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A shelf in the "subthreshold" photofission cross section

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The influence of a double-humped fission barrier on the photofission cross section far below the top of the barrier is considered. In the region about 2 MeV below the top of the outer barrier and at a cross section in the region of 10^{-9}–10^{-6} b the photofission is expected to become almost entirely isomeric or delayed fission. When this occurs a "shelf" should appear in the photofission cross section where the cross section changes with energy far more slowly than at higher γ-ray energy. The cross section on the shelf can therefore be orders of magnitude higher than otherwise expected. While the angular distribution near the top of the barrier is expected to have a well-defined and nonisotropic angular dependence, the angular distribution on the shelf is expected to be isotropic. The cross sections, although small, appear to be measurable with bremsstrahlung beams in the 3- to 5-MeV range using a several hundred μA electron beam. Successful experiments of this type should provide information on the shape of the inner and outer barriers at much lower excitation energies than possible by other techniques.

[NUCLEAR REACTIONS, FISSION Theory of photofission in the $E_\gamma = 2$–6 MeV region, a prominent shelf, experiments proposed.]

INTRODUCTION

Fission isomers have been discovered in a large number of heavy nuclei and enough information is now available to begin attempts to systematize and correlate[1–3] half-lives, excitation energies, etc., around the theory of shape isomerism proposed by Strutinsky.[4] Barrier shapes near the tops of the barriers are now fairly well known, but improvements in the theory will require more detailed information on the barrier shapes well below the tops. Since reactions which are highly specific in terms of parity, angular momentum, and excitation mechanism should provide the greatest insight, results from photofission below the (γ, n) threshold should be particularly valuable.

Rather detailed studies have been carried out on photofission near the top of the fission barrier.[5,6] In this paper we wish to draw attention to the usefulness of the photofission process in the region far below the (γ, n) threshold, and to the phenomena which one encounters there. We have concluded that experiments can be done with a sensitivity of at least 3×10^{-13} b, and that several interesting phenomena can be measured. From consideration of competition between fission and γ-ray decay for so-called class II states in the second minimum of the curve of potential energy vs deformation, we show that at low excitation energies the isomeric fission can become more probable than prompt fission. This occurs because as the excitation energy in the second minimum decreases, the width for decay of a class II state by fission through the outer barrier decreases much more rapidly than its width for γ-ray decay. When the γ-ray decay of the class II states becomes dominant, the photofission cross section depends almost exclusively on the penetration of the inner barrier and not on the depth of the potential minimum or the shape and penetration of the outer barrier. For isotopes of plutonium and uranium where the two barriers are thought to be of about equal height,[5] the prompt and isomeric fission will be shown to be about equal at a total photofission cross section in the range of 10^{-8} to 10^{-7} b which obtains near $E_\gamma = 4$ MeV. As one decreases the γ-ray energy still lower, the photofission cross section quickly becomes almost completely isomeric fission. Furthermore, the cross section starts to drop much more slowly as the energy is decreased and a "shelf" in the photofission cross section develops. While at higher energies, the angular distribution is usually nonisotropic with a $\sin^2\theta$ dependence, on the shelf the angular distribution becomes isotropic since a γ-ray cascade, which destroys the reference to the incoming photon beam, takes place before the isomer undergoes fission. When the γ-ray energy falls below the isomer excitation energy, the cross section begins to drop much more rapidly again. The point at which this occurs is probably at quite low γ-ray energy and the cross section at that energy often might be too small to measure.

It is worthwhile to point out that the detection of the shelf and the detection of isotropic fission is very strong evidence that an isomer exists even though one might not actually measure the half-life of the isomer. Such an experiment might, therefore, be useful for determining whether fission isomers actually exist in nuclei where the half-lives are shorter or longer than those which can be measured by conventional techniques.

DERIVATIONS OF CROSS SECTION RELATIONS

In this section we derive photofission cross sections for both prompt and isomeric fission and examine both cross sections separately and also their ratio. We will be deriving expressions mostly by simple extensions of the formalism developed by Lynn[7,8] for the examination and analysis of individual nuclear states in the context of the double-humped fission barrier.

