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Pion photoproduction at threshold on ^{6}Li and low-energy theorems

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The Low, Kroll-Ruderman, and Fubini-Furlan-Rossetti theorems are derived in the case of pion photoproduction on ^{6}Li. Particular care has been taken to be consistent with electromagnetic current conservation and Low's theorem in the extrapolation from soft to real pions. This is new compared to previous treatments of soft pion theorems in nuclear processes where one photon is present. Difficulties due to the spin-1 nature of ^{6}Li have been put forward.

[NUCLEAR REACTIONS Low, Kroll-Ruderman, Fubini-Furlan-Rossetti theorems for ^{6}Li$(\gamma, \pi^+)^6$He.]

I. INTRODUCTION

In the past few years, considerable attention has been dedicated to the application of low-energy pion theorems to nuclear processes, mainly elastic pion scattering and pion photoproduction at threshold on nuclei.[1] These theorems, based to a considerable extent on the partially conserved axial current (PCAC) hypothesis, were first derived in elementary particle physics.[2] Strictly speaking, they are only applicable to off-shell amplitudes involving soft (zero four-momentum) pion(s). However, correction procedures have been proposed and used to relate these soft pion results to the real world.[3]

Of particular interest is the study of pion photoproduction at threshold on a nucleon,[4] where soft pion techniques as well as extrapolation from soft to real pions have to meet the requirements imposed by electromagnetic gauge invariance, in particular by Low's theorem.[5] This important aspect has been emphasized by De Baenst[4] in his derivation of the Kroll-Ruderman (KR)[6] and the Fubini-Furlan-Rossetti (FFR)[7] theorems. In constrast, we consider that *little care has been taken in the past to satisfying electromagnetic current conservation when low energy pion theorems were applied to nuclear processes.*[1,8]

For further reference, let us state Low's theorem in the case of pion photoproduction $\gamma + N_1 \to \pi + N_2$ on nucleon[9] or nucleus: the two lowest terms —of order k^{-1} (infrared divergent term) and of order zero in k—of the series expansion, in powers of the photon four-momentum k, of the scattering amplitude depend only on the on-shell N_1-N_2-π coupling constant and on the electromagnetic constants (charges and anomalous magnetic moments) of the participating nucleons or nuclei N_1 and N_2. These two lowest order terms are obtained from the Born diagrams (Fig. 1) and gauge invariance requirements, and are not directly dependent on excited nuclear intermediate states, rescattering terms, distortion of the pion wave by strong interactions, etc. Furthermore, as far as these two lowest order terms are concerned, nuclei can be considered like elementary objects, just like nucleons.

In the present article, we will discuss photon and pion low energy theorems for pion photoproduction at threshold in the particular case of the transition between the ground states of ^{6}Li and ^{6}He, $\gamma + {}^6\text{Li} \to {}^6\text{He} + \pi^+$. Considering the nuclei as elementary objects, as requested by the soft photon theorem, we selected the very systematic procedure of De Baenst[4] for deriving the theorems of Low,[5,9] KR,[6] and FFR.[7] This permits us to display explicitly the approximation procedure used in the extrapolation from soft to real pions. The derivation is carried through in detail because, in contradistinction to the case of pion photoproduction on a nucleon ($J^P = \frac{1}{2}^+$), ^{6}Li and ^{6}He have 1^+ and 0^- quantum numbers, respectively; this introduces novel features which may give rise to theoretical difficulties when the extrapolation is done, as we shall point out in the following. We consider these issues particularly important because the recent measurement of this process made by a Louvain-Saclay collaboration[10] seems to be in disagreement with the existing theoretical prediction.[1,8,11] This, in fact, has motivated the present work.

The organization of the paper is as follows. After defining notations and kinematics in Sec. II, we compute in Sec. III the Born diagrams (Fig. 1); imposing on them electromagnetic current conservation, we deduce Low's theorem and we find that the anomalous electric quadrupole moment contribution, which is a term of order k in the

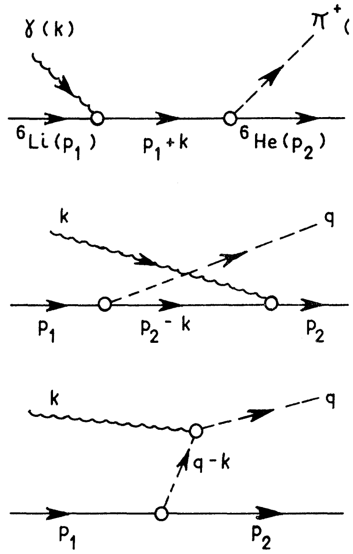

FIG. 1. The three Born diagrams in $\gamma(k)+{}^6\mathrm{Li}(p_1) \to {}^6\mathrm{He}(p_2)+\pi^+(q)$.

scattering amplitude and therefore is not determined by Low's theorem, could be very important even at threshold. The KR theorem is derived in Sec. IV as a consequence of Low's theorem. At this point we will point out why, due to the spin-1 nature of ^{6}Li, the extrapolation from soft to real pion is more ambiguous than in the case of the spin-$\frac{1}{2}$ target. In Sec. V, using PCAC, we derive the FFR theorem to improve the KR result. As a by-product we find that the recoil form factor[12] $F_R(t)$ for the $^6\mathrm{He} \to {}^6\mathrm{Li}$ transition is likely to be small. Then the slope of the differential cross section at threshold can be exactly known up to and including terms of order m_π. Section VI is devoted to a discussion of our results, their comparison with experimental data,[10] and previous theoretical works.[1,8,11] Our main conclusion is that *soft pion results cannot be directly compared with pion photoproduction on ^6Li.* Finally, we present an extrapolation procedure from $q^2=0$ to $q^2=m_\pi^2$ *compatible with electromagnetic current conservation and Low's theorem.* This is not the case for the extrapolation methods used in Refs. 1, 8, and 11.

