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Particle-phonon interactions in ^{248}Cf and ^{249}Cf†

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The extent of particle-photon mixing in ^{248}Cf and ^{249}Cf has been determined using the ^{249}Cf(d, t) and ^{249}Cf(d, d') reactions. $K^\pi = 2^-$ bands were identified at 593 and 1477 keV in ^{248}Cf. Since the only $K^\pi = 2^-$ neutron two quasiparticle configuration expected below 2 MeV in ^{248}Cf is $\{\frac{9}{2}^-[734]; \frac{5}{2}^+[622]\}_{2-}$, the second $K^\pi = 2^-$ band is considered to be predominantly of phonon character. In ^{249}Cf a $K^\pi = \frac{5}{2}^+$ band at 145 keV, known from decay scheme studies, receives a measurable population in the (d, d') reaction. The phonon admixture in the $\frac{5}{2}^+$ state which results from the $\{\frac{9}{2}^-[734] \otimes 2^-$ phonon$\}$ configuration was determined to be $(29 \pm 5)\%$. The collective admixture in the $\frac{5}{2}^+$ state was also calculated from an octupole particle-hole interaction and was found to be 35%, in good agreement with the experimental value. The anomalously low energy of the $\frac{5}{2}^+$ state could not be entirely reproduced by this calculation.

[NUCLEAR REACTIONS ^{249}Cf(d,d'), $E_d = 15$ MeV; ^{249}Cf(d,t), $E_d = 12$ MeV; measured $\sigma(\theta)$. ^{248}Cf, ^{249}Cf deduced levels, J, π, extracted phonon contributions. Enriched target. Calculated particle-phonon mixing.]

I. INTRODUCTION

In recent years a great deal of progress has been made both experimentally[1-3] and theoretically[4,5] in studies of single-particle states in actinide nuclei. From these studies a microscopic description of one quasiparticle states has emerged which can be used to predict the energies and wave functions of these states. As a result of the insight gained in the course of these studies it is now possible to observe anomalies which are caused by residual interactions not included in the central potential.

One of these anomalies has been noted in the α-decay studies[6] of ^{253}Fm. The $\frac{5}{2}^+[622]$ state was identified at 145 keV in ^{249}Cf, whereas the expected[2] energy is 450 keV. On the other hand, the $\frac{7}{2}^+[624]$ state has recently been observed[7] at 380 keV, in agreement with the theoretical predictions.[2] A similar reversal in the energies of the $\frac{5}{2}^+[622]$ and $\frac{7}{2}^+[624]$ states was also observed[8-10] in ^{247}Cm, which is isotonic with ^{249}Cf.

One of the residual interactions which could cause lowering of the $\frac{5}{2}^+[622]$ state in these nuclei is the particle-phonon interaction. The present paper describes a direct measurement by charged-particle reaction spectroscopy of the extent of this mixing in ^{248}Cf and ^{249}Cf. The results of these experiments are compared with the collective admixture calculated with an octupole particle-hole interaction.

II. EXPERIMENTAL PROCEDURE

The present experiments were performed with the deuteron beam obtained from the Argonne FN tandem Van de Graaff accelerator. The ^{249}Cf target was prepared[11] in the Argonne electromagnetic isotope separator by the deposition of 200-eV ^{249}Cf ions on a 40-μg/cm^2 carbon film. The target area was 1-×3-mm and its thickness was 50 μg/cm^2 as estimated from the separator beam current and collection time. A similar thickness value was also obtained from the results of deuteron elastic scattering experiments.

The ^{249}Cf(d, d') reaction was carried out at 15-MeV bombarding energy, and the deuterons scattered from the target were analyzed with an Enge split-pole magnetic spectrograph.[12] The analyzed particles were recorded on Kodak NTA-50 emulsion plates, which were later developed and scanned in $\frac{1}{4}$-mm wide strips by an automatic scanning device.[13] The solid angle of acceptance by the spectrograph was 2 msr, and under these experimental conditions the spectrograph had a resolution (full width at half-maximum) of ~7 keV. Spectra were measured at 90, 125, and 140° with respect to the beam direction. The intensities of the inelastic groups were measured relative to that of the elastic group and the intensities at all angles were normalized by the use of a NaI(Tl) monitor detector placed at 30°. Since the elastic group and the inelastic groups populating the ground state band were too intense to be counted in a long exposure (12 000 μC), three exposures of varying intervals were taken at each angle.

The ^{249}Cf(d, t) reaction was performed at 12-MeV deuteron energy and the emergent tritons were analyzed with the Enge split-pole magnetic spectrograph. The reaction spectra were measured at 90, 120, and 135° with respect to the beam direction. The absolute cross sections were determined relative to the elastic scattering cross section at 30° which was assumed to be entirely due to Rutherford scattering.

III. EXPERIMENTAL RESULTS

Figure 1 shows the (d, d') spectrum measured at 140° and Table I gives the differential cross sections at the three angles of measurement. An angular distribution of the deuterons elastically scattered from ^{249}Cf was measured at 10° intervals from 20° to 140°. The absolute cross section for elastic scattering at each angle was determined by assuming that the total cross section at 20° was due to the Rutherford scattering process. The differential cross sections for the inelastic groups were then obtained by intensity normalization of the three exposures.