The definition of parameters is facilitated by reference to Fig. 1 where the usual picture of potential energy vs deformation is shown. The inner well and the inner barrier are designated with the letter a; the outer well and the outer barrier are designated by the letter b. Nuclear states which are primarily associated with the ground state deformation will be referred to as class I states; their wave functions have a large amplitude in the first minimum as shown in the figure. Nuclear states associated primarily with the more deformed configuration will be referred to as class II states. A pure class I state has no amplitude in the second minimum and therefore can neither decay by fission nor emit γ rays to lower states in the second minimum. In a similar sense, a pure class II state cannot emit neutrons nor can it emit γ rays to lower class I levels. In the very weak coupling situation across barrier a, class II states couple weakly to neighboring class I states and derive a small amplitude in well a so that they can emit neutrons with small probability, $\Gamma_{n\mathrm{II}}$, and γ rays to lower class I states, $\Gamma_{\gamma\,\mathrm{a\,II}}$. Likewise, class I states can couple weakly to class II states and thereby fission promptly, $\Gamma_{f\mathrm{I}}$, or emit γ rays to a lower class II state $\Gamma_{\gamma\,\mathrm{b\,I}}$. Having defined these parameters, other width parameters which we will use will not require further explicit definition.

Lynn[7] has considered the coupling of class I and class II states and shows that the average fission width for class I states is given by

$$\overline{\Gamma}_{f\mathrm{I}}=\frac{\pi^2\overline{\mathrm{H}}'^2\overline{\Gamma}_{f\mathrm{II}}}{D_\mathrm{I}D_\mathrm{II}}, \qquad (1)$$

where $\overline{\mathrm{H}}'^2$ is the average matrix element coupling the class I and II states, $\overline{\Gamma}_{f\mathrm{II}}$ is the average fission width of the class II states and D_I and D_II are the spacing of class I and class II states, the average being taken over an energy $\Delta E\gg D_\mathrm{II}$. γ-ray widths for the class II states are also defined as

$$\overline{\Gamma}_{\gamma i\,\mathrm{aII}}=\frac{\pi^2\overline{\mathrm{H}'^2}\,\overline{\Gamma}_{\gamma\, i\,\mathrm{aI}}}{D_\mathrm{I}^2}, \qquad (2)$$

where $\overline{\Gamma}_{\gamma i\,\mathrm{aII}}$ is the average width for γ-ray decay of a class II state in the inner well to the level i. The average γ-ray decay width to the isomeric state (the ground state of well b) is defined as $\overline{\Gamma}_{\gamma\,\mathrm{0bII}}$ for a class II state.

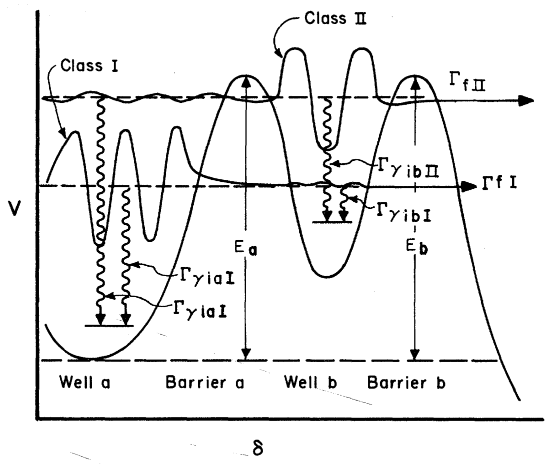

FIG. 1. The curve of potential energy vs deformation shows the inner and outer potential barriers with maximum potential energy E_a and E_b, respectively. Two wells in the potential energy exist, one associated with the ground state deformation, well a and the other associated with the more deformed unstable configuration well b. States primarily with the less deformed configuration are called class I states and those with the more deformed configuration are the class II states. The widths shown in the figure are defined in the text.

The derivation of the average prompt photofission cross section $\overline{\sigma(\gamma,pf)}$ can be conveniently broken into the average for the class I states and the average for the class II states. For the class I and class II states below the (γ,n) threshold, the average prompt fission cross section obtained from an average over Breit-Wigner resonance shapes can be written as

$$\overline{\sigma}_\mathrm{I}(\gamma,pf)=2\pi^2\lambda\!\!\!^{-}_\gamma{}^2 g_\gamma\frac{\overline{\Gamma}_{\gamma 0\mathrm{aI}}}{\overline{\Gamma}_\mathrm{I}}\frac{\overline{\Gamma}_{f\mathrm{I}}}{D_\mathrm{I}}, \qquad (3a)$$

