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π^+ + nucleus → K^+ + hypernucleus in the impulse approximation

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The impulse approximation and some crude nuclear and hypernuclear wave functions are used to calculate cross sections for some reactions of the type π^+ + nucleus → K^+ + hypernucleus for incident π^+ laboratory energies in the range 0.55–1.0 GeV. Target nuclei considered were ^{4}He, ^{7}Li, and ^{119}Sn. Cross sections (or cross sections per nucleon) were calculated for production of the ground states and some possible excited states of the resultant hypernuclei. Although some results are marginal, on the whole the calculated cross sections appear to be too small for this to be a useful production method for hypernuclei at the pion energies available at present meson factories.

[NUCLEAR REACTIONS Production of hypernuclei, π^+ on ^{4}He, ^{7}Li, ^{119}Sn, T_π =0.55–1.0 GeV.]

I. INTRODUCTION

The purpose of the present paper is to estimate the cross sections for reactions of the form

$$\pi^+ + \text{nucleus} \rightarrow K^+ + \text{hypernucleus} \tag{1}$$

for incident pion laboratory kinetic energies in the range 0.55–1.0 GeV. The motivation for the lower end of the pion energy range is to see if production of hypernuclei using the P^3 pion beam at Clinton P. Anderson Meson Physics facility (LAMPF)[1] might be possible. This beam is the only "meson factory" (i.e., high intensity) pion beam with an energy high enough to make reaction (1) energetically possible. Calculations were done at higher incident pion energies for comparison and to see what might result in the event of a linac being used in the P^3 channel.

Since this work is of an exploratory nature being merely an attempt to obtain an order of magnitude estimate for the differential and total cross sections for the reactions (1), only very crude models were used. The impulse approximation[2] was used to express the T matrix elements for the reaction (1) in terms of the t matrix elements for associated production $\pi^+n \rightarrow K^+\Lambda$ from a free neutron, which was considered as resulting from the exchange of a K^* meson.[3] Simple models for the target initial state nuclear wave function and final state hypernuclear wavefunction were used to reduce the T matrix element to a tridimensional integral over $d\vec{p}_n$($\vec{p}_n$ being the struck neutron momentum in the laboratory system) of the product of the nuclear wave function, the hypernuclear wavefunction and the $\pi^+n \rightarrow K^+\Lambda$ t matrix element. The integral was evaluated by pulling the t matrix element out of the integral and evaluating it with the value for $\vec{p}_n$ given by Sidorov[4] obtained by assuming the energy transferred to the hypernucleus is received by the Λ; namely, $\vec{p}_n = -[1-r]\vec{p}_y$, where $\vec{p}_y$ is the momentum of the hypernucleus in the laboratory and r is the square root of the ratio of the Λ mass to the mass of the hypernucleus.

The normalization of E/m particles cm^{-3} was used to express the angular distributions $d\sigma/d\Omega_K$ of the emergent kaons as

$$\frac{d\sigma}{d\Omega_K} = \frac{10^4\hbar^2 m_\pi m_K M_Y}{4\pi^2 p_\pi} \frac{p^2_K}{(E_\pi + M_A)p_K - E_K p_\pi \cos\theta_K} \times \sum_{s,s'} |T_{fi}|^2 \tag{2}$$

in the laboratory system. Here m_π, m_K, M_A, and M_Y denote the masses of the incident pion, emergent kaon, target nucleus, and hypernucleus, respectively, in MeV, p_π, p_K, E_π, and E_K denote the momenta and energies of the pion and kaon in MeV/c, respectively; θ_K is the angle between the pion and kaon momenta; $\sum_{s,s'}$ represents the average over the initial spin of the target nucleus and final spin of the hypernucleus and T_{fi} is the matrix element $T_{fi} = \langle K^+ {}^A_\Lambda Z | \hat{T} | \pi^+ {}^A Z\rangle$ of the transition operator $\hat{T}$ between an initial state consisting of a free π^+ meson and a free target nucleus ^AZ and a final state consisting of a free K^+ meson and a free hypernucleus $^A_\Lambda Z$. With these units $d\sigma/d\Omega_K$ as given by Eq. (2) is in μb/sr.

The differential and total cross sections were calculated for the target nuclei ^{4}He, ^{7}Li, and ^{119}Sn. The details of the calculations are discussed in Secs. II, III, and IV, respectively. A discussion of the results and the conclusions obtained in the present work are presented in Sec. V.