II. NOTATIONS AND KINEMATICS

We begin this section by the kinematics of reaction

$$\gamma(k,\epsilon)+{}^6\mathrm{Li}(p_1,\eta) \to {}^6\mathrm{He}(p_2)+\pi^+(q). \tag{1}$$

As usual, letters in parentheses denote the momenta and the polarizations of particles and nuclei. They satisfy the following conditions:

$$\begin{aligned} &k^2=0\,, \quad p_1^{\,2}=m_1^{\,2}\ (m_1=5601.6\ \mathrm{MeV})\,,\\ &p_2^{\,2}=m_2^{\,2}\ (m_2=5605.6\ \mathrm{MeV})\,,\\ &q^2=m_\pi^{\,2}\,, \quad \epsilon\cdot k=\eta\cdot p_1=0\,. \end{aligned} \tag{2}$$

Our metric is defined by $a\cdot b=a^0b^0-\vec{a}\cdot\vec{b}\equiv a^\mu b_\mu$, and the states are normalized covariantly: $\langle \vec{p}', \alpha' | \vec{p}, \alpha\rangle=(2\pi)^3 2p_0\delta^3(\vec{p}'-\vec{p})\delta_{\alpha',\alpha}$.

The Lorentz and parity invariant matrix element $\mathfrak{M}$ for Eq. (1) can be written as

$$\begin{aligned} \mathfrak{M}(\gamma+{}^6\mathrm{Li}\to{}^6\mathrm{He}+\pi^+) &\equiv \epsilon^\mu\eta^\nu\mathfrak{M}_{\mu\nu}\\ &\equiv -e\epsilon^\mu\langle \pi^+, {}^6\mathrm{He}|J_\mu^{\mathrm{em}}(0)|{}^6\mathrm{Li}\rangle\\ &= -\langle {}^6\mathrm{He}|\,j_{\pi^-}(0)|{}^6\mathrm{Li},\gamma\rangle\\ &= \epsilon^\mu\eta^\nu(A_1 g_{\mu\nu}+A_2 p_{1\mu}q_\nu+A_3 q_\mu q_\nu\\ &\qquad +A_4 p_{1\mu}k_\nu+A_5 q_\mu k_\nu)\,. \end{aligned} \tag{3}$$

$J_\mu^{\mathrm{em}}(x)$ is the electromagnetic current, such that $\Box\mathfrak{A}_\mu(x)=eJ_\mu^{\mathrm{em}}(x)$, with $\mathfrak{A}_\mu(x)$ the electromagnetic field, and $j_\pi(x)$ is the source of the pion field $\phi_\pi(x)$: $(\Box+m_\pi^{\,2})\phi_\pi(x)=j_\pi(x)$. The A_i's, $i=1,\ldots,5$, are functions of two independent kinematic invariants chosen to be $\nu\equiv p_1\cdot k/m_1^{\,2}$ and $\nu_1\equiv q\cdot k/m_1^{\,2}$ because they are well suited for deriving the Low theorem (Sec. III): $A_i\equiv A_i(\nu,\nu_1)$. Gauge invariance, i.e., $k^\mu\eta^\nu\mathfrak{M}_{\mu\nu}=0$, implies the two following relations:

$$A_1+p_1\cdot kA_4+q\cdot kA_5=0\,, \tag{4a}$$

$$p_1\cdot kA_2+q\cdot kA_3=0\,. \tag{4b}$$

In terms of $\mathfrak{M}$, the differential cross section in the center of mass system (c.m.s) is given by

$$\frac{d\sigma}{d\Omega}=\frac{|\vec{q}\,|}{|\vec{k}\,|}\frac{1}{(2\pi)^2}\frac{1}{16W^2}\left(\frac{1}{6}\sum_{\mathrm{pol}}|\mathfrak{M}|^2\right), \tag{5}$$

where W is the total c.m.s energy $W^2\equiv(p_1+k)^2$, and the sum is over the photon and ^{6}Li polarizations. Note that the final state Coulomb correction has been neglected in Eq. (5).

At threshold ($\vec{p}_2=\vec{q}=0$), $\sum_{\mathrm{pol}}|\mathfrak{M}|_{\mathrm{th}}^{\,2}=2\,|A_1|_{\mathrm{th}}^{\,2}$ (this is easily seen if use is made of Coulomb gauge $\vec{\epsilon}\cdot\vec{k}=0$). Then the slope of the c.m.s differential cross section near threshold is obtained:

$$\begin{aligned} \frac{|\vec{k}\,|}{|\vec{q}\,|}\frac{d\sigma}{d\Omega}\bigg|_{\mathrm{th}} &= \frac{1}{(2\pi)^2}\frac{1}{16(m_2+m_\pi)^2}\,\tfrac{1}{3}\,|A_1|_{\mathrm{th}}^{\,2}\,\frac{b/|\vec{q}\,|}{\exp(b/|\vec{q}\,|)-1}\\ &\equiv\left[\frac{a_{^6\mathrm{Li}}}{4\pi}\right]\frac{b/|\vec{q}\,|}{\exp(b/|\vec{q}\,|)-1}\,, \end{aligned} \tag{6}$$

where the final state Coulomb correction[13] has been included in the last factor of the second member, $b=2\pi\alpha Z\overline{m}c\simeq 12.5$ MeV/c with $\alpha=e^2/4\pi \simeq 1/137$ (e is the proton charge), $Z=2$ for ^{6}He, and $\overline{m}=m_\pi m_2/(m_\pi+m_2)$ is the reduced mass of the ^{6}He

$+\pi^+$ system. Observe that the Coulomb correction is the one obtained in the approximation of point charges and has a simple analytical form because the pion production near threshold is a pure S wave.