The energies of the excited states populated in the ^{249}Cf(d, d') reaction are also included in Table I. Many of the spin-parity assignments are based primarily on radioactive decay studies and these will be discussed in detail in the next section. The ratio $d\sigma(90°)/d\sigma(125°)$ was not used for l-value assignments because the contribution from multiple-step excitations, which may be important, are not yet understood.

The ^{249}Cf(d, t) spectrum measured at 120° is displayed in Fig. 2. The energies and differential cross sections of the $K^{\pi}=2^-$ and 7^- bands are listed in Table II. Other bands populated in this reaction will be published[14] separately in a discussion of two quasiparticle states in the Cf nuclei.

IV. DISCUSSION

A. Level assignment in ^{249}Cf

Several of the states populated in the present (d, d') experiments have been previously identified in the ^{253}Fm α-decay[6] and ^{249}Es electron capture decay.[7] The ground state of ^{249}Cf is known[15] to be the $\frac{9}{2}^-[734]$ Nilsson state on the basis of its α-decay properties In addition, the spins and parities of the 145- and 813-keV states were deduced[7] from the measured multipolarities of γ rays deexciting these levels to be $\frac{5}{2}^+$ and $\frac{5}{2}^-$, respectively. In the present (d, d') experiments the rotational bands built on the 145- and 813-keV states receive measurable population which indicates that these bands contain collective admixtures. On the other hand, we do not observe any population to the $\frac{7}{2}^+[624]$ and $\frac{1}{2}^+[620]$ single-particle states which are known to be at 379.5 and 417 keV, respectively.

In addition to the rotational bands at 145 and 813 keV, several other bands are also populated in the deuteron inelastic scattering experiments (see Fig. 3). In our (d, d') experiments[16] on ^{246}Cm we observe almost equal population to the $K^{\pi}=0^-$, 1^-, 2^-, and 3^- octupole bands. Thus one would expect similar population to these vibrational states coupled with the ^{249}Cf ground state. All the observed levels were analyzed on the assumption that they belong to collective states coupled to the $\frac{9}{2}^-[734]$ neutron state. The K quantum number of a band was deduced from the level spacings

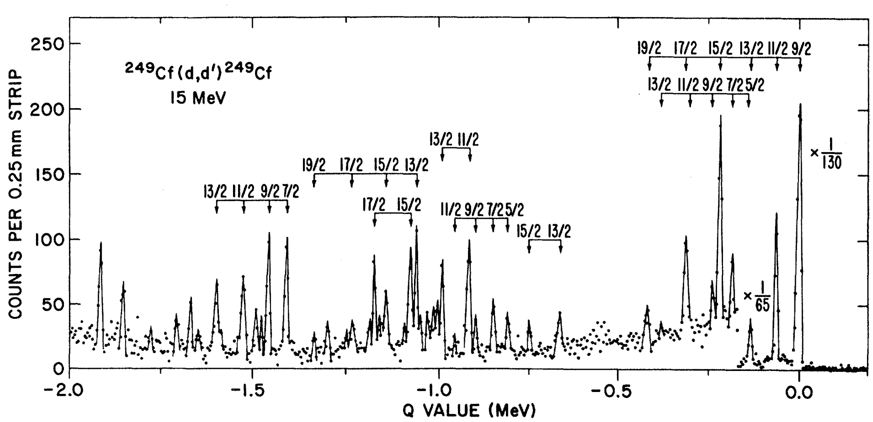

FIG. 1. Deuteron spectrum from the ^{249}Cf(d,d') reaction observed at 140°. The spectrum was measured with a split-pole magnetic spectrograph. Although the $\frac{5}{2}^+$ and $\frac{11}{2}^+$ members of the $\frac{5}{2}^+$ band at 145 keV are obscured by more intense peaks from the ground-state band, their positions are indicated.

TABLE I. Summary of ^{249}Cf(d,d') data.

