Level structure of ¹⁰⁶Pd from the decay of ¹⁰⁶Rh^g \dagger

S. T. Hsue and H. H. Hsu

Department of Physics, University of Northern Iowa, Cedar Falls, Iowa 50613

F. K. Wohn, W. R. Western, and S. A. Williams

Ames Laboratory-USAEC and Department of Physics, Iowa State University, Ames, Iowa 50010 (Received 2 December 1974)

The low-spin level structure of $106Pd$ was studied using high-resolution large volume Ge(Li) detectors for γ -ray singles, coincidence, and angular correlation measurements. The measurements allowed the resolution of most of the discrepancies found among previous studies. In particular, the use of two $Ge(Li)$ detectors for coincidences and angular correlations allowed unambiguous placement of previously multiply placed γ rays and resolved discrepancies in energy level or J^{π} assignments. The experimental level structure and $B(E2)$ ratios are compared with predictions of an anharmonic spherical vibrator model and a rotor-vibrator model; we find that the evidence strongly supports the latter of the two models.

RADIOACTIVITY ¹⁰⁶Pd [from ¹⁰⁶Rh^{ϵ} decay]; measured E_{γ} , I_{γ} , γ - γ coin angular correlations; Ge(Li) detectors; deduced I_B , logft, J^{π} for ¹⁰⁶Pd levels theory: energy levels and $B(E2)$ ratios, anharmonic spherical vibrator model and rotor-vibrator model.

I. INTRODUCTION

The level structure of ^{106}Pd is of considerable interest because this nucleus has been cited as one of the better examples of quadrupole vibrations about a spherical equilibrium shape. The low-spin excited states have been studied through
the decays of both $^{106}Ag^{f1-3}$ and $^{106}Rh^{f}$, $^{3-10}$ since the decays of both $^{106}Ag^{g_1-3}$ and $^{106}Rh^{g_1-10}$ since each has a ground-state spin and parity of 1^{\dagger} .

In spite of the number of studies, several important discrepancies remain in the properties of the low-spin level structure of ^{106}Pd . For example, there is disagreement on the existence of a level at 2i89 keV and on the spin and parity of the 2306-keV level. Also, in the three most recent high-resolution Ge(Li) studies of the decay of 106 Rh^{$s-10$} there are many γ -ray transitions placed more than once in the proposed decay schemes. There is also a large variance among the γ -ray transition intensities reported in the different studies. One possible reason for these discrepancies is that the previous coincidence measurements were done with Ge(Li)-Nal(T1) detector systems. Consequently, there were coincidence gate windows containing more than one γ ray in the relatively complex decay of $^{106}Rh^g$, resulting in ambiguous and possibly misleading coincidence information.

The purpose of the present work was to clarify many of these discrepancies by using high-resolution large-volume Ge(Li) detectors for coincidence and singles measurements. Coincidence measurements made with the large-volume detectors in a two-parameter manner were found to provide unambiguous cascade information for many of the previously multiply placed transitions. Angular correlation measurements made with large-volume Ge(Li) detectors resulted in the determination of the spins of some of the excited states. The measurements reported here resolve many of the discrepancies in the previously reported level schemes of ¹⁰⁶Pd.

We have initiated a study of the level structure of 106 Pd via two models. Our first thought was to try to understand this structure through the anharmonic vibration model of Brink, de Toledo Piza, and Kerman¹¹ which seemed more suited than the previously used model of Ferreira, Castilho, and Navarro¹² since we have both energy levels and reduced quadrupole transition ratios. This model is in the spirit of regarding 106 Pd as a spherical vibrator but agreement between theory and experiment is poor. Consequently we have also studied this nucleus via the breathing mode vibrator-asymmetric rotor model (with some vibrator-asymmetric rotor model (with some
extensions) of Davydov and Chaban.¹³⁻¹⁵ The agreement between theory and experiment is very much improved in this model. The results of these theoretical efforts are reported in the latter part of this paper.

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II. EXPERIMENTAL PROCEDURE AND RESULTS

The 30-sec activity of $^{106}Rh^g$ was obtained by the decay of ¹⁰⁶Ru ($T_{1/2}$ =1 yr). The radioactive source, carrier-free 108 Ru in dilute HCl solution with better than 99% purity, was obtained from International Chemical and Nuclear Corporation. Approximately 100 μ C of the activity were used for the singles and coincidence experiments, and approximately 50 μ C of ¹⁰⁶Ru in liquid form were placed in a cylindrical Lucite capsule for angular correlation studies.

A. γ -ray singles measurements

A 60-cm' Ge(Li) detector with 2.5-keV resolution at 1.33 MeV and peak-height to Comptonplateau ratio of 34 was used for the singles measurements with an absorber of 6.4 mm of Al. For the singles data, γ -ray peak centroids and areas were determined by fitting with a skewed-Gaussian fit function. The nonlinearity of the electronics system was determined from a calibration run using standard γ -ray calibration sources.

To determine the extent of possible contamina-

tion, two singles measurements were performed. The first measurement was made 6 months after receiving the radioactive source, and the second measurement was made 24 months later. From the relative γ -ray intensities, contributions were determined from activities with half-lives different from the 1-yr half-life of ¹⁰⁶Ru. Traces of ¹³⁴Cs, ¹⁴⁴Ce, and ¹⁵⁴Eu were observed. Several other sources of spurious peaks were also eliminated. The single- and double-escape peaks of high-energy γ rays were identified by energy difference and by intensity ratio, using single- and double-escape to photopeak intensity ratios for the detector, well established by previous investigations in the Ames Laboratory. Peaks at 1134, 1706, and 2001 keV were identified as pure sum peaks. Summing contributions to all crossover transitions were then calculated and eliminated.

