

# Nuclear rearrangement energy\*

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The concept of rearrangement energy in nuclear particle removal is carefully defined by specifying several energies associated with the process and its analysis. Connection is made between the present definition and closely related concepts apt to be confused with "rearrangement energy" so defined. Remarks are made concerning the implications of rearrangement to analysis and interpretation of experimental data.

[NUCLEAR STRUCTURE Nuclear rearrangement energy theoretically defined and differentiated from related energies. Reaction theories examined regarding rearrangement.]

## I. INTRODUCTION

The phrase "rearrangement energy" has been much used and its meaning much discussed in that literature involved with the relationship between single-particle energies and particle removal. There, nevertheless, appears to remain some confusion which I hope will be reduced by the following formal discussion of the subject. It appears that the difficulties may stem from the use of similar words for closely related but distinct concepts. This is true of the terminology applied to both theoretical and experimental aspects of the phenomena.

In Sec. II, three distinct energies are defined and relationships among them are developed. The rearrangement energy is defined as the difference between two of these energies. The other differences are also examined. In Sec. III the ideas of Sec. II are applied to several special cases, and contact is made with rearrangement related phenomena treated in the literature. These include removal energy shift discussed by Koltun<sup>1</sup> and Brandow,<sup>2</sup> the "rearrangement energy" of Brueckner and Goldman,<sup>3</sup> and the "rearrangement energy" of Weisskopf<sup>4</sup> and of Meldner and Perez.<sup>5</sup> Section IV includes remarks concerning the implication of rearrangement to the analysis of experimental data in different direct reaction models. The conclusions are then summarized in the last section.

## II. ENERGY DEFINITIONS AND RELATIONSHIPS

Let us define three energy quantities associated with particle removal. One is a purely theoretical, or model, quantity; one is an experimental quantity; and one is a hybrid between model and physical quantities. In a particular limit all three

may become equal. These energies are

(a) Single-particle energy  $\epsilon_\kappa$ . This energy is the eigenvalue of a one-body Hamiltonian  $H_0 = T + U$ , which is associated with a one-body eigenfunction  $\phi_\kappa(r)$ . Obviously, both  $\epsilon_\kappa$  and  $\phi_\kappa(r)$  are model quantities depending on the form chosen for the one-body potential  $U$ . In the following discussion we shall think of  $\epsilon_\kappa$  as being associated with the highest filled level (Fermi level) in an independent particle model based on  $H_0$ . The quantity  $\epsilon_\kappa$  is thus negative.

(b) Threshold separation energy  $\mathcal{E}_s$  ( $\equiv -\epsilon_s$ ).<sup>6</sup> This is an experimental quantity and is defined as the energy difference between the ground state of an  $(A-1)$ -body system  $|n_0\rangle$  and the ground state of the neighboring  $A$ -body system  $|\Phi\rangle$ ,

$$\mathcal{E}_s = E_{(A-1) \text{ g.s.}} - E_{A \text{ g.s.}} \quad (2.1)$$

It is positive ( $\epsilon_s$  is negative), and is associated with the exact eigenfunctions of the exact Hamiltonian

$$H|\Phi\rangle = E|\Phi\rangle. \quad (2.2)$$

(c) Centroid energy  $\mathcal{E}_c^\kappa$  ( $\equiv -\epsilon_c^\kappa$ ). This is the hybrid quantity, taken to be the expectation value of the exact Hamiltonian with respect to that configuration obtained by removing the model single-particle state from the exact state  $\Phi$ :

$$\mathcal{E}_c^\kappa = \langle \Phi | a_\kappa^\dagger (H - E_\Phi) a_\kappa | \Phi \rangle. \quad (2.3)$$

Here  $H$ ,  $|\Phi\rangle$ , and  $E_\Phi$  are physical quantities, while  $a_\kappa$  annihilates the model state  $\phi_\kappa$ .