Excitation energy (keV)	$\frac{d\sigma}{d\Omega}$ (μb/sr) 90°	125°	140°	Assignment J^π
0	$(8.97\pm0.45)\times10^4$	$(1.91\pm0.10)\times10^4$	$(1.24\pm0.07)\times10^4$	$\frac{9}{2}-$
61 ± 1	$(2.06\pm0.21)\times10^3$	$(1.31\pm0.14)\times10^3$	$(1.21\pm0.13)\times10^3$	$\frac{11}{2}-$
135 ± 2	640 ± 64	330 ± 33	340 ± 34	$\frac{13}{2}-$
186 ± 3	17 ± 9	13 ± 2	8 ± 3	$\frac{7}{2}+$
220 ± 2	41 ± 6	44 ± 5	30 ± 3	$\frac{15}{2}-$
241 ± 3	19 ± 5	8 ± 2	6 ± 2	$\frac{9}{2}+$
315 ± 3	23 ± 7	22 ± 3	19 ± 4	$\frac{17}{2}- + \frac{11}{2}+$
384 ± 4	*a*	*a*	2 ± 1	$\frac{13}{2}+$
425 ± 4	6 ± 3	7 ± 2	8 ± 2	$\frac{19}{2}-$
668 ± 2	15 ± 5	9 ± 2	8 ± 3	$\frac{13}{2}-$
751 ± 3	5 ± 3	5 ± 1	4 ± 1	$\frac{15}{2}-$
812 ± 2	19 ± 6	10 ± 2	7 ± 2	$\frac{5}{2}-$
851 ± 2	12 ± 4	5 ± 1	8 ± 1	$\frac{7}{2}-$
902 ± 3	8 ± 2	5 ± 2	4 ± 2	$\frac{9}{2}-$
920 ± 2	31 ± 3	17 ± 2	19 ± 3	$\frac{11}{2}+$
962 ± 3	4 ± 2	4 ± 1	2 ± 1	$\frac{11}{2}-$
992 ± 2	18 ± 6	13 ± 4	13 ± 2	$\frac{13}{2}+$
1007 ± 4	*a*	6 ± 2	7 ± 2	
1031 ± 5	12 ± 4	*a*	6 ± 2	
1063 ± 2	42 ± 7	20 ± 4	16 ± 2	$\frac{13}{2}+$
1078 ± 3	21 ± 5	16 ± 4	17 ± 2	$\frac{15}{2}+$
1115 ± 4	4 ± 1	4 ± 1	1 ± 1	
1145 ± 3	17 ± 4	12 ± 4	11 ± 2	$\frac{15}{2}+$
1164 ± 4	*a*	7 ± 3	5 ± 1	
1178 ± 3	20 ± 3	19 ± 2	13 ± 2	$\frac{17}{2}+$
1192 ± 4	15 ± 3	6 ± 2	4 ± 1	
1218 ± 4	18 ± 3	5 ± 2	*a*	
1236 ± 3	17 ± 5	6 ± 1	5 ± 1	$\frac{17}{2}+$
1251 ± 5	10 ± 3	2 ± 1	3 ± 1	
1304 ± 5	*a*	3 ± 1	4 ± 1	
1340 ± 4	*a*	4 ± 1	3 ± 1	$\frac{19}{2}+$
1415 ± 3	*b*	19 ± 3	18 ± 2	$\frac{7}{2}+$
1463 ± 3	22 ± 8	23 ± 3	17 ± 2	$\frac{9}{2}+$
1482 ± 4	8 ± 5	7 ± 2	4 ± 1	
1498 ± 4	7 ± 4	9 ± 2	6 ± 1	
1530 ± 3	16 ± 3	19 ± 2	14 ± 2	$\frac{11}{2}+$
1541 ± 4	10 ± 2	8 ± 2	4 ± 1	
1603 ± 3	19 ± 4	17 ± 4	11 ± 2	$\frac{13}{2}+$

TABLE I (*Continued*)

Excitation energy (keV)	$\frac{d\sigma}{d\Omega}$ (μb/sr) 90°	125°	140°	Assignment J^{π}
1648 ± 4	4 ± 2	6 ± 3	4 ± 1	
1674 ± 4	8 ± 3	8 ± 3	6 ± 1	
1709 ± 4	*a*	6 ± 2	6 ± 2	
1914 ± 3	20 ± 4	21 ± 4	18 ± 2	

[a] Insufficient population of this state for identification in the spectrum.
[b] Obscured by an impurity group in the 90° spectrum.

between the successive members of the band using the equation[17]

$$\frac{E_{K+1}-E_K}{E_{K+2}-E_{K+1}}=\frac{K+1}{K+2}\,. \quad (1)$$

The K values thus determined and the rotational constants ($\hbar^2/2\mathscr{I}$), are shown in Fig. 3. In general, the vibrational bands built on a single-particle state have rotational constants similar to that of the base state.[18] The near equality of the rotational constants of the excited bands to that of the ground state band supports the K assignments.

The phonon admixture in the 145-keV ($K^{\pi}=\frac{5}{2}^+$) state arises from the mixing between the $\{\frac{9}{2}^-[734]\otimes 2^-\text{ phonon}\}_{5/2^+}$ state and the $\frac{5}{2}^+[622]$ single-particle state. One way to determine the phonon contribution is to observe the intensities of all the $K^{\pi}=\frac{5}{2}^+$ bands populated in the (d,d') reaction. Since we did not identify all the $\frac{5}{2}^+$ rotational bands in the present work, we have to determine the phonon admixture by a different method. Two rotational bands with $K^{\pi}=\frac{13}{2}^+$ and $\frac{5}{2}^+$ are obtained by coupling the 2^- phonon with the $\frac{9}{2}^-[734]$ Nilsson state. From angular momentum conservation arguments it can be shown that $\frac{5}{2}^+$ and $\frac{13}{2}^+$ bands should receive equal population, except for the Q dependence of the reaction. Furthermore, since there are no single-particle bands with $K^{\pi}=\frac{13}{2}^+$ near 1-MeV excitation energy the cross section for the transition to this band should give the total phonon strength. Thus after accounting for Q dependence the ratio of the corrected (d,d') intensity of the 145-keV band to that of the $\frac{13}{2}^+$ band at 1063 keV will give the fractional phonon admixture in the 145-keV band.

