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Relativistic effects in bound-state form factors*

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The formalism for bound-state form factors in a three-dimensional relativistic theory is used to expose the theoretical limitations of recent work on meson exchange currents. The pair excitation and recoil emission currents used in phenomenological calculations are found to have little theoretical support. The constraints imposed by the Ward identity are examined and the singularity structure of the form factor in relativistic and nonrelativistic theories is compared.

[NUCLEAR REACTIONS Electromagnetic form factors, relativistic effects.]

I. INTRODUCTION

Starting with meson-exchange current contributions to magnetic moments[1] of two and three nucleon systems, the electromagnetic processes for which exchange effects have been calculated have increased to include radiative neutron capture,[2-4] electrodisintegration,[5] and elastic electron scattering.[6-8] While a certain amount of success has been claimed for these calculations they must, however, be regarded as phenomenological in nature. Standard practice has been to construct a two-body current operator from a few Feynman diagrams usually selected by arguments of interaction range. The two-body operator is then evaluated between wave functions the dynamical content of which is only tenuously related to the dynamical origin of the two-body currents. While such a phenomenology may be successful when the calculated exchange corrections are small, its success must not be allowed to obscure the need for a more critical examination of its underlying theoretical basis now that quantitative predictions are being sought for processes, such as elastic electron scattering at large momentum transfer,[6-8] where the exchange corrections dominate.

In this paper the theory of electromagnetic bound state form factors is examined from the point of view of a three-dimensional relativistic approach. The primary theme of the discussion is that the nucleon-nucleon dynamics, nucleon structure, and the bound state electromagnetic current operator are all intimately connected and that to go beyond a phenomenological description of the bound state form factor requires a consistent theoretical treatment of all aspects of the problem. While such a statement of principle may be generally accepted, we show in this paper how an examination of the relativistic formalism exposes the limitations of present exchange current phenomenology.

The starting point of the discussion will be the Bethe-Salpeter equation. For nucleon-nucleon scattering, procedures for reducing this four-dimensional equation to a three-dimensional relativistic equation[9,10] are known. Although the reduction is not unique, it is possible to improve systematically the agreement of the three-dimensional equations with the Bethe-Salpeter equation and the importance of relativistic effects in low-energy nucleon-nucleon scattering has been investigated, at least in simple models.[11,12] Such numerical experiments have not been carried out for the bound state problem. To do numerical calculations in sufficiently realistic models where the hadronic and electromagnetic interactions have been treated consistently is probably not feasible at present. However a new, more theoretically grounded phenomenology may be a realistic goal.

The formal expression for the current operator to be evaluated between Bethe-Salpeter bound state wave functions in a fully relativistic theory was given by Mandelstam[13] and, as it is likely to be unfamiliar, it is reviewed in Sec. II A. Due to the requirement of gauge invariance, part of the current operator is dictated by the choice of kernel used in the Bethe-Salpeter equation for the wave function. This intimate connection between the current operator and the bound state dynamics is a feature of the relativistic theory that is not contained in nonrelativistic approaches based on phenomenological local potentials. Clearly, any effort to include relativistic effects in the current operator will be highly suspect if the same relativistic effects cannot be built into the bound state dynamics. The current operator may also have contributions which are gauge invariant independent of the interaction kernel. It is expected that these currents require physical input (parameters) not already contained in the kernel and they may be treated phenomenologically as, for example, in Ref. 6.

In Sec. II B the reduction procedure developed by Gross[10,14] for the two-body wave function and form factor is outlined. The main feature of this reduction is that the only contribution to the relative-energy integration appearing in the relativistic theory which is retained is that which constrains one of the constituents of the two-body system to be on its mass shell. The other constituent is off mass shell and may propagate with positive or negative frequency. Gross has derived coupled equations for wave functions containing positive and negative energy off-shell particles and it is pointed out in Sec. II B that the pair excitation current[7,8] of conventional exchange current calculations can be interpreted as a perturbative solution of the coupled equations.