In examining the relationships among these quantities, we shall work with an exact Hamiltonian of

the form:

$$H = H_0 + H_1, \quad (2.4a)$$

$$H_0 = \sum_{\alpha} \epsilon_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}, \quad (2.4b)$$

$$H_1 = \frac{1}{2} \sum_{\alpha\beta\gamma\delta} (\alpha\beta | V | \gamma\delta) a_{\alpha}^{\dagger} a_{\beta}^{\dagger} a_{\delta} a_{\gamma} - \sum_{\alpha\gamma} (\alpha | U | \gamma) a_{\alpha}^{\dagger} a_{\gamma}, \quad (2.4c)$$

where  $U$  is the one-body potential providing  $\epsilon_{\alpha}$  and  $\phi_{\alpha}$ . We then obtain the following energy difference which we label  $\delta_1$ :

$$\epsilon_s - \epsilon_{\kappa} = \frac{\langle n_0 | j_{\kappa} | \Phi \rangle}{\langle n_0 | a_{\kappa} | \Phi \rangle} \equiv \delta_1. \quad (2.5a)$$

$$j_{\kappa} = [H_1, a_{\kappa}]. \quad (2.5b)$$

Note that since  $\epsilon_{\kappa}$  is arbitrary (by virtue of the choice of single particle potential  $U$ )  $\delta_1$  can be of either sign. If  $U$  is "badly" chosen, i.e., has little to do with the structure of  $|\Phi\rangle$ , then  $\delta_1$  will be large.

We shall next consider the centroid energy which can be expressed as

$$\mathcal{E}_c^{\kappa} = \sum_n \frac{(E_n - E_{\Phi}) |\langle n | a_{\kappa} | \Phi \rangle|^2}{\langle \Phi | a_{\kappa}^{\dagger} a_{\kappa} | \Phi \rangle}. \quad (2.6)$$

The hole configuration is distributed among the eigenstates  $|n\rangle$  according to the spectroscopic amplitude as follows:

$$\langle n | a_{\kappa} | \Phi \rangle = \frac{-\langle n | j_{\kappa} | \Phi \rangle}{E_n + \epsilon_{\kappa} - E_{\Phi}}. \quad (2.7)$$

With this, we see that

$$\mathcal{E}_c^{\kappa} = -\epsilon_{\kappa} - \frac{\langle \Phi | a_{\kappa}^{\dagger} j_{\kappa} | \Phi \rangle}{\langle \Phi | a_{\kappa}^{\dagger} a_{\kappa} | \Phi \rangle}. \quad (2.8)$$

A second energy difference  $\delta_2$  can now be defined,

$$\epsilon_{\kappa}^{\kappa} - \epsilon_{\kappa} = \frac{\langle \Phi | a_{\kappa}^{\dagger} j_{\kappa} | \Phi \rangle}{\langle \Phi | a_{\kappa}^{\dagger} a_{\kappa} | \Phi \rangle} = \delta_2. \quad (2.9)$$

(In Sec. III we shall show that in the Hartree-Fock scheme  $\delta_2$  vanishes.)

Finally, we examine the third energy difference  $\delta_3$ , which is a combination of  $\delta_1$  and  $\delta_2$ ,

$$(\epsilon_s - \epsilon_{\kappa}^{\kappa}) = \frac{\sum_n (E_n - E_{n_0}) |\langle n | a_{\kappa} | \Phi \rangle|^2}{\langle \Phi | a_{\kappa}^{\dagger} a_{\kappa} | \Phi \rangle} \equiv \delta_3; \quad (2.10a)$$

$$\delta_3 = \delta_1 - \delta_2 \geq 0. \quad (2.10b)$$

It is clear that  $\delta_3$  is greater than or equal to zero

by the expression above. It should also be noted that

$$\delta_3 = \mathcal{E}_c^{\kappa} - \mathcal{E}_s > 0, \quad (2.11)$$

where the inequality follows immediately if  $a_{\kappa}|\Phi\rangle$  is not an eigenstate, but is distributed over states having  $E_n > E_{n_0}$ .

In summary we have found

$$\epsilon_s - \epsilon_{\kappa} = \frac{\langle n_0 | j_{\kappa} | \Phi \rangle}{\langle n_0 | a_{\kappa} | \Phi \rangle} \equiv \delta_1, \quad (2.12a)$$

$$\epsilon_{\kappa}^{\kappa} - \epsilon_{\kappa} = \frac{\langle \Phi | a_{\kappa}^{\dagger} j_{\kappa} | \Phi \rangle}{\langle \Phi | a_{\kappa}^{\dagger} a_{\kappa} | \Phi \rangle} = \delta_2, \quad (2.12b)$$

$$\epsilon_s - \epsilon_{\kappa}^{\kappa} = \delta_1 - \delta_2 = \delta_3 \geq 0. \quad (2.12c)$$

The quantities defined here are schematically represented in Fig. 1.