The above assumption that the K_0+2 and K_0-2 bands receive equal population can be applied to other bands identified in the present work. Two pairs of $K_0\pm\nu$ bands have been identified in ^{249}Cf, namely, the ($\frac{7}{2}^+$, $\frac{11}{2}^+$) pair which is formed by the $\frac{9}{2}^-\pm 1^-$ coupling and the ($\frac{5}{2}^-$, $\frac{13}{2}^-$) pair from the γ-vibrational band built on the ^{249}Cf ground state. The ratios of the (d,d') cross sections (with Q dependence removed) are found to be $\sigma_{13/2^-}/\sigma_{5/2^-}=0.51$ and $\sigma_{11/2^+}/\sigma_{7/2^+}=0.47$. Since there are no $\frac{13}{2}^-$ or $\frac{11}{2}^+$ single-particle orbitals around 1 MeV in excitation, these higher-K bands should receive all of the transition strength expected for a $\frac{13}{2}^-$ or $\frac{11}{2}^+$ phonon band. It thus appears that our assumption that the $K_0+\nu$ and $K_0-\nu$ bands receive equal population is not correct. This could be due to a j dependence of the (d,d') reaction cross sections similar to the j dependence seen in transfer reaction studies. We have found in the ^{249}Cf(d,t) reaction that at 12 MeV $\sigma_{7^-}(1577)/\{\sigma_{2^-}(593)+\sigma_{2^-}(1477)\}=0.52$ and, furthermore, this ratio changes with the bombarding energy.[14] The measured cross section for the $\frac{13}{2}^+$ band at 1063 keV could thus be low because of this effect. Using the experimentally determined cross section ratios for the $K_>$ and $K_<$ bands in ^{249}Cf, we shall assume that the $\frac{13}{2}^+$ band is populated with 0.49 the intensity of an equivalent $\frac{5}{2}^+$ band in the (d,d') reaction. By equivalent, we mean a band at the same energy consisting of

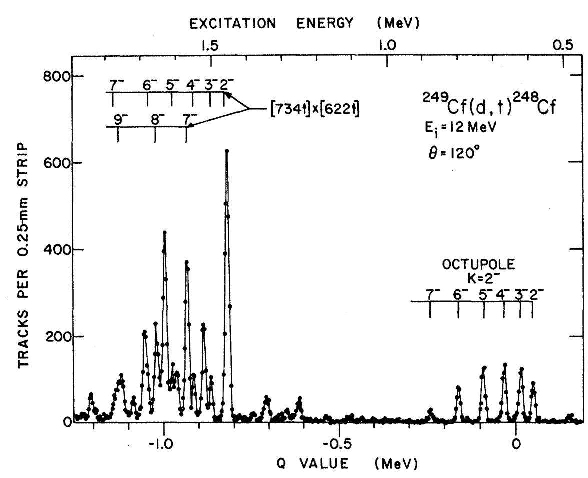

FIG. 2. Triton spectrum from the ^{249}Cf$(d,t)^{248}$Cf reaction observed at 120°. The spectrum was measured with a split-pole magnetic spectrograph.

TABLE II. Differential cross sections of bands in ^{248}Cf populated in the (d,t) reaction.

K^π	J^π	Excitation energy (keV)	$\frac{d\sigma}{d\Omega}(\mu b/sr)$ 90°	120°	135°
2^-	2^-	593 ± 1	13.2 ± 1.8	11.3 ± 0.9	10.7 ± 1.0
	3^-	630 ± 1	13.3 ± 1.5	15.3 ± 1.0	16.5 ± 1.8
	4^-	677 ± 1	17.8 ± 2.3	16.0 ± 1.0	19.4 ± 1.7
	5^-	735 ± 1	11.6 ± 1.7	16.5 ± 0.9	18.1 ± 1.7
	6^-	806 ± 1	6.7 ± 1.0	10.2 ± 0.5	11.0 ± 1.7
	7^-	885 ± 2	2.0 ± 0.7	3.7 ± 0.4	
	8^-	979 ± 4		0.9 ± 0.3	
2^-	2^-	1477 ± 4	7.8 ± 1.7		5.2 ± 2.2
	3^-	1509 ± 3	7.8 ± 1.2	11.8 ± 1.0	12.5 ± 2.5
	4^-	1557 ± 3	7.7 ± 1.3	12.9 ± 1.1	12.7 ± 2.6
	5^-	1621 ± 3	8.2 ± 1.3	13.7 ± 1.2	20.3 ± 1.7
	6^-	1686 ± 4	6.1 ± 1.3	11.2 ± 1.4	6.3 ± 1.3
7^-	7^-	1577 ± 1	34.3 ± 2.1	48.0 ± 1.9	55.4 ± 4.6
	8^-	1663 ± 2	13.6 ± 1.5	18.8 ± 2.5	28.8 ± 1.9
	9^-	1781 ± 5	4.4 ± 0.8	10.1 ± 1.0	8.7 ± 1.2

the single component $\frac{9}{2}^-[734]$ coupled to the 2^- phonon.