II. ^{4}He TARGET

The reason for starting with the ^{4}He target is twofold: the calculations are simple and this is so far the only example of a particle stable hypernuclear excited state. The problem still present[5] for ascribing the γ-ray lines of 1.09 MeV and 1.42 MeV to ${}^4_\Lambda$He* or ${}^4_\Lambda$H* makes it important to look at the possibility of producing excited states ${}^4_\Lambda$He*.

Using the impulse approximation, the T matrix for the reaction $\pi^+ + {}^4\text{He} \to K^+ + {}^4_\Lambda\text{He}$ can be expressed as

$$T_{fi} = 2\langle K^+ {}^4_\Lambda\text{He} | \hat{t} | \pi^+ {}^4\text{He}\rangle\,, \tag{3}$$

where the factor 2 arises from the fact that after antisymmetrizing the wave functions only the production on one single neutron must be considered.

A Gaussian form was assumed for the ^{4}He ground state wave function; i.e.,

$$\psi_\alpha = N \exp\left(-\tfrac{1}{2}\beta \sum_{i=2<j}^{5} r_{ij}^2\right) X_\alpha\,, \tag{4}$$

where $r_{ij} = |\vec{r}_i - \vec{r}_j|$ are the internuclear distances, $\beta = 9/32R_\alpha^2$ is determined by fitting the electron-helium scattering experiments,[6] X_α is the singlet spin function appropriate for the $J^\pi = 0^+$ ground state of ^{4}He, and N is the normalization constant.

The final ${}^4_\Lambda$He wave function was assumed to have the form

$$\psi_\Lambda = N' u(\vec{r}_3, \vec{r}_4, \vec{r}_5) v(|\vec{r}_2 - \tfrac{1}{3}(\vec{r}_3 + \vec{r}_4 + \vec{r}_5)|) X_\Lambda \tag{5}$$

by considering the process $\pi^+ n \to K^+ \Lambda$ as occurring on neutron No. 2. Here X_Λ is the singlet spin function appropriate to the $J^\pi = 0^+$ ground state of ${}^4_\Lambda$He and N' is the normalization constant. The ${}^4_\Lambda$He* wave function was assumed to have this same form but with $J^\pi = 1^+$ so that X_Λ was a triplet spin function. The function "u" is a correlation wave function for the core nucleus ^{3}He and was assumed to have a Gaussian form:

$$u = \exp\left(-\tfrac{1}{2}\alpha \sum_{i=3<j}^{5} r_{ij}^2\right), \tag{6}$$

whose parameter α, given by $\alpha = 1/3R_3^2$, is fixed by fitting the spin averaged rms nuclear core radius. The function "v" is a variational wave function for the Λ-^{3}He correlation, whose form was taken to be the same as that of von Hippel[7]:

$$v(r) = \exp(-a_1 r^2) + C \exp(-a_2 r^2)\,. \tag{7}$$

The parameters a_1, a_2, and C were determined by a variational calculation of either the known binding energy of the ${}^4_\Lambda$He ground state, or the assumed binding energies for the possible ${}^4_\Lambda$He excited states.

These assumed simple forms for the wave functions allowed all the integrations to be performed analytically. For ${}^4_\Lambda$He production the following expression resulted for the matrix element T_{fi} in the laboratory system

$$T_{fi} = 2(2\pi)^{-9} N N' [128\pi^7/\beta(4\beta + 3\alpha)^2]^{3/2} \{(12\pi^2\beta/3\beta + 2a_1)^{3/2} \exp[(3m/3m + m_\Lambda)^2 (3\beta/3\beta + 2a_1 - 1) q^2/4a_1] + C(12\pi^2\beta/3\beta + 2a_2)^{3/2} \exp[(3m/3m + m_\Lambda)^2 (3\beta/\beta + 2a_2 - 1) q^2/4a_2]\} A\,. \tag{8}$$

Here A is the spin independent part of $t = A + B\sigma \cdot \hat{n}$, and q is the magnitude of the momentum transfer $\vec{q} = \vec{p}_\pi - \vec{p}_K$. In the case of ${}^4_\Lambda$He* production only the spin-dependent part B contributes to the cross section. The explicit expressions for A and B using the exchange of a K^* meson are given by Hoff.[8]

Figures 1 and 2 show the angular distributions of the K^+ mesons and Table I shows the total cross sections for the production of the ground state and of the excited states of ${}^4_\Lambda$He with $B_\Lambda = 1.33$ and 0.33 MeV, respectively.