The energies and intensities of the γ rays observed in the singles run are listed in Table I. A total of 78 γ rays were observed. The γ -ray transitions listed with an upper limit to the intensity are transitions observed in the coincidence measurements described in the following section.

Some comments are necessary concerning the relative intensities in Table I. There are considerable variances in the relative intensities among the different studies, $3,7-9$ even for some of the more intense transitions. For example, the reported intensity of the 873-keV transition ranges from 19.0 ± 1.2^7 to $25.0 \pm 1.3.^9$ The probable reason for this variation is that the most intense transition used for intensity normalization has an energy of 511.8 keV. This photopeak usually contains some annihilation contribution; the amount of annihilation contribution depends on the detector used and its environment. In the previous studies, no mention was made of making corrections for the annihilation contribution. This could cause an overestimation of the β branching to the 512-keV level. In this work special effort has been made to determine this annihilation contribution by fitting the 512-keV photopeak as a doublet. The annihilation contribution was determined to be $(5.1 \pm 0.7)\%$ of the doublet intensity. The 0.7% uncertainty is the root-mean-square deviation from the 5.1% mean value of several fits to the doublet. Since the uncertainty in the annihilation contribution depends most strongly on the width of the annihilation peak relative to the 511.8-keV photopeak and the difference in the centroids of the two peaks, the several fits were done with these quantities changed by ± 1 standard deviation in both of these two quantities $(±0.25 \text{ keV}$ in the relative width and ± 0.10 keV in the centroid difference). Figure 1 shows the unresolved 511.0- and 511.8-keV peaks and the fit obtained with the expected centroid difference and peak widths. The annihilation correction reduced the β branching to the 512-keV

FIG. 1. γ -ray doublet composed of 511.0-keV annihilation and 511.8-keV photons. The fit shown indicates an annihilation contribution of 5.1% of the total intensity.

FIG. 2. γ -ray doublet composed of 1044.8- and 1050.5keV photons. The fit shown indicates a 1044.8-keV intensity of 0.7% of the total intensity.

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level by approximately 10% . The intensities of the other γ -ray transitions in the present work are in general higher than in the previous studies where the annihilation correction was apparently not made.

The weak γ ray at 1044.8 keV has an especially significant role in the decay of ^{106}Rh , as will be revealed in the discussion of the level at 1557 keV presented in Sec. III. Figure 2 shows the 1044.8- 1050.5-keV γ -ray doublet. The fit to the data of Fig. 2 corresponds to an intensity of 0.007 for the 1044.8-keV γ ray relative to the 1050.5-keV γ ray. The minimum and maximum values of this relative intensity were 0.005 and 0.009; these values were the extremes obtained from all variations of the photopeak shape, background shape, and centroid difference. Thus an intensity of 0.007 \pm 0.002 (relative to the 1050.5-keV γ ray) was used for the 1044.8-keV γ ray.

The 1600.7-keV transition observed by Strutz, Strutz, and Flammersfeld⁸ is identified in this work as the single-escape peak of the 2112-keV γ ray. Of the doublets observed by Strutz, Strutz, and Flammersfeld at 717, 1180, 1789, 1927, 2193, and 2308 keV, we see evidence that only the 717-keV peak should be a doublet. In a singles run using a

TABLE II. γ -ray coincidences in the decay of $^{106}Rh^s$.

Gating transition (keV)	Coincident transition ^a (keV)		
428	512, 533, 622, (716), 1062		
512	428, 434, (524), 533, 616, 622, 648,		
	(717), 874, 909, 1050, 1062, 1090,		
	(DE2112), 1109, 1115, 1181, 1195,		
	1315, 1487, 1601, (SE2112), (1730),		
	1766, 1797, 1927, 1988, 2112, 2193,		
	2316, 2366, 2390, 2406, 2542, 2571,		
	(2651), 2709		
616	434, 512, 533, 874, 909, 1115, 1150		
	1181, 1371, 1496, (1577), (1909)		
622	428, 476, 512, 533, 1109, 1305		
874	512, (525), 533, 616, (648), 1128		
1050	512, 533, 681, 716, (909), 1062		
1062	(428), 512, 622, 1051, 1562		
1128	(578), 874, 909, 1181, 1496		
1195	512, 533		
1496	512, 616, 1128		
1562	(681), (716), 1062		
1766	512		
1797	512		
1988	512		
2112	512		
2366	512		
2406	512		

^a Weak or possible coincidences are enclosed by parentheses.

1-cm' Ge(Li) detector with a resolution of 1.4 keV at 622 keV, the 717-keV peak had a width which was nearly twice the width expected for a photopeak of this energy. Since the 717-keV intensity is relatively weak and the photopeak efficiency of the $1-cm^3$ Ge(Li) detector is quite low at this energy, the statistics were poor and this data did not provide convincing evidence that the 717-keV peak is a doublet. However, our coincidence data, which are presented in the next section, definitely indicate that the '717-keV peak is a doublet. From our singles and coincidence data there was no evidence that any of the other peaks should be a doublet. The 1476-, 1522-, 1601-, 1893-, and 2031-keV transitions reported by Marsol, Rahmouni, and Adrisson' as photopeaks are identified in this work as either single- or double-escape peaks of high-energy γ rays.