$$\overline{\sigma}_\mathrm{II}(\gamma,pf)=2\pi^2\lambda\!\!\!^{-}_\gamma{}^2 g_\gamma\frac{\overline{\Gamma}_{\gamma 0\mathrm{aII}}}{\overline{\Gamma}_\mathrm{II}}\frac{\overline{\Gamma}_{f\mathrm{II}}}{D_\mathrm{II}}, \qquad (3b)$$

where $\lambda\!\!\!^{-}_\gamma$ is the wavelength of the γ ray divided by 2π, g_γ is the statistical factor for the γ-ray-induced reaction which is equal to $\frac{3}{2}$ for a dipole transition induced in an even-even target,[9] and Γ_II is the total width for decay of a class II state. Upon substituting Eq. (1) into Eq. (3a) and Eq. (2)

into Eq. (3b) and adding the resulting expressions together to obtain the fission from both class I and class II states we find

$$\bar{\sigma}(\gamma, pf) = 2\pi^2 \lambda\!\!\!^{-2} g\left(\frac{\bar{\Gamma}_{\gamma 0 a \mathrm{I}}}{\bar{\Gamma}_{\mathrm{I}}}\right)\left(\frac{\pi^2 \overline{H''^2}}{D_{\mathrm{I}}^2}\right)\left(\frac{\bar{\Gamma}_{f\mathrm{II}}}{D_{\mathrm{II}}}\right)\left(1+\frac{\bar{\Gamma}_{\mathrm{I}}}{\bar{\Gamma}_{\mathrm{II}}}\right), \tag{4}$$

where the subscript γ on $\lambda\!\!\!^{-}$ and g have been dropped. Blatt and Weisskopf[10] show that for a recycling system of levels with a spacing D, one can write $2\pi\bar{\Gamma}/D = P$ where P, the barrier penetration, has been written by Hill and Wheeler[11] as $P = \{1 + \exp[2\pi(V-E)/\hbar\omega]\}^{-1}$ for an inverted parabolic barrier. The parameter V is the height of the barrier, E is the excitation energy, and $\hbar\omega$ is related to the curvature of the barrier in the neighborhood of its peak. For situations well below the top of the barrier, the factor $(\pi^2 H''^2/D_{\mathrm{I}}^2)$ can be rewritten in terms of a barrier penetration factor also. In Lynn's very weak coupling approximation relating well-defined states on both sides of the inner barrier, we have from Eq. (2)

$$\frac{\Gamma_{\gamma i a \mathrm{II}}}{D_{\mathrm{II}}} = \left(\frac{\pi^2 H'^2}{D_{\mathrm{I}}^2}\right)\left(\frac{D_{\mathrm{I}}}{D_{\mathrm{II}}}\right)\left(\frac{\Gamma_{\gamma i a \mathrm{I}}}{D_{\mathrm{I}}}\right).$$

The strength function on one side is related to the strength function on the other by the penetration factor for the inner barrier so that

$$2\pi\left(\frac{\pi \overline{H''^2}}{D_{\mathrm{I}}^2}\right)\left(\frac{D_{\mathrm{I}}}{D_{\mathrm{II}}}\right) = \exp[-2\pi(E_{\mathrm{a}} - E_\gamma)/\hbar\omega_{\mathrm{a}}], \tag{5}$$

where E_{a} and $\hbar\omega_{\mathrm{a}}$ are the barrier's height and curvature, respectively, for the inner barrier. Upon substitution of Eqs. (5) in Eq. (4) we obtain a useful expression for prompt photofission

$$\overline{\sigma(\gamma, pf)} = \frac{2\pi^2 \lambda\!\!\!^{-2} g}{4\pi^2}\left(\frac{\bar{\Gamma}_{\gamma 0 a \mathrm{I}}}{\bar{\Gamma}_{\mathrm{I}}}\right)\left(1+\frac{\bar{\Gamma}_{\mathrm{I}}}{\bar{\Gamma}_{\mathrm{II}}}\right)\left(\frac{D_{\mathrm{II}}}{D_{\mathrm{I}}}\right) \times \exp[2\pi(E_\gamma - E_{\mathrm{a}})/\hbar\omega_{\mathrm{a}} + 2\pi(E_\gamma - E_{\mathrm{b}})/\hbar\omega_{\mathrm{b}}], \tag{6}$$

where E_{b} and $\hbar\omega_{\mathrm{b}}$ are the barrier height and curvature, respectively, for the inner barrier. To write an expression similar to Eq. (6) for isomeric or delayed fission, $\overline{\sigma(\gamma, dF)}$, we write as before separate expressions for the class I and II states