III. ^{7}Li TARGET

The choice of the ^{7}Li target is of interest due to the fact that special attention should be paid to the cross sections for producing the $\frac{1}{2}$ ground state and the $\frac{3}{2}$ excited state of ${}^7_\Lambda$Li, since the difference in binding energies of these two states gives a direct measure of the spin dependence of the Λ-N interaction.[9]

TABLE I. Total cross sections for the production of the ${}^4_\Lambda$He ground state and the $B_\Lambda = 1.33$; 0.33 MeV ${}^4_\Lambda$He excited states.

T_π (MeV)	σ (μb) ${}^4_\Lambda$He ground state	σ (μb) $B_\Lambda = 1.33$ MeV	σ (μb) $B_\Lambda = 0.33$ MeV
615	0.11	2.17×10^{-5}	9.16×10^{-6}
620	0.15	6.35×10^{-5}	2.93×10^{-5}
630	0.22	2.01×10^{-4}	1.13×10^{-4}
650	0.46	6.60×10^{-4}	3.89×10^{-4}
700	1.35	2.90×10^{-3}	1.76×10^{-3}
800	3.70	1.21×10^{-2}	6.28×10^{-3}
1000	7.50	2.94×10^{-2}	1.94×10^{-2}

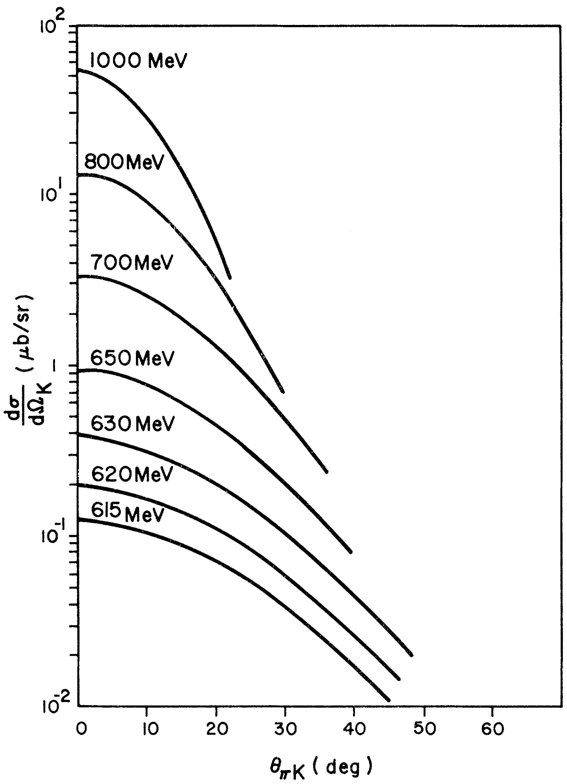

FIG. 1. Angular distributions $d\sigma/d\Omega_K$ for the reaction $\pi^+ + {}^4He \to K^+ + {}^4_\Lambda He$.

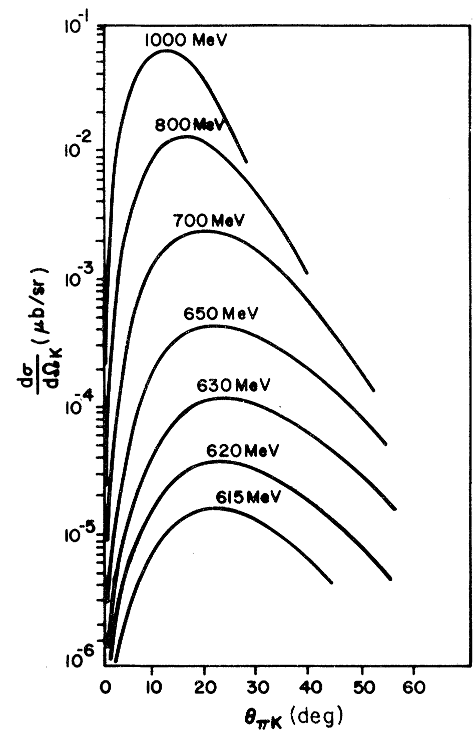

FIG. 2. Angular distributions $d\sigma/d\Omega_K$ for the production of ${}^4_\Lambda He^*$ with $B_\Lambda = 1.33$ MeV.