B. γ -ray coincidence measurements

For the coincidence measurements, a second $60-cm³$ Ge(Li) detector of similar quality was also used. The two detectors were placed in a 180' geometry. Ortec constant-fraction timing

FIG. 3. γ -ray spectra gated by (a) the 1050-keV photopeak, (b) the 1062-keV photopeak, and (c) the Compton plateau above the 1062-keV photopeak.

was used with a full-width time gate of 60 nsec. Concidence events were recorded in a bufferedmemory tape system in a 4096×4096 -word format. Coincidence and coincidence -background gates were digitally selected afterwards from the coincidence profile (i.e., the spectrum of all events in the gating detector that are coincident with any event in the other detector). The recorded events were searched to obtain coincidence and coincidence-background gated spectra. An advantage of such a system is that even part of a doublet could be selected as a gate. Coincidence spectra were obtained using as gating transitions all of the

transitions evident in the coincidence profile.

The results of the Ge(Li)-Ge(Li) coincidence measurements are presented in Table II. An example of the spectra yielding these results is shown in Fig. 3. The 2242-, 2308-, 2439-, 2525-, and 2820-keV transitions were not observed in the coincidence profile, indicating that they are ground-state transitions depopulating levels that are populated mainly by direct β branching.

As indicated by Fig. 3 and in Table II, the γ ray at '717 keV is a doublet. The decay scheme of Fig. 4 gives the placement of the two components: a 717.3-keV transition from the 1229-keV level

FIG. 4. Decay scheme of $^{106}Rh^g$, showing transitions among (a) lower energy levels and (b) higher energy levels. Intensities are normalized to 10² and 10⁴, respectively, for β and γ transitions.

Cascading γ rays ^a (kev)	A ₂	A_4	Spin sequence
$874 - (616) - 512$	-0.17 ± 0.11	0.46 ± 0.14	$0(2)2(1, 2)2(2)0^{b}$
1050-512	0.02 ± 0.05	0.05 ± 0.07	$2(1,2)2(2)0^{\circ}$
1195-512	0.28 ± 0.20	0.82 ± 0.21	0(2)2(2)0
2112-512	0.44 ± 0.40	0.73 ± 0.51	0(2)2(2)0
$874 - (616) - 512$	-0.14 ± 0.09	0.44 ± 0.11	$0(2)2(1, 2)2(2)0^{b}$
874-616	-0.07 ± 0.07	0.22 ± 0.09	$0(2)2(1, 2)2^{b}$
874-1128	0.41 ± 0.12	1.07 ± 0.15	0(2)2(2)0

TABLE III. Angular correlation results.

^a Transitions underlined indicate the gated transition. Transitions in parentheses are unobserved.

^b The result on the 874-616 cascade gives a mixing ratio of $\delta \leq 8$. The two results on the 874-(616)-512 cascade are consistent with $Q \sim 1$.

^c The mixing ratio determined from the 1050-512 cascade is $\delta = 0.21 \pm 0.08$.

to the 512 -keV level and a 715.7 -keV transition from the 2278-keV level to the 1562-keV level. The 715.7-keV component was found to be in coincidence with the 1050-keV γ ray and tentatively found to be in coincidence with the 428-keV and 1562-keV γ rays. Analysis of the spectrum of Fig. 3(a) indicates that about half the doublet intensity is due to the 715.7-keV transition. As will be discussed in the decay scheme section, equal division of the total doublet intensity is consistent with the expected zero β branching to the 1229-keV level.

The coincidence results reported in earlier studies^{7,8,10} were limited, since these studies were confined to the more intense transitions and used Nal(T1) as one of the coincidence detectors. Our capability to obtain coincidence spectra with the separate components of the 616-622-keV doublet and 1050-1062-keV doublet is particularly helpful in allowing unique placement of the γ -ray transitions in the decay scheme. In the previous studies by Strutz, Strutz, and Flammersfeld' and Ardisson and Marsol¹⁰ there were many transitions multiply placed in the proposed decay scheme. Of these multiply placed transitions, the 578- and 1181-keV transitions were observed by us to be in coincidence with the 1128-keV gate and therefore should be placed above the 1128-keV level. The 2308 and 2525-keV transitions were not observed in the coincidence profile, indicating that they are transitions to the ground state. The 1927- and 2193-keV peaks, each of which was observed by Strutz, Strutz, and Flammersfeld' as a doublet, were observed by us to be in coincidence with the 512-keV gate and therefore should be placed above the 512 keV level.

C. Angular correlation measurements

Two true-coaxial Ge(Li) detectors (20 cm' with 2.5-keV resolution at 1.33 MeV and 60 cm' with 3.5-keV resolution} were used in the angular correlation experiment. Each detector had an absorber of 6.⁴ mm of Lucite. Ortec constant fraction timing was used, with a time gate of 100 nsec. Photopeak and Compton background coincidence spectra were collected simultaneously and were routed into two different portions of a Nuclear Data 2048-channel analyzer. Coincidence spectra were collected at seven angles ranging from 90 to 270° in intervals of 30° . The Compton background coincidence spectrum was subtracted from the photopeak coincidence spectrum in such a way that the photopeak used as gate would disappear. Solid-angle correlations for the Ge(Li} detectors were obtained from the calculated values of Cam_l
and Van Lehn.¹⁶ and Van Lehn.

