Excitation of giant magnetic and spin-isospin dipole states in radiative π capture on ¹⁴N and ¹⁰B[†]

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The photon spectra in the capture of stopped pions on ¹⁴N and ¹⁰B were measured in the 50-150-MeV region with a high-resolution pair spectrometer. The total radiative capture branching ratios are 2.13 ± 0.21 and $2.27\pm0.22\%$, respectively. The spectrum corresponding to the first 13 MeV excitation in each of the residual nuclei ¹⁴C and ¹⁰Be is dominated by the transition to the analog of a giant *M*1 state of the target nucleus. The ground state transitions in both nuclei are resolved experimentally. The measured branching ratio for the extremely weak ¹⁴C (g.s.) transition is $(3\pm2)\times10^{-5}$. There is evidence for selective excitation of the analogs of the giant dipole spin-isospin states of ¹⁴N, of which the 3⁻ component appears to be the strongest. In ¹⁰B the transition strength to the giant resonance region is more fragmented. An analysis is presented that employs an impulse-approximation Hamiltonian with amplitudes taken directly from the fundamental process on the nucleon, $\pi^- + p \rightarrow n + \gamma$, and shell-model wave functions obtained using realistic interactions in the 1s, 1p, and 2s-1d shells. Also, a calculation for the ¹⁴C (g.s.) transition from 1s capture using the "elementary-particle soft-pion" ansatz is presented.

NUCLEAR REACTIONS (π, γ) on ¹⁰B and ¹⁴N with stopped π^- ; measured photon spectrum (50-150 MeV) and radiative branching ratio using pair spectrometer of 2 MeV resolution. Shell-model calculations compared with measurements.

I. INTRODUCTION

Within the last several years the (π, γ) reaction with stopped pions was found to be a good probe of nuclear structure. Among the measurements which demonstrated this were those on targets of ³He (Ref. 1), ⁴He (Ref. 2), ⁶Li (Ref. 3), ¹²C (Ref. 4), ¹⁶O (Ref. 5), and ²⁰⁹Bi (Ref. 6) in which the photon spectrum between 50 and 150 MeV was measured with a pair spectrometer of 2-MeV resolution. These data and their interpretations have established the general features of this reaction, which can be summarized briefly as follows: (1) The total radiative branching ratio for the direct transitions producing high-energy photons on nuclei with $A \ge 4$ ranges from 1% (²⁰⁹Bi) to 4.4% (⁶Li), with most measured values near 2%. (2) The largest fraction (70-90%) of the photons are associated with quasifree capture on a proton, i.e., $\pi^- + A - (A - 1) + n + \gamma$, which produces a continuum spectrum with a maximum between 110 and 120 MeV, falling off sharply at the high-energy end near 135 MeV and extending down below 50 MeV. (3) Strong and selective excitations of unbound

states in the energy region of the giant dipole resonance (GDR) built on the target nucleus were observed in ¹²C. Since the (π^-, γ) transition operator contains the nucleon spin, these excitations have generally been interpreted⁷ as spin-isospin dipole vibrations characterized by L = 1, S = 1, $J^{\pi} = 0^{-}, 1^{-}, 2^{-}, T = 1, \text{ and } T_{z} = +1 \text{ in the SU(4)}$ classification⁸ of giant resonances. These spinisospin vibrations are distinct from the isospin modes (L=1, S=0, T=1) excited in E1 photoexcitation which involve no spin change. In ¹²C both 1⁻ and 2⁻ states were strongly excited in (π^-, γ) (Ref. 4). The identification of the 1^- component as a spin-isospin vibration mode is not without ambiguity, however, since its energy coincides with the energy of the 1^- states observed in E1 photoexcitation. In other nuclei, e.g. ¹⁶O, no narrow resonance-like peaks were observed in the GDR region, whereas they clearly exist in photoexcitation reactions.

(4) Transition strengths to the particle-stable states and low-continuum states (below the GDR) on targets of ³He, ⁶Li, ¹²C, and ¹⁶O exhibit one strong dominating transition. The transitions

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 ${}^{6}\text{Li}(\pi^{-}, \gamma)^{6}\text{He}(\text{g.s.})^{3}$ and ${}^{3}\text{He}(\pi^{-}, \gamma)^{3}\text{H}(\text{g.s.})^{1}$ have served as test cases in the theoretical analyses. For the heavier targets the level density in the residual nuclei ${}^{12}\text{B}$ and ${}^{16}\text{N}$ is so large compared to the 2-MeV experimental resolution that the strong transition could not be assigned to a single state. Theoretical arguments favored the identification⁵ of much of the strength with the 1⁺ and 2⁻ ground states, respectively.

(5) The general utility of the (π^-, γ) reaction for structure studies on nuclei with A > 16 has not been firmly established. In the ²⁴Mg and ⁴⁰Ca data⁵ sharp lines were not observed. However, the recently completed study on ²⁰⁹Bi $(\pi^-, \gamma)^{209}$ Pb (Ref. 6) shows some evidence for excitation of a sharp line.

Theoretical interpretations of these data have proceeded along two lines. Partially conserved axial-vector current (PCAC) and soft-pion theorems have been applied⁹ to calculate transitions in ³He and ⁶Li. In the soft-pion limit, the (π, γ) reaction is governed by the matrix element of the weak axial-vector current, and thus it can be related to Gamow-Teller β decay and the axial-vector matrix elements of μ capture. By introducing assumptions about the dependence of the form factors on momentum transfer, several authors⁹ predicted (π^{-}, γ) rates from the experimental weakinteraction matrix elements. Such calculations for ³He and ⁶Li compare^{1,3} reasonably well with the data. A limitation of this approach is that it holds only for 1s capture. For nuclei with $4 \leq A$ \leq 40, *p*-state capture accounts for more than 50% of π absorption.

The second approach, which has a wider applicability to nuclear structure studies, makes use of an impulse-approximation (IA) Hamiltonian determined directly from the fundamental photopion production process $\pi^- + p = n + \gamma$. This is applied without adjustment of parameters to calculate the (π^-, γ) rates in complex nuclei described by shellmodel wave functions. In ⁶Li this has led to excellent agreement³ with the most recent data.

To make further advances in (π^-, γ) nuclear structure studies, several factors seemed important. First, there was need for measurements on several additional transitions where a single nuclear state was isolated experimentally. ¹⁰B and ¹⁴N are the only nuclei where this is possible with a resolution of 2 MeV. The first excited state of the residual nucleus ¹⁴C is at 6.1 MeV; in ¹⁰Be the first and second excited states are at 3.4 and 6.0 MeV. Furthermore, since the target ¹⁰B has $J^{\pi} = 3^+$, whereas all other light nuclei studied had $J^{\pi} = 0$, $\frac{1}{2}^+$, or 1⁺, the dependence on angular momentum could be further investigated.

Second, the further clarification of the role of

giant M1 states in (π^-, γ) reactions was of interest. In 1963 Kurath¹⁰ suggested that in light nuclei there exists a concentration of magnetic dipole transition strength between T = 0 ground states and excited T = 1 states similar to the well-known concentration of E1 strength in the GDR. The most direct observation of such giant M1 states was expected¹⁰ to be in 180° electron scattering, and indeed prominent M1 transitions have been observed¹¹ in ⁶Li, $^{10}B,\ ^{12}C,\ ^{14}N,\ ^{20}Ne,\ ^{24}Mg,\ ^{28}Si,$ and other nuclei. In recent years Mukhopadhyay¹² suggested that μ capture from atomic 1s orbits exhibits concentration of transition strength to the giant M1 states because the dominant part of the transition operator resembles the Gamow-Teller (GT) interaction. Experimental verification in μ capture has been limited and must necessarily be indirect, e.g., through observation of secondary and tertiary γ and β rays, since the neutrino emitted in the primary transition cannot be detected. Radiative pion capture, however, provides an excellent method for further exploring these giant M1states since radiative π -capture transitions from l = 0 atomic orbits are essentially governed by the same GT matrix elements $\int \overline{\mathfrak{o}} \tau^+$ that appear in μ capture and β decay. A complication arises in π capture, not existing in μ capture, in that π 's are captured predominantly from p orbits in light nuclei (~90% for ¹⁴N). In this case the \bar{q} -dependent terms of the interaction make large contributions¹³ to the radiative capture rate. However, this effect does not significantly change the above results because, when the pion momentum operator $\vec{q} = -i\vec{\nabla}$ operates on the 2*p*-pion wave function, it yields both a monopole (essentially the GT operator) and a quadrupole term. The contribution of the quadrupole term is negligible¹³ (precisely for the same reasons that the momentum-dependent terms are negligible for 1s absorption). As a result, the role of the GT operator is much greater than expected from the 10-20% 1s state capture probabilities. Thus the (π, γ) reaction appeared to be a promising means for observing the analogs of the well-known¹¹ giant M1 states in ¹⁰B at 7.48 MeV and in ^{14}N at 9.2 and 10.4 MeV.

A third area of interest was the (π^-, γ) excitation of collective states in the GDR region. The comparison of the distribution of (π^-, γ) transition strength with that of photoexcitation and electron scattering might elucidate the spin-isospin structure of the GDR. The spin-isospin modes of ¹⁰B have $J^{\pi} = 1^-$, 2^- , 3^- , 4^- , and 5^- and in ¹⁴N have $J^{\pi} = 0^-$, 1^- , 2^- , and 3^- . We hoped to obtain evidence for some of these higher spin components which cannot be easily observed in other reactions. Our study presents some evidence for such excitations in ¹⁴N.