Some new theoretical results arising from the formalism of Sec. II are given in Sec. III. The coupling of a photon with a constituent particle satisfies the Ward identity, and in Sec. III A it is shown that in a three-dimensional theory with relativistic kinematics the off-shell constituents must have both positive and negative energy propagators if the Ward identity is to be preserved. In the nonrelativistic limit the Bethe-Salpeter and Gross equations go over to the Lippmann-Schwinger equation. By considering the nonrelativistic limit of the various time orderings for the bosons exchanged by the bound state constituents in a relativistic theory it is shown in Sec. III B that the recoil emission current[7] is contained in the nonrelativistic limit of the Bethe-Salpeter theory. This is in agreement with the lowest order perturbation theory result obtained by Thompson and Heller.[15]

The analytic properties of the form factor are examined in Sec. III C. In the Gross and nonrelativistic theories the form factor has only the anomalous singularity, while the Bethe-Salpeter form factor has additional normal singularities. We have found that other three-dimensional reductions, in particular the Blankenbecler-Sugar quasipotential approach,[9] can lead to form factors which also have a normal singularity.

The results are discussed in Sec. IV.

II. FORMALISM

A. Bethe-Salpeter formalism

In this section the Bethe-Salpeter formalism for the electromagnetic current of a two-body bound state is reviewed. The bound state constituents will be referred to as nucleons even though spin will for the most part be ignored. The exact nature of the coupling of the boson mediating the nucleon-nucleon interaction will not play any role and will remain unspecified.

The Bethe-Salpeter equation for the wave function of two nucleons in a bound state of four-momentum D and with relative momentum p is

$$\psi_D(p) = iG(\tfrac{1}{2}D - p)G(\tfrac{1}{2}D + p)\int d^4l\, K(p, l; D)\psi_D(l), \tag{2.1}$$

where G is the nucleon propagator and K is the interaction kernel. The nucleon propagator G contains both positive and negative frequency parts and, in general, includes all self-energy contributions. It is convenient to define the vertex function

$$X_D \equiv G^{-1}(\tfrac{1}{2}D - p)G^{-1}(\tfrac{1}{2}D + p)\psi_D(p). \tag{2.2}$$

The equation for X_D is

$$X_D(p) = i\int d^4l\, K(p, l; D)G(\tfrac{1}{2}D - l)G(\tfrac{1}{2}D + l)X_D(l). \tag{2.3}$$

The electromagnetic current J_μ for a two-body bound state (deuteron) is given by (see Fig. 1)

$$J_\mu = \int \frac{d^4p\, d^4l}{(2\pi)^8}\, \overline{X}_{D'}(p)G(\tfrac{1}{2}D' - p)G(\tfrac{1}{2}D' + p)\Gamma_\mu \times G(\tfrac{1}{2}D - l)G(\tfrac{1}{2}D + l)X_D(l) \tag{2.4a}$$

$$= \int \frac{d^4p\, d^4l}{(2\pi)^8}\, \overline{\psi}_{D'}(p)\Gamma_\mu\psi_D(l), \tag{2.4b}$$

where Γ_μ is the relativistic current operator.[13] The operator Γ_μ can be split up into two parts: a disconnected piece Γ_μ^0 shown in Fig. 2(a) and a connected piece consisting of diagrams that are two-nucleon irreducible, examples of which are shown in Figs. 2(b) and 2(c). The disconnected piece is always present and the approximation obtained by retaining only this contribution to Γ_μ will be called the relativistic impulse approximation. The diagrams contributing to the connected part of Γ_μ are of two types. First there are those diagrams which are expected to be gauge invariant independent of the kernel, e.g., Fig. 2(b). These are genuine exchange currents and to determine their presence requires physical input in addition

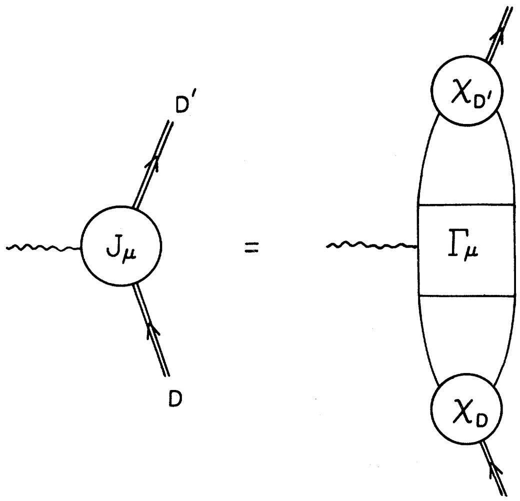

FIG. 1. Schematic representation of the two-body bound state electromagnetic current.