The results of these angular correlation measurements are summarized in Table III. These surements are summarized in Table III. The
results have been reported previously.¹⁷ The 1194-512-keV cascade indicates that the spin of the 1706-keV level is 0 rather than 1 as assigned by Rao and Fink.³ The 2114-512-keV cascade indicates the spin of the 2624-keV level to be 0 rather than $(2, 1)$ as assigned by Ardisson and Marsol.¹⁰ Zero spin assignments for levels at 1706 and 2624 keV are in agreement with the recent angular cor-
relation study by Avignone and Pinkerton.¹⁸ relation study by Avignone and Pinkerton.¹⁸

HI. DECAY SCHEME

The decay scheme for $^{106}Rh^s$ based on the results of this work is shown in Fig. 4. Of the 78γ rays

listed in Table I, all but 5 were placed in the scheme; these 5 are the 532.8-, 647.7-, 908.7-, 977.4-, and 2484.0-keV transitions. Coincidence relationships are illustrated by the use of closed circles placed at the heads or tails of the transitions. The absolute γ -ray transition intensities per 10^4 β decays are indicated in parentheses after the energies. Also shown in Fig. 4 are $\log ft$ values and β -branching values per 100 β decays.

Level energies determined from a weighted average of all transition energies into or out of the respective levels and the statistical uncertainties in the level energies are presented in Table IV, along with $\log ft$ values and β -branching information deduced from transition intensity imbalances. For the β branching and log ft cal-

TABLE IV. Level energies for excited states in ¹⁰⁶Pd and the corresponding β branching and $\log ft$ values for $^{106}{\rm Rh}^{\it g}$ decay.

Level energy	β branching	
(keV)	$\binom{0}{0}$	$\log ft$
0.0	\pm 1.4 ^a 80.5	5.17
511.81 ± 0.08	6.5 ± 1,1	5.97
1128.11 ± 0.17	± 0.13 0.69	6.54
1133.7 ± 0.5	9.6 ±1.0	5.39
1228.9 ± 0.4	< 0.001	>9.2
1556.6 ± 0.5	< 0.005	>8.3
1562.20 ± 0.12	1.77 ± 0.17	5.78
1706.49 ± 0.10	0.066 ± 0.008	7.07
2001.6 ± 0.3	0.45 ± 0.04	5.94
2242.4 ± 0.4	< 0.014	>7.2
2278.08 ± 0.09	0.040 ± 0.004	6.66
2308.59 ± 0.16	0.049 ± 0.005	6.53
2438.68 ± 0.12	0.0211 ± 0.0020	6.71
2499.89 ± 0.10	0.029 ± 0.003	6.48
2525.7 ± 0.3	0.008 ± 0.004	7.01
2624.17 ± 0.23	0.090 ± 0.008	5.79
2704.8 ± 0.5	0.0120 ± 0.0014	6.52
2783.3 ± 0.4	0.0010 ± 0.0002	7.47
2820.3 ± 0.3	0.0014 ± 0.0002	7.23
2827.69 ± 0.18	0.0076 ± 0.0009	6.47
2877.40 ± 0.11	0.027 ± 0.003	5.82
2901.91 ± 0.25	0.031 ± 0.007	5.69
2917.26 ± 0.12	0.0179 ± 0.0017	5.90
3027.4 ± 1.2	0.0002 ± 0.0001	7.5
3036.6 ± 0.4	0.0031 ± 0.0006	6.34
3054.7 ± 0.3	0.0103 ± 0.0012	5.77
3082.6 ± 0.3	0.0021 ± 0.0002	6.38
3162.8 ± 0.4	0.0006 ± 0.0001	6.68
3220.53 ± 0.22	0.008 ± 0.004	5.27
3320.2 ± 0.5	0.0006 ± 0.0002	5.90

^a The ground-state β branching and its uncertainty are deduced from the γ -ray intensity per 100 decays of 10.5 \pm 0.3 for the 616.3- and 622.2-keV transitions measured by Odru (Ref. 20).

culations, a Q_β of 3.55 MeV¹⁹ and a ground-state β branching of 80.5% were used. In the recent $A = 106$ compilation, ¹⁹ $\beta_{\epsilon s}$ (the ground-state β branching) was assigned an adopted value of 79.3% which was deduced from a γ -ray intensity per 100 decays of ¹⁰⁶Rh^{ℓ} of 20.6 ± 0.6 for the 511.8-keV γ ray reported by Odru.²⁰ Odru also reported an absolute intensity of 10.5 ± 0.3 per 100 decays for the 616.3- and 622.2-keV γ rays. Our value of β_{rs} of 80.5 ± 1.4 is deduced from the latter result of Odru and our decay scheme, including internal conversion contributions; the uncertainty of 1.4 includes our uncertainty in the total γ ray plus internal conversion branching to the ground state. Our choice of the 616.3- plus 622.2-keV γ -ray absolute intensity rather than the 511.8-keV value is based upon a probable annihilation contribution to the 511.8-keV value of Odru. The photon intensity ratio of 20.6/10.5 from Odru is about 9% greater than our intensity ratio for the corresponding γ rays. Since an annihilation contribution of about 9% is quite feasible, we feel our choice of the 616.3- plus 622.2-keV value is more appropriate for the determination of β_{gs} .