The first simple model considered for the ^{7}Li target was that of a neutron with which the π^+ interacts plus an inert ^{6}Li core. No antisymmetrization with respect to identical nucleons was carried out, the production of the ${}^7_\Lambda$Li hypernucleus being calculated under the assumption that the process $\pi^+ n \to K^+\Lambda$ occurs on one single neutron which is distinguishable with respect to the other neutrons in the ^{6}Li core. The n-^{6}Li interaction was assumed to be represented by a square well of depth D and radius R_0 and the internal ${}^7_\Lambda$Li wave function was written

$$\psi_i(\vec{R}) = A_S j_1(\alpha R) \sum_{M_L=-1}^{+1} C(1\tfrac{1}{2}\tfrac{3}{2}; M-M_L, M_L) X_{nc}(\tfrac{1}{2}; M-M_L)\, Y_1^{M_L}(\hat{R}), \qquad R \leq R_0$$

$$= B_S h_1^{(1)}(i\beta R) \sum_{M_L=-1}^{+1} C(1\tfrac{1}{2}\tfrac{3}{2}; M-M_L, M_L) X_{nc}(\tfrac{1}{2}; M-M_L)\, Y_1^{M_L}(\hat{R}), \qquad R \geq R_0 . \tag{9}$$

Here R denotes the n-^{6}Li relative coordinate, $\alpha = [2\mu(D-B_n)/\hbar^2]^{1/2}$, and $\beta = (2\mu B_n/\hbar^2)^{1/2}$, where B_n is the separation energy necessary to remove the neutron from the ^{7}Li nucleus and μ is the reduced mass for the n-^{6}Li relative motion. The ^{6}Li spin 1 and the neutron spin $\frac{1}{2}$ have been coupled to form a spin of $\frac{1}{2}$, represented in Eq. (9) by the neutron-core spin function X_{nc}. The parameters $D = 56.9$ MeV and $R_0 = 2.33$ fm were determined by fitting the rms radius of ^{7}Li[10] and the neutron separation energy of ^{7}Li. Another simple model used for the ^{7}Li nucleus was of $\alpha + d + n$ cluster model.[11] The ^{7}Li wave function was written

$$\psi_i = N \exp(-\alpha s^2/2) \exp(-\beta t^2/2) \exp\left(-\frac{\gamma}{2} \sum_{i=2<j}^{5} r_{ij}^2\right) z \exp(-\delta z^2/2) \sum_{M_L=-1}^{+1} C(1\tfrac{1}{2}\tfrac{3}{2}; M-M_L, M_L) X_{nc}(\tfrac{1}{2}; M-M_L) Y_1^{M_L}(\hat{z}), \tag{10}$$

where r_{ij} is the distance between the ith nucleon and the jth nucleon in the α cluster, "s" is the relative n-p distance in the deuteron cluster, t is the α-d relative coordinate and z is the n-^{6}Li relative coordinate. The parameters α, β, and γ are given by Kopaleishvili *et al.*[12] from an analysis of the ^{6}Li nucleus with an $\alpha+d$ cluster model. The parameter δ can be expressed in terms of the rms radius of ^{7}Li by $\delta=90/49R^2_7$ and it is determined by fitting the experimental value[10] of R_7.

Similar wave functions were used for the hypernucleus $_\Lambda$Li^7. First, the Λ-^{6}Li relative motion was assumed to be represented by a square well of depth D' and radius R_0', and the $^7_\Lambda$Li wave function was written

$$\psi_f(\vec{R})=A_s'j_0(\alpha'R')\sum_{m'}C(1\tfrac{1}{2}\tfrac{1}{2};m-m',m')X_c(1;m-m')\,X_\Lambda(\tfrac{1}{2};m'),\quad R'\le R_0'$$

$$=B_s'h_0^{(1)}(i\beta'R')\sum_{m'}C(1\tfrac{1}{2}\tfrac{1}{2};m-m',m')X_c(1;m-m')X_\Lambda(\tfrac{1}{2};m'),\quad R'\ge R_0' \tag{11}$$

with R' denoting the Λ-^{6}Li relative coordinate, $\alpha'=[2\mu'(D'-B_\Lambda)/\hbar^2]^{1/2}$, and $\beta'=(2\mu'B_\Lambda/\hbar^2)^{1/2}$, where μ' is the reduced mass for the Λ-^{6}Li relative motion and $B_\Lambda=5.57$ MeV[13] is the separation energy necessary to remove the Λ particle from the ground state of the $^7_\Lambda$Li hypernucleus.