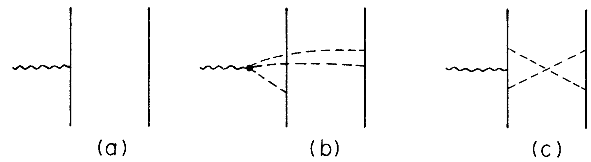

FIG. 2. (a) Disconnected part of the current operator. (b) Exchange current diagram. (c) Interaction current diagram.

to the nucleon-nucleon interaction.[6] Secondly, there are diagrams which must be combined with the relativistic impulse approximation to obtain a gauge invariant result. These are interaction currents. For example, if the kernel K contains the two-boson-exchange crossed ladder graph, the diagram shown in Fig. 2(c) must be included in Γ_μ. In general, a diagram will contribute to the interaction current if, when the photon line is removed, the resulting diagram is contained in the kernel K.

For the purposes of this paper it will be sufficient to consider only the ladder approximation to the Bethe-Salpeter equation. In this case there are no interaction currents (at least for isoscalar boson exchange which will be assumed here). In this approximation it is also appropriate to use the bare nucleon propagators, that is, the nucleons are treated as point particles. In a relativistic theory, nucleon structure cannot be properly accounted for by merely multiplying the body form factor by the free nucleon form factor as is the practice in phenomenological nonrelativistic calculations. The dynamics responsible for the nucleon's electromagnetic structure must be consistently incorporated into the nucleon propagators and the Bethe-Salpeter interaction kernel in order to preserve gauge invariance and unitarity. For a more detailed discussion of how constituent structure may be incorporated in the Bethe-Salpeter framework see Ref. 16.

Specializing to the Breit frame, the kinematics of which are shown in Fig. 3, the current in relativistic impulse approximation is

$$J_\mu = ie\int \frac{d^3p\,dp_0}{(2\pi)^4}\bar{X}_{D'}(p-\tfrac{1}{2}D')G(D'-p)\Lambda_\mu G(D-p) \times X_D(p-\tfrac{1}{2}D)G(p)\,, \qquad (2.5)$$

where Λ_μ is the photon-nucleon coupling: $\Lambda_\mu = (p_1+p_2)_\mu$ for scalar particles and $\Lambda_\mu = \gamma_\mu$ for spinor particles. The propagators are $G(p) = (p^2 - M^2 + i\epsilon)^{-1}$ for scalar particles and $G(p) = (\gamma\cdot p + M + i\epsilon)^{-1}$ for spinor particles.

B. Three-dimensional relativistic formalism

In this section the reduction of the relativistic impulse approximation from the four-dimensional form to a three-dimensional form is considered. In the p_0 plane the propagator $G(p)$ in the integrand of (2.5) has singularities at $p_0 = \pm[(\vec{p}^{\,2}+M^2)^{1/2} - i\epsilon]$ and following Gross[10,14] the three-dimensional reduction is achieved by keeping only the contribution to the p_0 integral from the positive energy pole of $G(p)$; that is, the spectator particle is placed on mass shell. With this approximation Eq. (2.5) becomes

$$J_\mu = e\int \frac{d^3p\,\bar{X}_{D'}(p-\frac{1}{2}D')N(D'-p)\Lambda_\mu N(D-p)X_D(p-\frac{1}{2}D)}{(2\pi)^3 2E_{\vec{p}}[(D_0-E_{\vec{p}})^2 - E_{\vec{q}/2}{}^2 + i\epsilon][(D_0-E_{\vec{p}})^2 - E_{\vec{q}/2}{}^2 + i\epsilon]}\,, \qquad (2.6)$$

where $E_{\vec{p}} = (\vec{p}^{\,2}+M^2)^{1/2}$ and the numerators of the nucleon propagators are 1 for scalar particles and $N(D-p) = (D_0 - E_p)\gamma_0 - (\vec{d}-\vec{p})\cdot\vec{\gamma} + M$ for spinor particles. The three-dimensional Gross equation for the vertex function is

$$X_D(p-\tfrac{1}{2}D) = \int \frac{d^3l}{2E_{\vec{l}}}\frac{K_L(p,l;D)N(D-l)X_D(l-\frac{1}{2}D)}{[(D_0-E_{\vec{l}})^2 - E_{\vec{d}-\vec{l}}{}^2]}\,. \qquad (2.7)$$

