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No-resonance condition for a $^1S_0$ separable ΛN potential with suppression effects

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Two-channel ΛN potentials each of whose matrix elements is a single nonlocal separable are discussed. A simpler condition than obtained previously is derived for the Λ- channel shape such that the ΛN scattering length and effective range agree with the values obtained from a meson-theoretic potential both with the Σ channel unsuppressed and fully suppressed and which at the same time guarantees the absence of an unwanted ΛN resonance below the Σ-channel threshold due to a bound state in the uncoupled Σ channel. A specific nonlocal separable shape is exhibited which obeys this condition for a range of values of its parameters when the most recent $^1S_0$ meson-theoretic potential scattering lengths and effective ranges of Brown, Downs, and Iddings are used as input.

[NUCLEAR REACTIONS YN potential, ΛN scattering length and effective range, two-channel separable potential, $\Lambda N \rightarrow \Sigma N$ suppression, ΛN resonance.]

I. INTRODUCTION

In a number of recent papers[1-4] it has been stressed that for application to some hypernuclear bound states, it makes more sense to use a phenomenological ΛN potential that reproduces features, at or near the ΛN threshold, of a meson theoretic potential (MTP) with the Σ channel suppressed, than to use one that reproduces the MTP values of the ΛN phase shift at energies far above this threshold. Within this philosophy, investigations of $^1S_0$ nonlocal separable (NLS), two-channel, phenomenological, ΛN potentials whose relative momentum space matrix elements have the form

$$\langle \vec{k}'_X | V_{XY} | \vec{k}_Y \rangle = p_{XY} \lambda_{XY} v_X(k'_X) v_Y(k_Y) \tag{1}$$

have been carried out.[1,2,4] Here V_{XY} is the potential energy operator responsible for $YN \rightarrow XN$, where $X, Y = \Lambda$ or Σ; k_Y is the relative YN momentum ($\hbar = c = 1$); λ_{XY} is the strength parameter for V_{XY}; p_{XY} is a pure number used to introduce suppression effects which are present in hypernuclei[5]; e.g., $p_{\Lambda\Lambda} = p_{\Sigma\Sigma} = 1$, $p_{\Lambda\Sigma} = \epsilon$, and ϵ is varied from $\epsilon = 1$ down to $\epsilon = 0$ to suppress $\Lambda - \Sigma$ conversion.

In Refs. 1, 2, and 4 the $^1S_0$ ΛN scattering length a_ϵ and effective range $r_{0\epsilon}$ calculated from an NLS potential of the Eq. (1) type were fitted to the values of the scattering length and effective range, respectively, at $\epsilon = 1$ (i.e., no $\Lambda N \leftrightarrow \Sigma N$ suppression) and $\epsilon = 0$ (i.e., full $\Lambda N \leftrightarrow \Sigma N$ suppression) obtained from the MTPs of Brown, Downs, and Iddings (BDI).[6,7] The results of these earlier works were threefold. First,[4] a simple condition on the shape $v_\Lambda(k_\Lambda)$ was obtained that had to hold to guarantee unitarity; i.e., $\lambda_{\Lambda\Sigma}^2 > 0$. Second,[2] a more complicated condition on the shape $v_\Lambda(k_\Lambda)$ was obtained that had to hold if a nonexistant ΛN resonance below the ΣN threshold[8] was to be eliminated. Third, for both the MTP input of Refs. 6 and 7 and for a variety of specific one-parameter shapes $v_\Lambda(k_\Lambda)$, the no-resonance condition was violated; i.e., every NLS $^1S_0$ potential investigated previously that fit the BDI MTP values of a_ϵ and $r_{0\epsilon}$ at $\epsilon = 0$ and $\epsilon = 1$, produced an unwanted resonance.

In the present work two new results are presented. First, a simple form of no-resonance condition is obtained. This condition is combined with the unitarity condition previously obtained to yield an acceptable range of values for the slope-to-value ratio of v_Λ^2 as a function of k_Λ^2 evaluated at $k_\Lambda^2 = 0$. Second, an explicit example of a shape $v_\Lambda(k_\Lambda)$ that yields the BDI Ref. 7 values for a_0, a_1, r_{00}, and r_{01} and meets the no-resonance condition is displayed.