Spin-parity assignments shown in Fig. 4 for the levels of ¹⁰⁶Pd have been made utilizing results from reaction studies,²¹ neutron-capture γ -ray studies,²² the log ft values calculated from the β branching according to the rules proposed by Raman and Gove,²³ and the assumption that the observed γ rays have multipolarities of E1, E2, $M1$, or $E2/M1$. In addition, we have incorporated the spin assignments of Okano and Kawase²⁴ for the levels at 2828, 2877, 3055, 3083, 3163, 3221, and 3320 keV; their assignments were made from angular correlation measurements of transitions from the higher-excited states of ¹⁰⁶Pd.

There are several features of the proposed decay scheme which should be emphasized. No γ -ray transition was placed more than once in the decay scheme, in contrast with the schemes proposed by Strutz, Strutz, and Flammersfeld⁸ and Ardisson and Marsol.¹⁰ Specific conclusions drawn from the present work, which primarily concern discrepancies among the previous studies, are discussed below.

1229 -keV level

The level at $1229 \,\mathrm{keV}$ with a spin and parity of 4^+ is expected to have negligible β branching from the 1^+ ground state of 106 Rh. In our level scheme, the intensity imbalance of γ rays into or out of this level was less than the uncertainty in the intensities of the γ rays involved. Consequently, we assigned an upper limit of 0.001 per 100 β decays for the β branching to this level. This result,

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which is consistent with expectations, was obtained with half of the intensity of the 715.7-717.3 keV doublet assigned to the 717.3-keV transition. If the existence of the 715.7-keV transition were ignored and all the intensity were assigned to the 717.3-keV transition, then the 1229-keV level would have a β feeding per 100 β decays of 0.006 \pm 0.002 and a corresponding log ft of 8.5 which is definitely not consistent with a third-forbidden β transition. Thus our approximate assignment of $(50\pm25)\%$ of the intensity to each component of the 715.7-717.3-keV doublet is consistent with both our coincidence data and the level scheme.

1SS7-ke V level

The 3' level at ¹⁵⁵⁷ keV was also found to have negligible β branching, as expected for a secondforbidden β transition. Of the three transitions depopulating this level, the 1044.8-keV transition (which we found to have 0.007 ± 0.002 of the intensity of the 1050.5-keV transition) is the strongest. The 428.5-keV transition has essentially the same energy as a stronger transition at 428.4 keV; hence we could not directly determine its intensity. The much weaker 327.7-keV transition could not be detected because it lay just below the Compton edge of the 511.8-keV transition (which is more intense by a factor of about 4×10^4). However, the relative intensities of these three transitions can be obtained from either the ¹⁰⁶Rh^m decay¹⁹ or the
¹⁰⁶Ag^m decay.¹⁹ Since either decay gives essenti $^{106}\text{Ag}^m$ decay. 19 Since either decay gives essential ly the same relative intensities, we were able to reliably determine the intensity of these three transitions in the 106 Rh^{s} decay. The resulting intensity imbalance to the 1557-keV level was much less than its uncertainty of 0.005 per 100 decays; hence we indicated this value and the corresponding $\log ft$ of 8.3 as limiting values in Table IV and in Fig. 4.

2189-ke V level

A level at 2189 keV, depopulated by 2189- and 1062-keV transitions, was first proposed by Rao and Fink.³ Recently Avignone and Pinkerton¹⁸ proposed a spin and parity of 2^+ for such a level based on an angular correlation between 1062- and 1128 keV γ rays. However, the presence of this level was disputed by Hattula and Luikkonen,⁷ Strutz, was disputed by Hattuna and Bankkonen, better
Strutz, and Flammersfeld,⁸ and Ardisson and Marsol,¹⁰ who found no 2189-keV transition and placed the 1062-keV transition between the 2624 and 1562-keV levels. These results were based on coincidence data obtained with a Ge(Li)-NaI(TI) system. The discrepancy is resolved by our Ge(Li)-Ge(Li) coincidence study with more definitive gates. The γ -ray spectrum coincident with

the 1062-keV transition is shown in Fig. 3. It is clear that this transition is in coincidence with the 1050- and 1562-keV transitions (but not with the 1128-keV transition) and should thus be placed above the 1562-keV level. Also, no 2189-keV γ ray was observed in our singles measurements. We therefore conclude there is no basis for a level at 2189 keV.

2306-ke V level

Avignone and Pinkerton¹⁸ proposed a spin-parity of 0' for a 2306-keV level based on a 1178-1128 keV angular correlation and $(1, 2, 3)^+$ for the 2308keV level based on a 1796-512-keV angular correlation. The 2306 -keV 0^+ state is different from the 2305.7 -keV 4^{\pm} level found in the study of the the 2305.7-keV 4^* level found in the study of the 2305.7-keV 4^* level found in the study of the high-spin states,^{2,25-29} since the 4-2-0 spin sequence is not consistent with the large A_4 measured. In the present work, with the γ -ray energies more precisely determined, the energy sums of the 1180-1128-keV cascade and the 1797-512 keV cascade agree to within 0.5 keV, suggesting that they originate from one level at 2308.59 keV. If these two cascades originate from this level, as we propose, then the presence of the 2308.47 keV transition to the ground state would limit the spin of this level to 1 or 2. We conclude there is no evidence for a level at 2306 keV in the decay of $106Rh.$