Since the $\frac{5}{2}^+$ and $\frac{11}{2}^+$ members of the $\frac{5}{2}^+[622]$ band are obscured by the more intense groups of the ground-state band the intensities of these levels could not be measured. By assuming that the intensity pattern for the band is given by the squares of the appropriate Clebsch-Gordan coefficients for the $l=3$ transitions the cross sections for these states (which should contain 37% of the $l=3$ strength to this band) were estimated. This approach does not take into account the effects of multiple-step processes. However, it can be deduced from our ^{246}Cm(d, d') data[16] that the contribution of such processes is not significant in comparison to the $l=3$ direct transition. From the present data we obtain the phonon admixture in the $\frac{5}{2}^+$ state at 145 keV to be $(29 \pm 5)\%$, where the indicated error includes only the statistical contribution.

The reliability of the procedure used to calculate the cross sections of the obscured members

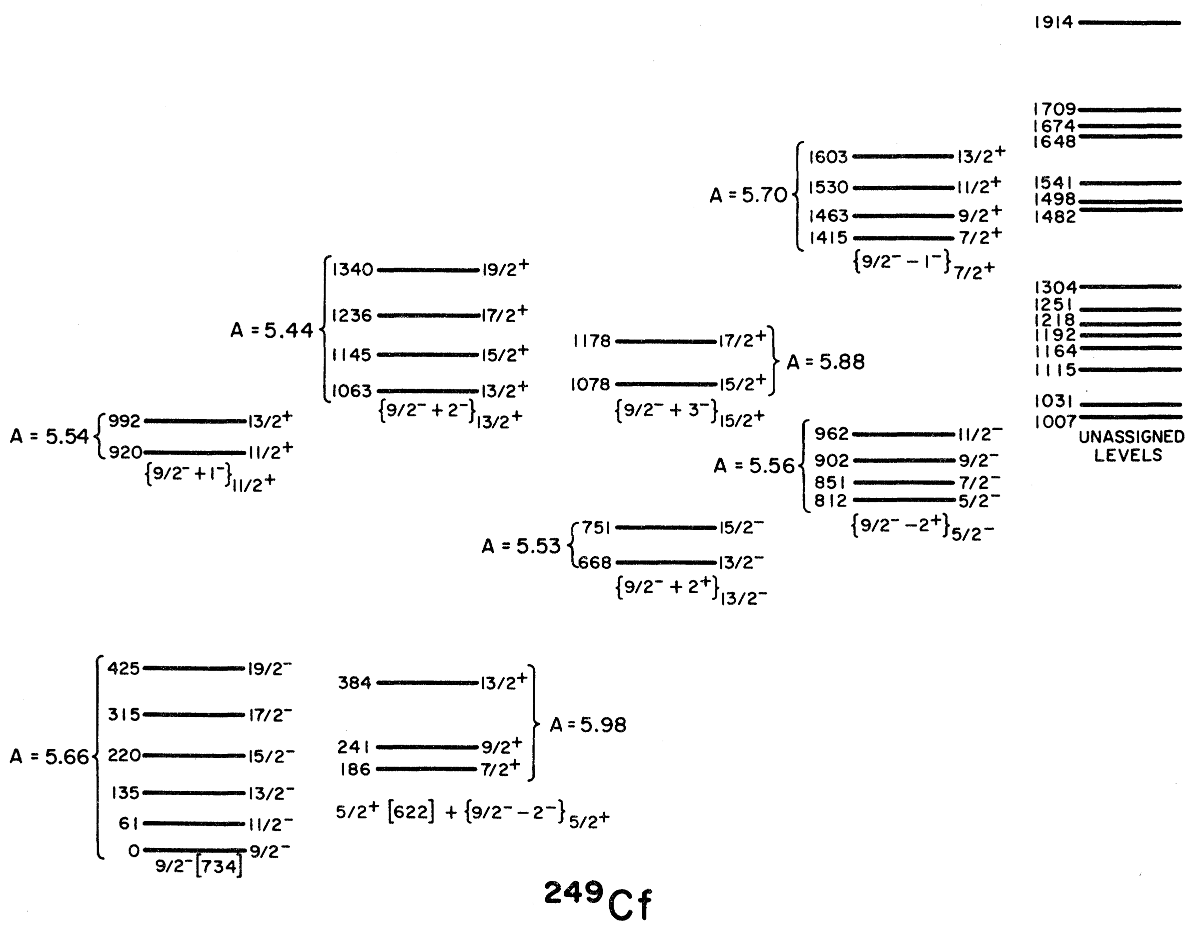

FIG. 3. Energy levels of ^{249}Cf observed in the ^{249}Cf(d,d') reaction. Only collective states built on the $\frac{9}{2}^-$ ground state are populated and the rotational constant for each band is denoted by A.