FIG. 1. Plan view of the experimental setup. The insert shows the $e^+ - e^-$ pair spectrometer and range-telescope geometry. The trigger for an event is $\pi 1 \times \pi 2 \times \pi 3 \times \overline{\pi s} \times \overline{\pi c} \times (A \times B)_i \times (A \times B)_i$, $i \neq k, k \pm 1$.

II. EXPERIMENT

The experiment was performed in the stopped- π^- channel of the Lawrence Berkeley Laboratory 184 inch cyclotron (Fig. 1). A π^- beam of 180 MeV/c, extracted from an internal Be production target by the cyclotron fringe field, is brought to a focus 10 m from production by a quadrupole-dipole-quadrupole magnet system. Maximum achieved beam intensity was $2 \times 10^6 \pi^{-}/\text{sec}$ with a circular spot size of ~8 cm diam and $\Delta p/p \approx 13\%$ (full width). The π^- were brought to rest in targets of liquid nitrogen (15.2-cm-diam 5.1-cmlong cylindrical flask) and 91.4%-enriched ¹⁰B power $(11.3 - \times 13.9 - \times 5.1 - \text{cm}^3 \text{ parallelepiped},$ 941-g mass). Typical stopping rates were (2-3) $\times 10^{5}$ /sec. The photons were detected in a 180° pair spectrometer (Fig. 1, also Refs. 3 and 5) employing a 3% radiation length gold foil (0.22 g/cm^2) converter. The momenta of the $e^+-e^$ pair were determined by measuring their trajectories in a magnetic field $(B_{max} \approx 8.3 \text{ kG})$ with three wire spark chambers. Each chamber consisted of four wire planes with seven magnetostrictive wire delay-line readouts. The wire spacing was 0.1 cm and the wire angles with the horizontal midplane of the magnet were +12, -12, -12, and 0° . A PDP-15 computer was used on line to record the data onto magnetic tape and to monitor the performance of the spark chambers. The acceptance (conversion $\times \Delta \Omega / 4\pi \times detection$ efficiency) of the spectrometer as a function of photon energy [Fig. 2(a)] was determined with a Monte Carlo calculation which includes the experimental geometry, a field map, pair-production cross sections, energy loss due to radiation and ionization, and multiple scattering in the converter and chambers. Numerous runs with a liquid hydrogen target were taken during the course of the experiment to check the performance of the spectrometer. A spectrum is shown in Fig. 2(b). The 129.41-MeV photon of the $\pi^- + p + n + \gamma$ reaction gives the instrumental line shape, and it is seen that 2-MeV resolution [full width at half-maximum (FWHM)] was achieved. The peak of the line shape is shifted downward by ~2 MeV from the photon energy due to energy loss of the $e^+ - e^-$ pair in the converter and spark chambers. The charge-exchange capture $\pi^- + p + n + \pi^0$; $\pi^0 + 2\gamma$ provides, via the Panofsky ratio, a check on the relative acceptance in



FIG. 2. (a) Efficiency of the pair spectrometer as a function of photon energy; $\eta = \text{conversion probability} \times (\Delta \Omega/4\pi) \times \text{detection efficiency.}$ (b) Photon spectrum of π capture on hydrogen. The distortion of the rectangular shape of the π^0 spectrum is due to the reduction of efficiency at the low-energy end.

the region $54.9 \le E_{\gamma} \le 83.0$ MeV. The modification of the rectangular π_0 spectrum by the acceptance curve can be observed.

The 5–10% good events of the total triggers were selected with an off-line pattern recognition program; the different classes of background events are described in Ref. 5. The efficiency for finding good events was determined by examining 50 000 triggers by eye in a direct display of the spark chamber coordinates. The program detection efficiency was $53 \pm 3\%$ at 130 MeV with a small additional bias against lower-energy events. The spectrometer acceptance at 130 MeV is $\eta(130) = (\text{conversion} \times \Delta \Omega / 4\pi) \times (\text{detection efficiency}) = (4.15 \times 10^{-5}) (0.532) = (2.21 \pm 0.12) \times 10^{-5}.$

The number of pions stopping in each target was obtained in two ways. First, the fraction of incident π 's stopping in the target was determined from target in/out measurements. In this way π 's stopping in the target walls as well as geometric and electronic inefficiencies are taken into account; also, this method was checked with equivalent-geometry CH₂ targets and measured CH₂ range curves. Second, the stopping fraction was calculated from the equivalent CH₂ stopping power of the targets and measured CH₂ range curves. The two methods were generally in agreement to within 6%.

The radiative branching ratio for a single peak or the entire spectrum (total radiative branching ratio) is determined by use of the expression

$$R_{\gamma} = \frac{N_{\gamma}(1-l)te^{\mu x}}{\pi_{\rm in}\,\epsilon(1-\delta)\eta(130)}$$

 N_{γ} is the number of counts in the spectrum after eliminating, through target cuts, events originating outside the target; *l* represents the small fraction of counts resulting from radiative in-flight transitions; t is the unfolding factor which multiplies $N_{\gamma}(1-l)$ to give the number of photons expected with a uniform spectrometer acceptance at the value $\eta(E_{\gamma} = 130 \text{ MeV})$. For a single peak, $t = \eta(130)/\eta(E_{\gamma})$. For R_{γ} (total) it is determined by folding the pole-model distribution function (Sec. III A) with the spectrometer acceptance and line shape (Fig. 1) and comparing the result with the spectrum. The fraction of the photons with energies below 50 MeV, and thus not observed in the pair spectrometer, is 3-5% as given by the pole model. $e^{\mu x}$ corrects for the attenuation of photons in the target, scintillation counter, and spark chamber between the converter foil and origin. $\pi_{in}\epsilon(1-\delta)$ is the number of pionic atoms formed as determined from the particles passing through the three upstream counters of the telescope (π_{in}) , the π -stopping fraction ϵ , and the small corrections for nonradiative in-flight interactions (estimated $\sim 1\%$).

III. EXPERIMENTAL RESULTS

A. General

In-flight subtraction. The photon spectra for π^- capture on ¹⁴N and ¹⁰B are displayed in Figs. 3 and 4. From the raw data Figs. 3(a) and 4(a) one sees



FIG. 3. Photon energy spectrum from π capture on ¹⁴N. (a) Raw data for stopped- π capture. (b) Photon spectrum for pions with mean energy of 44 MeV used for in-flight background subtraction. (c) Spectrum after in-flight subtraction. The solid line is the spectrum calculated from the one-pole diagram (Δ =13 MeV) representing quasifree capture (Ref. 19). Evidence for resonance excitation at $E_{\gamma} \approx 118$ MeV can be seen. (d) Fit to the data with pole model + Breit-Wigner + four lines. The transition to ¹⁴C(g.s.) is seen to be extremely weak. The strongest transition is to the analog of the giant *M*1 state of ¹⁴N at 9.17 MeV.

that there are a few counts at energies above the kinematically allowed region for stopped- π reactions. The trajectory reconstructions indicate that these photons emanate from the target, and we identify them with in-flight radiative capture (REX) $\pi^- + A_z \rightarrow A_{z-1} + \gamma$ and in-flight charge exchange (CEX) $\pi^- + A_z \rightarrow A_{z-1} + \pi^0$, $\pi^0 \rightarrow 2\gamma$. From the range curve data we can see that π 's with kinetic energies up to ~35 MeV were entering both the ¹⁴N and ¹⁰B targets. For $T_{\pi} = 35$ MeV, REX photons up to ~173 MeV can be produced.



FIG. 4. Photon spectrum from capture of stopped pions on ¹⁰B. (a) Raw data. (b) Spectrum after small in-flight subtraction. The solid line is the spectrum calculated from the one-pole diagram (Δ =13 MeV) representing quasifree capture (Ref. 19). (c) Fit to the data with pole model + five lines. Dashed lines show the contributions of the first three states of ¹⁰Be. The strongest transition is to the 2⁺/₂ state which is the analog of the giant *M*1 state in ¹⁰B at 7.48 MeV.

CEX photons from the decay of a 35-MeV π^{0} range from 32 to 140 MeV.

The in-flight spectrum [Fig. 3(b)] at 90° to the beam was measured with the ¹⁴N target at $T_{\pi} = 44 \pm 7$ MeV (the lowest energy consistent with not having π 's stop in the target). It is rather featureless, in agreement with the expected dominance of the CEX reaction as discussed below. By normalizing this in-flight spectrum to the stopped- π spectra between 140 and 150 MeV, we find a 10±3 and 6±2% subtraction necessary for ¹⁴N and ¹⁰B, respectively. The resulting spectra are shown in Figs. 3(c) and 4(b).