2624-ke V level

The spin of this level is found to be 0 from our 2112-512-keV angular correlation. However, Ardisson and Marsol¹⁰ ruled out the 0 spin assignment because of a 2624-keV transition to the ground state and also a 1489-keV transition to the 1133 keV 0^+ state. In the present work the 1489-keV transition was observed to be in coincidence with the 512-keV transition and should be placed between the 2001- and 512-keV levels. We did not observe a 2624-keV transition in our singles measurement. If the spin of this level were greater surement. If the spin of this level were greater
than 0, as stated by Ardisson and Marsol,¹⁰ theı it would be impossible to explain the large positive A_4 observed in this work and that of Avignone tive A_4 observed in this work and that of Avigno
and Pinkerton.¹⁸ We conclude therefore that the level at 2624 keV has a spin zero.

IV. THEORETICAL INTERPRETATION

Previously $106Pd$ has been regarded as a quadrupole vibrator about a spherical equilibrium shape as indicated by some of the previous studies^{7,18} which compared the energy levels with those predicted by the quadrupole vibration model of Fer-
reira, Castilho, and Navarro.¹² Hattula and Liu reira, Castilho, and Navarro.¹² Hattula and Liuk-

FIG. 5. Comparison between the predictions of two models (1 and 2) and the experimental. levels below about 3.3 MeV. The models and state labels are discussed in text. For model 2, the parameter values are as follows: $\mu = 0.39$; γ =25.2° and $\hbar \omega$ =2174 keV for the first sequence (S=1); γ =22.2° and $\hbar \omega$ =1927 keV for S=2; γ =24.9° and $\hbar \omega$ =2302 keV for $S=3$.

konen' found very close agreement with the oneand two-phonon levels in their comparison. Avignone and Pinkerton¹⁸ extended the comparison with this model by including certain three- and fourphonon levels, They found that the model did not give satisfactory level splittings or, in some cases, level orderings, However, due to the establishment of several more 0' levels of nearly equal spacing, they concluded that the evidence that ¹⁰⁶Pd exhibits characteristic vibrational properties had been strengthened.

The identification of high-phonon quadrupole vibrations in 106 Pd is difficult to make from energy considerations alone, especially when the energy deviations from the higher-phonon states are as large as Avignone and Pinkerton found. Transition rates should also be taken into account to establish vibrational characteristics of these

levels. The 2' level at ¹⁵⁶² keV, which Avignone and Pinkerton assumed was a three-phonon state, has a transition to the ground state. Furthermore, the 1050-keV transition from this level to the 2^+ one-phonon level at 512 keV was found in this work to be 91% M1. Thus the 1562-keV level cannot be regarded as a pure three-phonon state.

In order to extend the comparison of 106 Pd levels with a quadrupole-vibration model, we have used the anharmonic vibrational model of Brink, de the anharmonic vibrational model of Brink, de
Toledo Piza, and Kerman.¹¹ The theoretical ener gies obtained from this model are compared with experimental levels in Fig. 5. A typical weighted least-squared fitting technique involving the one-, two-, and three-phonon levels yielded the parameters $\hbar \omega = 584$ keV, $\epsilon_0 = -0.125$, $\epsilon_2 = -0.64$, ϵ_4 $= 0.067$. Our agreement with experiment is neither better nor worse than that obtained by Avignone

γ -ray transitions (indicated by energies in keV)	Experimental results	Theoretical results ^a	Theoretical expressions ^b
(622/512) (616/512) (716/512)	0.63 ± 0.14 ° 0.96 ± 0.14 ^c 1.55 ± 0.23 ^c	0.9 1.2 1.95	$2+t_{0}$ $2 + t_2$ $2+t_{A}$
(328/429)	0.33 ± 0.07 ^d	0.10	$14+15t_2-t_4$ $5(7+4t_2+3t_4)$
(616/1128) (428/1050)	$0.37 \pm 0.06^{\circ}$	0.79	$100(2+t_2)$ $7(35+7t_0+10t_2+18t_4)$
(616/1128) (578/1194)	6.33 ± 1.24 ^e	6.0	$\frac{2+t_2}{1+t_2}$

TABLE V. Experimental and theoretical $B(E2)$ ratios.

 $t_0 = -1.1$, $t_2 = -0.8$, $t_4 = -0.05$.

Theoretical expressions taken from Brink, de Toleda Piza, and Kerman (Ref. 11).

 \textdegree Ratios taken from Coulomb excitation study of Robinson et al. (Ref. 30).

^d The $E2$ component and $B(E2)$ ratio were obtained from Inoue (Ref. 31).

[~] Average values of several independent works were used, with the intensities from Strutz, Strutz, and Flammersfeld (Ref. 8), Marsol, Rahmouni, and Ardisson (Ref. 9), and the present work; the mixing ratio of the 1050-keV transition from Hattula and Liukkonen (Ref. 7), Klema and McGowan (Ref. 32), and the present work.

and Pinkerton.¹⁸ In the model of Brink, de Toledo Piza, and Kerman the anharmonic terms in the Hamiltonian make the $B(E2)$ ratios differ from the predictions of a pure harmonic quadrupole vibrator. The deviations from the harmonic results are expressed by the parameters t_0 , t_2 , t_4 . Our best weighted least-squared fit to the known $B(E2)$ ratios is given in Table V. Also given in Table V are the theoretical expressions used together with the values of t_0 , t_2 , and t_4 which were found.