Clearly, if it were the case that  $H$  equaled  $H_0$ , then  $j_{\kappa}$  would vanish as would all of the energy differences  $\delta$  and we would have the equality of the three energies  $\epsilon$ .

We are now in a position to define the rearrangement energy. By judicious choice of  $U$  we can minimize  $\mathcal{E}_c^{\kappa}$  with respect to a variation of  $\phi_{\kappa}$ , and also make  $\delta_2$  vanish bringing the model single-particle energy equal to the negative of the centroid energy. We then define the corresponding  $\delta_3 (= \delta_1)$  as the rearrangement energy. This represents the positive energy difference between the hole "energy" and the  $(A-1)$  ground state energy. This definition of rearrangement energy corresponds to that of Weisskopf<sup>4,7</sup> in considering the relaxa-

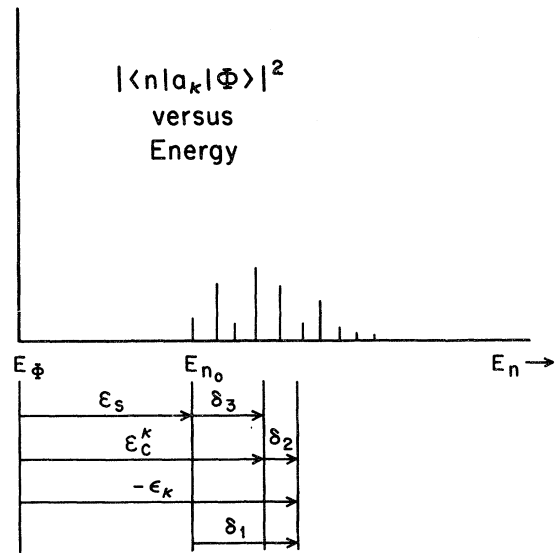


FIG. 1. Energies and energy differences defined in the text.

tion of the hole configuration into the ground state. [Note that the hole configuration will actually evolve into a set of states of the  $(A-1)$  system.]

At this point let us consider the relationship between the Fermi energy ( $\epsilon_F$ ) and the average energy per particle ( $\epsilon_A$ ). If we associate  $\epsilon_F$  with  $\epsilon_c^F$ , we find  $\epsilon_s > \epsilon_F$  from Eq. (2.11). This result is at variance with a recently published comment of Harada.<sup>8</sup> The relationship between  $\epsilon_s$ , i.e.,  $\Delta E/\Delta A$ , and  $\epsilon_{av}$ , i.e.,  $E(A)/A$ , depends on the details of nuclear binding. For heavy nuclei  $\epsilon_{av}$  is generally less than  $\epsilon_s$  and no definite inequality between  $\epsilon_{av}$  and  $\epsilon_F$  follows from Eq. (2.11). For light nuclei, however,  $\epsilon_{av}$  is often greater than  $\epsilon_s$ . In that case

$$\epsilon_{av} > \epsilon_s > \epsilon_F. \quad (2.13)$$

For these cases the average energy particle must be greater than the Fermi energy. This is an extension of an apparent paradox discussed some time ago by Bethe<sup>9</sup> who commented on the equality of  $\epsilon_{av}$  and  $\epsilon_F$ .

### III. EXAMPLES AND ILLUSTRATIONS

In this section we evaluate the expressions developed in the preceding section for certain assumed forms for  $|\Phi\rangle$  and  $H$ . We also relate our quantities to those already studied in the literature. We have already indicated that if  $H=H_0$ , then

$$\epsilon_\kappa = \epsilon_c^K = \epsilon_s. \quad (3.1)$$

#### A. $\Phi$ as Slater determinant

Let us assume that  $|\Phi\rangle$  is a single Slater determinant. It follows that

$$\begin{aligned} \delta_2 = \sum_{\beta\gamma} [(k\beta|V|k\gamma) - (k\beta|V|\gamma k)] \langle \Phi | a_\beta^\dagger a_\gamma | \Phi \rangle \\ - (k|U|k). \end{aligned} \quad (3.2)$$

Therefore if  $U$  is chosen as the Hartree-Fock potential  $\delta_2$  vanishes.