The cross sections for in-flight processes on nuclei at these low beam energies have not been measured. Clearly¹⁴ they do not exceed $Z \times (\text{free}$ proton cross section). These are for CEX¹⁵ $\sigma_0({}^{14}N) = 7 \times 5.25 = 36.8 \text{ mb and for REX}{}^{16} \sigma_v({}^{14}N)$ = 7×1 mb. Here we have used nucleon cross sections at T_{π} = 15 MeV which we estimate to be the average energy for π 's interacting in flight in the ¹⁴N target. To calculate from these cross sections the expected in-flight contributions to the total spectrum, we assumed that the CEX photon distribution was rectangular and that the REX spectrum was similar to the stopped- π spectrum. To check the shape of the CEX spectrum, a calculation¹⁷ was performed for the in-flight photon spectrum at 90° to the beam using π^0 angular distributions of $1 \pm \cos \theta$. These produced shapes very close to rectangular. Little is known about the shape of the in-flight REX spectrum. However, since we calculate that its contribution is small, our assumption should not lead to a significant error in the estimate of the total in-flight contribution. With these assumptions, we estimate upper limits for the in-flight contributions to the stopped- π spectrum of 13% for CEX and 4.3% for REX.

It is well known¹⁴ that $Z \times (\text{free proton cross} \text{section})$ overestimate the nuclear cross sections, since no account is made of pion attenuation and binding energy effects. In a measurement at 70 MeV on ¹²C Hilscher *et al.*¹⁸ found a reduction factor of 48% on $Z \times (\text{free proton cross section})$ for the CEX reaction on ¹²C. This factor is consistent with the cross sections deduced from our in-flight data at $T_{\pi} = 44$ MeV to within the uncertainty (up to factor of 2) resulting from our lack of knowledge on the π^0 energy distribution. Thus we conclude that our in-flight subtraction is in reasonable agreement with the expected in-flight contribution.

Quasifree capture. In nuclei ranging from ³He to ²⁰⁹Bi, the quasifree component $[\pi^- + A \rightarrow (A-1) + n + \gamma]$ is well described phenomenologically by the pole model.¹⁹ Details of the model and the ex-

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E_{γ} (MeV)	Г (MeV)	$E_x(^{14}C)^{a}$ (MeV)	$E_x(^{14}N)^{b}$ (MeV)	J^{π}	R_{γ} (expt.) (× 10 ⁻⁴)	R_{γ} (theory) ^{c,d} (× 10 ⁻⁴)
138.1	0	0	2,31	0+	0.3 ± 0.2	1.02 ± 0.14
131.1	0	7.0 ± 0.1	9.17	2^+	7.7 ± 0.9^{e}	24.3 ± 2.7
129.8	0	8.32	10.43	2^{+}	4.0 ± 0.6	1.2 ± 0.2
126.9	0	11.3	13.75	1+	5.1 ± 0.7	4.9 ± 0.7
118.2 ± 1.0	2.4 ± 0.5	20.0 ± 1.0	22.2	(3 -) f	20.5 ± 2.0 g	195 ± 22 ^h
Pole ($\Delta = 13$.	5 MeV)				176 ± 18	
Total					213 ± 21	$227\pm25~^{\rm i}$

TABLE I. Energies and branching ratios for transitions in the ${}^{14}N(\pi^-,\gamma)$ reaction with stopped pions.

^a Energies from previous work (Ref. 21) except for the 7.0 MeV state and BW at 20 MeV. ^b Energies for analog states in ¹⁴N.

^c Obtained from $R_{\gamma} = [C_s \omega_s \lambda_{\gamma}(1s)/\lambda_a(1s)] + [C_p \omega_p \lambda_{\gamma}(2p)/\lambda_a(2p)]$ with $\omega_p = 0.90 \pm 0.03$, $\omega_s = 1 - \omega_p$, $C_s = 0.5$, $C_p = 1.4$ (text), $\lambda_a(1s) = 4.48 \pm 0.30 \text{ keV}/\hbar$, and $\lambda_a(2p) = 2.1 \pm 0.3 \text{ eV}/\hbar$ (Ref.

42); uncertainties indicated are due to x-ray data only.

Assumed $(p_{3/2}, p_{1/2})^{-2}$ configurations for positive parity states. For some states $(sd)^2$ excitations are important (Sec. IV B).

^e If we assume a single line at $E_x = 7.3$ MeV, $10^4 R_y = 10.3 \pm 1.7$.

^f Dominant J^{π} value expected from theory (text).

^g Fit with $\Delta = 13.5$ MeV [Fig. 3(d)]; separation of pole and BW not well defined; other fits show variations of 30% are possible.

 $^{\rm h}$ 0⁻, 1⁻, 2⁻, 3⁻, and 4⁻ states based on $1\hbar\,\omega$ excitations (text).

ⁱ Sum of strength to p^{-2} and $1\hbar\omega$ excitations (text).

pression for the spectrum are given in Ref. 20. The normalization and the average excitation energy $(E^* = \Delta - \Delta_{\min}; \Delta_{\min} = M_{A-1} + m_n - M_A)$ of the recoil nucleus are not specified in this model, so our procedure has been to determine them by fitting to the data between 70 and 110 MeV. This region should be free of nuclear resonances. For ^{14}N ($\Delta_{min} = 8.5$ MeV) values $E^* = 4.5 - 5.5$ MeV and for ${}^{10}\text{B}$ ($\Delta_{\min} = 7.9 \text{ MeV}$) values $E^* = 5.1 - 6.1 \text{ MeV}$ give good descriptions of the data.

In the spectrum of ¹⁴N one can see a resonancelike peak at ~20 MeV in 14 C which is the region of the GDR. A similar resonance cannot be clearly discerned in the spectrum of ¹⁰B. To extract a value for the branching ratio to the GDR region in the ¹⁴N spectrum, a Breit-Wigner (BW) form superimposed on the pole-model continuum was fit to the data. Clearly the extracted transition fractions (Table I) in such an analysis are model dependent, since the separation between the pole model and resonance excitation is not well defined. A shell-model analysis and more discussion of the giant resonance region in ¹⁴C is presented in Sec. IVD.

B. Results on ¹⁴N

The total and partial radiative branching ratios determined for ¹⁴N are given in Table I. The quasifree fraction of the total radiative branching ratio R_{γ} =2.13 \pm 0.21% as given by the pole model

 $[\Delta = 13.5, \text{ Fig. 3(d)}]$ is $83 \pm 2\%$. An additional 10% of the strength is attributed to GDR excitation as described by the BW, leaving only 7% for the particle-stable and low-continuum states.

 $E_r = 0 - 13 MeV$. The data on the first 13-MeV excitation of ¹⁴C, after subtraction of the polemodel and BW contributions as shown in Fig. 3(d), are shown on an expanded scale in Fig. 5. From previous work²¹ it is known that at least 15 levels occur in this region. Of these, it is possible to identify two with little ambiguity in the (π^-, γ) reaction. The ground state is separated by 6.09 MeV from the first excited state; the measured branching ratio is $(3 \pm 2) \times 10^{-5}$. A strong transition is observed at E_{γ} =131 MeV, and a single-line fit yields $E_x = 7.0 \pm 0.3$ MeV, which agrees closely with the 7.01 ± 0.01 MeV measured²¹ previously for the 2_1^+ state.

To analyze the remaining transition strength we must be guided by previous experiments and theory. Previous (π, γ) studies on ⁶Li (Ref. 3) and ¹²C (Ref. 4), together with the present work, show that the strongest transitions are to states whose analogs in the target nucleus have the largest M1matrix elements with the ground state. Specifically, in the earlier studies the dominating transitions were to ⁶He (g.s., 0^+) and ¹²B (g.s., 1^+) which are the $T_s = +1$ analogs of the 3.56-MeV state in ⁶Li and 15.1-MeV state in ¹²C. Both states are observed strongly in 180° electron scattering and have large measured M1 matrix



FIG. 5. Photon spectrum and level diagram for (π^-, γ) transitions identified (tentatively) with particle-stable and lowcontinuum states of ¹⁴C. The spectrum shown is after subtraction of the pole model and BW contribution [Fig. 3(d)]. The strong *M*1 transitions observed in 180° electron scattering are identified. The *M*1 transition to ¹⁴N(13.8 MeV) has not yet been looked for in 180° electron scattering.

elements. In ¹⁴N(e, e') the largest observed²² M1 rates are to 2⁺ states at 9.17 and 10.43 MeV. The analogs in ¹⁴C occur at 7.0 and 8.3 MeV, respectively. Our data are consistent with population of both of these states with the 7.0-MeV state strongest (Fig. 5).

Additional transition strength is seen at 10–13 MeV excitation. Unfortunately, the ¹⁴N (*e*, *e'*) studies were not extended to this excitation region. Preliminary results¹¹ on ¹⁴C(*e*, *e'*) indicate that a 1⁺ state at 11.3 MeV has considerable *M*1 strength to the ¹⁴C ground state. These data establish the J^{π} , but do not guarantee a large *M*1 matrix element for the analog state in ¹⁴N, since different ground states are involved. A 1⁺ T = 1 state in ¹⁴N was identified²³ at 13.72 MeV in the ¹⁵N(³He, α)-¹⁴N reaction and perhaps this is the analog of the state observed in ¹⁴C(*e*, *e'*).

Taking into account these various results, we fit the data on the first 14-MeV excitation in 14 C with four lines at 0.0, 7.0, 8.3, and 11.3 MeV. The results are displayed in Fig. 5 and the corresponding branching ratios given in Table I.