The determination of the parameters D' and R_0' was carried out in a little different way, since no experimental value for the rms radius of $^7_\Lambda$Li is available. A variational calculation was done for B_Λ in the $^7_\Lambda$Li hypernucleus assuming the Λ-^{6}Li interaction to be represented by a rigid potential taken to have the shell model form.[14] A trial function e^{-aR^2} was used and the parameter "a" was determined by fitting the binding energy of the $^7_\Lambda$Li hypernucleus. The relation $\langle R^2\rangle^{1/2}=(3/4a)^{1/2}$ was then used together with the S-wave boundary condition relation to determine the parameters $D'=22.79$ MeV and $R_0'=2.30$ fm of the Λ-^{6}Li square well.

The $\alpha+d+\Lambda$ cluster model for the hypernucleus $^7_\Lambda$Li was also used to write

$$\psi_f=N'\exp(-\alpha s^2/2)\exp(-\beta t^2/2)\exp\left(-\tfrac{1}{2}\gamma\sum_{i=2<j}^{5}r_{ij}^2\right)\exp(-az'^2)\sum_{m'}C(1\tfrac{1}{2}\tfrac{1}{2};m-m',m')X_c(1,m-m')X_\Lambda(\tfrac{1}{2},m') \tag{12}$$

with z' denoting the Λ-^{6}Li relative coordinate. In the $^7_\Lambda$Li* excited state, the spin coupling is given by

$$\sum_{m'}C(1\tfrac{1}{2}\tfrac{3}{2};m-m',m')X_c(1,m-m')X_\Lambda(\tfrac{1}{2},m').$$

Using the square well wave functions, the following expression for the matrix element T_{fi} in the laboratory system was obtained

$$T_{fi}=(\tfrac{3}{4})^{1/2}(2\pi)^{-3}\langle X_\Lambda;S'|\hat{t}|X;S\rangle I(q), \tag{13}$$

where $I(q)$ is a function of the magnitude of the momentum transfer obtained by integration over the magnitude p_R and cosine of the polar angle μ_R of the Λ-^{6}Li relative momentum.

The double integration in Eq. (13) was done numerically by introducing the transformations $\mu_R=2\xi-1$ and $p_R=p_0\eta/(1-\eta)$ to reduce the integrations to the interval 0–1. Then a subroutine was used to divide the interval into N subintervals and perform a three-point Gaussian quadrature over each subinterval. A value of $N=30$ provided results accurate to within 1%. All the numerical work was performed on the XDSΣ7 at the University of Wyoming Computer Services Center.

Using the cluster wavefunctions, all the integrations were performed analytically and the matrix element T_{fi} in the laboratory system was calculated to be (with $m_n=m$)

$$T_{fi}=-\pi iNN'(\pi^4/2^6\gamma^3\alpha\beta a)^{3/2}[6\pi^3a/\delta^4(\delta+2a)]^{1/2}$$
$$\times\frac{3mq}{a(6m+m_\Lambda)}\exp\left[-\left(\frac{6m}{6m+m_\Lambda}\right)^2\frac{q^2}{4a}(1+\delta/\delta+2a)\right]$$
$$\times\langle X_\Lambda;S'|\hat{t}|X;S\rangle\,. \tag{14}$$

The average over spins were calculated in a straightforward way and the following expressions were obtained

$$\sum_{s,s'}|\langle X_\Lambda;s'|\hat{t}|X;S\rangle|^2=\tfrac{1}{3}|A|^2+\tfrac{1}{27}|B|^2,\quad ^7_\Lambda\text{Li ground state},$$
$$=\tfrac{8}{27}|B|^2,\quad ^7_\Lambda\text{Li excited state}. \tag{15}$$

TABLE II. Total cross sections, per neutron, for the production of the $B_\Lambda = 5.57$; 4.37, and 0.77 MeV states of ${}^7_\Lambda$Li with the square well (SW) model and with the cluster (αdn) model.