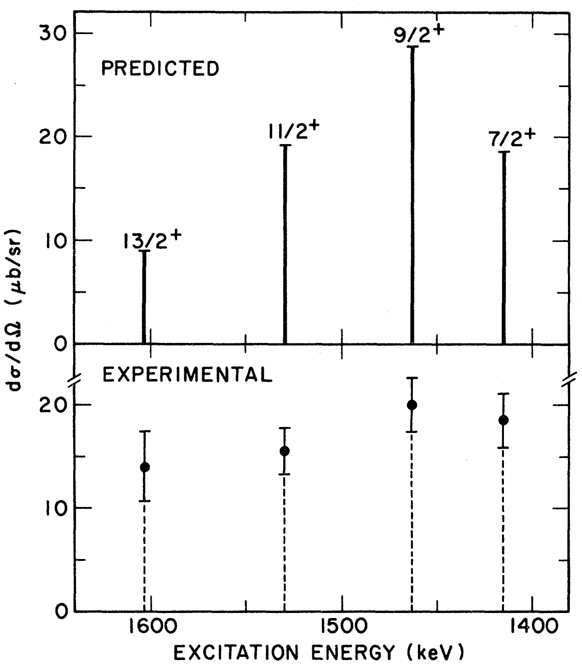

FIG. 4. Comparison of the experimental cross sections (dashed bars) with the calculated values (solid bars) for the $K^\pi = \frac{7}{2}^+$ band at 1415 keV. The calculated values are normalized at the $\frac{7}{2}^+$ level, and the experimental cross sections represent an average of the 125 and 140° values.

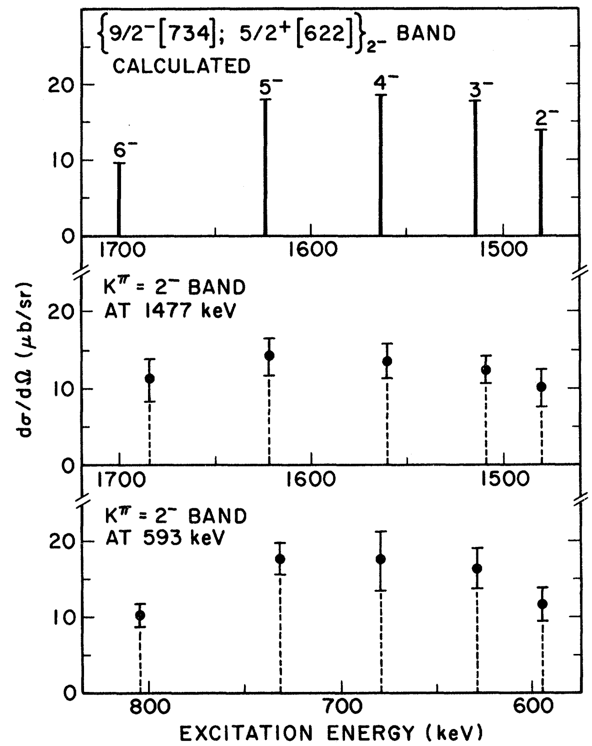

FIG. 5. Comparison of the experimental cross sections (dashed bars) with the calculated values (solid bars) for the two $K^\pi = 2^-$ bands at 120°.

of the $\frac{5}{2}^+[622]$ band may be further checked by examining the population of the $K^\pi = \frac{7}{2}^+$ band at 1415 keV. Assuming only $E3$ excitation, one can calculate the relative intensities of the members of the band from the squares of the respective Clebsch-Gordan coefficients. The intensities thus calculated are compared with the experimental intensities in Fig. 4. The fair agreement between the experimental and calculated intensities clearly indicates that these states are predominantly populated by $E3$ transitions.

B. Level assignments in ^{248}Cf

The ^{249}Cf(d, t) reaction will populate only those two quasiparticle states which contain the $\frac{9}{2}^-[734]$ neutron state as one component. From the single-particle spectra[6,7] of ^{249}Cf one expects only one $K^\pi = 2^-$ neutron two quasiparticle state in ^{248}Cf below 2-MeV excitation energy. In our (d, t) spectrum we identify two bands with $K^\pi = 2^-$ at energies 593 and 1477 keV. The fact that the 593- and 1477-keV bands contain $\{\frac{9}{2}^-[734];\ \frac{5}{2}^+[622]\}_{2^-}$ components is also deduced from the excellent agreement between the observed cross sections and the cross sections calculated for that two quasiparticle configuration (Fig. 5). Since only one 2^- two quasiparticle state is expected in this energy region, the other component of the two observed bands must be a 2^- phonon configuration.