We now define the different vertex functions necessary for the next three sections. Using Lorentz covariance, parity properties, and electromagnetic current conservation, we take the following invariant decomposition for the relevant electromagnetic vertex functions in the limit of zero momentum transfer:

$$\langle \pi^+(q)\,|J_\mu^{\rm em}(0)\,|\,\pi^+(q')\rangle = +(q+q')_\mu\,, \qquad (7)$$

$$\langle {}^6\mathrm{He}(p_2)\,|J_\mu^{\rm em}(0)\,|{}^6\mathrm{He}(p_2')\rangle = +2(p_2+p_2')_\mu\,, \qquad (8)$$

$$\langle {}^6\mathrm{Li}(p_1,\eta)\,|J_\mu^{\rm em}(0)\,|{}^6\mathrm{Li}(p_1',\eta')\rangle = -3\{(p_1+p_1')_\mu[(\eta\cdot\eta') + (\tau/2m_1^2)(\eta'\cdot p_1)(\eta\cdot p_1')] - (1+\kappa)[\eta'_\mu(\eta\cdot p_1') + \eta_\mu(\eta'\cdot p_1)]\}\,, \qquad (9)$$

with[14]

$$\mu = 3(1+\kappa)\,, \qquad (10)$$

$$Q = \frac{3}{m_1^2}(\tau-\kappa)\,, \qquad (11)$$

the total magnetic dipole and electric quadrupole of ^{6}Li, respectively. We use natural units, i.e., $\hbar = c = 1$. Then, from experimental data,[15] $\kappa = 0.6358$ and $\tau = -21(\pm 10\%)$.

For the pion-nucleus vertex function we use[12]:

$$\langle {}^6\mathrm{He}(p_2)\,|j_{\pi^-}(0)\,|{}^6\mathrm{Li}(p_1,\eta)\rangle = \eta\cdot(p_2-p_1)g(t) \qquad (12)$$

with $t = (p_1-p_2)^2$ and

$$g(m_\pi^2) \equiv g\,, \qquad (13)$$

the on-shell π-^{6}Li-^{6}He coupling constant. Similarly, for the relevant matrix elements of the axial vector current $J_5^{-\mu}(0)$, we use the following decomposition[12]:

$$\langle {}^6\mathrm{He}(p_2)|J_5^{-\mu}(0)|{}^6\mathrm{Li}(p_1,\eta)\rangle = i\eta^\mu F_A(t) + i(p_2-p_1)^\mu[\eta\cdot(p_2-p_1)]\frac{1}{m_\pi^2}F_P(t) + i(p_1+p_2)^\mu[\eta\cdot(p_2-p_1)]\frac{1}{2m_1m_\pi}F_R(t) \qquad (14)$$

and

$$\langle \pi^+(q)\,|J_5^{+\mu}(0)|\,0\rangle = -iq^\mu f_\pi\,. \qquad (15)$$

The expressions $F_{A,P,R}(t)$ are the axial, pseudoscalar, and recoil form factors, respectively. If time reversal invariance holds, these form factors are real. Finally, we close this section by giving the threshold values of several kinematic variables:

$$|k|\Big|_{\rm th} = \frac{(m_2+m_\pi)^2-m_1^2}{2(m_2+m_\pi)} = m_\pi[1+O(\mu)]\,, \qquad (16)$$

$$\nu\Big|_{\rm th} \equiv \frac{p_1\cdot k}{m_1^2}\Big|_{\rm th} = \frac{(m_2+m_\pi)^2-m_1^2}{2m_1^2} = \mu + O(\mu^2)\,, \qquad (17)$$

$$\nu_1\Big|_{\rm th} \equiv \frac{q\cdot k}{m_1^2}\Big|_{\rm th} = \mu\frac{(m_2+m_\pi)^2-m_1^2}{2(m_2+m_\pi)m_1} = \mu^2 + O(\mu^3)\,, \qquad (18)$$

where $\mu = m_\pi/m_1 \simeq 0.025$. Observe that μ^2 ($\simeq 6\times 10^{-4}$) and $(m_2-m_1)/m_1$ ($\simeq 7\times 10^{-4}$) have the same order of magnitude.

Now we are ready to derive the photon and pion low-energy theorems in the next three sections, having in mind that correction terms of order μ^2 and higher cannot be separated from correction terms due to the nucleus mass differences.

III. PHOTON LOW-ENERGY THEOREM

Electromagnetic current conservation together with the well-known result of quantum field theory that all terms of order k^{-1} in the amplitude come only from the external line insertion of the photon are the two needed ingredients to prove Low's theorem[5] in particle physics. This ensures the general validity and model independent nature of the results derived from this theorem. However, when the process involves more than three particles[9] besides the photon (this is *not* our case), the terms of order zero in k not only depend on physical independently measurable quantities but also on some derivatives of the amplitude without photon. Note that the theorem has been also proved in potential scattering for local,[16] nonlocal,[17] and velocity-dependent[18] potentials. An essential feature of the theorem, in its different versions, is that the rescattering terms and off-shell effects never directly appear in the first two terms in the expansion of the amplitude in powers of the photon four-momentum.