The propagators for the off-mass-shell nucleons still contain both positive and negative frequency parts. For the moment considering only scalar particles, the explicit separation of the propagator into positive and negative frequency parts is

$$\tilde{G}(D-p) = [(D_0 - E_{\vec{p}})^2 - E^2_{\vec{d}-\vec{p}} + i\epsilon]^{-1}$$
$$= \frac{1}{2E_{\vec{d}-\vec{p}}}\left(\frac{1}{D_0 - E_{\vec{p}} - E_{\vec{d}-\vec{p}} + i\epsilon} - \frac{1}{D_0 - E_{\vec{p}} + E_{\vec{d}-\vec{p}} + i\epsilon}\right)$$
$$= G^+ + G^- . \tag{2.8}$$

Using (2.8) and (2.7), define the wave function ψ^+ (ψ^-) containing a positive (negative) frequency off-shell particle which satisfies the coupled equations[14]

$$\psi^+ = (G_1^+)^{-1}\int \frac{d^3p}{2E_{\vec{p}}}(K_L^{++}\psi^+ + K_L^{+-}\psi^-), \tag{2.9a}$$

$$\psi^- = (G^-)^{-1}\int \frac{d^3p}{2E_{\vec{p}}}(K_L^{--}\psi^- + K_L^{-+}\psi^+). \tag{2.9b}$$

The deuteron current is

$$J_\mu = e\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}}(\bar{\psi}^+\Lambda_\mu^{++}\psi^+ + \bar{\psi}^+\Lambda_\mu^{+-}\psi^- + \bar{\psi}^-\Lambda_\mu^{-+}\psi^+ + \bar{\psi}^-\Lambda_\mu^{--}\psi^-), \tag{2.10}$$

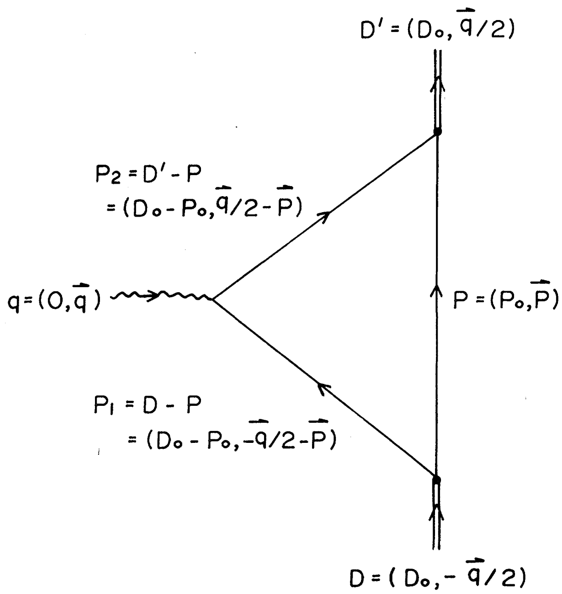

FIG. 3. Kinematics of the relativistic impulse approximation in the Breit frame.

where the superscripts + (−) indicate that for a spinor theory the operator is evaluated for a positive (negative) energy spinor. If the interaction connecting positive and negative energy states is considered to be a perturbation, (2.9a) and (2.9b) may be approximated by

$$\psi^+ = (G^+)^{-1}\int \frac{d^3p}{2E_{\vec{p}}}K_L^{++}\psi^+ , \tag{2.11a}$$

and

$$\psi^- = (G^-)^{-1}\int \frac{d^3p}{2E_{\vec{p}}}K_L^{-+}\psi^+ . \tag{2.11b}$$

The current in this approximation is

$$J_\mu = e\int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}}\bar{\psi}^+ \times\left[\Lambda_\mu^{++} + \int \frac{d^3p}{2E_{\vec{1}}}(\Lambda_\mu^{+-}K_L^{-+} + K_L^{+-}\Lambda_\mu^{-+})\right]\psi^+ . \tag{2.12}$$

The first term in the integral of (2.12) may be identified with the usual nonrelativistic current and the remaining terms with the "pair excitation" current in the conventional treatment of exchange currents.[1] In terms of time ordered diagrams the pair excitation current is shown in Fig. 4. In the region where only small nucleon momenta contribute to the integral (2.12), for example in the region of small momentum transfer, the approximation of (2.11) and (2.12) may be valid (see Ref. 17). However, when the pair excitation terms become comparable to the nonrelativistic impulse approximation, as in the form factor at large momentum transfer,[7,8] there is no reason to expect the perturbative solution (2.11) of the coupled equations (2.9) to be adequate.