The value of t_2 is very well defined, being nearly independent of the weighting assigned to each experimental $B(E2)$ ratio. The parameter t_0 is less well defined, but was consistently between -0.95 and -1.2 . The parameter t_4 expresses the deviation from harmonicity for the two-phonon 4' state to the one-phonon 2' state transition relative to the one-phonon 2' state to ground-state transition. This parameter is poorly determined by the data. Various weightings yielded values between the -0.05 given in Table V and $+0.35$.

Within the framework of this model, which retains only linear terms, t_0 , t_2 , and t_4 must be small in comparison with 2.0. We do not find this to be the case for t_0 and t_2 ; the large sizes required for these parameters suggest the inapplicability of this linearized model to 106 Pd. Even discounting the large sizes of t_0 and t_2 , the model is disappointing in its agreement with experiment. This is particularly so in view of the necessity of adding new parameters for the quadrupole transitions in addition to those required for the energy levels. We are forced to conclude that ^{106}Pd is not a simple anharmonic quadrupole vibrator.

One was led to consider this model by the almost degenerate 0^+ , 2^+ , 4^+ triplet near 1150 keV. If, however, one mentally omits the $0⁺$ level at 1134 keV, the structure of the first five levels strongly resembles the structure of the rotation-vibration model of Davydov and Chaban.¹³⁻¹⁵ This model of Davydov and Chaban.¹³⁻¹⁵ This model consists of breathing-mode vibrations (β vibrations) coupled with asymmetric rotor motion. The rotation causes centrifugal stretching of the vibrator which couples the motions. One makes the small oscillation approximation by expanding the β -vibrator potential (including the centrifugal term} about a dynamic equilibrium deformation whose value depends upon the rotor energy. This approximation limits the range of validity of one of the three model parameters, namely the nonadiabaticity parameter μ . This parameter is a measure of the stiffness of the β vibrator to centrifugal stretching; when $\mu = 0$, the β vibrations and rotations are completely uncoupled. For μ much in excess of 0.6 the next order term in the potential expansion seems too large to be safely ignored. Thus effectively, $0 \le \mu \le 0.6$. The rotor asymmetry is described by a second model parameter, an angle γ , which ranges from 0 to 30'. The last model parameter is an over-all energy scale factor $\hbar\omega$.

The states of the model are labeled by I , the angular momentum; by N , an ordinal number which labels the multiple occurrences of rotor states with the same value of I ; and n , which is an ordinal number labeling the β bands. For example, 221 would denote the second angular momentum 2 level of the first (ground state) β band. All states

considered have positive parity.

The 0^+ ground state, 2^+ (512 keV), 2^+ (1128 keV), 4^+ (1229 keV), and 3^+ (1557 keV) levels are typical of this model for $\gamma \gtrsim 20^{\circ}$ and their relative spacing is mostly independent of μ . The value of μ very strongly influences the location of the 0' firstexcited β bandhead and to some extent the spacing of the rotor levels (again 2^+ , 2^+ , 3^+ , 4^+ , etc.) built upon this bandhead. The first-excited 0' level is found experimentally at 1134 keV . This level cannot be the β bandhead for several reasons. First of all, it would require a value of μ greatly in excess of 1.0 to be this low in energy. That in turn would greatly compress the 2^+ , 2^+ , 3^+ , 4^+ sequence above this 0^+ in comparison with experiment. Also the $B(E2)$ transitions from this level suggest that it is not the excited β bandhead. Therefore, we have extended the model by assuming the existence of more than one rotor-vibrator sequence. By a sequence, we mean a full set of rotor levels together with ground-state and excitedstate β vibrations. This of course implies the assumption of additional degrees of freedom and we speculate as to their origin in the conclusions of this paper. For this work we were content to assume their existence. In this extension of the model each level is labeled by S (in addition to I, N, and n) to denote in an ordinal manner the sequence to which the state belongs. In ¹⁰⁶Pd we have identified three sequences with sequence heads at 0.0, 1134, and 1706 keV.

The comparison of this extended model with the experimental levels is given in Fig. 5. The parameter values for each of the three sequences are also given. We have taken μ to be the same for all three sequences but γ and $\hbar\omega$ were allowed to be different. Also, each sequence head was merely assigned the experimental value. Figure 5 shows 21 excited levels of which 15 were used to determine the parameters. Thus, we have effectively 9 parameters and by performing a standard weighted least-squares fit we find a weighted root-meansquare (rms) average of the deviations between theory and experiment of 20 keV. As a further measure of confidence one can also consider the rms average of the magnitude of the difference between theoretical and experimental energies divided by the experimental energies. The value of this quantity is 2.3% . In the fitting procedure we first identified the ground-state sequence including its first excited β bandhead. Then we identified separately the second sequence. In this case the excited β bandhead at 3137 keV is to be compared to the experimental level at 3082.6 keV. We then noted that to two significant figures μ was the same for both sequences. For this reason, and because we could not identify the excited β band-

TABLE VI. The ratios of $B(E2)$ transition probabilities using the expression of Davidson and Davidson (Ref. 33).

 a See text for discussion of \langle 2nd sequence | 1st sequence) overlap factor.