Let us next evaluate  $\delta_3(\delta_1)$ , the rearrangement energy, under the assumption that  $\Phi$  is a Slater determinant and  $U$  is the Hartree-Fock potential. The quantity  $\delta_1$  differs from zero to the extent that two-hole-one-particle components exist in  $|n_0\rangle$ . Let us calculate the strength of these components to lowest order (assuming  $|n_0\rangle$  is predominantly a one-hole state),

$$\langle n_0 | 2h1p \rangle \approx \frac{\langle \Phi | j_\kappa^\dagger | 2h1p \rangle}{\epsilon_k - \epsilon_{n_1} - \epsilon_{h_2} + \epsilon_p}. \quad (3.3)$$

This gives the following positive expression for

$\delta_1$ :

$$\begin{aligned} \delta_1 &= \sum_{2h1p} \frac{\langle 2h1p | j_\kappa | \Phi \rangle}{\mathcal{E}_{2h1p} - \mathcal{E}_\kappa} \\ &= \sum_{2h1p} \frac{\langle 2h1p | V a_\kappa | \Phi \rangle^2}{\mathcal{E}_{2h1p} - \mathcal{E}_\kappa}. \end{aligned} \quad (3.4)$$

Furthermore, this expression indicates an obvious interpretation for  $\delta_3$ , the rearrangement energy. The quantity  $(\mathcal{E}_c^K - \mathcal{E}_s)$  is a difference between an "approximate" and an exact energy for  $E_{n_0}$  if  $a_\kappa |\Phi\rangle$  is used to approximate wave function for  $|n_0\rangle$ . In Eq. (3.4) we have obtained the second order perturbation form for this difference.

#### B. Removal energy shift

In the event that the exact  $|\Phi\rangle$  is not a simple Slater determinant the use of the Hartree-Fock potential will lead a nonzero  $\delta_2$ . In other words, one can calculate, using Eq. (2.9), a difference between the hole "energy" and the model single-particle energy. Koltun has referred to the former as the mre (mean removal energy), and the difference between this and the model single-particle energy as proportional to "removal energy shift." Koltun following Brandow has shown that a self-consistent potential can be chosen which will provide model single-particle energy identical with the mre, so that  $\delta_2$  vanishes.

If the state  $\Phi$  is not a *single* Slater determinant then it will be possible to add, as well as remove, a particle from the state  $\kappa$ . This possibility underlies the discussion of Baranger,<sup>10</sup> who introduces a double centroid energy composed of one contribution for removal, and one for addition:

$$\mathcal{E}_c^{K \text{ remove}} = -\epsilon_\kappa - \frac{\langle \Phi | \{a_{\kappa_2}^\dagger, j_\kappa\} | \Phi \rangle}{N(\kappa)} + \frac{\langle \Phi | j_\kappa a_{\kappa_1}^\dagger | \Phi \rangle}{N(\kappa)}, \quad (3.5a)$$

$$\mathcal{E}_c^{K \text{ add}} = -\epsilon_\kappa - \frac{\langle \Phi | j_\kappa a_{\kappa_1}^\dagger | \Phi \rangle}{N(\kappa) - 1}. \quad (3.5b)$$

He defines a double centroid,

$$\mathcal{E}_c^{KB} = N(\kappa) \mathcal{E}_c^{K \text{ remove}} + [1 - N(\kappa)] \mathcal{E}_c^{K \text{ add}}, \quad (3.6)$$

and finds

$$\mathcal{E}_c^{KB} = -\epsilon_\kappa - \langle \Phi | \{a_{\kappa_2}^\dagger, j_\kappa\} | \Phi \rangle. \quad (3.7)$$

In this formulation one might define another difference between centroid and single-particle energy,

$$\delta_2^B = \langle \Phi | \{a_{\kappa_2}^\dagger, j_\kappa\} | \Phi \rangle. \quad (3.8)$$

Baranger argues that there is an appropriate choice of a single-particle potential which mini-

mizes  $\mathcal{E}_c^{\kappa^B}$  and causes  $\delta_2^B$  to vanish.

In a different context this same effect has been discussed by chemists with regard to electronic level energies.<sup>11</sup> In their language it is the energy difference between Hartree-Fock energies and that of "natural transition orbitals."

In each case the energy difference  $\delta_2$  is purely a function of one's model and represents no physical process. It should not be confused with the rearrangement energy defined above.