GDR region. The GDR built on the ¹⁴N ground state has been studied through photoexcitation²⁴ and radiative proton capture ¹³C(p, γ)¹⁴N (Ref. 25). The total photoabsorption cross section shows a rather smooth energy dependence compared to other 1p shell nuclei,²⁶ e.g., ¹²C and ¹⁶O, with a peak near 22 MeV and a considerable tail at higher energies. The ¹³C(p, γ)¹⁴N excitation function shows a broad structure in the region $18 \leq E_x \leq 24$ MeV with prominent peaks at $E_x = 22.5$ and 23.0 MeV. The analogs in ¹⁴C are expected at $E_x = 20.1$ and 20.6 MeV. These energies are close to the 20 ± 1 MeV for the position of the BW peak determined in this experiment.

In the ¹³C(p, γ_0)¹⁴N study, the γ_0 angular distribution was thought²⁵ to be consistent with $J^{\pi} = 2^{-1}$ for most of the observed giant electric dipole strength. The results of our shell-model calculations (Sec. IV) indicate that the strongest transitions in the ¹⁴N(π^- , γ)¹⁴C reaction are to 3⁻ states, with some strength also to 2⁻ states. If indeed the giant 3⁻ states are seen in the present experiment, some major 2⁻ and 3⁻ components of the GDR are nearly degenerate, since the measured excitation energies in the two experiments are so close. This differs from ¹²C, where the major 1⁻ and 2⁻ components are separated by about 3.5 MeV and could be resolved in the ¹²C(π^- , γ) experiment.⁴

C. Results on ¹⁰B

The total and partial radiative branching ratios determined for ¹⁰B are given in Table II. The quasifree fraction of the total radiative branching ratio $R_{\gamma} = 2.27 \pm 0.22\%$ as given by the pole model is $87 \pm 4\%$.

 $E_x = 0-13 \text{ MeV}$. The data on the first 13-MeV excitation of ¹⁰Be, after subtraction of the polemodel contribution as shown in Fig. 4(c), are displayed on an expanded scale in Fig. 6. Previous experiments²⁷ established that the first three states of ¹⁰Be have $J^{\pi} = 0^+$, 2_1^+ , and 2_2^+ and the energies are 0, 3.37, and 5.96 MeV, respectively; the analogs in ¹⁰B are at 1.74, 5.11, and 7.477

Ε _γ (MeV)	$E_x ({}^{10}\text{Be}) a$ (MeV)	$E_x ({}^{10}\text{B}) {}^{b}$ (MeV)	J^{π}	R_{γ} (expt.) (× 10 ⁻⁴)	R_{γ} (theory) ^{c,d} (× 10 ⁻⁴)
137.4	0	1.74	0+	2.5 ± 0.4	3.6 ± 0.7
134.1	3.37	5.11	2^+	4.4 ± 0.7	8.5 ± 1.7
131.6	5,96	7.48	2^{+}	10.5 ± 1.3	16.9 ± 2.7
130.2	7.55	8.89	2^{+})		6.5 ± 1.0
128.9 ± 0.7	8.6 ± 0.7	~ 9.7	(4^+) e	10.6 ± 1.6 f	2.7 ± 0.4
127.4 ± 0.7	$\textbf{10.1} \pm \textbf{0.7}$	~ 11.5	(3+))		1.4 ± 0.2
Pole ($\Delta = 13$.)	0 MeV)			198 ± 23	
Total				227 ± 22	

TABLE II. Energies and branching ratios for transitions in the ${}^{10}B(\pi^-,\gamma)$ reaction with stopped pions.

^a Energies and J^{π} of lowest four states are from previous work (Ref. 27).

^b Energies on analog states in ¹⁰B.

^c Obtained from $R_{\gamma} = [C_s \omega_s \lambda_{\gamma}(1s) / \lambda_a(1s)] + [C_p \omega_p \lambda_{\gamma}(2p) / \lambda_a(2p)]$ with $\omega_p = 0.80 \pm 0.05$, $\omega_s = 0.80 \pm 0.05$

=1- ω_p , C_s =0.5, C_p =1.4 (discussed in text), $\lambda_a(1s)$ =1.68±0.12 keV/ \hbar (Ref. 42), and $\lambda_a(2p)$

= $0.32 \pm 0.06 \text{ eV}/\hbar$ (Ref. 42); uncertainties indicated are due to x-ray data only.

^d Assuming $(p_{3/2}, p_{1/2})^6$ configurations.

^e Dominant J^{π} values expected from theory (Sec. IV C).

^f Values for individual states cannot be determined reliably from our data; values corresponding to curves of Fig. 5 are $R_{\gamma}(8.6) = 0.049 \pm 0.01\%$ and $R_{\gamma}(10.1) = 0.058 \pm 0.010\%$.

MeV. The three additional particle-stable states²⁷ (1⁻, 0⁺, 2⁻) of ¹⁰Be are within 0.3 MeV of the 2_2^+ state. With our resolution of 2 MeV the ground state can be resolved and evidence for its population is clearly seen in the spectrum. The strongest transition occurs at $E_{\gamma} \approx 132$ MeV, and a single-line fit yields $E_x = 6.0 \pm 0.3$ MeV, which agrees closely with the previous value for the 2_2^+ state. The analog of this state is strongly exicted in ¹⁰Be(*e*, *e'*)²⁸ and its *M*1 strength is by far the largest measured in ¹⁰B. The calculations by Mukhopadhyay¹² for μ capture using the Cohen-Kurath wave functions indicate that this 2⁺ state receives 62% of the transition strength to states with $(p_{1/2}, p_{3/2})^6$ configuration. Thus it seems reasonable to assume that the peak at ~132 MeV is mostly due to the 2⁺₂ state and that the other three states make much smaller contributions. The data also show population of the 2⁺₁ state, since only it can account for the observed filling in of counts be-



FIG. 6. Photon spectrum and level diagram for (π^-, γ) transitions identified (tentatively) with particle-stable and lowcontinuum states of ¹⁰Be. The spectrum shown is after subtraction of the pole-model contribution [Fig. 4(c)]. The first three levels of ¹⁰Be are clearly resolved. The transition strength to higher levels is not resolved, but two lines were sufficient to fit the data. The *M*1 transition to the 7.5-MeV state dominating 180° electron scattering is indicated on the level diagram; the analog state is seen to dominate the (π^-, γ) spectrum.

tween the ground state and the 2^+_2 state. A fit of three lines with energies fixed at the first three ¹⁰B states gives a good description to the data. The three extracted branching ratios (Table II) are expected to be quite free of uncertainties due to background and level population ambiguities. and thus they should provide good test cases for theoretical calculations.

Additional transition strength is observed to states between 8 and 12 MeV. Not much is known experimentally about levels in this region. The calculations by Mukhopadhyay¹² for μ capture predict relatively strong excitation (21%) of a 3⁺ state at 8.9 MeV and weaker (4.1%) excitation of a 4⁺ state at 10.8 MeV (energies are theoretical estimates). Our calculation (Sec. IV) for π capture predicts that the strongest excitations to this region are a 2^+ state and a 4^+ state. Other states are predicted to be much weaker in both μ and π capture. Noting these results, we fit two lines to the remaining transition strength, allowing both the energies and intensities to vary (Table II).

GDR region. The GDR region of ¹⁰B has been investigated by photoexcitation²⁹ and radiative proton capture ${}^{9}\text{Be}(p,\gamma){}^{10}\text{B},{}^{30}$ but not in (e, e'). The photoabsorption²⁹ cross sections show two peaks at 20.1 ± 0.1 and $23.1\pm0.1~MeV.~$ The analogs in ¹⁰Be are expected at ~18.7 and 21.7 MeV. Regarding the spin-parity structure, little is known, except that since ¹⁰B has a 3⁺ ground state, states

where

(2)

2. Pionic orbits

A great advantage of the (π, γ) reaction with stopped pions is that the π 's initial state is a bound atomic orbit which can be studied through the pionic x-ray spectra. The pionic wave functions can be obtained by solving³⁶ the Klein-Gordon equation in the potential generated by the nucleus. Such solutions show that the hydrogenic wave functions are distorted by the strong interaction of the π^- with the nucleus and the finite nuclear size. However, since both the hydrogenic and the distorted s- and p-wave functions vary relatively slowly inside the nucleus, one can use hydrogenic wave functions and an appropriate scale factor $C_{nl}, \Lambda_{\gamma}(nl, \text{ optical potentials}) = C_{nl}\Lambda_{\gamma}(nl, \text{ hydro-}$ genic). The C_{nl} can be determined from

$$C_{nl} = \frac{\langle NL \mid \phi^{\pi}(\text{opt. pot.}) \mid NL \rangle^{2}}{\langle NL \mid \phi^{\pi}(\text{hydrogenic}) \mid NL \rangle^{2}}, \qquad (3)$$

seen strongly in E1 photoabsorption must have $J^{\pi} = 2^{-}$, 3⁻, or 4⁻. The ¹⁰B(π, γ) data show little resolved structure in the GDR and no clear separation between quasifree and resonance capture can be ascertained. Since other 1p shell nuclei clearly show strong (π, γ) transitions to the GDR region, it seems probable that this also occurs in ¹⁰B, but that there is greater fragmentation of the strength.

IV. SHELL-MODEL STUDIES A. General

Calculation of the radiative π -capture transition probabilities requires essentially three ingredients: the effective interaction responsible for the transition, specification of the bound pion wave function, and appropriate nuclear wave functions. To deduce branching ratios from the transition rates, one also needs pionic capture schedules and strong absorption level widths. Each of these subjects is discussed below.