The compositions of the two $K^\pi = 2^-$ states were determined from the distribution of the (d, t) reaction strength to these bands assuming that they exhaust the total cross section for the removal of a neutron from the $\frac{5}{2}^+[622]$ orbital. The ratio R

TABLE III. Values of R from the ^{249}Cf$(d,t)^{248}$Cf reaction:

$$R = \sum_{I_f} \left(\frac{d\sigma}{d\Omega}\right)_{\exp}^{I_f} \Big/ \sum_{I_f} \left(\frac{d\sigma}{d\Omega}\right)_{\rm DWBA}^{I_f}.$$

	$\theta = 90°$	$\theta = 120°$	$\theta = 135°$
R_{593}	0.648 ± 0.087	0.484 ± 0.029	0.506 ± 0.051
R_{1477}	0.396 ± 0.069	0.359 ± 0.032	0.324 ± 0.057
$\frac{R_{593}}{R_{593} + R_{1477}}$ [a]	0.620 ± 0.106	0.574 ± 0.045	0.609 ± 0.083
Average phonon admixture in the 593-keV band		$40.7 \pm 4.5\%$	

[a] This quantity gives the two quasiparticle strength to the the 593-keV $K^\pi = 2^-$ band.

of the experimental cross sections to those calculated using a distorted-wave Born approximation (DWBA) computer code[19] was determined at the three angles of measurement (see Table III). The quantity $R_{593}/(R_{593}+R_{1477})$ then gives the two quasiparticle strength to the $K^{\pi}=2^-$ band at 593 keV as (59.3± 4.5)% and the remaining 40.7% to the rotational band at 1477 keV.

C. Theoretical calculation of phonon admixtures

We calculate the composition of the low-lying $\frac{5}{2}^+$ state in ^{249}Cf making use of the experimental data for ^{248}Cf discussed in the preceding section. The ^{249}Cf(d, t) data allow us to describe the $K^{\pi}=2^-$ states as linear combinations of the following wave functions

$$|\psi_1^{2-}\rangle=\{\tfrac{9}{2}^-[734];\tfrac{5}{2}^+[622]\}_{2^-} \tag{2}$$

and

$$|\psi_2^{2-}\rangle=\text{all other components of the } 2^- \text{ phonon.} \tag{3}$$

We expect[20] that the major component in $|\psi_2^{2-}\rangle$ is the proton two quasiparticle configuration $\{\frac{7}{2}^+[633];\frac{3}{2}^-[521]\}_{2^-}$; this, however, does not enter into our calculation. We assume that configurations (2) and (3) are mixed by a 2^- octupole particle-hole interaction.[21] The interaction energy for ^{248}Cf is given by the matrix element

$$V_{1,2}=V_0[\langle\tfrac{9}{2}^-|f(r)Y_3{}^2(\theta,\phi)|\tfrac{5}{2}^+\rangle\langle\psi_2^{2-}|f(r)Y_3{}^{-2}(\theta,\phi)|0\rangle[\{N_{5/2}(1-N_{9/2})\}^{1/2}+\{N_{9/2}(1-N_{5/2})\}^{1/2}], \tag{4}$$

where $|0\rangle$ denotes the phonon vacuum and the occupation probabilities N have been calculated for the ^{248}Cf ground state. The value of V_0 and the exact nature of the radial dependence $f(r)$ will be the same in our analysis of both ^{248}Cf and ^{249}Cf.

If we denote the energies of configurations (2) and (3) by E_1 and E_2, and our experimental energies in ^{248}Cf as E_L and E_H, then we can derive the following expressions

$$E_{H,L}=\tfrac{1}{2}(E_1+E_2)\pm\tfrac{1}{2}[(E_1-E_2)^2+4V_{1,2}{}^2]^{1/2} \tag{5}$$

and

$$\frac{a_L{}^2}{a_H{}^2}=\frac{E_1-E_L}{E_2-E_L}. \tag{6}$$

In the above equations $V_{1,2}$ is the interaction energy as given by Eq. (4) and a_L and a_H denote phonon admixtures in the lower and higher states, respectively. Using the experimental values, $E_L=0.593$ MeV, $E_H=1.477$ MeV, $a_L=\sqrt{0.4}$, and $a_H=\sqrt{0.6}$, we calculate

$$E_1=0.947 \text{ MeV},$$
$$E_2=1.123 \text{ MeV},$$
$$V_{1,2}=0.433 \text{ MeV}. \tag{7}$$

Assuming that there is little change in $|\psi_2\rangle$ as we go from ^{248}Cf to ^{249}Cf, we can determine the phonon mixing in the lowest state of ^{249}Cf. For the $\frac{5}{2}^+$ state in ^{249}Cf the two configurations are

$$|\phi_1\rangle=\tfrac{5}{2}^+[622] \tag{8}$$

and

$$|\phi_2\rangle=\tfrac{9}{2}^-[734]\otimes|\psi_2^{2-}\rangle. \tag{9}$$

These two configurations interact via the same matrix element as the two 2^- states in ^{248}Cf, apart from their dependence on the occupation probabilities. In this case the expression for the interaction matrix element is

$$V_{1,2}=V_0[\langle\tfrac{9}{2}^-|f(r)Y_3{}^2(\theta,\phi)|\tfrac{5}{2}^+\rangle\langle\psi_2{}^{-2}|f(r)Y_3{}^{-2}(\theta,\phi)|0\rangle][(N_{5/2}^{9/2})^{1/2}(N_{9/2}^{5/2})^{1/2}-(1-N_{5/2}^{9/2})^{1/2}(1-N_{9/2}^{5/2})^{1/2}], \tag{10}$$

where $N_{5/2}^{9/2}$ denotes the occupation probability of the $\frac{5}{2}^+[622]$ orbital in the single-particle state $\frac{9}{2}^-[734]$.