III. THEORETICAL DEVELOPMENTS

A. Ward identity

In the relativistic theory the photon-nucleon vertex function Λ_μ and the nucleon propagator are related by the Ward identity

$$(p_1 - p_2)_\mu \Lambda^\mu(p_1, p_2) = G^{-1}(p_1) - G^{-1}(p_2). \tag{3.1}$$

Using the kinematics of the Breit frame shown in

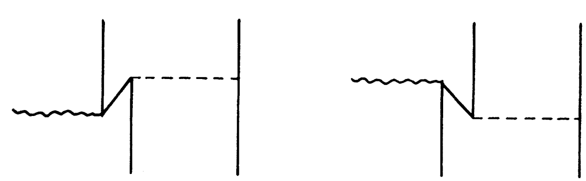

FIG. 4. Pair excitation diagrams.

Fig. 3 and the expression for the Gross propagator $\tilde{G}$ (for scalar particles) it is seen that

$$\tilde{G}^{-1}(p_1) - \tilde{G}^{-1}(p_2) = -2\vec{q}\cdot\vec{p}$$

and

$$(p_1 - p_2)_\mu \Lambda^\mu = -2\vec{q}\cdot\vec{p},$$

that is, the Ward identity is satisfied by $\tilde{G}$. It is also easily verified that it is satisfied for spinor particles and in an arbitrary frame. Consequently the proof of conservation of the deuteron electromagnetic current is the same in the Gross formalism as it is in the Bethe-Salpeter formalism.

For the positive frequency part of the Gross propagator G^+

$$[G^+(p_1)]^{-1} - [G^+(p_2)]^{-1} = -4\vec{q}\cdot\vec{p} + 2(D_0 - E_{\vec{p}}) \times (E_{\vec{q}/2+\vec{p}} - E_{\vec{q}/2-\vec{p}}),$$

so that the Ward identity is not satisfied. This is a warning that an attempt to include relativistic effects by merely calculating the current with a positive frequency wave function obtained from an equation that incorporates only relativistic *kinematics* is in danger of violating gauge invariance. In the limit of very large nucleon mass, D_0 and $E_{\vec{p}}$ may be expanded in powers of (p/M) and the Ward identity is regained for positive frequency propagators up to terms of order $(p/M)^4$.

B. Time ordering and the nonrelativistic limit

In the nonrelativistic limit, that is, when M becomes very large, the Bethe-Salpeter formalism reduces to the nonrelativistic Lippmann-Schwinger formalism. The three-dimensional relativistic formalism of Sec. II B incorporates those pieces of that Bethe-Salpeter formalism which survive in the nonrelativistic limit. The Bethe-Salpeter formalism has contributions with particles propagating in all possible time orderings. It is of interest to ask which time orderings survive in the nonrelativistic limit. For large M the negative frequency parts of the nucleon propagators are $O(p^2/M^2)$ compared to the positive frequency parts, so it is sufficient to keep only nucleons propagating forward in time. This is done by using the three-dimensional formalism of Sec. II B with the propagator G^+ for the off-mass-shell nucleon. Consider the expression for the current corresponding to the diagram shown in Fig. 5(a) where Eq. (2.7) has been used to insert explicitly the boson exchange kernel. What is important in determining which time orderings are present are the particle propagators and, ignoring all possible spin complications, the product Π of propagators for Fig. 5(a) is

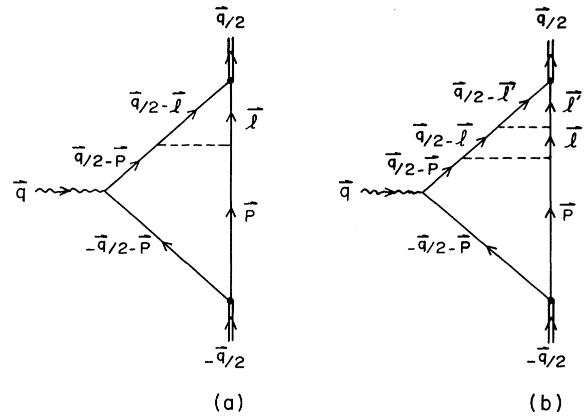

FIG. 5. Impulse approximation explicitly showing (a) one insertion of the one-boson-exchange (ladder) kernel and (b) two insertions of the one-boson-exchange kernel.