### C. Brueckner rearrangement

One source of confusion surrounds the relationship of rearrangement energy and the rearrangement potential  $V_N$  of Brueckner and Goldman (BG).<sup>3</sup> In using the Brueckner techniques one must work with model wave functions and a model Hamiltonian  $\mathcal{H}$ . Let us use the BG single-particle Hamiltonian including the rearrangement term  $V_N$  to find single-particle energies and wave functions,

$$(T + U^{BG} + V_N^{BG})\phi_\lambda = \epsilon_\lambda \phi_\lambda. \quad (3.9)$$

We shall use a basis of these states below. Let us use the following form for the many-body Hamiltonian:

$$\mathcal{H} = \mathcal{H}^{(1)} + \Delta\mathcal{H}, \quad (3.10a)$$

where

$$\mathcal{H}^{(1)} = T + \bar{K}. \quad (3.10b)$$

$$\bar{K} = +\frac{1}{2} \sum (\alpha\beta | K(\Phi) | \gamma\delta) a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma, \quad (3.10c)$$

$$\Delta\mathcal{H} = \sum_i \frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_i} (a_i^\dagger a_i - \langle\Phi|a_i^\dagger a_i|\Phi\rangle). \quad (3.10d)$$

In  $\mathcal{H}^{(1)}$ , the two-body  $K$  matrix is that one characteristic of the density of  $\Phi$ , and

$$\langle\Phi|\mathcal{H}|\Phi\rangle = \langle\Phi|\mathcal{H}^{(1)}|\Phi\rangle. \quad (3.11)$$

The additional term  $\Delta\mathcal{H}$  is required to change the strength of the effective two-body interaction  $K$  when  $\mathcal{H}$  operates on a state with density differing from that of  $\Phi$ .

We next consider the centroid energy within the Brueckner context:

$$\begin{aligned} \mathcal{E}_c^{\lambda,B} &= \langle\Phi|a_\lambda^\dagger(\mathcal{H} - E_\Phi)a_\lambda|\Phi\rangle \\ &= \sum (E_n - E_\Phi) |\langle n|a_\lambda|\Phi\rangle|^2, \end{aligned} \quad (3.12a)$$

$$\mathcal{E}_c^{\lambda,B} = \langle\Phi|a_\lambda^\dagger(\mathcal{H}^{(1)} - E_\Phi)a_\lambda|\Phi\rangle + \langle\Phi|a_\lambda^\dagger\Delta\mathcal{H}a_\lambda|\Phi\rangle. \quad (3.12b)$$

The last term on the right hand side is given by

$$\langle\Phi|a_\lambda^\dagger\Delta\mathcal{H}a_\lambda|\Phi\rangle = -\frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_\lambda} \quad (3.13)$$

and is attractive. It represents an extra binding energy arising from the increased two-body potential to be used when  $\mathcal{H}$  operates on the hole configuration rather than state  $\Phi$ .

We treat the first term in Eq. (3.12b) as we did Eq. (2.3), giving

$$[\mathcal{H}^{(1)}, a_\lambda] = -\epsilon_\lambda a_\lambda - j_\lambda^a - j_\lambda^b, \quad (3.14a)$$

$$j_\lambda^a = [\bar{K} - U, a_\lambda], \quad (3.14b)$$

$$j_\lambda^b = [-V_N, a_\lambda]. \quad (3.14c)$$

It can then be shown that

$$\langle\Phi|a_\lambda^\dagger(\mathcal{H}^{(1)} - E_\Phi)a_\lambda|\Phi\rangle = -\epsilon_\lambda + (\lambda|V_N|\lambda). \quad (3.15)$$

The term  $(\lambda|V_N|\lambda)$  is repulsive. It is associated with the rearrangement potential. Combining Eqs. (3.15) and (3.13) we obtain

$$\mathcal{E}_c^{\lambda,B} = -\epsilon_\lambda + (\lambda|V_N|\lambda) - \frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_\lambda}. \quad (3.16)$$

Furthermore, it can be shown that

$$(\lambda|V_N|\lambda) = \frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_\lambda}. \quad (3.17)$$

Therefore, with the single-particle states obtained using  $V_N$  we have  $\epsilon_c^{\lambda,B} = \epsilon_\lambda$ , and  $\delta_2$  vanishes.