II. NO-RESONANCE CONDITION

It was shown in Ref. 1 that if the Eq. (1) potential form is used to calculate the four parameters a_0, a_1, r_{00}, and r_{01}, the four equations obtained could be used to determine the three strength parameters $\lambda_{\Lambda\Lambda}$, $\lambda_{\Lambda\Sigma}$, and $\lambda_{\Sigma\Sigma}$ as functions of the shapes $v_\Lambda(k_\Lambda)$ and $v_\Sigma(k_\Sigma)$ and to obtain a relation that $v_\Lambda(k_\Lambda)$ by itself had to satisfy; namely,

$$\tfrac{1}{2} r_{00} - A_2 - u_r(\alpha_0 + A_0) = 0\,, \tag{2}$$

where $\alpha_\epsilon \equiv 1/a_\epsilon$, u_r is defined by the $k_\Lambda \rightarrow 0$ expansion

$$v_\Lambda^2(k_\Lambda) = u_0 + u_2 k_\Lambda^2 + \cdots \equiv u_0(1 + u_r k_\Lambda^2 + \cdots)\,, \tag{3}$$

and the A_l are defined by the $k_\Lambda \to 0$ expansion

$$[-g^P_{\Lambda\Lambda}/v_\Lambda^2(k_\Lambda)] = A_0 + A_2 k_\Lambda^2 \cdots , \tag{4}$$

where $g^P_{\Lambda\Lambda}$ is the Cauchy principal value integral

$$g^P_{\Lambda\Lambda} = P\frac{2}{\pi}\int_0^\infty \frac{v_\Lambda^2(q)q^2dq}{q^2 - k_\Lambda^2}. \tag{5}$$

It was further shown in Ref. 2 that to avoid a ΛN resonance below the ΣN threshold due to a bound state in the uncoupled ΣN channel it was necessary that

$$\xi \equiv \frac{(\alpha_1 + A_0)}{k_0^2(\alpha_0 + A_0)(u_r - \chi)} > 1 , \tag{6}$$

where

$$\chi = \tfrac{1}{2}(r_{00} - r_{01})/(\alpha_0 - \alpha_1) \tag{7}$$

and k_0^2 is the value of k_Λ at the threshold of the ΣN channel; i.e., $k_0^2 = 2\mu_\Lambda(M_\Sigma - M_\Lambda)$, M_Y being that Y hyperon mass and μ_Λ being the ΛN reduced mass.

It was pointed out in Ref. 4 that unitarity required $\xi > 0$, which from the definition (6), Eqs. (4) and (5), which imply $A_0 < 0$, and the BDI values of the scattering parameters, which imply

$$\alpha_0 < \alpha_1 < 0 \quad \text{and} \quad \chi < 0 , \tag{8}$$

means u_r must satisfy

$$\chi < u_r . \tag{9}$$

With the Ref. 7 input and the mass values (in MeV) $M_N = 939.0$, $M_\Lambda = 1115.0$, $M_\Sigma = 1193.0$, we find (to four figures) $\chi = -1.652\ \text{fm}^{-2}$ and $k_0^2 = 2.032\ \text{fm}^{-2}$.

What we shall do now is replace the no-resonance condition (6), which involves both A_0 and u_r with a simpler condition that involves u_r alone. For numerical work we shall use the BDI Ref. 7 values for the scattering parameters, namely,

$$\begin{aligned} a_0 &= -1.42\ \text{fm}, \quad r_{00} = 3.64\ \text{fm}, \\ a_1 &= -1.77\ \text{fm}, \quad r_{01} = 3.18\ \text{fm}, \end{aligned} \tag{10}$$

and the above values of the baryon masses.

First we note that all $v_\Lambda(k_\Lambda)$ such that $u_r = 0$ cannot satisfy $\xi > 1$. For $u_r = 0$, the definition (6) may be combined with Eq. (7) to yield

$$\xi = \left[\frac{\alpha_1 + A_0}{\alpha_0 + A_0}\right]\frac{(-1)}{k_0^2\chi}. \tag{11}$$

From the above numerical values of k_0^2 and χ, $0 < -1/(k_0^2\chi) < 1$, while from $A_0 < 0$ and relation (8) the bracketed ratio in Eq. (11) also lies between 0 and 1. So in this case $0 < \xi < 1$ rather than $\xi > 1$.

In addition, we can see from the definition (6) that as u_r increases from zero ξ decreases in magnitude. Thus, $u_r \geq 0$ cannot yield $\xi > 1$. Combining this result with Eq. (9) limits u_r to the range $\chi < u_r < 0$.