	σ (μb) $B_\Lambda = 5.57$ MeV		σ (μb) $B_\Lambda = 4.37$ MeV		σ (μb) $B_\Lambda = 0.77$ MeV	
T_π (MeV)	SW	αdn	SW	αdn	SW	αdn
580	0.11	0.07	1.20×10^{-4}	2.36×10^{-5}	1.04×10^{-5}	0.96×10^{-6}
600	0.27	0.24	2.90×10^{-4}	1.60×10^{-4}	8.08×10^{-5}	1.41×10^{-5}
620	0.44	0.54	1.08×10^{-3}	4.57×10^{-4}	2.16×10^{-4}	5.75×10^{-5}
650	0.74	1.17	2.20×10^{-3}	1.39×10^{-3}	5.17×10^{-4}	1.88×10^{-4}
700	1.34	2.76	4.50×10^{-3}	3.88×10^{-3}	1.16×10^{-3}	7.54×10^{-4}
800	2.23	6.52	9.54×10^{-3}	1.25×10^{-2}	2.95×10^{-3}	2.71×10^{-3}
1000	3.84	16.31	1.82×10^{-2}	3.26×10^{-2}	7.79×10^{-3}	1.45×10^{-2}

Figure 3 shows the angular distributions $d\sigma/d\Omega_K$, per neutron, for the production of the ${}^7_\Lambda$Li ground state, obtained with the square well model. Table II shows the total cross sections, per neutron, for the production of the ${}^7_\Lambda$Li ground state and the ${}^7_\Lambda$Li* excited state for a Λ separation energy of 4.37 and 0.77 MeV, respectively.

IV. ^{119}Sn TARGET

The calculation of the cross sections for producing the ${}^{119}_\Lambda$Sn hypernucleus was undertaken due to the importance of obtaining examples of heavy hypernuclei, since only light hypernuclei up to $A=15$ have been identified experimentally. If the value of B_Λ were known for several specific heavy hypernuclei, instead of the usual extrapolation of B_Λ to the limit $A \to \infty$ to obtain D (the well depth of Λ in nuclear matter), it would be possible to obtain a two-parameter fit to the B_Λ versus $A^{-2/3}$ curve, the second parameter being related to the effective mass of the Λ[9].

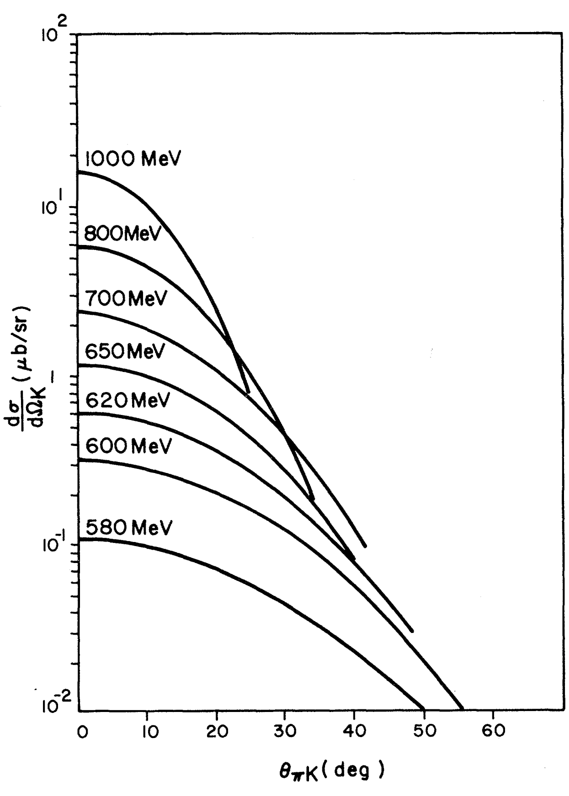

FIG. 3. Angular distributions $d\sigma/d\Omega_K$ for the reaction $\pi^+ + {}^7\text{Li} \to K^+ + {}^7_\Lambda\text{Li}$ with a square well model.