$$\Pi = [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})[(E_{\vec{p}} - E_{\vec{l}})^2 - \omega_{\vec{p}-\vec{l}}^{\,2}](D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1}, \tag{3.2}$$

where $\omega_{\vec{k}} = [\vec{k}^2 + \mu^2]^{1/2}$ and μ is the mass of the exchanged boson. First separate the boson propagator into pieces of positive and negative frequency (forward and backward in time),

$$[(E_{\vec{p}} - E_{\vec{l}})^2 - \omega_{\vec{p}-\vec{l}}^{\,2}]^{-1} = (2\omega_{\vec{p}-\vec{l}})^{-1}[(E_{\vec{p}} - E_{\vec{l}} - \omega_{\vec{p}-\vec{l}})^{-1} - (E_{\vec{p}} - E_{\vec{l}} + \omega_{\vec{p}-\vec{l}})^{-1}], \tag{3.3}$$

so that Π will be written as a sum of two terms $\Pi = \Pi_- + \Pi_+$ when Π_- (Π_+) has a forward (backward) going boson. Consider Π_-:

$$\Pi_- \sim [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})(E_{\vec{l}} - E_{\vec{p}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1}.$$

Using partial fractions, Π_- can also be written

$$\Pi_- \sim [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{l}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1}$$
$$+ [(E_{\vec{l}} - E_{\vec{p}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1}. \tag{3.4}$$

Examining the energy denominators it is seen that the first term of (3.4) corresponds to the time ordering shown in Fig. 6(a). To interpret the second term of (3.4), begin by explicitly inserting the interaction kernel again as shown in Fig. 5(b). Taking the positive frequency part of the boson propagator, the second term of (3.4) becomes

$$[(D_0 - E_{\vec{l}'} - E_{\vec{q}/2-\vec{l}'})(E_{\vec{l}'} - E_{\vec{l}} - \omega_{\vec{l}-\vec{l}'})(E_{\vec{l}} - E_{\vec{p}} - \omega_{\vec{p}-\vec{l}}) \times (D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{l}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1}. \quad (3.5)$$

With repeated use of partial fractions (3.5) can be written

$$[(D_0 - E_{\vec{q}/2-\vec{p}} - E_{\vec{l}'} - \omega_{\vec{p}-\vec{l}} - \omega_{\vec{l}'-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{l}} - \omega_{\vec{p}-\vec{l}}) \times (D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1} + \text{other terms}. \quad (3.6)$$

The first term corresponds to absorption of the first boson (momentum $\vec{p} - \vec{l}$) after emission of a second boson (momentum $\vec{l}' - \vec{l}$). The "other terms" of (3.6) can be broken up into specific time ordered pieces by repeated use of the wave equation and partial fractions. The wave function, being the solution of a homogeneous integral equation, contains an inifinite iteration of the interaction kernel so that the second term of (3.4) corresponds to infinitely many different time orderings in *all* of which the boson of momentum $\vec{p} - \vec{l}$ propagates simultaneously with other (virtual) bosons before it is absorbed.

Similarly, using partial fractions,

$$\begin{aligned}\Pi_+ \sim{}& [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{p}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2-\vec{p}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1} \\ &+ [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{p}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2+\vec{p}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{p}} - E_{\vec{q}/2+\vec{p}})]^{-1} \\ &- [(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2-\vec{l}} - \omega_{\vec{p}-\vec{l}})(D_0 - E_{\vec{l}} - E_{\vec{q}/2+\vec{p}} - \omega_{\vec{p}-\vec{l}})(E_{\vec{l}} - E_{\vec{p}} - \omega_{\vec{p}-\vec{l}})]^{-1}. \quad (3.7)\end{aligned}$$

The first term of (3.7) corresponds to the time ordering shown in Fig. 6(b) and the second term to Fig. 6(c). As before, repeated use of the wave equation and partial fractions shows that the last term corresponds to time orderings in which the boson of momentum $\vec{p} - \vec{l}$ propagates simultaneously with other virtual bosons.