But we can do better than this. The inequality (6) and $(\alpha_1 + A_0)/(\alpha_0 + A_0) < 1$ yield $k_0^2(u_r - \chi) < 1$ or $u_r < \chi + k_0^{-2}$. A *necessary* set of limits on u_r for both unitarity and our no resonance condition to hold is then

$$\chi < u_r < \chi + k_0^{-2} , \tag{12a}$$

or, with the numerical input described above

$$-1.652\ \text{fm}^{-2} < u_r < -1.162\ \text{fm}^{-2}. \tag{12b}$$

We also note that $(\alpha_1 + A_0)/(\alpha_0 + A_0) > k_0^2(u_r - \chi)$ implies $\xi > 1$ and in addition $(\alpha_1 + A_0)/(\alpha_0 + A_0) > \alpha_1/\alpha_0$. Thus, a *sufficient* set of limits on u_r for both unitarity and our no-resonance condition to hold is

$$\chi < u_r < (\alpha_1/\alpha_0)k_0^{-2} \tag{13a}$$

or, with our numerical input

$$-1.652\ \text{fm}^{-2} < u_r < -1.259\ \text{fm}^{-2}. \tag{13b}$$

Of course for $\chi + (\alpha_1/\alpha_0)k_0^{-2} < u_r < \chi + k_0^{-2}$ it may (or may not) happen that $\xi < 1$ and an unwanted resonance exists.

Equations (13) provide a simple check on any shape $v_\Lambda(k_\Lambda)$ which one might try to use in Eq. (1). For multiparameter shapes $v_\Lambda(k_\Lambda)$ there will be a large region in the parameter space for which Eq. (2) is satisfied, but Eqs. (13) should narrow this region considerably as indicated by the example given in the next section.

III. EXAMPLE

The most straightforward explanation[1] of why the Eq. (1) NLS potential with one-parameter shapes used in earlier works produces a resonance that is not produced by the MTP to which it is matched, is that the MTP includes a hard core, whereas the NLS potential contains no such short range repulsion. Thus, by choosing the NLS potential parameters to give the same attractive behavior as the MTP at $k_\Lambda = 0$, we make it too attractive at higher values of k_Λ where the hard core of the MTP comes into play.

There are two ways out of this bind. One way is to replace the single separable term in the Eq. (1) matrix element of $V_{\Lambda\Lambda}$ by a sum of separable terms such as $\lambda_1 v_1(k'_\Lambda)v_1(k_\Lambda) + \lambda_2 v_2(k'_\Lambda)v_2(k_\Lambda)$ with $\lambda_1 < 0$ and $\lambda_2 > 0$ so that the first term represents an attraction and the second term a repulsion.[9] An alternative is to use a multiparameter shape $v_\Lambda(k_\Lambda)$ instead of a one-parameter shape, so that we still have freedom to modify the high k_Λ behavior of the ΛN phase shift even after fitting the BDI low k_Λ behavior. We follow the second simpler alternative.

We choose for our shape $v_\Lambda(k_\Lambda)$ the form

$$v_\Lambda(k_\Lambda) = (\beta_\Lambda^2 + k_\Lambda^2)^{-1} - x(\beta_c^2 + k_\Lambda^2)^{-1}, \quad \beta_c > \beta_\Lambda. \tag{14}$$

The second term represents a "core potential" (i.e., one whose effect is to be large at high values of k_Λ) addition to the commonly used Yamaguchi[10] potential shape of the first term, so that its range β_c^{-1} is chosen to be less than the range β_Λ^{-1}. The third parameter x is left free to give us the needed flexibility of an exploratory calculation.

From Eq. (3) for this shape we obtain

$$u_r = -2(1/\beta_\Lambda^4 - x/\beta_c^4)/(1/\beta_\Lambda^2 - x/\beta_c^2). \tag{15}$$

We note that we satisfy the very weak no-resonance condition $u_r < 0$ only for the regions

$$x < 0, \quad 0 < x < (\beta_c/\beta_\Lambda)^2, \quad (\beta_c/\beta_\Lambda)^4 < x. \tag{16}$$

We may now use Eqs. (14) and (15) along with Eqs. (4) and (5) to reduce Eq. (2) to

$$ax^2 + bx + c = 0, \tag{17}$$

where

$$a = (-r_{00}\beta_c^2 + 3\beta_c - 4\alpha_0)\beta_\Lambda^6,$$

$$c = (-r_{00}\beta_\Lambda^2 + 3\beta_c - 4\alpha_0)\beta_c^6,$$

and

$$b = b_1 + b_2 + b_3$$

with

$$b_1 = 2\beta_\Lambda^4\beta_c^4 r_{00},$$

$$b_2 = -4\beta_\Lambda^3\beta_c^3(\beta_\Lambda^2 + \beta_\Lambda\beta_c + \beta_c^2)/(\beta_\Lambda + \beta_c),$$

$$b_3 = 4\beta_\Lambda^2\beta_c^2(\beta_\Lambda^2 + \beta_0^2)\alpha_0.$$

In the limit $x \to 0$ we must get back to the results of Ref. 1 for a Yamaguchi shape, namely

$$r_{00}\beta_\Lambda^2 - 3\beta_\Lambda + 4\alpha_0 = 0.$$

So, we take that solution of Eq. (17) for which $x \to 0$ as $c \to 0$.