Again a square well model was assumed to represent the n-^{118}Sn relative motion and the internal ^{119}Sn wave function was written

$$\begin{aligned} \psi_i(R) &= A_S j_0(\alpha R) X_n(\tfrac{1}{2}, S), \quad R \le R_0, \\ &= B_S h_0^{(1)}(i\beta R) X_n(\tfrac{1}{2}, S), \quad R \ge R_0, \end{aligned} \tag{16}$$

where all the analogous relevant quantities involved here were already described in the discussions following Eqs. (9) and (11). The parameters $D = 23.38$ MeV and $R_0 = 9.35$ fm of the square well were determined by fitting the rms radius of ^{119}Sn, assumed to be given by $R = 1.3 \times (119)^{1/3}$ fm and the neutron separation energy of ^{119}Sn. A similar wave function was used for the Λ-^{118}Sn relative motion, and the parameters were determined by assuming $B_\Lambda = 30$ MeV for the ${}^{119}_\Lambda$Sn ground state and 6 fm for the rms radius of ${}^{119}_\Lambda$Sn.

The matrix element T_{fi} for the reaction $\pi^+ + {}^{119}\text{Sn} \to K^+ + {}^{119}_\Lambda\text{Sn}$ in the laboratory was then calculated from equations identical to those for the n-^{6}Li square well model but with the $\frac{3}{4}^{1/2}$ in Eq. (13) replaced by $\frac{1}{2}$. For simplicity only the production of excited S states was calculated. The average over spin thus yielded in all cases consider

$$\sum_{S,S'} |\langle X_\Lambda; S' | \hat{t} | X; S\rangle|^2 = |A|^2 + |B|^2 . \tag{17}$$

Table III shows the total cross sections per neutron for the production of the $1s$ ground state and the $2s$, $3s$, and $4s$ excited states. The angular distribution $d\sigma/d\Omega_k$ per neutron for ${}^{119}_\Lambda$Sn ground state production is shown in Fig. 4.

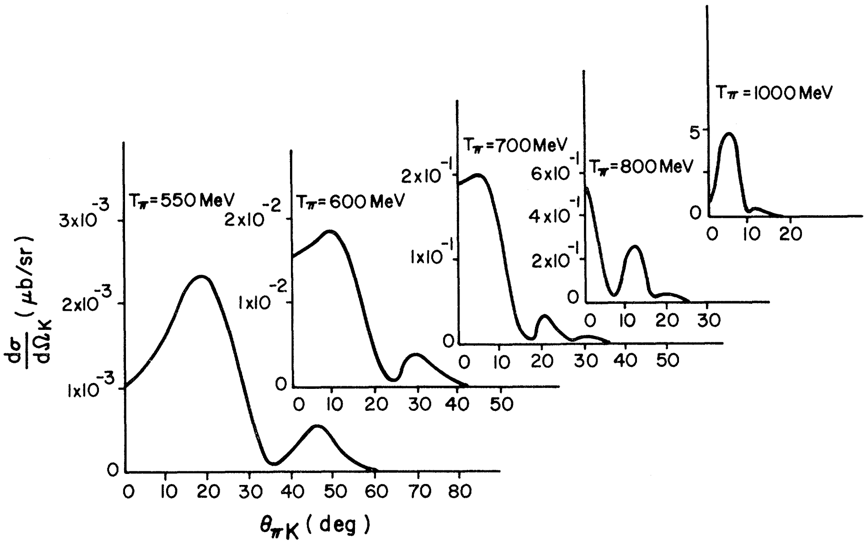

FIG. 4. Angular distributions $d\sigma/d\Omega_K$ for the reaction $\pi^+ + {}^{119}\mathrm{Sn} \to K^+ + {}^{119}_{\Lambda}\mathrm{Sn}$.

V. DISCUSSION

Figure 1 shows that in the case of production of the ${}^4_\Lambda$He ground state, the majority of the K^+ mesons are emitted at small angles, and with increasing energy of the incident pions the cone about the forward direction becomes narrower. The angular distributions in the ${}^4_\Lambda$He excited state production (Fig. 2) have the peak moved to about 20°. The total cross sections for producing the ${}^4_\Lambda$He ground state are low, being of the order of tenths of a microbarn in the LAMPF π^+ energy range (615–650 MeV). The total cross sections for producing ${}^4_\Lambda$He excited states are ~10^3 times smaller in the LAMPF region and ~500 times smaller for energies 700–1000 MeV. Obviously, a different model for the $\pi^+ n \to K^+\Lambda$ interaction could introduce some changes in the relative ${}^4_\Lambda$He*/${}^4_\Lambda$He production rate, but the total rate for producing the ${}^4_\Lambda$He hypernucleus is expected to remain at the same order of magnitude. Considering the difficulties inherent with a ^{4}He target, the actual situation at LAMPF is not satisfactory for the production of the ${}^4_\Lambda$He hypernucleus.