In the nonrelativistic limit

$$\begin{aligned}\Pi = -\{&[\vec{q}^2/8M_D - E_B - \vec{l}^2/2M - (\tfrac{1}{2}\vec{q} - \vec{l})^2/2M][(\vec{p} - \vec{l})^2 + \mu^2][\vec{q}^2/8M_D - E_B - \vec{p}^2/2M - (\tfrac{1}{2}\vec{q} - \vec{p})^2/2M] \\ &\times [\vec{q}^2/8M_D - E_B - \vec{p}^2/2M - (\tfrac{1}{2}\vec{q} + \vec{p})^2/2M]\}^{-1}; \quad (3.8)\end{aligned}$$

that is, just the Lippmann-Schwinger form with an instantaneous Yukawa interaction. The nonrelativistic limit of the terms of Π corresponding to the time orderings of Figs. 6(a) and 6(b) is

$$\begin{aligned}-\{&[\vec{q}^2/8M_D - E_B - \vec{l}^2/2M - (\tfrac{1}{2}\vec{q} - \vec{l})^2/2M]\,\omega_{\vec{p}-\vec{l}}(\omega_{\vec{p}-\vec{l}} + E_B) \\ &\times [\vec{q}^2/8M_D - E_B - \vec{p}^2/2M - (\tfrac{1}{2}\vec{q} - \vec{p})^2/2M][\vec{q}^2/8M_D - E_B - \vec{p}^2/2M - (\tfrac{1}{2}\vec{q} + \vec{p})^2/2M]\}^{-1}. \quad (3.9)\end{aligned}$$

This is not quite the Lippmann-Schwinger form. The other time orderings give a nonvanishing contribution in the nonrelativistic limit which combines with (3.9) to give the Lippmann-Schwinger result (3.8). Even though the interaction of the nonrelativistic theory is instantaneous it is obtained as the nonrelativistic limit of diagrams in which many bosons may be "in the air" at the same time.

In the conventional treatment of the relativistic effects,[1,7] the recoil emission current, which is the nonrelativistic limit of the time ordering of Fig. 6(c), is introduced as an explicit correction to the nonrelativistic theory. The analysis of this section on how the Lippmann-Schwinger theory arises as the nonrelativistic limit of the relativistic theory indicates that inclusion of the recoil emission current in nonrelativistic calculations is double counting. This result was also obtained by Thompson and Heller[15] in a perturbation theory analysis.

C. Analyticity

The bound state form factor is an analytic function of q^2 with singularities in the time-like region ($q^2 > 0$). In the Bethe-Salpeter theory the form factor develops normal singularities[18] corresponding to thresholds for physical states that can couple to a time-like photon and a deuteron-antideuteron pair. For the relativistic impulse approximation, the first normal singularity is at $q^2 = 4M^2$ and arises because the singularities of $G(D' - p)$ and $G(D - p)$ pinch the integration contours in (2.5). There will be further normal singularities from

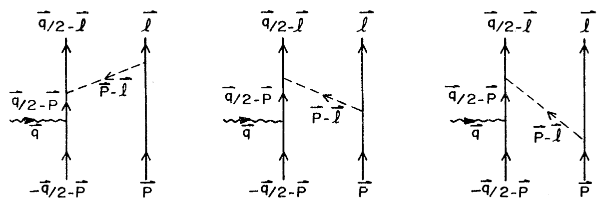

FIG. 6. Time-ordered contributions contained in the relativistic impulse approximation.

intermediate states containing a nucleon-antinucleon pair plus bosons. In addition to the normal singularities, the relativistic impulse approximation also has an anomalous singularity[18,19] which arises due to the confluence of singularities from all three nucleon propagators in the region of integration. For a loosely bound state the anomalous singularity occurs at a smaller value of q^2 than the normal singularities. In fact the anomalous singularity is at $M_D^2(4M^2 - M_D^2)/M^2 \ll 4M^2$ and it is expected to dominate the form factor for small spacelike momentum transfer.

The Gross three-dimensional approximation retains only the contribution to the relativistic impulse approximation in which the spectator nucleon is on mass shell. The form factor in this approximation will have the anomalous singularity at the same value of q^2 as the relativistic impulse approximation (see Ref. 14). All the normal singularities will, however, be absent. In the nonrelativistic limit the anomalous singularity survives and the Lippmann-Schwinger form factor has a singularity at $q^2 = 16E_BM$, where E_B is the deuteron binding energy. For $E_B \ll M$, this is essentially the same value of q^2 as the position of the anomalous singularity of the relativistic theory.

