$(\pi, \pi N)$ puzzle: Effects of pion charge exchange

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The ratio of cross sections for nucleon knockout by pions, e.g. $\sigma[^{12}C(\pi^-, \pi N)^{11}C]/\sigma[^{12}C(\pi^+, \pi N)^{11}C]$, is shown to be insensitive to pion charge exchange either before or after the knockout collision.

[NUCLEAR REACTIONS $^{12}C(\pi^\pm, \pi N)$, $T_\pi = 50-300$ MeV; ratio of $\sigma(T_\pi)$'s; pion charge exchange effects.]

Recent activation measurements[1] of the cross sections $\sigma^\pm$ for the removal of a neutron from ^{12}C by $\pi^\pm$ have shown that the ratio $\mathcal{R} = \sigma^-/\sigma^+$ varies with incident pion energy in a characteristic way. In particular, at the (3, 3) resonance $\mathcal{R} = 1.55 \pm 0.10$, in contrast with the impulse approximation prediction of 3 and the earlier measured value of ≈1.[2] The energy variation has been successfully interpreted[3] in a simple semiclassical model as being due to the energy dependence of the probability for charge exchange as the recoil nucleon leaves the nucleus. Given the importance of nucleon charge exchange in determining $\mathcal{R}$, one might naturally ask about the effects of a pion charge exchange, both before and after the knockout collision. In this communication we consider these effects and show that the changes they make in the predicted values of $\mathcal{R}$ are negligible.

It turns out, as might have been expected,[4] that the initial and final state interactions of the pion charge exchanging with the nucleus *also* work to reduce the value for $\mathcal{R}$ from the $I_{\pi N} = \frac{3}{2}$ dominance value of 3. The question to be answered is "by how much and with what energy dependence?"

To estimate this quantitatively let P_N be the probability that the recoil nucleon undergoes a charge exchange as it emerges from the nucleus, and $Q_N = 1 - P_N$ the probability that it does not. Likewise, let $P_\pi(Q_\pi)$ and $P'_\pi(Q'_\pi)$ be the probabilities that the initial and final pions charge-exchange (or not) before arriving at or leaving the position of the "hard" pion-nucleon collision. Then, with $x_i = P_i/Q_i$, the ratio $\mathcal{R}$ in this model becomes

$$\mathcal{R} = \frac{\sigma_{\pi^+ p} + \sigma_{\pi^-, \mathrm{el}} x_N + \sigma_{\pi^-, \mathrm{ex}} x'_\pi}{\sigma_{\pi^- p} + \sigma_{\pi^+ p}(x_N + x'_\pi) + \sigma_{\pi^0 p} x_\pi} . \quad (1)$$

When $x_\pi = x'_\pi = 0$ this formula reduces to Eq. (1) of I. The x_i are, of course, energy dependent. An expression for P_N was given and discussed at length in I.

The probabilities P_π and Q_π can be found by solving the 3×3 set of coupled linear differential equations for pion transport (see e.g. Ref. 5). For $N = Z$, we find

$$P_\pi = \tfrac{1}{3}(1 - e^{-(3/2)y}),$$
$$Q_\pi = \tfrac{1}{6}(2 + 3e^{-(1/2)y} + e^{-(3/2)y}), \quad (2)$$

where, for incident pion energy T_π,

$$y = A\rho_0 \sigma_{\pi, \mathrm{ex}}(T_\pi) d_\pi(T_\pi). \quad (3)$$

Here $\rho_0 = \frac{3}{4}\pi R^3$ is the (uniform) nuclear density, $\sigma_{\pi, \mathrm{ex}}$ is the cross section for πN charge exchange, and d_π is the distance the pion travels in nuclear matter. Formulas for P'_π and Q'_π are the same but with y evaluated at the outgoing pion energy T'_π.