If we had assumed  $\Phi$  to be a Slater determinant of single-particle states obtained from a potential lacking  $V_N$ ,

$$(T + U^{BG})\chi_\eta = \tilde{\epsilon}_\eta \chi_\eta, \quad (3.18)$$

we would have found

$$\langle\Phi|a_\eta^\dagger(\mathcal{H}^{(1)} - E_\Phi)a_\eta|\Phi\rangle = -\tilde{\epsilon}_\eta. \quad (3.19)$$

Then the centroid and  $\tilde{\epsilon}_\eta$  differ as shown:

$$\mathcal{E}_c^{\eta,B} = -\tilde{\epsilon}_\eta - \frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_\eta}, \quad (3.20a)$$

$$\epsilon_c^{\eta,B} - \tilde{\epsilon}_\eta = \frac{\delta\langle\Phi|\bar{K}|\Phi\rangle}{\delta n_\eta}. \quad (3.20b)$$

In other words,  $\delta_2$  is  $\delta\langle\Phi|\bar{K}|\Phi\rangle/\delta n_\eta$ . This difference between centroid energy and the single-particle energy arises from the "poor" choice of single-particle potential—one lacking  $V_N$ . The energy shift  $\delta\langle\Phi|\bar{K}|\Phi\rangle/\delta n_\eta$  seems to be of the  $\delta_2$  ("removal energy shift") type discussed above. This energy should not be considered a rearrangement energy,<sup>12</sup> as defined in this paper, for it is not related to the difference between the  $A-1$  particle ground state  $|n_0\rangle$  and the hole configuration  $a_\lambda|\Phi\rangle$ .

It can be made to vanish by the proper choice of single-particle potential.

#### D. Rearrangement energy and reaction time

Meldner and Perez<sup>5</sup> have defined a rearrangement energy which is identical to the one used in this paper. Their definition is in terms of centroids (differing from ours), related to measured quantities:

$$\mathcal{G}_c^{\text{MP}}(\tau) = \frac{\sum_n C_n(\kappa, \tau)(E_n - E_\Phi)}{\sum_n C_n(\kappa, \tau)}, \quad (3.21)$$

where  $C_n(\kappa, \tau)$  is the observed strength for going to state  $|n\rangle$  by the removal of a particle from state  $\kappa$  which takes place over a time  $\tau$  ( $\tau \rightarrow 0$ , sudden;  $\tau \rightarrow \infty$  adiabatic). The weights  $C_n(\kappa, \tau)$  can be taken as  $\sigma_n(\kappa, E_{\text{avail}})$  which is the cross section for going to state  $|n\rangle$ , with the removal of a particle from state  $\kappa$  involving available energy  $E_{\text{avail}}$  ( $E_{\text{avail}} \rightarrow 0$ , adiabatic,  $E_{\text{avail}} \rightarrow \infty$ , sudden). Clearly as  $E_{\text{avail}} \rightarrow 0$ , only the state  $|n_0\rangle$  can be formed so that

$$\mathcal{G}_c^{\text{MP}}(E_{\text{avail}} \rightarrow 0) = E_{n_0} - E_\Phi = \mathcal{G}_s. \quad (3.22)$$

On the other hand, for  $E_{\text{avail}} \rightarrow \infty$  it can be argued that

$$\sigma_n(\kappa, E_{\text{avail}} \rightarrow \infty) \propto |\langle n | a_\kappa | \phi \rangle|^2, \quad (3.23)$$

so that

$$\mathcal{G}_c^{\text{MP}}(E_{\text{avail}} \rightarrow \infty) = \mathcal{G}_c^K. \quad (3.24)$$

The difference between the MP centroids at the two energy (time) extremes MP call the rearrangement energy. This difference is  $(\epsilon_s - \epsilon_c^K)$ , and is the same as our definition of rearrangement energy. In Sec. IV, I will consider the effect of the speed of the reaction from a different point of view.

#### IV. REARRANGEMENT AND THE ANALYSIS OF EXPERIMENTAL DATA

The concept of rearrangement arises in the experimental context where it is observed that the strength with which various final target states are populated in particle exchange, or knockout, reactions depends on the total energy involved in the reaction, or the "speed" of the reaction. Meldner and Perez following the general time dependent approach of Pauli<sup>13</sup> have discussed this. The intuitive picture applied to the process is as follows: In a slow, or adiabatic, removal of a nucleon from an  $A$ -body system the reaction would lead to the ground state of the  $A - 1$  system. Whereas, for rapid removal it would lead to a one-hole configuration, and hence to a distribution of eigenstates of the  $A - 1$  system. I shall examine this situation

in some detail and explore the relationship between experimentally obtained spectroscopic factors, various approximate reaction models, and the phenomena of rearrangement or redistribution.