1. (π, γ) interaction

Using the CGLN³¹ photopion production amplitude, Delorme and Ericson³² write the effective Hamiltonian with the pion in the $\phi_{I}^{n}(\mathbf{r})$ atomic orbit as

$$\mathcal{K}_{\rm eff} = \left(1 + \frac{m_{\pi}}{m_{p}}\right) \sum_{j,\lambda}^{A} e^{-i\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}_{j}} H_{\lambda}(j)\phi_{i}^{n}(\vec{\mathbf{r}})\delta(\vec{\mathbf{r}}-\vec{\mathbf{r}}_{j}),$$
(1)

 $H_{\lambda}(j) = 2\pi i t^{+}(j) [A\vec{\sigma}_{j} \cdot \hat{\epsilon}_{\lambda} + B(\vec{\sigma}_{j} \cdot \hat{\epsilon}_{\lambda})(\vec{q} \cdot \vec{k}) + C(\vec{\sigma}_{j} \cdot \vec{k})(\vec{q} \cdot \hat{\epsilon}_{\lambda}) + iD\vec{q} \cdot (\vec{K} \times \hat{\epsilon}_{\lambda}) + E(\vec{\sigma}_{j} \cdot \vec{q})(\vec{q} \cdot \hat{\epsilon}_{\lambda})]$

accounts for nearly all the transition strength in s-state capture; for *p*-state capture the terms linear in \tilde{q} make large contributions.¹³ The E term, quadratic in \overline{q} , is not expected to make significant contributions to s- and p-state capture.

in the notation of Ref. 13. The first term in $H_{\lambda}(j)$

The effective coupling constants A, B, C, D, and E are linear combinations of the electric and magnetic multipole amplitudes contributing to the $\gamma + n \rightarrow \pi^- + p$ cross section at low energies. Threshold values have been given by numerous authors.³³ Although the most recent solutions (1972-73) are all based on the tables of Berends, Donnachi, and Weaver,³⁴ (1967), there are still some discrepancies in B, C, D, and E. Our calculations were performed with the values of Maguire and Werntz, including the sign change in the D term.³⁵ The effect of using other values was investigated for several transitions and was found to be small.

where *NL* are the shell-model single-particle states. Values of $C_{1s} = 0.5$ and $C_{2b} = 1.4$ were used for both ¹⁰B and ¹⁴N. These values were determined by Maguire and Werntz³⁵ for ¹²C by comparing pion wave functions based on the optical model of Krell and Ericson³⁶ with hydrogenic wave functions.

3. Nuclear wave functions

The nuclear wave functions were calculated with standard shell-model techniques (coefficient of fractional parentage and Racah algebra) using harmonic-oscillator wave functions with $\hbar \omega = 14$ MeV. The (π^-, γ) rates are known^{13,35} to be sensitive to the value of $\hbar \omega$ adopted. The present value, derived from considerations of the energy spectrum of the nuclei around ¹⁶O, is consistent with electron scattering experiments which yield $\langle r^2
angle^{1/2}$ = 2.58 fm ($\hbar \omega$ = 13.7 MeV) for the 1*p*-shell harmonic-oscillator length parameter. The single-particle energies³⁷ were taken from experiment. Positive parity states were calculated in a $(1p_{3/2} 1p_{1/2})^n$ space with n = -2 for ¹⁴N and n = 6 for ¹⁰B. Higher shell admixtures play an important role for some states (e.g., 2_1^+ in ¹⁴N) as will be discussed. For negative parity states (calculated only for ¹⁴N), one particle was promoted from the 1s to 1p shell or from the 1p to 2s - 1d shells (Fig. 7). It is expected that if the ground states of ^{10}B and ^{14}N are well described by $(1p_{3/2}, 1p_{1/2})^n$, more complex excitations will not affect the total π^- capture rate to either positive or negative parity states, but may affect the distribution of strength among the different states.

The basis states were allowed to mix with a realistic two-body interaction obtained from the bare G-matrix elements of Kuo and Lee,³⁸ a somewhat modified version of the earlier matrix elements of Kuo and Brown.³⁹ The bare interaction, however, requires fairly large Hilbert spaces. Therefore some calculations were also performed with the effective interaction of Cohen and Kurath⁴⁰ with their set of single-particle energies; some comparisons are given below.

4. Branching ratios

The transition probabilities are given by

$$\Lambda_{\gamma}(nl; J_{i} - J_{f}) = \frac{k}{\pi} \frac{1}{2J_{i} + 1} \frac{1}{2l + 1} S_{i \to f} , \qquad (4a)$$

$$S_{i \to f} = \int \frac{d\hat{k}}{4\pi} \sum_{m's} |\langle J_f M_f | \mathcal{H}_{eff} | J_i M_i \rangle|^2 .$$
 (4b)

The branching ratio R_{γ} , i.e., the number of pho-

tons per stopped pion, is related to $\Lambda_{\gamma}(nl)$ as follows:

$$R_{\gamma} = \sum_{n,l} \frac{\Lambda_{\gamma}(nl)}{\Lambda_{a}(nl)} \omega(nl) , \qquad (5)$$

where $\Lambda_a(nl)$ are the total absorption rates and $\omega(nl)$ are the probabilities for absorption from orbit nl. The latter are restricted by the condition $\sum_{n,l} \omega(nl) = 1$ which expresses the fact that the nuclear absorption lifetimes are much shorter than the free-pion lifetimes $(10^{-12} \ll 2.8 \times 10^{-8} \text{ sec})$. It is generally assumed³⁵ that the ratio $\Lambda_{\gamma}(nl)/\Lambda_a(nl)$ depends only on l, not n. In light nuclei, capture occurs only from l=0 and l=1 orbits; thus the quantity

$$R_{\gamma} = R_s + R_p = \frac{\Lambda_{\gamma}(1s)}{\Lambda_a(1s)} \omega_s + \frac{\Lambda_{\gamma}(2p)}{\Lambda_a(2p)} \omega_p \tag{6}$$

is compared to experiment. The quantities $\omega_s = \sum_n \omega(ns)$ and $\omega p = \sum_n \omega(np)$ have not been obtained for ¹⁰B or ¹⁴N, but extrapolating from⁴¹ ⁶Li, ⁹Be, ¹²C, and ¹⁶O it appears that $\omega_s = 0.20$ ± 0.05 for ¹⁰B and $\omega_s = 0.10 \pm 0.03$ for ¹⁴N are reasonable and these were used ($\omega_p = 1 - \omega_s$). The total absorption rates were taken⁴² to be: for ¹⁴N: $\lambda_a(1s) = 4.48 \pm 0.30 \text{ keV}/\hbar = (6.82 \pm 0.46)10^{18}$ sec⁻¹ and $\lambda_a(2p) = 2.1 \pm 0.3 \text{ eV}/\hbar = (3.19 \pm 0.46)10^{15}$ sec⁻¹; for ¹⁰B: $\lambda_a(1s) = 1.68 \pm 0.12 \text{ keV}/\hbar$ $= (2.55 \pm 0.18)10^{18} \text{ sec}^{-1}$ and $\lambda_a(2p) = 0.32 \pm 0.06$ eV/ $\hbar = (0.487 \pm 0.091)10^{15} \text{ sec}^{-1}$.

B. Positive parity states of ¹⁴N

Although the eigenstates were originally obtained in the j-j coupling scheme, it is more instructive to examine the positive parity states in the LS



FIG. 7. Model space for positive and negative parity excitations used in the shell-model calculations for ^{14}N .

representation. The states of interest are:

¹⁴ N(g.s.):	1+,	$T=0\rangle=0.1636 \left L=0 \ S=1\rangle+0.9564 \left L=2 \ S=1\rangle+0.2420 \right L=1 \ S=0\rangle;$
¹⁴ C(g.s.):	0+,	$T=1\rangle = 0.7980 L=0 S=0\rangle + 0.6027 L=1 S=1\rangle;$
5.1 MeV:	$ 2_{1}^{+},$	$T=1\rangle = 0.9068 L=2 S=0\rangle - 0.4216 L=1 S=1\rangle;$
7.1 MeV:	1+,	$T=1\rangle = L=1 S=1\rangle;$
12.2 MeV:	$ 0_{2}^{+},$	$T=1\rangle = -0.6027 L=0 S=0\rangle + 0.7980 L=1 S=1\rangle;$
12.8 MeV:	22+,	$T=1\rangle = 0.4216 L=2 S=0\rangle + 0.9068 L=1 S=1\rangle$.

We note that the Kuo-Lee interaction breaks the Wigner supermultiplet symmetry and selection rules. The Wigner supermultiplet predicts pure L=2 for the ¹⁴N ground state, L=0 for the 0⁺ T=1state, and L = 2 for the 2_1^+ state, which would explain the hindrance of the ${}^{14}C - {}^{14}N \beta$ decay and the fact that the 2_1^+ state exhausts all the M1 sum rules.^{10,40} Our wave functions predict $\log ft = 5.5$, which is greater than the typical Gamow-Teller value $(\log ft \sim 3)$ but smaller than the experimentally observed²¹ $\log ft = 9.01$. This is not catastrophic, since corrections to the transition operator (exchange currents, second-order forbiddenness, relativistic effects) and expansion of the Hilbert space [e.g. $(sd)^2$ admixtures], which are normally small, become crucial in the case of the present hindered transition.