We are now in a position to derive the soft photon theorem for $\gamma + {}^6\mathrm{Li} \to {}^6\mathrm{He} + \pi^+$. First we compute the Born diagrams (B) of Fig. 1 which exhibit the following polar singularities:

$$\text{a } {}^6\text{Li pole at } (p_1+k)^2 = m_1^2 \text{ or } p_1\cdot k = 0\,, \qquad (19\text{a})$$

$$\text{a } {}^6\text{He pole at } (p_2-k)^2 = m_2^2 \text{ or } p_2\cdot k = 0\,, \qquad (19\text{b})$$

$$\text{a pion pole at } (q-k)^2 = m_\pi^2 \text{ or } q\cdot k = 0\,. \qquad (19\text{c})$$

At these poles, using Eqs. (7)–(13), we can evaluate the residues of the invariant functions A_i defined in Eq. (3). Then we have:

$$A_{1B}=eg\left[-\tfrac{3}{2}(1+\kappa)\frac{q\cdot k}{p_1\cdot k}\right], \qquad (20a)$$

$$A_{2B}=eg\left(\frac{2}{p_2\cdot k}-\frac{3}{p_1\cdot k}\right), \qquad (20b)$$

$$A_{3B}=eg\left(\frac{1}{q\cdot k}-\frac{2}{p_2\cdot k}\right), \qquad (20c)$$

$$A_{4B}=eg\left[\frac{3}{4}\frac{m_1^2-m_2^2+m_\pi^2}{m_1^2}(1-\kappa)\frac{1}{p_1\cdot k}+\frac{3}{2}\frac{\tau}{m_1^2}\frac{q\cdot k}{p_1\cdot k}\right], \qquad (20d)$$

$$A_{5B}=eg\left[-\frac{1}{q\cdot k}+\tfrac{3}{2}(1+\kappa)\frac{1}{p_1\cdot k}\right]. \qquad (20e)$$

Now we make the plausible assumptions that $\mathfrak{M}$, in Eq. (3), can be expanded in power series of k and that the Born terms—Eqs. (20)—are the only ones with polar singularities as defined in Eqs. (19). We split the amplitude $\mathfrak{M}$ into its terms of order k^{-1} and zero in k, $\mathfrak{M}_L$, and the higher order terms, $\mathfrak{M}_R$:

$$\mathfrak{M}=\mathfrak{M}_L+\mathfrak{M}_R . \qquad (21)$$

Correspondingly, we have

$$A_i=A_{iL}+A_{iR} , \qquad (22)$$

with $A_{iR}=O(k)$ if $i=1, 2, 3$, and $A_{iR}=O(k^\circ)$ if $i=4, 5$. Using gauge invariance—Eqs. (4a) and (4b)—we get the full expression for the A_{iL}'s:

$$A_{1L}=A_{1B}+eg\left[1-\frac{3}{4}\frac{m_1^2-m_2^2+m_\pi^2}{m_1^2}(1-\kappa)\right], \qquad (23a)$$

$$A_{4L}=eg\left[\frac{3}{4}\frac{m_1^2-m_2^2+m_\pi^2}{m_1^2}(1-\kappa)\frac{1}{p_1\cdot \kappa}\right], \qquad (23b)$$

$$A_{iL}=A_{iB} \quad \text{when } i=2,3,5 . \qquad (23c)$$

This is Low's theorem for $\gamma+{}^6\text{Li}\rightarrow{}^6\text{He}+\pi^+$. Observe that Eq. (4b), together with Eqs. (20b) and (20c), state the exact equality of the electric charge of ^{6}Li with the sum of the electric charges of ^{6}He and π^+. The A_{iL}'s are model independent and are expressed in terms of the independently measurable physical quantities: electrical charges, magnetic moment of ^{6}Li, and the on-shell π-^{6}Li-^{6}He coupling constant. If we were to approximate $\mathfrak{M}$ by $\mathfrak{M}_L$, the best place would be at the smallest value of $|\vec{k}|$, i.e., when the final pion is produced at rest (threshold) as seen in Eq. (16). However, there are no good criteria to measure the degree of accuracy of this approximation because there is no definite scale on which we could measure $|\vec{k}|_{\text{th}}$. If we choose m_1, we would conclude that $|\vec{k}|_{\text{th}}/m_1\simeq\mu\simeq 0.025$ is a small number but[12] we could as well choose $R\simeq 2.4$ fm, the charge radius of ^{6}Li. In this last case the parameter of expansion $|k|_{\text{th}}\cdot R\simeq 1.7$ becomes very large. Bypassing this difficulty, we find the following soft photon result for the slope of the differential cross section at threshold:

$$\frac{a_{^6\text{Li}}}{4\pi}=\left[\frac{g^2}{4\pi}6.33\times10^{-3}+O(|\vec{k}|_{\text{th}})\right]\ \mu\text{b/sr} , \qquad (24)$$

where we have used Eqs. (6), (10), (16), and (23a).

To have a partial idea about the size of the neglected terms, we consider at this point the full Born terms ($\overline{B}$) which are obtained from Eqs. (20) after imposing gauge invariance, Eqs. (4a) and (4b). We get:

$$A_{1\overline{B}}=A_{1B}+eg\left[1-\frac{3}{4}\frac{m_1^2-m_2^2+m_\pi^2}{m_1^2}\times(1-\kappa)-\tfrac{3}{2}x\frac{\tau}{m_1^2}(k\cdot q)\right], \qquad (25a)$$

$$A_{5\overline{B}}=A_{5B}-\frac{3(1-x)}{2}\frac{\tau}{m_1^2}, \qquad (25b)$$

$$A_{i\overline{B}}=A_{iB} \quad \text{when } i=2,3,4 . \qquad (25c)$$

x is an unknown parameter in Eqs. (25a) and (25b). The only difference between Eqs. (23a)-(23c) and Eqs. (25a)-(25c) is the inclusion of some terms of order k in $\mathfrak{M}$ proportional to τ/m_1^2, the anomalous quadrupole moment of ^{6}Li.