From ^{248}Cf data, we set

$$\langle\psi_2^{2-}|E|\psi_2^{2-}\rangle=1.123 \text{ MeV}. \tag{11}$$

Since the $\frac{9}{2}^-[734]$ configuration is the ground state of ^{249}Cf we have

$$\langle\phi_2|E|\phi_2\rangle=1.123 \text{ MeV}. \tag{12}$$

From the single-particle spectra of the neighboring odd-mass nuclei we estimate[2] the unperturbed energy of the $\frac{5}{2}^+[622]$ orbital relative to the $\frac{9}{2}^-[734]$ configuration to be 0.45 MeV. Hence

$$\langle\phi_1|E|\phi_1\rangle=0.45 \text{ MeV}. \tag{13}$$

Using the values of the occupation probabilities for ^{248}Cf and ^{249}Cf and $V_{1,2}$ for ^{248}Cf we obtain

$$V_{1,2} \text{ (in } ^{249}\text{Cf})=0.351 \text{ MeV}. \tag{14}$$

From these matrix elements we determine the energy and wave function of the lowest $\frac{5}{2}^+$ state as

$$E_{5/2^+}=0.30\text{ MeV} \tag{15}$$

and

$$|\psi_{5/2^+}\rangle=0.920\tfrac{5}{2}^+[622]+0.392\{\tfrac{9}{2}^-[734]\otimes|\psi_2^{2^-}\rangle\}. \tag{16}$$

The same mixing also applies to the ^{249}Cf ground state $\frac{9}{2}^-$. Here we have

$$|\phi_1\rangle=\tfrac{9}{2}^-[734] \tag{17}$$

and

$$|\phi_2\rangle=\tfrac{5}{2}^+[622]\otimes|\psi_2^{2^-}\rangle. \tag{18}$$

The interaction matrix element between these configurations is again given by Eq. (10). Using the $V_{1,2}$ value of 0.351 MeV we obtain

$$E_{9/2^-}=-0.08\text{ MeV} \tag{19}$$

and

$$|\psi_{9/2^-}\rangle=0.978\tfrac{9}{2}^-[734]+0.209\{\tfrac{5}{2}^+[622]\otimes|\psi_2^{2^-}\rangle\}. \tag{20}$$

From Eqs. (15) and (19) we have

$$E_{5/2^+}-E_{9/2^-}=0.38\text{ MeV}, \tag{21}$$

which is still considerably larger than the experimental value of 0.145 MeV but is some improvement over the 0.45-MeV estimate that one obtains with just a pairing force calculation.

The operator for the phonon transition which causes the population of states in deuteron inelastic scattering experiments is of the form

$$O_{2^-}=a_{2^-}^\dagger+a_{2^-}, \tag{22}$$

where $a^\dagger$ and a are the phonon creation and annihilation operators. Using the wave functions from Eqs. (16) and (20) we obtain

$$|\langle\tfrac{5}{2}^+|O_{2^-}|\tfrac{9}{2}^-\rangle|^2=|(0.978)(0.392)+(0.920)(0.209)|^2=0.331. \tag{23}$$

As mentioned before, the $\frac{13}{2}^+$ band should be a pure phonon band as there are no $\frac{13}{2}^+$ single-particle states in this region. The calculated energy of this $\frac{13}{2}^+$ state relative to the ground state is 1.20 MeV, in good agreement with the observed energy of 1.06 MeV. The (d, d') transition probability to this state is

$$|\langle\tfrac{13}{2}^+|O_{2^-}|\tfrac{9}{2}^-\rangle|^2=(0.978)^2=0.956. \tag{24}$$

Accordingly, the $\frac{5}{2}^+$ band at 145 keV should have $0.331/0.956=0.35$ of the population of the band at 1063 keV. This is in good agreement with the measured value of 0.29 ± 0.05.

In conclusion, we find that the same matrix element that describes the mixing of the two $K^\pi=2^-$ states in ^{248}Cf also gives an accurate description of the particle-phonon mixing in the low-lying states of ^{249}Cf. We note that this particle-phonon mixing does not resolve the problem of the relatively low excitation energy of the $\frac{5}{2}^+$ band relative to the $\frac{9}{2}^-$ ground state. It should be emphasized that a large increase in the matrix element of Eq. (14) does not solve this problem as the energies of both the $\frac{9}{2}^-$ and the $\frac{5}{2}^+$ states are shifted in the same direction. An unrealistically large increase of a factor of 2 in this matrix element gives energy difference of 270 keV between the two bands. The energy of the $K^\pi=\frac{5}{2}^+$ band remains to be explained.

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