For the ^{7}Li target, again the emergent kaons are forward peaked in the ${}^7_\Lambda$Li ground state production and peaked at about 30° in the ${}^7_\Lambda$Li* production, following the shapes of the "A" and "B" parts of the $\pi^+ n \to K^+\Lambda$ interaction. The total cross sections are still of the order of tenths of microbarns, in the LAMPF π^+ energy range. Since the calculations involved here are the cross sections per neutron, and at best the contributions of the four neutrons in the ^{7}Li target add coherently the total cross section could be as large as 16 times the values given in Table II. The feasibility of producing ${}^7_\Lambda$Li in this way cannot be ruled out. The ${}^7_\Lambda$Li/${}^7_\Lambda$Li* relative production rate is in the range 1000–300 for T_π = 580–650 MeV (LAMPF region) and 300–200 for T_π = 700–1000 MeV. But now not only is this ratio dependent on the model assumed for the $\pi^+ n \to K^+\Lambda$ interaction but also this ratio is dependent on the particular coupling scheme used to write Eqs. (9) and (10). Furthermore, the inert core plus struck neutron model of ^{7}Li, represented here by a neutron in a square well or an αdn system, is clearly a cruder approximation than the Gaussian wave function assumed for the ^{4}He target. Even maintaining the coupling scheme used here, the average over spins would change if the Sidorov approximation[4] is avoided, since all components Y_1^M in $\psi_i(\vec{R})$ could contribute to T_{fi} instead of only Y_1^0.

The total cross sections for producing the ${}^{119}_\Lambda$Sn are even smaller, but again it should be noted that only cross sections per neutron were calculated in the present work. One positive thing with this

TABLE III. Total cross sections, per neutron, for the production of the $1s$ ${}^{119}_\Lambda$Sn ground state and the $2s$, $3s$, and $4s$ ${}^{119}_\Lambda$Sn excited states.

T_π (MeV)	σ_{1s} (μb)	σ_{2s} (μb)	σ_{3s} (μb)	σ_{4s} (μb)
550	1.9×10^{-3}	1.6×10^{-2}	2.4×10^{-2}	4.8×10^{-2}
600	6.8×10^{-3}	2.3×10^{-2}	5.5×10^{-2}	1.4×10^{-1}
700	3.3×10^{-2}	0.9×10^{-1}	7.0×10^{-1}	2.3×10^{1}
800	4.8×10^{-2}	2.3×10^{-1}	2.2×10^{1}	1.2×10^{2}
1000	3.4×10^{-1}	2.1×10^{1}	1.2×10^{2}	2.7×10^{2}

target is that the production of excited states is favored with respect to the ground state production.

Although the calculations involved in this work are crude, at least these estimates show that the cross sections for producing hypernuclei through the reaction π^+ + nucleus → K^+ + hypernucleus are in general small. Obviously, an increase in the incident π^+ energy to the GeV region would make this method a very practicable one. The actual situation at LAMPF (pion energy <660 MeV) does not appear to favor the production of hypernuclei by the process investigated here. A better calculation of the cross sections should be done, with the avoidance of some of the assumptions used in the present work. The final state interactions of the K^+ mesons and the antisymmetrization with respect to identical nucleons should be taken into account, along with more realistic models for the $\pi^+ n \to K^+ \Lambda$ interaction, and for the nuclear and hypernuclear wave functions. In addition, the effects of (off shell) multiple scattering of the incident pion by the nucleons in the target before the $\pi^+ n \to K^+ \Lambda$ interaction takes place should be investigated, perhaps in the spirit of an absorptive separable representation of $\pi^+ n$ scattering.[15] Any of these neglected effects might be sufficient to give, at least for some of the heavier targets, an order of magnitude increase in the size of the calculated cross sections, indicating a feasible experimental situation. Of course, the presence or absence of such effects themselves is some justification for attempting preliminary experimental work.

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