In this case, the slope of the differential cross section at threshold is found to be proportional to

$$\frac{a_{^6\text{Li}}}{4\pi}=\frac{g^2}{4\pi}(6.33+0.27x+0.003x^2)10^{-3}\ \mu\text{b/sr} , \qquad (26)$$

where we have used Eqs. (6), (10), (11), (16), and (25a). This model-dependent result indicates that the anomalous electric quadrupole moment might drastically change the soft photon result [Eq. (24)]. This is due to the magnitude of τ.

IV. KROLL-RUDERMAN THEOREM

In the previous section we have seen that even at threshold, contributions from $\mathfrak{M}_R$, defined in Eq. (21), might not be negligible [cf. Eq. (26)]. Because the pion has finite mass, it is impossible to take the limit $k\rightarrow 0$ and thus $\mathfrak{M}_L$ is not as accurate as we would like. Indeed, the photon has to provide at least the energy to create the pion mass.

Since at threshold $|\vec{k}|_{\text{th}}=m_\pi[1+O(\mu)]$ [Eq. (16)], we may do a series expansion in terms of m_π of the relevant part of the scattering amplitude A_1. Using

Eqs. (12), (13), (16), (17), (22), and (23a) we find

$$A_1\big|_{\text{th}} = eg(0)[1+O(m_\pi)]\,. \qquad (27)$$

$g(0)$ is the off-shell ^{6}Li-^{6}He-π coupling constant at zero transfer momentum. The slope of the differential cross section at threshold is then obtained from Eqs. (6) and (27):

$$\frac{a_{^6\text{Li}}}{4\pi} = \frac{g^2(0)}{4\pi}\frac{e^2}{4\pi}\frac{1}{12 m_1{}^2}[1+O(m_\pi)]$$

$$= \frac{g^2(0)}{4\pi}[7.5\times 10^{-3} + O(m_\pi)] \quad \mu\text{b/sr}\,. \qquad (28)$$

Equation (27) is the KR theorem. In a world where the pion mass could be neglected, A_1 would be exactly given by Eq. (27).

When the pion photoproduction is on nucleons (spin-$\frac{1}{2}$), the Kroll-Ruderman theorem can also be derived[19] as a consequence of electromagnetic current conservation and analytical properties of the scattering amplitude. If, as for nucleons, this method could be used in photoproduction on ^{6}Li (spin-1), then by using Eqs. (4a), (20a), (20d), and (20e) we could know exactly the invariant function A_1 at the unphysical point $\nu_1 = \nu = 0$. However, independently of problems raised by the compositeness of ^{6}Li, analyticity properties are not as simple as for nucleons and the KR theorem cannot be derived in this way: A_1 is *not* determined at $\nu_1 = \nu = 0$ because it has a simple pole at $\nu = 0$ and its residue is a function of ν_1. We cannot make a series expansion in terms of ν and ν_1 around that point and, within its neighborhood, limits and extrapolations on A_1 must be carefully done.

Up to now we have been interested in photon low energy results. The KR theorem, the exact result in the zero-mass pion limit, has been obtained from the soft photon theorem. In the next section, we will consider the unphysical soft pion limit $q_\mu \to 0$, which implies $q^2 = 0$, $\nu = (m_2{}^2 - m_1{}^2)/2m_2{}^2$, $\nu_1 = 0$. Note that these last three variable values are nearby the mentioned ambiguous point $q^2 = m_\pi{}^2$, $\nu = \nu_1 = 0$.

V. FUBINI-FURLAN-ROSSETTI THEOREM

The matrix elements of the axial vector current in Eq. (14) can be related[2,3,4,7] to the soft pion photoproduction amplitude (i.e., $\mathfrak{M}$ defined in Eq. (3), extrapolated to the unphysical limit $q_\mu = 0$) by means of current algebra and the PCAC hypothesis, or alternatively by using the modified PCAC hypothesis[2] in the presence of electromagnetic interactions; for the latter, we have

$$[\partial_\lambda + ie\mathcal{A}_\lambda(x)]J_5^{-\lambda}(x) = m_\pi{}^2 f_\pi \phi_{\pi^-}(x)\,, \qquad (29)$$

with f_π as defined in Eq. (15). The resulting equation,

$$\lim_{q_\mu\to 0}\mathfrak{M}(q) = \frac{-i}{f_\pi}\lim_{q_\mu\to 0}[e\epsilon^\nu\langle{}^6\text{He}|J_{5\nu}^-(0)|{}^6\text{Li}\rangle + \epsilon^\nu q^\lambda T_{\nu\lambda}]\,, \qquad (30)$$

with $\epsilon^\mu T_{\mu\lambda} = \langle{}^6\text{He}|J_{5\lambda}^-(0)|\gamma{}^6\text{Li}\rangle$, is a supplementary condition imposed on $\mathfrak{M}$.