$$\bar{\sigma}_{\mathrm{I}}(\gamma, df) = 2\pi^2 \lambda\!\!\!^{-2} g \frac{\bar{\Gamma}_{\gamma 0 a \mathrm{I}}}{\bar{\Gamma}_{\mathrm{I}}}\frac{\bar{\Gamma}_{df\mathrm{I}}}{D_{\mathrm{I}}} \tag{7a}$$

and

$$\bar{\sigma}_{\mathrm{II}}(\gamma, df) = 2\pi^2 \lambda\!\!\!^{-2} g R \frac{\bar{\Gamma}_{\gamma 0 a \mathrm{II}}}{\bar{\Gamma}_{\mathrm{II}}}\frac{\bar{\Gamma}_{\gamma \mathrm{b} \mathrm{II}}}{D_{\mathrm{II}}}, \tag{7b}$$

where $\bar{\Gamma}d_{f\mathrm{I}}$ is the average width for delayed fission of the class I states and can be written by simple extension of Lynn's expressions as

$$\bar{\Gamma}_{df\mathrm{I}} = \frac{\pi^2 \overline{H''^2}}{D_{\mathrm{I}}}\frac{\bar{\Gamma}_{\gamma \mathrm{b} \mathrm{II}}}{D_{\mathrm{II}}} R, \tag{8}$$

where $\Gamma_{\gamma \mathrm{b} \mathrm{II}}$ is the total radiation width of a class II state in well b. The quantity R is the rate of isomer decay by delayed fission compared to the total isomer decay rate which might be considerably less than 1 if the inner barrier is more penetrable than the outer barrier. Equations (7a) and (7b) can be added together using the appropriate substitutions described earlier to give the total average delayed fission cross section as

$$\overline{\sigma(\gamma, df)} = \frac{2\pi^2 \lambda\!\!\!^{-2} g R}{2\pi}\frac{\bar{\Gamma}_{\gamma 0 a \mathrm{I}}}{\bar{\Gamma}_{\mathrm{I}}}\frac{\bar{\Gamma}_{\gamma \mathrm{b} \mathrm{II}}}{D_{\mathrm{II}}}\left(1+\frac{\bar{\Gamma}_{\mathrm{I}}}{\bar{\Gamma}_{\mathrm{II}}}\right) \times \exp[2\pi(E_\gamma - E_{\mathrm{a}})/\hbar\omega_{\mathrm{a}}]\frac{D_{\mathrm{II}}}{D_{\mathrm{I}}}. \tag{9}$$

The expectations on photofission cross sections in this paper are based on Eqs. (6) and (9).

PHOTOFISSION THRESHOLD SHAPES

As pointed out in the Introduction, prompt photofission will decrease as excitation decreases much faster than delayed (isomer) photofission. The ratio of these two types of fission can be found, of course, from the ratio of Eqs. (6) and (9). The resulting expression is

$$\frac{\sigma(\gamma, pf)}{\sigma(\gamma, df)} = \frac{1}{R}\left(\frac{D_{\mathrm{II}}}{\bar{\Gamma}_{\gamma \mathrm{b} \mathrm{II}}}\right)\frac{\exp}{2\pi}[-2\pi(E_{\mathrm{b}} - E_\gamma)/\hbar\omega_{\mathrm{b}}]. \tag{10}$$

This ratio depends only on the competition between the penetration of barrier b and γ decay in well b as pointed out in the Introduction. The quantity $\bar{\Gamma}_{\gamma \mathrm{b} \mathrm{II}}/D_{\mathrm{II}}$ is relatively constant with γ-ray energy compared to the exponential term, which is related to the outer barrier penetration. At some value for E_γ not too far below the barrier the prompt and delayed photofission became equal. From Eq. (10), this occurs when

$$E_\gamma - E_{\mathrm{b}} = \frac{\hbar\omega_{\mathrm{b}}}{2\pi}\ln\left(2\pi R\frac{\Gamma_{\gamma \mathrm{b} \mathrm{II}}}{D_{\mathrm{II}}}\right). \tag{11}$$