The calculated ground state magnetic dipole and electric quadrupole moments are $0.34\mu_N$ and 13 mb, respectively; the experimental²¹ values are $0.40361\mu_N$ and 16 ± 7 mb. Cohen and Kurath⁴⁰ obtain $\mu = 0.331\mu_N$. We also calculated B(M1) and B(E2) rates which are presented in Table IV together with the experimental results. The calculated B(M1) rates to the 2_1^+ and 2_2^+ states are 4.877 and 0.041 $(e\hbar/2mc)^2$, respectively, while the experimental rates are 1.44 and 1.53, respectively. The Cohen-Kurath calculation predicts 4.846 $(e\hbar/2mc)^2$ for the transition to the 2_1^+ state. These discrepancies with experiment can be accounted for in terms of sd-shell excitations (see below).

The calculated radiative π -capture rate to the five positive parity states listed above are presented in Table IV. Comparing to the experimental branching ratios (Table I) we see that the calculation describes qualitatively the main features of the spectrum: the weakness of the ground state transition, that the 2_1^+ state is strongest and that the 1_1^+ state has appreciable strength. The good quantitative agreement with experiment on the 1_1^+ state is particularly significant, since the studies described below indicate that there is only a 1% $(sd)^2$ component in this state. We note that the (π^-, γ) rate to the 1_1^+ state does not exhibit a close correlation with ¹⁴N *M*1 rates: for example, the theoretical ratios $R_{\gamma}(1_1^+)/R_{\gamma}(2_1^+) = 0.2$ is much larger than the corresponding $B(M1)(1_1^+)/B(M1)(2_1^+)$ = 0.016. This feature is expected for some states since the (π^-, γ) operator is more complex than the *M*1 operator. Regarding the ground state transition, it is not surprising that the theoretical value $(1.02\pm0.2)\times10^{-4}$ is larger than the measured value $(0.3\pm0.2)\times10^{-4}$, since the *ft* value was also overestimated. Again, one must bear in mind that small components in the wave function can have large effects on highly hindered (π^-, γ) transitions.

The overestimate of transition strength to the 2_1^+ state can be explained in terms of $(sd)^2$ excitations. Such admixtures to states in ¹⁴N were recently calculated⁴³ in a weak-coupling scheme involving the low-lying eigenstates of the p^{-2} , p^{-4} , and $(sd)^2$ model spaces diagonalized separately in the SU(3) basis. This calculation explains the properties of most of the states below 13 MeV in ¹⁴N. It predicts that the 1⁺ ground state, 0_1^+ T = 1, and 1_1^+ T = 1 states contain very small $(sd)^2$ admixtures, i.e. 4, 4, and 1% respectively. This explains why there is no essential discrepancy between theory and experiment (see Table I) for these states. However, the 2_1^+ , 2_2^+ , and 0_2^+ states contain large $(sd)^2$ admixtures, i.e. 49, 56, and 98%. Since the (π^-, γ) transition operator is in the impulse approximation a one-body operator, and the ¹⁴N ground state is 96% p^{-2} , the $(sd)^2$ admixtures will merely spread the strengths appearing in Table I to more states. Thus, the total strength to the 2_{1}^{+} state $R_{\gamma} = (0.96) (24.3 \times 10^{-4})$ will be divided mainly into two fragments. Taking the above admixtures, this means 51% will go to the 2_1^+ state giving $R_{\gamma} = 11.9 \times 10^{-4}$ and 44% will go to the 2^{+}_{2} state giving $R_{\gamma} = 10.3 \times 10^{-4}$. These results are much closer to the measured distribution of strength between these two states, i.e., $R_{\gamma}(2_1^+)$ $=(7.7\pm0.9)\times10^{-4}$ and $R_{\gamma}(2_2^+)=(4.0\pm0.6)\times10^{-4}$. The summed strength 22×10^{-4} is still higher than the experimental sum $(11.7 \pm 1.1) \times 10^{-4}$.

In the μ -capture reaction ¹⁴N(μ^- , ν_{μ})¹⁴C, similar discrepancies exist between p^{-2} calculations^{44,45} and the measured⁴⁶ transition rate to the 2_1^+ state.

Using Cohen and Kurath wave functions, Mukhopadyay⁴⁴ obtains a value ~2×10⁴ sec⁻¹ for the *s*-state μ -capture rate, which is about twice the experimental value⁴⁶ (1±0.3)×10⁴ sec⁻¹. Thus, by assuming^{44,45}~50% (*sd*)² in the 2⁺₁ state and ~4% (*sd*)² in the ¹⁴N ground state, one removes the discrepancy with experiment on the 2⁺₁ state in both μ and π capture. Also, we note that in μ capture the ¹⁴C(g.s.) and 2⁺₂ states are predicted to have negligibly weak transition strength and that the 1⁺₁ state has ~7% of the 2⁺₁ state strength. This distribution of strength in a p^{-2} space correlates closely with our calculations for the (π^-, γ) reaction.

C. Positive parity states in ¹⁰B

The positive parity states which could produce strong (π^-, γ) transitions are those with *M*1 transitions to the 3⁺ T = 0 ground state of ¹⁰B (i.e., 2⁺ $T = 1, 3^+, T = 1, 4^+, T = 1$) and 0⁺ T = 1 and 1⁺ T = 1 states. The number of p^6 shell-model components are 10, 14, 7, 4, 7, and 9, respectively. Although the calculation was performed in the j-jcoupling scheme, it is more instructive to present the wave functions in the SU(3) scheme, which here coincides with the Wigner supermultiplet scheme:

- $| 3^{+} T = 0 \rangle = 0.868 | [42]2, 1 \rangle_{1} + 0.285 | [42]2, 1 \rangle_{2} 0.327 | [42]3, 1 \rangle + 0.001 | [42]4, 1 \rangle + 0.150 | [411]3, 0 \rangle \\ 0.041 | [33]3, 0 \rangle + 0.143 | [321]1, 2 \rangle 0.103 | [321]2, 2 \rangle + 0.050 | [321]2, 1 \rangle + 0.047 | [222]0, 3 \rangle , \\ | 2_{1}^{+} T = 1 \rangle = -0.019 | [42]2, 0 \rangle_{1} + 0.822 | [42]2, 0 \rangle_{2} 0.272 | [411]1, 1 \rangle + 0.339 | [33]1, 1 \rangle$
 - $+ 0.032 | [411]3, 1 \rangle 0.297 | [33]3, 1 \rangle + 0.037 | [321]1, 2 \rangle + 0.006 | [321]1, 1 \rangle \\+ 0.112 | [321]1, 1 \rangle_{2} + 0.055 | [321]2, 2 \rangle 0.041 | [321]2, 1 \rangle + 0.160 | [321]2, 1 \rangle_{2}$
 - $+0.034|[321]2,0\rangle+0.009|[222]0,3\rangle$,

 $| 3^{+} | T = 1 \rangle = 0.909 | [42]3, 0 \rangle - 0.284 | [411]3, 1 \rangle + 0.205 | [33]3, 1 \rangle - 0.041 | [321]1, 2 \rangle + 0.005 | [321]2, 2 \rangle + 0.084 | [321]2, 1 \rangle_{1} + 0.207 | [321]2, 1 \rangle_{2} ,$

 $|4^{+}T=1\rangle = -0.617|[42]4,0\rangle + 0.619|[411]3,1\rangle - 0.345|[33]3,1\rangle + 0.248|[321]2,2\rangle$

 $| 0^{+} T = 1 \rangle = 0.846 | [42] 0, 0 \rangle + 0.005 | [411] 1, 1 \rangle + 0.500 | [33] 1, 1 \rangle + 0.033 | [321] 1, 1 \rangle_{1} + 0.170 | [321] 1, 1 \rangle_{2} - 0.060 | [321] 1, 1 \rangle + 0.001 | [222] 0, 0 \rangle .$

For the higher excited 2^+ states we get

- 2_3^+ : $E_x = 7.0$ MeV C_i : -0.422, 0.126, -0.263, -0.671, -0.098, -0.240, -0.317, 0.191, -0.179, 0.150, 0.114, 0.041, -0.085, -0.071,

where the C_i are the coefficients of each basis vector in the same order as for the 2_1^+ state. The basis states are indicated in the standard notation $|[f]L, S\rangle$, where [f]=permutation symmetry of the spatial wave function. {In SU(3) notation⁴⁷ [42]=(2, 2), [411]=(3, 0), [33]=(0, 3), [321] =(1, 1), [222]=(0, 0).} The above eigenstates do not have a simple structure in the Wigner supermultiplet scheme, partly because the additional quantum number required to distinguish the states in case of degeneracy, e.g., the two [42] L=2states, does not have physical meaning. The M1, E2, and (π^-, γ) rates obtained from the above functions are given in Tables III and IV.

The magnetic and quadrupole moments of the 3^+ T = 0 ground state are $1.9\mu_N$ and + 0.056 b, respectively. The experimental values are $1.8\mu_N$ and + 0.086 b. Thus we expect the ¹⁰B ground state wave function to be fairly reliable.

We find that nearly all M1 strength is exhausted below 12 MeV (Table III) with 91% of the strength going to three states $(2_2^+, 2_3^+, 3_1^+)$ out of a possible 25. In the SU(3) basis it is not easy to see why the 2_2^+ state exhausts most of the M1 sum rule (47%, Table III) while the 2_1^+ state is weak (1%). Cohen and Kurath obtain $1.812\mu_N$ for the ground state and B(M1) values of 2.786, 0.665, 1.521, and 0.244 $(e\hbar/2mc)^2$ for the 2_2^+ , 2_3^+ , 3_1^+ , and 4_1^+ states, respectively.