Any two values in the octant of the $\beta_c^{-1} - \beta_\Lambda^{-1}$ plane defined by $\beta_\Lambda^{-1} > \beta_c^{-1} > 0$ may be used in Eq. (17) and a corresponding value of x obtained. Provided that the value of x is real, this means the NLS potential of Eq. (1) may be made to match the BDI data of Eq. (10). Only for a part of that octant will Eq. (16) be satisfied and only for a more limited region of the $\beta_c^{-1}\beta_\Lambda^{-1}$ plane will either Eqs. (12) or Eqs. (13) be satisfied.

We carried out the indicated calculation by varying β_Λ^{-1} from 0.1 to 1.0 fm in steps of 0.1 fm. For each value of β_Λ^{-1} we varied β_c^{-1} over the range of $\beta_c^{-1} < \beta_\Lambda^{-1}$ in steps as small as 0.0001 fm. When β_Λ^{-1} was too small, x turned out to be complex for all $\beta_c^{-1} < \beta_\Lambda^{-1}$. When β_Λ^{-1} was too large, the no-resonance condition was violated for all $\beta_c^{-1} < \beta_\Lambda^{-1}$. We did find a range $0.27 \text{ fm} \lesssim \beta_\Lambda^{-1} \lesssim 0.49$ fm for each value of which a range of values β_c^{-1} existed such that either or both Eqs. (12) and (13) held. The results of some typical calculations are shown in Table I.

TABLE I. Typical no-resonance results for the Eq. (14) shape $v_\Lambda(k_\Lambda)$.

$1/\beta_\Lambda$ (fm)	$1/\beta_c$ (fm)	x	u_r (fm^{-2})	ξ
	0.2371	1.430	−1.382	1.793
0.29	0.2406	1.386	−1.259	1.230
	0.2500	1.290	−1.172	1.001
	0.1740	3.362	−1.364	1.685
0.35	0.1773	3.304	−1.260	1.234
	0.1871	2.943	−1.171	1.000
	0.1071	11.21	−1.354	1.635
0.41	0.1101	10.36	−1.259	1.236
	0.1202	8.499	−1.169	1.001
	0.0358	115.6	−1.337	1.553
0.47	0.0382	106.3	−1.259	1.241
	0.0489	58.09	−1.166	1.001

The left most column of Table I shows four values of β_Λ^{-1} for which no-resonance results could be obtained. For each of these values we have listed three rows of results for the parameters β_c^{-1}, x, u_r, and ξ. The first row gives values of these parameters for the smallest value (to four figures) of β_c^{-1} (for the given value of β_Λ^{-1}) for which a solution exists; i.e., for any smaller value of β_c^{-1}, x becomes complex. The second row gives values of the parameters for which the no-resonance sufficiency inequality of Eq. (13) is just met. The third row gives the values of the parameters for which $\xi \gtrsim 1$. For each β_Λ^{-1} then the values given for β_c^{-1} in the first and third rows define the range of β_c^{-1} for which solutions exist that satisfy the original no-resonance inequality of Eq. (6) and the BDI input of Eq. (10). Each such range is extremely small, being less than 0.02 fm in each case, so that only in a very narrow strip of the $\beta_\Lambda^{-1} - \beta_c^{-1}$ plane are acceptable solutions to be found. Another interesting point is that the no-resonance sufficiency condition $u_r < -1.259$ fm gives a much narrower range of β_c^{-1} than is allowed by the original no-resonance condition $\xi > 1$ which itself gives values of u_r that lie much closer to the no-resonance necessity condition of Eq. (12), $u_r < -1.162$ fm. All of the solutions shown obey the unitarity condition $-1.652 < u_r$.

It is possible therefore to find a small region of the $\beta_\Lambda^{-1} - \beta_c^{-1}$ plane for which the shape $v_\Lambda(k_\Lambda)$ given in Eq. (14) can be part of a NLS potential of the Eq. (1) type that obeys unitarity, yields the low-energy scattering parameters of Eq. (10), and does not give rise to a ΛN resonance below the ΣN channel threshold due to a bound state in the uncoupled ΣN channel.

The drawback of using the particular shape $v_\Lambda(k_\Lambda)$ given in Eq. (14) is that it contains more free parameters than one would like. A more acceptable situation would be to find a two-parameter shape $v_\Lambda(k_\Lambda)$ which has the same desirable properties as the shape given in Eq. (14). A search for just such a $v_\Lambda(k_\Lambda)$ is under way.

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[10]See B. F. Gibson and D. R. Lehman, Phys. Rev. C 10, 888 (1974), and references cited there for use of the Yamaguchi shape for the ΛN potential.