At this point it is instructive to calculate $(E_\gamma - E_{\mathrm{b}})$. The value for D_{II} has been measured[12] to be about 500 eV and $\Gamma_{\gamma \mathrm{b} \mathrm{II}}$ according to Lynn[8] is about 8meV. If we consider a nucleus where $R=1$ (no tunneling from the isomeric state into well a) and assume that $\Gamma_{\gamma \mathrm{b} \mathrm{II}}/D_{\mathrm{II}}$ changes slowly, we find $E_{\mathrm{b}} - E_\gamma \simeq 1.5\hbar\omega_{\mathrm{b}}$(MeV). For a typical value of $\hbar\omega_{\mathrm{b}}$ of 1 MeV the two types of fission become equal at about 1.5 MeV below the top of barrier b. Measurements in the region below 4.5 MeV will be dominated therefore by isomeric fission.

Over most of the energy region either Eqs. (6) or (9) will dominate. Furthermore, in a particular region where only one equation dominates, certain simplifications can be made which facilitate the evaluation and interpretation of the cross section.

Consider first the factor $1+(\overline{\Gamma}_{\mathrm{I}}/\overline{\Gamma}_{\mathrm{II}})$ which is common to both Eqs. (6) and (9). The parameter $\overline{\Gamma}_{\mathrm{II}}=\overline{\Gamma}_{\gamma\,\mathrm{b\,II}}+\overline{\Gamma}_{f\mathrm{II}}+\overline{\Gamma}_{\gamma\,\mathrm{a\,II}}$. At higher energies ($E_\gamma > 4.5$ MeV) $\overline{\Gamma}_{f\mathrm{II}}\gg\overline{\Gamma}_{\gamma\mathrm{bII}}$, $\overline{\Gamma}_{\gamma\mathrm{a\,II}}$, $\overline{\Gamma}_{\mathrm{I}}$ so that $\overline{\Gamma}_{\mathrm{I}}/\overline{\Gamma}_{\mathrm{II}} \ll 1$. Therefore, at higher energies the same factor can be adequately approximated by unity. However, at low energies (below about 4.5 MeV) $\overline{\Gamma}_{f\mathrm{II}}<\overline{\Gamma}_{\gamma\,\mathrm{bII}}$ as pointed out earlier. In addition, owing to the greater density of class I states compared with class II states and the greater energy associated with class I transitions, the statistical model[10] tells us that $\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}\gg\overline{\Gamma}_{\gamma\,\mathrm{a\,II}}$, $\overline{\Gamma}_{\gamma\,\mathrm{b\,II}}$. Therefore, the factor becomes at lower energies $\overline{\Gamma}_{\mathrm{I}}/\overline{\Gamma}_{\mathrm{II}}$.

Since prompt fission dominates at higher energy where the factor is unity and delayed fission dominates at lower energy where the factor becomes $\overline{\Gamma}_{\mathrm{I}}/\overline{\Gamma}_{\mathrm{II}}$, we can rewrite Eq. (6) and Eq. (9) with the respective approximations. The accuracy is adequate since the major energy dependence is in the exponential factors which cause variations of 10^3 to 10^6 per MeV. Equations (6) and (9) become

$$\overline{\sigma}(\gamma, pf)=\frac{2\pi^2\lambda\!\!\!^{-2}g}{4\pi^2}\,\frac{\overline{\Gamma}_{\gamma 0\mathrm{a\,I}}}{\overline{\Gamma}_{\mathrm{I}}}\left(\frac{D_{\mathrm{II}}}{D_{\mathrm{I}}}\right)\times\exp[2\pi(E_\gamma-E_{\mathrm{a}})/\hbar\omega_{\mathrm{a}}+2\pi(E_\lambda-E_{\mathrm{b}})/\hbar\omega_{\mathrm{b}}] \tag{12a}$$

$$\overline{\sigma}(\gamma, df)=\frac{2\pi^2\lambda\!\!\!^{-2}gR}{2\pi}\,\frac{\overline{\Gamma}_{\gamma 0\mathrm{a\,I}}}{D_{\mathrm{I}}}\exp[2\pi(E_\gamma-E_{\mathrm{a}})/\hbar\omega_{\mathrm{a}}]. \tag{12b}$$