The comparison with experimental (π, γ) branching ratios (Table II) is most significant on the lowest three states in ¹⁰Be, since the experimental branching ratios should be quite accurate. For these three states, 0_1^+ , 2_1^+ , and 2_2^+ , the calculated relative distribution of strength 1/2.4/4.7 is in excellent agreement with the experimental relative branching ratios of $1/(1.8\pm0.41)/(4.2\pm0.8)$. The calculated absolute values are too high by factors of 1.4 to 1.9. The over-all theoretical normalization is affected by the choice of $\hbar\omega$ (radial *p*-shell wave function), distortion factors C_s and C_p , and the 1s and 2p strong-absorption level widths which are used to obtain branching ratios from transition rates. The cumulative error from the uncertainties in these quantities could account for discrepancies of the size obtained.

The μ -capture reaction ${}^{10}\text{B}(\mu^-, \nu_{\mu}){}^{10}\text{Be}$ was studied with the Cohen-Kurath model by Mukhopadyhay.¹² Unfortunately the μ -capture measurements have not been performed. Comparing the predicted distribution of μ -capture strength with the R_{γ} of Table IV, one sees that the 2^+_2 state dominates both reactions. A significant difference is obtained for the 3^+_1 (T=1) state, estimated to be between 7 and 9 MeV in ${}^{10}\text{Be}$. It is the second strongest state in μ capture with 34% of the 2^+_2 state strength, but weakly excited in (π^-, γ) with 9% of the 2^+_2 state branching ratio.

D. Negative parity states in ¹⁴N

Within the chosen model space there are 19, 45, 53, 41, and 24 nonspurious shell-model components in the 0⁻, 1⁻, 2⁻, 3⁻, and 4⁻ T = 1 subspaces, respectively.⁴⁸ The calculated wave functions for the 0⁻, 1⁻, and 2⁻ states have already been checked in a study³⁷ of the ¹³C(p, γ)¹⁴N reaction. The experimental E1 spectrum is described fairly well. The main concentration of strength is predicted in 2⁻ states at 21.3 MeV and 1⁻ states at 21.5 MeV in ¹⁴N. In the ¹³C(p, γ_0)¹⁴N reaction²⁵ the peaks are seen at 22.5 and 23.0 MeV in ¹⁴N, and much of the strength was associated with 2⁻ states. The calculations also predict 2⁻, 1⁻, and 0⁻ strength at higher energies.

In comparing photonuclear and (π^-, γ) transitions to negative parity states in the GDR region, one must bear in mind that the E1 operator of photoexcitation does not explicitly depend on the spin and has strong "non-spin-flip" matrix elements (e.g., $p_{3/2} + d_{5/2}$ etc.). However, the (π^-, γ) operator depends on the nucleon spin (except for the very weak *D* term) and has strong "spin-flip" matrix elements (e.g., $1p_{3/2} + 1d_{3/2}$). Thus one expects the collective state observed in photoexcitation to contain predominantly non-spin-flip excitations, while the collective states observed

TABLE III. M1 and E2 transition rates ${}^{14}N[1^+ T=0(g.s.) \rightarrow J^{\pi} T=1]$ and ${}^{10}B[3^+ T=0(g.s.) \rightarrow J^{\pi} T=1]$. Configurations p^{-2} (¹⁴N) and p^6 (¹⁰B) are assumed.

	Theory	Expt. ^a		Theory	_ /_ \
J^{π}	E _x (MeV)	B(M 1) $(e\hbar/2mc)^2$	B(M1) $(e\hbar/2mc)^2$	% of sum	B(E2) $e^2 \mathrm{fm}^4$
			¹⁴ N		
01	1.5	$\textbf{0.019} \pm \textbf{0.003}$	0.096	2	
2^{+}_{1}	6.5	1.44 ± 0.17	4.877	93	0.919
1_{1}^{+}	8.5		0.077	1	2.679
0^{+}_{2}	13.7	0.05	0.140	3	
2^+_2	14.2	1.53 ± 0.19	0.041	1	0.356
			¹⁰ B		
2_{1}^{+}	3.0	0.10	0.065	1	0.026
2^{+}_{2}	4.7		2.702	47	0.013
2^+_3	6.9		1.838	32	0.223
3_{1}^{+}	7.4		0.697	12	0.287
3_{2}^{+}	11.0		0.144	3	0.447
3^{+}_{3}	12.9		0.065	1	0.190
4_{1}^{+}	9.0		0.013	0	0.771

^a Reference 21 for ¹⁴N; Ref. 27 for ¹⁰B.

in the (π^-, γ) reaction must contain predominant spin-flip excitations. For a $J^{\pi} = 1^+$ target such as ¹⁴N, the (π^-, γ) reaction can have strong transitions to 3⁻ states though the dipole operator, i.e., an operator with L = 1, S = 1, J = 2, and T = 1. Such states cannot be excited in photoabsorption via E1transitions.

The energy separations of isospin and spin-isospin dipole vibrations are generally not well known. For the simpler case of ¹⁶O we calculated the excitation energies using the Kuo-Brown interaction, with the result: $E_x \approx 23.0$ MeV for the ordinary GDR (isospin wave), $E_x \approx 27.0$ MeV for the 1⁻ T = 1 spin-isospin GDR, and $E_x \approx 21.0$ MeV for the 2⁻ T = 1 spin-isospin resonance. The situation in ¹⁴N is more complex because the above three vibrations are mixed by recouplings arising from the Pauli principle.

The calculated branching ratios for ¹⁴N are presented graphically in Fig. 8. The strongest states are: $J^{\pi}(R_{\gamma} \text{ in } \%) E_x$ in MeV: 3⁻(0.06)14.7; 2⁻(0.10)16.0; 3⁻(0.06)16.7; 3⁻(0.12)17.0; 3⁻(0.25)17.5; 1⁻(0.07)18.1; 2⁻(0.05)18.8; 2⁻(0.12)19.7; and 2⁻(0.06)23.9. We see that the strongest transitions due to 1⁻, 2⁻, and 3⁻ final states are predicted at 18.1, 19.7, and 17.5 MeV, respectively. As in photoexcitation, the calculated energies are lower by several MeV

 E_r^{a}

(MeV)

 J^{π}

T

 $\Lambda \gamma (1s)$

 $(10^{16} \text{ sec}^{-1})$

than the peaks in the spectra [Fig. 8(a)]. The total radiative branching ratio to all negative parity states is 1.95%, with 0.06% to 0^- states, 0.42% to 1^- states, 0.66% to 2^- states, 0.73% to 3^- states, and 0.07% to 4^- states.

A more complete comparison of the calculated distribution of negative parity excitations with the (π^-, γ) data is given in Fig. 8(b). To obtain the curves on this figure we (a) assigned each theoretical level a BW shape with full width at half-maximum = 1 MeV, (b) shifted all E_x up by 2.5 MeV, (c) folded the theoretical spectrum $(R_s + R_p \text{ vs } E_\gamma)$ with the instrumental line shape and detection efficiency (Fig. 2), and (d) normalized the theoretical spectrum to the number of stopped pions. The resulting spectrum was multiplied by 0.4 to approximately fit the data in the GDR region. Thus the figure corresponds to $R_\gamma = 0.4 \times 1.95 = 0.78\%$ for the calculated negative parity states.

The factor of 0.4 is somewhat arbitrary. Clearly a factor <1 is needed since the calculated branching ratio of 1.95% to $1\hbar\omega$ excitations is almost equal to the measured $2.13 \pm 0.21\%$ for all transitions. One expects appreciable strength to other excitations, e.g., $0\hbar\omega$, $2\hbar\omega$, and quasifree (QF). If the $1\hbar\omega$ states are associated with the BW contribution obtained in the fit to the data with the pole model +BW ($R_{\rm y} = 0.21 \pm 0.02\%$, Ta-

 R_{p} (× 10⁻⁴) R_{v}^{b}

 $(\times 10^{-4})$

TABLE IV. Theoretical (π^-, γ) transition rates and branching ratios for 1s and 2p capture in ¹⁴N and ¹⁰B. For ¹⁰B only the strong states are included.

 $\Lambda_{\gamma}(2p)$ (10¹² sec⁻¹) R_{s} (× 10⁻⁴)

				¹⁴ N			
0+	1	0.0	0.3963	0.1856	0.291	0.730	1.02
0^{+}_{2}	1	12.2	0.1621	0.1875	0.119	0.738	0.86
2_{1}^{+}	1	5.0	7.5624	4.7509	5.561	18.707	24.33
2^+_2	1	12.7	0.1811	0.2661	0.133	1.048	1.18
1_{1}^{+}	1	7.0	0.8281	1.0789	0.609	4.248	4.86
				¹⁰ B			
01	1	0.0	0.0707	0.1426	0.277	3.278	3.56
2^{+}_{1}	1	2.6	0.1985	0.3351	0.778	7.706	8.48
2^+_2	1	4.3	1.4949	0.4777	5.862	10.986	16.85
2^+_3	1	6.5	0.4400	0.2052	1.725	4.719	6.45
4^+_1	1	8.6	0.1741	0.0881	0.623	2.025	2.71
4^+_3	1	14.1	0.2906	0,1397	1.140	3.212	4.35
31	1	7.0	0.1148	0.0431	0.450	0.992	1.44

^a Theoretical value for the energy in the residual nucleus (¹⁴C and ¹⁰Be) relative to ground state. The identification with experimental levels (Tables I and II) is on the basis of J^{π} .