In view of the approximations made in I, we assume $d_\pi = \lambda_{\pi N}$, the mean free path for pions in nuclear matter. Then

$$\lambda_{\pi N}^{-1} = A\rho_0(\sigma_{\pi N} + \sigma_{\mathrm{abs}}), \quad (4)$$

the first term involving scattering interactions, $\sigma_{\pi N} = \frac{1}{2}(\sigma_{\pi^+ p} + \sigma_{\pi^- p})$, and the second involving absorption processes in which the pion disappears. The latter cross section is not well known; we will use the σ_{abs} found from a fit to data on pion production by medium energy nucleons.[6] With Eq. (4),

$$y = \sigma_{\pi, \mathrm{ex}}/(\sigma_{\pi N} + \sigma_{\mathrm{abs}}), \quad (5)$$

which is roughly energy independent if $\sigma_{\pi, \mathrm{ex}}$ and $\sigma_{\pi N}$ are total cross sections. Note that $y \approx 0.2$ and thus P_π is small.

It is not quite appropriate to use total cross sections in Eq. (5), however. For, in the charge

TABLE I. The $x_i = P_i/Q_i$ and predicted $\mathcal{R}=\sigma^-/\sigma^+$ according to Eq. (1) for $^{12}C(\pi, \pi N)^{11}C^*$. For comparison we also give the $\mathcal{R}$ of Ref. 3, which has no pion charge exchange contributions, and the experimental ratios from Ref. 1 (interpolated where necessary).

T_π (MeV)	x_N	x_π	x'_π	$\mathcal{R}$ [Eq. (1)]	$\mathcal{R}$ [Ref. 3]	$\mathcal{R}_{exp}$
50	0.58	0.061	0.071	0.77	0.80	0.43 ± 0.13
100	0.48	0.032	0.049	1.11	1.11	1.05 ± 0.06
200	0.23	0.018	0.031	1.64	1.67	1.72 ± 0.15
300	0.09	0.011	0.019	1.84	1.88	1.77 ± 0.10

exchange reaction represented by the $\sigma_{\pi, ex}$, the recoil nucleon must not escape from the nucleus. That escape *is* required, on the other hand, by the scattering represented by $\sigma_{\pi N}$ in the denominator. These requirements can be met, in the (3, 3) dominance approximation, by setting

$$\sigma_{\pi, ex} = (1-h)\sigma^{tot}_{\pi, ex}, \quad \sigma_{\pi N} = h\sigma^{tot}_{\pi N},$$

$$h(T_\pi) = \tfrac{1}{4}\int_0^{\theta\,max} (1+3\cos^2\theta)\sin\theta\, d\theta, \tag{6}$$

$$\cos\theta_{max} = 1 - mQ/q^2,$$

where θ and q are the scattering angle and three-momentum in the πN center of mass and Q is the neutron removal energy (18.7 MeV for ^{12}C). The $\sigma^{tot}_{\pi,ex}$ and $\sigma^{tot}_{\pi N}$ are still not the free πN cross sections, however, as the reactions occur in the nuclear medium. They should incorporate a correction factor F which takes account of the Pauli exclusion principle and the average over the Fermi motion of the struck nucleon. We estimate F using a zero-temperature Fermi gas model (i.e., F is the ratio of curves B and A of Fig. 3 of Ref. 5). Thus, finally,

$$y = (1-h)F\sigma^{tot,free}_{\pi, ex}/(hF\sigma^{tot,free}_{\pi N} + \sigma_{abs}) \tag{7}$$

to be evaluated at T_π or $T'_\pi = T_\pi - T_N \approx \frac{2}{3} T_\pi$, as required.

We are now in a position to reevaluate $\mathcal{R}$ of Eq. (1). As in I, we fit the parameter β in P_N (which represents the somewhat uncertain Pauli principle correction to $\sigma_{N,ex}$) so that $\mathcal{R}$ agrees with experiment[1] at 180 MeV. We find a value of β some 10% smaller than before, but still within the expected range of values. Table I then shows predicted values of the x_i and $\mathcal{R}$ for various representative energies, and compares with the $\mathcal{R}$'s from I, where no pion charge exchange effects were considered. The changes in $\mathcal{R}$ are small; this happens because x_π and x'_π are small to begin with and, moreover, fall off with energy in a fashion similar to x_N.

In summary, we see that the pion charge exchange effects on $\mathcal{R}$, though in a direction which improves agreement with experiment, are small and basically do not change the conclusion that most, if not all, of the deviation of $\mathcal{R}$ from the impulse approximation value is explained by charge exchange of the recoil nucleon.

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