In the limiting procedure in Eq. (30), we can use the fact that ^{6}He (isospin $I=1$) and ^{6}Li (isospin $I=0$) have different masses. This implies that $\mathfrak{M}(q)$ and $\epsilon^\mu T_{\mu\lambda}$ have no pole at $q_\mu = 0$ and therefore Eq. (30) can be written as

$$\lim_{q_\mu\to 0}\mathfrak{M}(q) = -\frac{ie}{f_\pi}\lim_{q_\mu\to 0}\epsilon^\nu\langle{}^6\text{He}|J_{5\nu}^-|{}^6\text{Li}\rangle\,, \qquad (31)$$

with $p_2 = p_1 + k$. Using Eq. (3) [the A_i's are now functions of q^2: $A_i(q^2) \equiv A_i(q^2,\nu,\nu_1)$] and Eq. (14), we get from Eq. (31)

$$A_1(0,\bar\nu,0) = (e/f_\pi)F_A(0)\,, \qquad (32a)$$

$$A_4(0,\bar\nu,0) = (e/f_\pi)F_R(0)\frac{1}{m_1 m_\pi}\,, \qquad (32b)$$

with $\bar\nu \equiv (m_2{}^2 - m_1{}^2)/2m_1{}^2$. A Golberger-Treiman type relation[12,20] is easily obtained by applying PCAC to Eq. (14):

$$F_A(0) + \frac{m_2{}^2 - m_1{}^2}{2}\frac{1}{m_1 m_\pi}F_R(0) = f_\pi g(0) \qquad (33)$$

or, using Eqs. (32a) and (32b),

$$A_1(0,\bar\nu,0) + \tfrac{1}{2}(m_2{}^2 - m_1{}^2)A_4(0,\bar\nu,0) = eg(0)\,, \qquad (34)$$

with $g(0)$ as defined in Eq. (12).

With an off-shell pion, electromagnetic current conservation cannot any longer be expressed by Eqs. (4a) and (4b) and the Ward identity[21] has to be used, namely the $A_i(q^2,\nu,\nu_1)$'s must satisfy the more general conditions

$$A_1(q^2) + p_1\cdot k A_4(q^2) + q\cdot k A_5(q^2) = e\,g[(k-q)^2]\frac{m_\pi{}^2 - q^2}{m_\pi{}^2 - (q-k)^2}\,, \qquad (35a)$$

$$p_1\cdot k A_2(q^2) + q\cdot k A_3(q^2) = -eg[(k-q)^2]\frac{m_\pi{}^2 - q^2}{m_\pi{}^2 - (q-k)^2}\,. \qquad (35b)$$

In the soft pion limit, the amplitude $\mathfrak{M}(0)$ must satisfy Eqs. (32a), (32b), (35a), and (35b). Observe that Eq. (35a), when $q_\mu = 0$, is identical to Eq. (34). This shows the consistency of our procedure. From Eqs. (32a), (32b), (35a), and (35b), we want to improve the KR result [Eq. (27)] by including the terms of order m_π in the evaluation of

A_1. To proceed further we define

$$\mathfrak{M}(q) \equiv \mathfrak{M}_L(q) + \mathfrak{M}_R(q) ,$$
$$A_i(q^2) \equiv A_{iL}(q^2) + A_{iR}(q^2) . \tag{36}$$

These are a generalization of Eqs. (21) and (22) with the $A_{iL}(q^2, \nu, \nu_1)$'s for the off-shell pion defined as

$$A_{1L}(q^2) = eg(q^2)\left[1 - \frac{3}{2}(1+\kappa)\frac{q\cdot k}{p_1\cdot k} - \frac{3}{4}\frac{m_1^2 - m_2^2 + q^2}{m_1^2}(1-\kappa)\right] , \tag{37a}$$

$$A_{2L}(q^2) = eg(q^2)\left(\frac{2}{p_2\cdot k} - \frac{3}{p_1\cdot k}\right) , \tag{37b}$$

$$A_{3L}(q^2) = eg(q^2)\left[\frac{1}{(q+k)^2 - m_\pi^2} - \frac{2}{p_2\cdot k}\right] , \tag{37c}$$

$$A_{4L}(q^2) = eg(q^2)\left[\frac{3}{4}\frac{m_1^2 - m_2^2 + q^2}{m_1^2}(1-\kappa)\frac{1}{p_1\cdot k}\right] , \tag{37d}$$

$$A_{5L}(q^2) = eg(q^2)\left[-\frac{1}{(q+k)^2 - m_\pi^2} + \frac{3}{2}(1+\kappa)\frac{1}{p_1\cdot k}\right] . \tag{37e}$$

Equations (37a)–(37e) are identical to Eqs. (23a)–(23c) when $q^2 = m_\pi^2$.

Introducing Eqs. (36) and (37a)–(37d) into Eqs. (32a) and (32b), we get

$$A_{1R}(0,\bar\nu,0) = e\left\{\frac{F_A(0)}{f_\pi} - g(0)\left[1 - \frac{3}{4}\frac{m_1^2 - m_2^2}{m_1^2}(1-\kappa)\right]\right\} , \tag{38a}$$

$$A_{4R}(0,\bar\nu,0) = e\left[\frac{F_R(0)}{f_\pi}\frac{1}{m_1 m_\pi} + g(0)\frac{3}{2}\frac{1-\kappa}{m_1^2}\right] . \tag{38b}$$

From the definition of $\mathfrak{M}_L(q)$, it is readily found that

$$A_{1L}(q^2) + p_1\cdot k A_{4L}(q^2) + q\cdot k A_{5L}(q^2) = eg(q^2)\frac{m_\pi^2 - q^2}{m_\pi^2 - (q-k)^2} , \tag{39a}$$

$$p_1\cdot k A_{2L}(q^2) + q\cdot k A_{3L}(q^2) = -eg(q^2)\frac{m_\pi^2 - q^2}{m_\pi^2 - (q-k)^2} . \tag{39b}$$

Then Eqs. (35a), (35b), (36), (39a), and (39b) imply

$$A_{1R}(q^2) + p_1\cdot k A_{4R}(q^2) + q\cdot k A_{5R}(q^2) = e[g((q-k)^2) - g(q^2)]\frac{m_\pi^2 - q^2}{m_\pi^2 - (q-k)^2} , \tag{40a}$$

$$p_1\cdot k A_{2R}(q^2) + q\cdot k A_{3R}(q^2) = -e[g((q-k)^2) - g(q^2)]\frac{m_\pi^2 - q^2}{m_\pi^2 - (q-k)^2} . \tag{40b}$$