^b Results are for p^{-2} and p^6 configurations; the effects of $(sd)^2$ excitations are important for some states, as discussed in text.

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FIG. 8. Results of the shell-model calculation for $1\hbar\omega$ negative parity excitations in the reaction ${}^{14}N(\pi^-,\gamma){}^{14}C$ are compared to the data. (a) Branching ratios to the strongest states. The data are on an arbitrary scale. (b) Theoretical transition strength (× 0.4) folded with the instrumental resolution and efficiency. A level width of 1 MeV (BW shape) was assigned to each level. It is seen that the peak in the data at $E_x({}^{14}C) = 20$ MeV is predicted to arise from strong excitation of 3⁻ spin-isospin dipole states.

ble I), a reduction factor of 0.1 is needed.

The central question is how to separate resonance and QF capture. Our wave functions for the negative parity states are not orthogonal to the wave functions of the same J^{π} obtained by coupling the (A-1) nuclear wave functions with those of an unbound neutron moving in an optical potential. Therefore, our computed rates include a certain amount of QF cross section. However, it is difficult to see how these amounts can be large, since when one expands the distorted optical-potential wave functions for an unbound neutron in an harmonic oscillator basis, it is seen that the $1\hbar\omega$ components are small.⁴⁹

Other uncertainties in the calculations result from the use of pionic orbit distortion factors (Sec. IVA) derived from the bound-state calculations involving only *p*-shell nucleons, and from the use of harmonic-oscillator wave functions for unbound nucleons. It seems reasonable that errors here could lead to factors of 2 in the total $1\hbar\omega$ rates, but factors of 10 seem improbable.

It is clear that the excitation of continuum states

needs a more precise theoretical treatment. Nevertheless, from the above comparisons with the ¹⁴N data it seems reasonable to conclude the 3⁻ states are indeed strongly excited. A similar calculation⁵⁰ for the ⁶Li(π^- , γ)⁶He reaction also indicated that 3⁻ states would be strongest, and some evidence for a peak ($E_x \approx 23$ MeV) was seen in the data.^{3,51} These results, if corroborated by further investigations, demonstrate an attractive feature of the (π^- , γ) reaction, viz., that collective spinisospin dipole vibrations with $J_f = J_i + 2$ are preferably excited. Such states are difficult to observe in other reactions and cannot be formed in *E*1 photoexcitation.

V. CALCULATION OF 1s RADIATIVE CAPTURE TRANSITION TO THE ¹⁴C (g.s) IN THE "ELEMENTARY-PARTICLE SOFT-PION" ANSATZ

The ¹⁴N(π^- , γ)¹⁴C(g.s.) transition rate from 1sstate capture may be calculated following the treatment of Delorme⁵² for the similar ⁶Li(π^- , γ)-⁶He(g.s.) transition, which is also a 1⁺ \rightarrow 0⁺ transition. Delorme obtains the expression

$$\Lambda_{\gamma}(1s; 1^{+} \rightarrow 0^{+}) = \frac{(1.22e)^{2}}{6\pi^{2} f_{\pi}^{2}} C_{s}(Z\alpha m_{\pi}')^{3} k \left(1 - \frac{m_{\pi}}{4m_{i}}\right) \\ \times \frac{1}{4m_{i}^{2}} F_{A}^{2}(q^{2})$$
(7)

with $e^2 = 4\pi\alpha$, $f_{\pi} = 0.932 m_{\pi}$, k = photon momentum= 138.6 MeV, $q^2 = (\text{four-momentum transfer})^2$ = 0.489 fm⁻² for the ¹⁴N \rightarrow ¹⁴C transition, $m'_{\pi} = \text{re-duced pion mass}$, $m_i = \text{mass}$ (¹⁴N), Z = 7; $\hbar = c = 1$. The factor 1.22, which was 1.35 in Delorme's original calculation, comes from the more recent work of Ericson and Rho⁹; it represents the corrections for ρ exchange, incoherent rescattering, and nuclear intermediate states. The distortion of the pion wave function due to the strong interaction in the initial state is taken into account by the multiplicative factor $C_s = 0.50$ (Sec. IV A).

The axial-vector form factor for the pure $1^+ \rightarrow 0^+$ GT transition is determined by assuming⁹ that its variation with q^2 between 0 and 0.489 fm⁻² is the same as that of the electromagnetic form factor $F_M(q^2)$ of the *M*1 transition ¹⁴N(g.s.) $+ {}^{14}$ N(2.313 MeV), where the latter state is the analog of 14 C(g.s.). $F_A{}^2(0)$ is determined from the β -decay ft value⁵³ 1.052×10⁵ sec. Using the expression given by Delorme

$$\frac{F_A^2(0)}{4m_i^2} = \frac{2\pi^3 \ln 2}{3G^2 m_e^{-5} f t_{1/2}}$$
(8)

we obtain $F_A^2(0)/4m_i^2 = 1.978 \times 10^{-6}$.

Ensslin *et al.*⁵⁴ measured the ¹⁴N(*e*, *e'*)¹⁴N(2.313 MeV) transition and obtain the following parametrization of $F_M(q^2)^{55}$:

$$\times \left[(0.40 \pm 0.06) + (0.823 \pm 0.071)q^2 \right]$$
 (9)

which gives

 $F_M(489 \text{ fm}^{-2})/F_M(0) = 1.38 \pm 0.12$.

 $F_M(q^2) = 0.01226e^{-0.7627q^2}$

Taking this value fc $F_A(q^2 = 0.489 \text{ fm}^{-2})/F_A(q^2 = 0)$, we obtain

$$\Lambda_{\gamma}(1s; 1^+ \rightarrow 0^+) = 1.31 \times 10^{11} \text{ sec}^{-1}.$$

The radiative capture branching ratio for 1s capture then is

$$R_{s} = \frac{\Lambda_{\gamma} (1s; 1^{+} - 0^{+})}{\Lambda a(1s)} \omega_{s}$$
$$= \frac{1.31 \times 10^{11} \text{ sec}^{-1}}{6.82 \times 10^{18} \text{ sec}^{-1}} \times 0.1 = 1.9 \times 10^{-9}$$

This value is considerably smaller than the 2.9×10^{-5} obtained in the shell-model calculation (Sec. IV B). We note that the shell-model calculation yields $\log ft_2^1 = 5.5$ instead of the experimental value $\log ft_2^1 = 9$. We see that if the assumed variation of $F_A(q^2)$ with q^2 is correct, the soft-pion prediction gives an extremely small 1s radiative capture branching ratio. Thus the measured value $R_{\gamma} = (3 \pm 2) \times 10^{-5}$, if different from zero, must be explained in terms of *p*-state capture.

VI. CONCLUSIONS

We can draw from this study of the (π^-, γ) reaction with stopped pions on ¹⁴N and ¹⁰B some conclusions:

(1) The (π^-, γ) reaction on *p*-shell nuclei selectively excites the analogs of giant *M*1 states of the target. The data on ¹⁴N and ¹⁰B and those of previous studies on ⁶Li and ¹²C clearly demonstrate that the strongest (π^-, γ) transitions correspond to such excitations. The successful measurement and analyses of these transitions most clearly establish the (π^-, γ) reaction as a quantitative probe of nuclear structure.

(2) The calculations of the (π, γ) transition rates for 1s and 2p capture, in terms of an IA Hamiltonian and shell-model wave functions obtained using realistic interactions, yielded satisfactory agreement with our measurements. In ¹⁴N there is good agreement on the strongest transitions (2_1^+ and 1_1^+ states) if one includes the $(sd)^2$ admixtures in excited states required by other data on the same states. In ¹⁰B, branching ratios for three states could be measured accurately due to the wide level separations. The shell-model calculation in a p^6 vector space predicts the relative branching ratios to these three states correctly, but overestimates the absolute values by ~1.7.

(3) The interesting transition ¹⁴N(π^-, γ)¹⁴C(g.s.) is very weak, as was anticipated from the ~10⁶ hindrance of the β -decay between the same two states. The shell-model calculations overestimate both the (π^-, γ) and β rates although small values are predicted. The PCAC and soft-pion calculation for the 1s radiative capture yield a negligibly small branching ratio. Thus the observed strength $R_{\gamma} = (3 \pm 2) \times 10^{-5}$, if not equal to zero, must arise from *p*-state capture. The small experimental upper limit determines that even the *p*-state radiative transition rate is small; perhaps this transition can be used in future studies extending soft-pion theorems to *p*-state capture.

(4) The ¹⁴N data give evidence for excitations of spin-isospin dipole vibrations at 20 ± 1 MeV in ¹⁴C. The shell-model calculations suggest that the predominant contributions are from 2⁻ and 3⁻ states, with the 3⁻ states strongest. This contrasts with *E*1 photoexcitation where 1⁻ and 2⁻ states dominate and 3⁻ states cannot be excited. Thus the (π^-, γ) reaction, by exciting spin-isospin dipole vibrations with $J_f = J_i + 2$, provides complementary information in the study of continuum states in the GDR region.

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