The parameters of these equations must now be evaluated in order to estimate cross sections. We consider the case where $R=1$ and assume a dipole transition for the incoming γ ray so that the statistical factor[9] $g=\frac{3}{2}$. The parameter $D_{\mathrm{II}}/D_{\mathrm{I}}$ can be obtained from the application of statistical theory, we assume a form for the level density

$$\rho_{\mathrm{I}}(E)=\frac{C}{(E-\delta_{\mathrm{I}})}\exp[a_{\mathrm{I}}(E-\delta_{\mathrm{I}})]^{1/2}$$

and

$$\rho_{\mathrm{II}}(E)=\frac{C}{(E-E^*-\delta_{\mathrm{II}})}\exp[a_{\mathrm{I}}(E-E^*-\delta_{\mathrm{II}}]^{1/2} \tag{13}$$

which is somewhat simplified from that of Lang and LeCouteur,[13] where δ refers to the energy gap and E^* is the excitation of the isomer. Strutinsky[14] has shown that the parameter $a^{1/2}$ related to the single particle level density is about the same for the nucleus at its ground state and isomer deformation. This value is, therefore, taken to be $a=28$ MeV^{-1}. The value δ_{I} and δ_{II} were assumed to be equal for lack of hard information otherwise, and the value was taken to be 1.1 MeV in accordance with Newton's[15] analysis.

With the above-mentioned parameters in Eqs. (13) and assuming $E^*=2.5$ MeV, we find that the ratio of level densities changes by only a factor of 2 as E changes from 5.5 to 4 MeV. The ratio of level spacings near 5.5 MeV, where very weak coupling holds, has been measured for several nuclei and the typical value[12] of 30 was used tor $\rho_{\mathrm{I}}/\rho_{\mathrm{II}}$ independent of energy.

It is convenient next to evaluate the parameter $\overline{\Gamma}_{\gamma 0\mathrm{a\,I}}/D_{\mathrm{I}}$. Price *et al.*[16] have measured the average partial radiation width normalized to an energy of 4 MeV for neutron capture in ^{238}U. They find $\langle\Gamma_{ij}\rangle=0.31$ meV. Using a value[17] of 18 eV for the resonance spacing at 5 MeV and the mass and energy dependence of the single particle model,[10] we can solve for the constant C in the relationship $\langle\Gamma_{ij}\rangle$ eV $=CE_\gamma^3$ (MeV)$A^{2/3}D$ (eV). The result is $\Gamma_{\gamma 0\mathrm{a\,I}}/D_{\mathrm{I}}=7.0\times10^{-9}E_\gamma^3$ (MeV)$A^{2/3}=2.7\times10^{-7}E_\gamma^3$. The final parameter required for evaluating the cross section is $\overline{\Gamma}_{\mathrm{I}}/D_{\mathrm{I}}$. Below the neutron threshold $\overline{\Gamma}_{\mathrm{I}}\simeq\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}$ which can be written according to Blatt and Weisskopf[10] as

$$\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}/D_{\mathrm{I}}=K\int_0^{E_\lambda}dE_\gamma\frac{E_\gamma{}^3\exp[7.8(E_\gamma-E_\gamma-1.1)^{1/2}]}{(E_\lambda-E_\gamma-1.1)} \tag{14}$$

making use of the level density derived earlier in this paper. Evaluation of Eq. (14) for ^{238}U shows that $\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}/D_{\mathrm{I}}$ changes by very nearly a factor of 5 per MeV. The constant K can be determined by normalizing this dependence to its value at some particular energy where $\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}/D_{\mathrm{I}}$ is known. We take the measured parameters[18,17] determined from neutron capture measurements on ^{238}U near 5 MeV of $\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}=0.023$ eV and $D=18$ eV. A useful empirical expression can be written, therefore, of the form $\overline{\Gamma}_{\gamma\,\mathrm{a\,I}}/D_{\mathrm{I}}=4.1\times10^{-7}\exp(1.6E_\gamma)$. With the parameters now specified, we can evaluate Eqs. (12a) and (12b). The approximations set forth above are adequate since the dominant energy dependence is in the exponential barrier penetration factors. The expressions for prompt and delayed fission become

$$\overline{\sigma}_{(\gamma,pf)}=5.92\times10^3E_\gamma\exp[-1.6E_\gamma]\times\exp[2\pi(E_\gamma-E_{\mathrm{a}})/\hbar\omega_{\mathrm{a}}+2\pi(E_\gamma-E_{\mathrm{b}})/\hbar\omega_{\mathrm{b}}]. \tag{15a}$$

$$\bar{\sigma}_{(\gamma, df)} = 5.04\times10^{-4}E_\gamma R\exp[2\pi(E_\gamma - E_a)/\hbar\omega_a]. \tag{15b}$$

Results on the total cross section obtained by adding Eq. (15a) and (15b) are given in Fig. 2. In these calculations $\hbar\omega_b$ is held fixed at 1.0 MeV and $\hbar\omega_a$ is varied from 0.75 to 2.0 MeV to give the family of curves in the figure. One can see clearly the shelf and the bend where the prompt and delayed fission become equal. The sharp drop at the isomer excitation energy of 2.5 MeV represents the transition into the energy region where both inner and outer barriers are again effective in inhibiting fission. The various curves are displaced slightly from 2.5 for purposes of illustration.