Expanding the $A_{iR}(q^2)$'s in a double series in powers of ν and ν_1,[4] we have

$$A_{iR}(q^2) = \sum_{j,k=0} a_{jk}^{(i)}(q^2)\nu^j\nu_1^k + \delta_{i4}\,\tfrac{3}{2}eg(q^2)\frac{\tau}{m_1^2}\frac{\nu_1}{\nu} , \tag{41}$$

with, in particular,

$$a_{00}^{(i)}(q^2) = 0 \quad (i = 1, 2, 3) , \tag{42}$$

$$a_{10}^{(1)}(q^2) = -m_1^2 a_{00}^{(4)}(q^2) , \tag{43}$$

as a consequence of Eqs. (40a) and (40b). The addition to A_{4R} of a term proportional to τ has been discussed in Sec. III. What is left without proof in the present work is the convergence of the series expansion, Eq. (41). For the photoproduction at threshold of a pion on its mass shell ν and ν_1 are of order μ and μ^2, respectively, as seen from Eqs. (17) and (18). Of course, this is not sufficient for the series to converge; besides ν and $\nu_1 \ll 1$, the coefficients $a_{jk}^{(i)}(m_\pi^2)$ must not grow too fast in order for the series to converge.

From Eqs. (41) and (42), we have

$$A_{1R}(0,\bar\nu,0) = \sum_{j=1} a_{j0}^{(1)}(0)\bar\nu^j = a_{10}^{(1)}(0)\bar\nu + O(\mu^4) , \tag{44}$$

$$A_{4R}(0,\bar\nu,0) = a_{00}^{(4)}(0) + \sum_{j=1} a_{j0}^{(4)}(0)\bar\nu^j = a_{00}^{(4)}(0) + O(\mu^2) , \tag{45}$$

since numerically it just happens (see Sec. II) that

$$\bar\nu \equiv \frac{m_2^2 - m_1^2}{2m_1^2} \simeq \mu^2 .$$

Introducing Eqs. (38b) and (43) into Eq. (44), we have

$$a_{10}^{(1)}(0) = -\tfrac{3}{2}eg(0)(1-\kappa) - e\frac{F_R(0)}{f_\pi\mu} + O(\mu^2) , \tag{46}$$

which is the Fubini-Furlan-Rossetti theorem[7] for the pion photoproduction on ^{6}Li.

Now we are in position to give the threshold value for A_1 up to and including terms of order m_π. Using Eqs. (17), (18), (22), (23a), and (41) we get

$$A_1(m_\pi^2, \nu|_{\rm th}, \nu_1|_{\rm th}) = eg(m_\pi^2)[1-\tfrac{3}{2}(1+\kappa)\mu] + a_{10}^{(1)}(m_\pi^2)\mu + O(m_\pi^2), \quad (47)$$

which becomes, with the help of Eq. (46),

$$A_1|_{\rm th} = eg(0)(1-3\mu) - e\frac{F_R(0)}{f_\pi} + O(m_\pi^2), \quad (48)$$

to be compared with Eq. (27), the KR result. $F_R(0)$ is not known. However, from Eq. (46) we find $F_R/f_\pi = O(\mu)$. Since $g(0) \simeq 150$ (see below) we can neglect F_R/f_π, and we have from Eqs. (6) and (48) that

$$\frac{a_{^6\rm Li}}{4\pi} = \frac{g^2(0)}{4\pi}\frac{e^2}{4\pi}\frac{1}{12m_1^2}[1-8\mu+O(m_\pi^2)] = \frac{g^2(0)}{4\pi}[6.0\times10^{-3}+O(m_\pi^2)]\ \mu\text{b/sr}. \quad (49)$$

VI. DISCUSSION

Recently, a Louvain-Saclay collaboration[10] has made a measurement of pion photoproduction at threshold. They find

$$a_{^6\rm Li} = (0.073\pm0.002)a_p, \quad (50)$$

where $a_{^6\rm Li}$ was defined in Eq. (6) and a_p is defined by

$$\frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma_p}{d\Omega} \equiv \frac{a_p}{4\pi}.$$

Here σ_p is the cross section of the reaction $\gamma + p \to \pi^+ + n$ and from Ref. 22 we have

$$a_p/4\pi = 15.4\pm0.5\ \mu\text{b/sr}. \quad (51)$$

This gives

$$a_{^6\rm Li}/4\pi \simeq 1.12\ \mu\text{b/sr}. \quad (52)$$

Neglecting terms of order k and higher in Eq. (24), and taking the experimental result [Eq. (52)] into account, one gets

$$g_L^2/4\pi = 177. \quad (53)$$

The index L has been introduced to indicate the theoretical ingredients [Low's theorem and truncation of the series in $k|_{\rm th}$] used in the estimate of the on-shell ^{6}Li-^{6}He-π coupling constant.