The Gross prescription is not the only possible three-dimensional reduction. One alternative is the quasipotential or Blankenbecler-Sugar approach.[9] In this approximation the spectator nucleon is not constrained to be on mass shell and contributions to the relative energy integral in addition to that required to ensure the proper nonrelativistic limit are retained. The quasipotential form for the current[20] for scalar nucleons is

$$J_\mu = e\int \frac{d^3p}{(2\pi)^3}\bar{X}_{D'}(p - \tfrac{1}{2}D')g(p, D')\Lambda_\mu g(p, D)X_D(p - \tfrac{1}{2}D) \times\frac{[g^{-1}(p, D) - g^{-1}(p, D')]}{E_{\vec{q}/2+\vec{p}}^2 - E_{\vec{q}/2-\vec{p}}^2}\,, \qquad (3.10)$$

where g is the Blankenbecler-Sugar propagator,[9,12]

$$g(p, D) = \frac{E_{\vec{d}+\vec{p}} + E_{\vec{p}}}{2E_{\vec{d}+\vec{p}}E_{\vec{p}}}[D_0^2 - (E_{\vec{p}} + E_{\vec{d}+\vec{p}})^2]^{-1}, \qquad (3.11)$$

and X_D is the solution of the equation

$$X_D(p - \tfrac{1}{2}D) = g^{-1}(p, D)\int d^3l\, K(p, l; D)g(l, D)X_D(l - \tfrac{1}{2}D)\,. \qquad (3.12)$$

In addition to the anomalous singularity at $q^2 = M_D^2(4M^2 - M_D^2)/M^2$ the quasipotential form factor also has a normal singularity at $q^2 = 4M^2$, that is, its singularity structure matches that of the relativistic form factor more closely than does the form factor of the Gross approximation. The singularities corresponding to a nucleon-antinucleon pair plus bosons are absent reflecting the fact that the three-dimensional theories satisfy elastic unitarity at all energies in the scattering region.

IV. DISCUSSION

Implicit in recent work on meson exchange currents (for example Refs. 1–8) is the view that these are corrections to be added to nonrelativistic results grounded in a phenomenology based on local potentials. In this paper a different point of view has been adopted, namely, the relativistic Bethe-Salpeter theory was taken as the starting point and its reduction to the Lippmann-Schwinger theory in the nonrelativistic limit was traced out to identify explicitly the origin of the relativistic effects meson exchange currents are to incorporate. The three-dimensional reduction techniques have been known for some time, what is new here is the application to interpreting exchange current phenomenology.

In the Bethe-Salpeter formalism the current operator for a bound state depends on the choice of kernel in the equation for the wave function. The practical implication of this statement is that for processes where the large momentum components of the bound state wave function play a major role, it is necessary to know the *details* of the dynamics determining those large momentum components in order to construct the appropriate current operators. At present our knowledge of the nucleon-nucleon interaction is much too phenomenological to allow construction of reliable current operators to be used for large momentum transfer processes.

By examining the three-dimensional reduction of the Bethe-Salpeter formalism some limitations of the conventional exchange current calculations can be seen. First, the inclusion of negative energy nucleon propagation through the pair excitation current was shown in Sec. II B to be the perturbative solution of a more general coupled channel problem. Present calculations[7,8] fail to justify this approximation even though the calculated correction is large. Furthermore it was shown that when relativistic kinematics are used, negative energy propagation must be retained in order to preserve the Ward identity.

By explicitly separating the Bethe-Salpeter expression for the electromagnetic current into its various time ordered pieces and then taking the nonrelativistic limit, it was shown in Sec. III B that those pieces corresponding to the photon being absorbed by a bound nucleon while virtual bosons are present do not vanish nonrelativistically but are necessarily present if the Lippmann-Schwinger results are to be obtained. The conclusion is that the recoil emission current[2,15] is double counting something that is contained in the nonrelativistic theory.

The form factor in Gross approximation and in a nonrelativistic calculation has only the anomalous singularity, reflecting the fact that the deuteron is a loosely bound state. In the Blankenbecler-Sugar quasipotential approach the form factor has the normal singularity corresponding to a nucleon-antinucleon pair in addition to the anomalous singularity. The other normal singularities of the relativistic theory are absent in three-dimensional or nonrelativistic calculations.

In this paper all the effects of nucleon structure on the deuteron current have been ignored. At present it is possible to introduce structure in a way consistent with unitarity and gauge invariance only in simple models.[16] Calculations with models in which the nucleon is treated as a pion-nucleon bound state[21,22] may be able to show the importance of treating nucleon structure consistently in both the current operator and the bound state wave function. Presumably form factors in such models would have additional normal singularities reflecting the fact that both two- and three-body unitarity are satisfied in the scattering region.

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