As a concrete example, let us consider an  $(e, e'p)$  knockout reaction. We assume that the electron only interacts once with an  $A$ -body nucleus, and that this results in the removal of a proton from the single-particle state  $\phi_h(r)$ . Furthermore, let us assume that the removal occurs by means of a potential of the form  $V_0 \delta(\vec{r} - \vec{r}_e)$ . Let us also assume that resulting final states of our system can be asymptotically specified by the momentum of the removed particle  $k'_p$  and the eigenstate of the  $A - 1$  system  $|n\rangle$ . The amplitude for the knockout is then  $\langle k'_p k'_n | T | k_e \Phi \rangle$ , where we neglect recoil of the target. Clearly, energy conservation restricts the states  $|n\rangle$  which can be populated by the reaction. We indicate the exact scattering state of a proton and a  $(A - 1)$  system in state  $|n\rangle$ , by  $|n, k_p^{(-)}\rangle$ , and

$$\langle k'_p k'_n | T | k_e \Phi \rangle = \sum_{n'} M_{nn'}(k'_p, k'_e, k_e) \langle n' | a_h | \Phi \rangle, \quad (4.1)$$

where

$$M_{nn'}(k'_p, k'_e, k_e) = V_0 \int \psi_{n, k_p^{(-)}}^*(r) e^{i(k_e - k'_e) \cdot r} \phi_h(r) d^3r \quad (4.2a)$$

with

$$\psi_{n, k_p^{(-)}}^*(r) = \langle n k_p^{(-)} | \psi^\dagger(r) | n' \rangle, \quad (4.2b)$$

being that part of the wave function with the target in state  $|n'\rangle$  which asymptotically has the target in state  $|n\rangle$ , and a proton in momentum state  $k'_p$ .

The expression in Eq. (4.1) includes the spectroscopic amplitude  $\langle n' | a_h | \Phi \rangle$  which gives information about the relation of the eigenstates  $|n'\rangle$  and  $|\Phi\rangle$ . This is the quantity which was used in the discussion of earlier sections. The amplitude  $T$  gives a measure of the strength with which state  $|n\rangle$  is populated by the knockout. It depends on the energy of the reaction. The spectroscopic factor is, on the other hand, independent of the energy of the reaction. The matrix  $M_{nn'}$ , which connects these quantities, contains information concerning the redistribution of the strength in the course of the reaction.

The reaction can be viewed in the following way: (a) A hole is created in the single-particle state  $h$  with a well defined distribution into eigenstates  $|n\rangle$  of the  $A - 1$  system; (b) through final-state interactions between the  $A - 1$  system and the removed particle, the strength is redistributed among the states  $|n\rangle$  to achieve the final strength. For the case of adiabatic removal, all the strength

is sent to the ground state. At the other extreme, if the removed particle never interacts with the system, then the strength is essentially that associated with the hole configuration. One experimentally measures quantities related to  $\langle k'_p k'_n | T | k_e \Phi \rangle$ , and then with a reaction model for  $M_{nn'}$  tries to obtain spectroscopic information by unfolding the effects of the final state interactions. Schematically,

$$\langle n | a_h | \Phi \rangle = \sum M_{nn'}^{-1} \langle n' | T | \Phi \rangle, \quad (4.3)$$

where only states  $|n'\rangle$  having nonzero values for  $\langle n' | a_h | \Phi \rangle$  need be considered. The success or failure in obtaining reliable values for  $\langle n | a_h | \Phi \rangle$  thus depends on the model used to construct  $M_{nn'}$ .

We next examine various models for  $M_{nn'}$  and determine their ability to obtain valid spectroscopic factors.

#### A. Plane wave Born approximation

In this approximation

$$M_{nn'}(k'_p k'_e, k_e) \propto V_0 \delta_{nn'} \tilde{\phi}_h(k'_p + k'_e - k_e), \quad (4.4)$$

where  $\tilde{\phi}_h$  is the Fourier transform of the hole state. This model completely ignores redistribution. It assumes that the scattering state which asymptotically has the target in state  $|n\rangle$ , has the target in this state everywhere in space. Let us assume, for example, that the exact spectroscopic amplitudes were given by

$$\langle n | a_h | \Phi \rangle^{\text{exact}} = \delta_{n,n_0} \langle n_0 | a_h | \Phi \rangle. \quad (4.5)$$

Then, the reaction amplitude  $\langle n_0 | T | \Phi \rangle$  (the strength for going to  $|n_0\rangle$ ) would depend on the extent to which  $|n_0 k_p^{(-)}\rangle$  contained the state  $|n_0\rangle$  near the target. Since plane wave Born approximation completely ignores the redistribution, it yields a spectroscopic factor  $|\langle n_0 | a_h | \Phi \rangle|^2$  which is much too small.