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Extracted from intensity ratio times energy ratio factor. All others are measured.

 \textdegree This has the form $\langle 3r \text{d} \text{ sequence} | 2 \text{nd} \text{ sequence} \rangle$ / $\langle 3rd$ sequence | 1st sequence \rangle overlap factor ratio = 84.

head of the third sequence, we took μ to be the same for all three sequences in the final fitting procedure. The second 4, the 5 and 6 levels of the first sequence, the 3 and 61evels of the second sequence, and the 3 and 4 levels of the third sequence must be regarded as output of the theory as they were not used in the fitting procedure.

As a further test of the validity of this model for ¹⁰⁶Pd we have computed the ratios of $B(E2)$ reduced transition probabilities using the expression duced transition probabilities using the expres
given by Davidson and Davidson.³³ The result are given in Table VI. For six of the ratios in Table VI only transitions within the first sequence

are involved. The agreement between theory and experiment for these six ratios is very good—in fact, as good as is found when this model is applied to its more traditional regions of validity, plied to its more traditional regions of validity,
as, for example, in the rare earth region.³¹ We stress the fact that no new parameters are involved in this calculation. We must qualify that by saying that the model unambiguously predicts the ratios for transitions within a sequence. However, for transitions between levels of two sequences, as for example $S=1$ and $S=2$, our ignorance as to the role played by the other assumed degree of freedom can only be expressed as an overlap between the sequence states. For the three ratios in Table VI in which both numerator and denominator involve a second-sequence -tofirst-sequence transition, the agreement between theory and experiment is very good, indicating that the $\langle S = 2|S = 1 \rangle$ overlap factor appears to cancel out. Such is not the case, however, for the sole ratio in which both numerator and denominator involve a third-sequence-to-first-sequence transition. This ratio 518/1195 is very poorly predicted by theory indicating that our model also has difficulty with the 1706-keV level.

The remaining four ratios in Table VI involve mixed-sequence transitions of different types which are not expected to cancel out. The denominator of the 428/1050 ratio (2112-0112/2112-2111) involves a mixed-sequence transition. The denominator of the 1150/716 ratio (0121-2211/0121- 2112) also involves a mixed transition but not the same one. The third such mixed transition case is the $622/512$ ratio $(0112-2111/2111-0111)$ where the mixed transition is in the numerator. If we assume that the $\langle S = 2 | S = 1 \rangle$ overlap factor is sequence but not state dependent we find the factors

 $\langle 2|1 \rangle = 0.013 \pm 0.001$ from $428/1050$,

 $\langle 2|1 \rangle = 0.013$ from 1150/716,

 $\langle 2|1 \rangle = 0.10 \pm 0.03$ from 622/512.

The agreement of the first two and the strong disagreement between these two and the third is puzzling since the 0112 level is involved in both the first and third of these. There is little we can say regarding the other mixed-band transitions since we have only one of each type which precludes the above sort of comparison.

V. CONCLUSION

In assessing the comparative successes of the spherical-vibrator model and rotor-vibrator model we note that in the latter case we have fitted 15 levels with 9 parameters and then without further

parameters have calculated 10 unambiguous $B(E2)$ ratios. In the spherical-vibrator model case we have fitted seven experimental levels with four parameters and calculated six $B(E2)$ ratios by fitting three additional parameters. Over all we conclude. that the rotor-vibrator model as extended gives a better description of the character of the gives a setter description of the enaracter of states of ¹⁰⁶Pd than does the spherical-vibrat model. Although we make this assessment on the basis of the evidence presented above we have also kept in mind that the agreement of the rotorvibrator model with experiment could be further improved by fitting both energy levels and $B(E2)$ ratios simultaneously.

The extended rotor-vibrator model involves the assumption of additional degrees of freedom which permit the existence of more than one rotation β vibration sequence and these may be either of a particle nature or collective. At this point one can only speculate as to which of these would be the more fruitful assumption to follow in a more detailed calculation. Preliminary consideration leads us to believe that the collective approach is probably better. One might conceive that these excited sequences are built upon excited $0⁺$ -coupled particle states. The simplest assumption would be that of 0^+ -coupled pairs for both protons and neutrons. By examining the shell-model levels in this region one can conclude that there is a sufficiency of candidates, indeed more than enough, with inadequate experimental information to reduce the number to be considered to a manageable few. Further, if such 0 couplings occur, then at some modest excitation energy (perhaps 2 MeV or so) one could expect to see nonzero coupled states, in particular a 1^+ . While there are several possible candidates between 2 and 3 MeV there seem to be none which would have anappropriate strength magnetic-dipole transition to the ground state. Because of these difficulties the number of parameters that would of necessity be involved in a 0^+ coupled pair theory would approach the number of firm experimental data for $106Pd$. On the other hand, the assumption that these several sequences arise out of asymmetry (i.e., γ) vibrations would in fact reduce the number of parameters of the model from our present 9 to 4. Furthermore, a very simple calculation involving these γ vibrations suggests that such a model has the potential to both reproduce the experimental energy levels and to explain the "cross-sequence" $B(E2)$ transitions. The full development of such a model and its application to 106 Pd is presently being pursued.

The apparent success of this model in describing the states of ^{106}Pd is a very exciting development since previously it was thought that this model should only apply to rare earth or actinide

nuclei. This development suggests the need for continued interest both theoretical and experimental for nuclei near $A=100$.

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