) from Ref. 3. It is probably a complex excitation and remains unassigned in the level scheme.

The 4923 keV γ ray is weakly seen in thermal capture; its strength in resonances is surprisingly weak. On the basis of the resonance work alone we would hesitate to assign an $E1$ character to the transition. Its strength is such that it lies at the 10% level of the cumulative distribution for $\nu=7$, so it is just consistent with the $E1$ hypothesis. Its identification with the bandhead of the $\frac{3}{2}^- [512]$ particle state on the basis of the strong (d, p) excitation, however, seems unambiguous. We conclude that its weakness in the resonances of our experiment is a statistical fluctuation.

No evidence is seen in our experiment for the 1360 keV $\frac{1}{2}^-$ and 117.1 keV states proposed by Mulligan. We conclude that the 1360 keV state has positive parity and that the 117.1 has positive parity and/or spin $\geq \frac{5}{2}$. We do confirm the 1387 ($\frac{3}{2}^-$) and 1094 ($\frac{1}{2}^-$, $\frac{3}{2}^-$) assignments proposed by them, on the basis of the observed resonance strengths.

All of the other transitions of Table I are reported in the thermal capture work of Ref. 2, with the exception of the 3980.2 keV γ ray. Most, if not all, of the remaining transitions in Table I are clearly $E1$ in character on the basis of strength.

C. Reaction mechanism

It has been suggested that capture reactions near the position of single-particle size resonances in the neutron strength function would display some of the characteristics of direct transitions.¹⁴ In this event the radiative widths would be expected to show correlations with (d, p) cross sections, and the off-resonance capture amplitudes would show a similar correlation. The presence of the large ^{167}Er cross sections near thermal has prevented us from extracting non-resonant amplitudes through a multilevel interference analysis. We can, however, search for

(n, γ) - (d, p) correlations by comparing the resonance averages against (d, p) cross sections. Such an analysis has been carried out for the entries of Table I and the (d, p) cross sections of Tjøm and Elbek at 60° for the six transitions which proceed to $\frac{1}{2}^-$ or $\frac{3}{2}^-$ levels with well-established Nilsson assignments.

Although in the (d, p) population of these states we see variations of two orders of magnitude, corresponding to the single-particle expansion coefficients of these states, the (n, γ) average widths show variations of at most a factor of 2.

A quantitative calculation of the linear correlation coefficient between the average width and the $\sigma(d, p)$ at 60° shows a value $r=0.59$. This is consistent with a zero correlation at a significance level $P=0.88$. We conclude that ^{168}Er resonance capture proceeds via compound nucleus formation in a statistical reaction process.

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