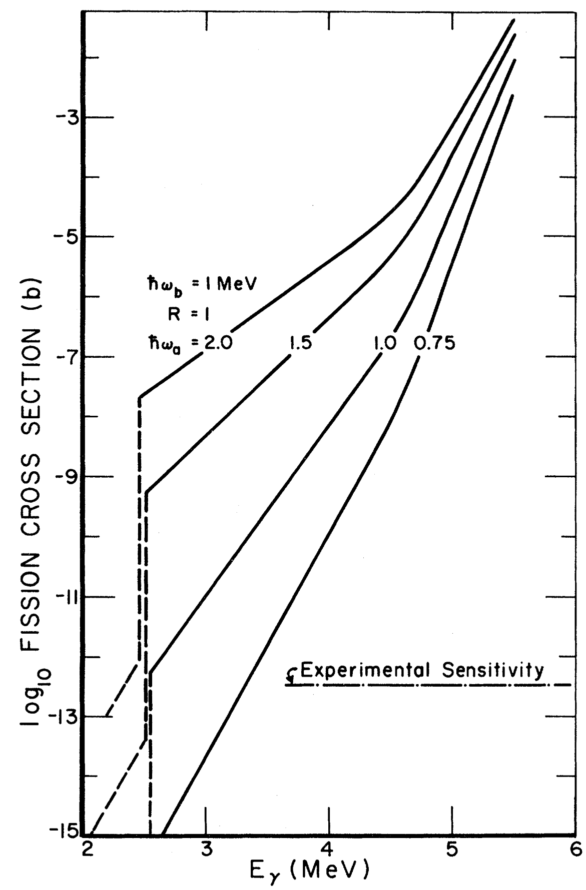

FIG. 2. The total photofission cross section calculated from Eqs. (15) is shown. The calculations are carried out for a value of $\hbar\omega_b = 1$ MeV and four different values of $\hbar\omega_a$ as indicated. In the calculations it is assumed that the isomer decays exclusively by fission ($R = 1$). At higher energies prompt fission dominates, while on the shelf the delayed fission dominates. The equation for prompt or delayed fission is used wherever one is dominant and the two curves are joined smoothly in between. The dashed line represents the rather abrupt fall of the cross section back to the smaller value characterized again by penetration of both barriers. A sensitivity attainable with existing techniques is shown by the horizontal dot-dashed line.

Of course only *average* cross sections have been calculated. Care must be taken in measurements to allow a resolution width sufficiently wide ($\geq$200 keV) to average over at least several resonances. Otherwise, at lower energies wide fluctuations in measured cross sections can be expected. The effects of such fluctuations are of course mitigated to some extent by the rapid change in fission cross section with energy which makes moderate accuracy acceptable for the purpose of extraction of useful nuclear parameters. Of course, photofission measurements in this energy range using neutron capture γ-ray line spectra are unlikely to be productive since the probability of the γ-ray energy matching a resonance in the photofission cross section is very low and erroneous photofission cross sections would almost certainly result.

EXPERIMENTS

Most of the cross section of interest in Fig. 2 is considerably smaller than has been measured before. The author is aware of no experiment more sensitive than that of Rabotnov *et al.*[5] on ^{238}U for which a cross section of 3×10^{-6} b was measured at 4.8 MeV. A sensitivity in the 10^{-11}–10^{-7} range is necessary to explore the phenomena discussed here. Using the Dickinson-Lent[19] relation for converting electron energy to bremsstrahlung, we conclude that a sensitivity for fission of 3×10^{-13}b might be obtained with a 1-mA beam, and a sandwich of mica track detectors and fission events in 100-mg/cm^2 thickness of a heavy element. While such a sensitivity might be difficult to achieve, useful experiments on cross section and even angular distributions appear possible when the cross sections are higher, i.e., in the 10^{-11}–10^{-10} range.