As we see in Sec. III, the ^{6}Li electric quadrupole moment contribution could modify appreciably this estimate of $g^2/4\pi$. Recalling that the undetermined parameter x in Eqs. (25a) and (26) is a measure of the strength of the anomalous electric quadrupole moment τ, we find for different values of x:

$$g_\tau^2(0)/4\pi = g_L^2/4\pi = 177,$$
$$g_\tau^2(1)/4\pi = 170, \quad g_\tau^2(-1)/4\pi = 184, \quad (54)$$
$$g_\tau^2(2)/4\pi = 162, \quad g_\tau^2(-2)/4\pi = 193.$$

We have used Eqs. (26) and (52) to compute these values. $g_\tau(x)$ is the estimate of ^{6}Li-^{6}He-π coupling constant obtained for a given value of the parameter x. Let us mention at this point that, in the case of the reaction $\gamma+p\to\pi^+ +n$, if one uses just the terms obtained from Low's theorem and the experimental value for the p-n-π coupling constant, one gets[4]

$$\left.\frac{a_p}{4\pi}\right|_L = 15.7\ \mu\text{b/sr},$$

in very good agreement with the experimental value given in Eq. (51). Observe that Eqs.(23a)–(23c) or Eqs. (25a)–(25c) contain more than the Born diagrams (Fig. 1) even if we have not considered other type of contributions (excited states, break-up, rescattering terms, etc.) directly. Indeed, electromagnetic current conservation forces us to take in account, partially at least, additional contributions.

If we want to use soft pion results, we must know the numerical value of $g(0)$, defined in Eq. (12). For that purpose[12] use is made of the ^{6}He β-decay data and the Goldberger-Treiman (GT) relation given in Eq. (33). We find

$$\frac{g^2(0)_{\rm GT}}{4\pi} \simeq \frac{1}{4\pi}\left[\frac{F_A(0)}{f_\pi}\right]^2 = 1445. \quad (55)$$

As we have shown in the last section, $F_R(0)/f_\pi = O(\mu)$ or $F_R(0) = O(m_\pi^2/m_1)$, since $f_\pi \simeq 0.96m_\pi$ as obtained from the decay width of $\pi\to\mu+\nu$. Observe that Delorme in Ref. 12 concluded that $F_R \simeq O(m_\pi)$, using an impulse approximation calculation. Introducing Eq. (55) into Eq. (28)—the KR result—we have

$$\frac{a_{^6\rm Li}}{4\pi} = [11.2+O(m_\pi)]\ \mu\text{b/sr (KR)}. \quad (56)$$

Furthermore, using Eq. (49) we get the value of $a_{^6\rm Li}$ according to the KR and FFR theorems:

$$\frac{a_{^6\rm Li}}{4\pi} = [8.7+O(m_\pi^2)]\ \mu\text{b/sr (KR+FFR)}. \quad (57)$$

Comparing Eqs. (56) and (57) with the experimental result given by Eq. (52), we conclude that terms of order m_π^2 and higher in the scattering amplitude *must be very important*, but there is no model-independent way to compute them. Notice that our estimate of the anomalous electric quadrupole moment contribution in Sec. III already showed that terms of order m_π^2 might be important.

However, if following Ref. 4 we assume the simplest extrapolation between $q^2=0$ and $q^2=m_\pi^2$, i.e., that Eq. (46) is valid at the physical point $q^2=m_\pi^2$,

$$a_{10}^{(1)}(m_\pi^2)=-\tfrac{3}{2}eg(m_\pi^2)(1-\kappa)-e\frac{F_R(m_\pi^2)}{f_\pi\mu}+O(\mu^2)\,, \tag{58}$$

we obtain

$$A_1\big|_{\rm th}=eg(m_\pi^2)(1-3\mu)-e\frac{F_R(m_\pi^2)}{f_\pi}+O(\mu^2)\,. \tag{59}$$

If we neglect the terms proportional to $F_R(m_\pi^2)$ and of order μ^2 in Eq. (58) and use Eqs. (6) and (52), we get

$$g_E^2/4\pi=186\,, \tag{60}$$

where g_E is the on-shell ^{6}Li-^{6}He-π coupling constant obtained from (1) the Louvain-Saclay collaboration result[10] [Eq. (52)], (2) the KR and FFR results and (3) the simplest extrapolation from $q^2=0$ to $q^2=m_\pi^2$ (Eq. 58) compatible with electromagnetic current conservation and Low's theorem. This last assumption, if valid, would indicate that the $A_1(q^2, \nu, \nu_1)$ amplitude has a strong variation in magnitude from $q^2=0$ to $q^2=m_\pi^2$. A_1 would then vary proportionately with $g(q^2)$. In this case, the expansion parameter for $g(q^2)$ and $a_{jk}^{(i)}(q^2)$, Eq. (11), would be $m_\pi^2R^2\simeq(1.7)^2$, as suggested in Ref. 12, rather than $\mu^2\simeq(0.025)^2$, where $R\simeq 2.43$ fm is the charge radius of ^{6}Li.

Comparing our results with previous works[1,8,11,12] that use soft pion theorems, we see that the main difference is that we satisfy electromagnetic current conservation and Low's theorem at each step of our work while previous treatments are taking little care of it. Take for example our Ref. 1 where in Eq. (C7), p. 121, the second term in the curly bracket is dropped without real justification. This second term is a local object, $[J_\mu^{\rm em}(0), \int d^3x\,\partial_\mu J_5^{-\mu}(\vec{x},0)]$, sandwiched between initial and final nuclei and depends just like the first term on the size of the nucleus. Therefore it cannot be dropped. Indeed the relations between threshold photoproduction and the off-shell coupling constant $g[(q-k)^2\simeq -m_\pi^2]$, obtained in Refs. 1, 8, 11, and 12 violate electromagnetic current conservation and Low's theorem.

In conclusion, we see that the main information which can be extracted from the Louvain-Saclay experiment[10] is an estimate of the on-shell ^{6}Li-^{6}He-π coupling constant

$$g^2/4\pi\simeq 150\text{–}200 \quad \text{if } -2\leqslant x\leqslant 2\,,$$

as we can see from Eqs. (53), (54), and (60), to be compared with the on-shell N-N-π coupling constant $g_{\pi NN}$[23]

$$g_{\pi NN}^2/4\pi\simeq 14.8\pm 0.3\,.$$

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