#### B. Distorted wave Born approximation

In this model

$$M_{nn'}(k'_p k'_e, k_e) \propto \delta_{nn'} \int \psi_{n k_p}^{*(-)}(r) e^{i(k_e - k'_e) \cdot r} \phi_h(r) d^3r. \quad (4.6)$$

No explicit redistribution is included but, within the framework of the optical model, collisions leading from the asymptotic target state to other target states are taken into account by the absorption of the imaginary potential. In this way a wave function which asymptotically has the target in state  $|n_0\rangle$  will have diminished probability of having it in this state near the origin. Consider again

the case that

$$\langle n | a_h | \Phi \rangle^{\text{exact}} = \delta_{n_0, n} \langle n_0 | a_h | \Phi \rangle. \quad (4.7)$$

The correct size of the spectroscopic factor can now be obtained through the effect of the imaginary part of the optical potential. If the hole configuration were spread among several states  $|n\rangle$ , then distorted wave Born approximation (DWBA) would have a chance of giving correct relative spectroscopic factors only if the off diagonal matrix elements in  $M_{nn'}^{\text{exact}}$  were small, and the DWBA (through the complex potential) were to approximate the diagonal ones. The former condition is most likely to hold at high energies where the probability of redistribution within a *finite* range of states is small. At very high energies the nature of the distortion will probably be primarily one of the absorption of plane waves. In that case the DWBA may do quite well.

#### C. Coupled-channel Born approximation

This model gives

$$M_{nn'}(k'_p k'_e, k_e) = \int \psi_{k'_p n, n'}^{cc}(r) e^{i(k_e - k'_e) \cdot r} \phi_h(r) d^3r. \quad (4.8)$$

Here, explicit redistribution is allowed, and one consequently has the best chance for obtaining reliable relative, as well as absolute, spectroscopic factors for those cases in which the off diagonal elements of  $M_{nn'}$  are not small.

In summary the extent to which the model  $M_{nn'}^{-1}$  approximates the exact one will determine the reliability of spectroscopic factors. Thus, even in an "adiabatic" reaction in which all the strength goes to the ground state, one may find spectroscopic factors which reflect the "sudden" hole strength distribution if the correct  $M_{nn'}^{-1}$  is employed,

$$\langle n | a_h | \Phi \rangle = M_{nn_0}^{-1} \langle n_0 | T | \Phi \rangle. \quad (4.9)$$

It is evident that, for very low energy reactions, the DWBA (diagonal treatment) will fail. This is not surprising.

The job for a reaction theory, regarding spectroscopic information, is to undo the physical rearrangement which has taken place. It must set the system back to the condition which prevailed at the creation of the hole. Clearly, in certain cases coupled-channel calculations offer the only hope for this. While  $\langle n | a_h | \Phi \rangle$  itself does not depend on the "speed" of the reaction,  $\langle n | T | \Phi \rangle$  does, and so too does the degree of difficulty in obtaining a reaction theory which provides reliable spectroscopic factors.

## V. CONCLUSION

We have tried to clarify the definition of rearrangement energy by defining three distinct energies: (a) single-particle model energy, (b) hole configuration energy, (c) threshold separation energy. We have established specific expressions which can be used to calculate the differences among these.

We have defined the rearrangement energy as one of these differences. This definition is in line with the concept of the relaxation of a hole configuration to the ground state, as suggested by Weisskopf. It is also the same as the definition more recently suggested by Meldner and Perez. We suggest that this energy may be viewed as the difference between an exact and an "approximate" energy for the ground state, and hence estimated by simple perturbation theory. We have shown

that the average energy/particle ( $\epsilon_{av}$ ) must be greater than the Fermi energy ( $\epsilon_F$ ) if  $\epsilon_{av} > \epsilon_s$  (the case for many light nuclei); while  $\epsilon_{av}$  may be smaller than  $\epsilon_F$  if  $\epsilon_{av} < \epsilon_s$  (the case for most heavy nuclei).

We have made a distinction between the rearrangement energy and the "removal energy shift" discussed by Koltun and others, and have shown that the BG rearrangement energy falls into this latter category.

Our discussion of the relationship between rearrangement and experimental spectroscopic factors has shown that reaction models must "undo" the rearrangement if correct spectroscopic factors are to be found.

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