The considerations of this paper point to a number of low energy photofission phenomena that might be worthy of investigation. The angular distribution from prompt dipole-excited photofission is a $\sin^2\theta$ function[20] around the direction of the incident γ-ray beam. However, in isomeric fission induced by γ rays at least one and usually more γ rays should be emitted in the radiative decay to the shape isomer. The orientation to the incident γ-ray beam is thereby lost and the fission is isotropic. Of course, if the fission is primarily isomeric and the isomer is long-lived, atomic effects might also disorient the nucleus causing isotropy of the fragments. In either case, isotropy will be a signature of isomeric fission.

If fission is detected at quite a low energy such as 3 to 4 MeV, the magnitude of the cross section could provide strong evidence, that the fission is isomeric even though no timing experiment (delayed coincidence) is carried out. If, in addition, the angular distribution is found to be isotropic, one probably has much stronger evidence that the photofission is primarily isomeric. Finally, a measurement of the energy dependence of the cross section in the 3- to 5-MeV range which showed the predicted shelf should provide hard evidence indeed that isomeric fission had been observed. Of course, the measurement of these phenomena in isotopes where no fission isomers have been found would demonstrate the existence of a fission isomer even though its half-life might be too short or too long to be seen by the conventional techniques.

The proposed experiments could also provide a more detailed picture of the shape of barriers a and b well below the top of the barrier. The slope of the curve in the regions of prompt or isomeric fission will measure $\hbar\omega$ for the two barriers. The absolute cross section will also provide information on the barrier height. The energy at which prompt and isomeric fission both contribute provides useful information on competition between γ-ray decay and fission for the class II states.

From a practical viewpoint, it might also be worthwhile to reconsider the role of photofission in nuclear reactors and in stellar processes, since the cross section is probably significantly higher than previously thought. For example, at 3 MeV, the cross section predicted here is 10^4 times higher than what one might have expected before isomeric fission was discovered.

CONCLUSIONS

It has been shown that in the competition between fission and γ decay in the second well, the γ decay is dominant well above the energy of the fission isomer. This dominance results in a shelf in the photofission cross section at low energy where the cross section can be parametrized independent of the shape and penetration of the outer barrier. One may expect the following phenomena to characterize very low energy photofission:
(1) The fission above the shelf should be primarily prompt fission.
(2) Above the shelf the photofission cross section angular distribution should be characteristic of the electromagnetic excitation process.
(3) The fission on the shelf should be primarily isomeric or delayed.
(4) The angular distribution of fission fragments on the shelf should be isotropic.
(5) Owing to the shelf the photofission cross section in the region of the shelf may be several orders of magnitude higher than one would expect for the single humped fission barrier.
(6) At the lower "edge" of the shelf the photofission cross section falls back to a very low value determined by penetration of the full barrier.

Measurements appear to be possible into the 10^{-13}-b range which appears to be sufficient sensitivity to measure the cross section even somewhat below the lower edge of the shelf. Such measurements might provide the following information.
(1) The slope of the shelf will be determined primarily by the value of the curvature parameter $\hbar\omega_a$ near the bottom of the second minimum.
(2) The magnitude of the cross section on the shelf might lead to a value for either the branching ratio for isomer decay or to a value for the photon strength function $\Gamma_{\gamma 0 a I}/D$.
(3) The slope of the cross section at an energy above the shelf might lead to a determination of $\hbar\omega_b$ well below the top of the outer barrier once $\hbar\omega_a$ is determined.
(4) The energy can be determined where the width in the second well for γ-decay and fission decay are equal.
(5) The magnitude of the photofission cross section measured with the electron beam just above the isomer excitation energy might permit a rough measure of the half-life of isomers which are too short or too long-lived to be detected by existing techniques.

Very low energy photofission experiments appear to be a fruitful source of information about the fission barrier in regions where other techniques are not so effective. By combining this information with that from other experiments, the details of the fission barrier shape might be reconstructed with an accuracy considerably exceeding that possible with existing techniques.

It might even be expected that the parametrization of a fission barrier with a parabolic shape as has been done in this paper is entirely inadequate. In this case the equations in this paper, through straightforward changes, would be converted to penetrabilities P_a and P_b of the inner and outer barriers and the derived barrier shapes expressed in terms of these quantities. Actual measurements will show whether this is necessary.

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