Galilean invariance of the pion-nucleon interaction*

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We examine the role of Galilean invariance in a nonrelativistic theory of pions and nucleons. We define a nonrelativistic pion-nucleon interaction h as one which in perturbation theory gives the same matrix for a physical reaction as the limit of small nucleon velocities of the relativistic matrix. The latter matrices are always Galilean invariant, but this does not require that h be Galilean invariant. If pion emission or absorption occurs from a nucleon moving in a potential, then the Galilean correction term can be shown to be ambiguously of order v/c or of order v^2/c^2. We show that h cannot always reproduce the nonrelativistic limit of a relativistic matrix. Finally, we suggest that pion production by nucleons on nuclei with excitation of giant dipole and quadrupole states may be particularly sensitive to the presence of a Galilean correction term in the production matrix.

[NUCLEAR REACTIONS Nonrelativistic form of pion-nucleon interaction.]

I. INTRODUCTION

By this time a considerable number of papers have been written about Galilean invariance (GI) of the pion-nucleon interaction. There are two reasons why we join the venture. The first one is that we believe that previous articles are incomplete, so that their interpretation may be difficult and even confusing. The second reason is that we would like to propose possible tests of Galilean invariance.

The relativistic pion-nucleon interaction can be of the following types[1]:
(a) pseudoscalar coupling

$$H_{\pi N}=ig\sum_{i=1}^{3}\int\bar{\psi}\gamma^5\tau_i\psi\phi_i d^3x; \quad (1)$$

(b) pseudovector coupling

$$H'_{\pi N}=G\sum_{i=1}^{3}\int\bar{\psi}\gamma^5\gamma^\mu\tau_i\psi\nabla_\mu\phi_i d^3x, \quad (2)$$

where $\bar{\psi}$, ψ are nucleon field operators and ϕ is that of the pion. The pseudoscalar interaction is preferred generally because it is a nonderivative one and is renormalizable. On the other hand, the pseudovector coupling is suggested by soft pion and partially conserved axial-vector current (PCAC) theories.[2] As pointed out by Dyson and by Foldy,[3] the two interaction Hamiltonians are equivalent to first order in the coupling constants if $G=g/2M$.

It has been pointed out by Barnhill[4] that the nonrelativistic (NR) reduction of the pseudoscalar Hamiltonian is not unique if it is carried out to first order in g with the neglect of relativistic terms of order M^{-2}. To this order, the NR limit is often written as[4]

$$H_{\pi N}=-\frac{g}{2M}\sum_i\int d^3x\Big[\psi^\dagger\vec{\sigma}\cdot\vec{\nabla}_\pi\tau_i\psi\phi_i -\lambda\frac{\mu}{2M}\psi^\dagger\vec{\sigma}\cdot(\vec{\nabla}_N-\overleftarrow{\nabla}_N)\tau_i\psi\phi_i\Big]. \quad (3)$$

This form, with $\lambda=1$ and $\mu\to q_0$ the pion energy, is obtained directly from Eq. (2). In Eq. (3) μ and M are pion and nucleon masses, respectively, $\vec{\nabla}_\pi$ operates only on ϕ whereas $\vec{\nabla}_N$ and $\overleftarrow{\nabla}_N$ operate on ψ and $\psi^\dagger$, respectively. The nonuniqueness resides in λ which is an arbitrary number; it was shown by Barnhill that the NR interactions with different values of λ are related by canonical transformations. Thus, if the NR wave functions are transformed by the same unitary operator, the physical results remain unchanged despite the "ambiguity." The latter comes about from the different order in which the *two* required Foldy-Wouthuysen transformations can be carried out. The normal first-order transformation[5] gives to order g

$$H_{\pi N}=-\frac{g}{2M}\sum_i\int d^3x\,\psi^\dagger(\vec{\sigma}_D\cdot\vec{\nabla}_\pi\tau_i\phi_i+\gamma_5\tau_i\dot{\phi}_i)\psi, \quad (4)$$

where $\dot{\phi}\equiv\partial\phi/\partial t$ and $\vec{\sigma}_D$ is the Dirac spin operator. This semirelativistic form shows the equivalence of the interactions given by Eqs. (1) and (2). If both initial and final nucleons are free ones, then the result of the nonrelativistic reduction of the last term of Eq. (4) is equal to the last term of Eq. (3) if $q_0=\mu$ and $\lambda=1$. This value of λ is said to make the NR interaction Hamiltonian Galilean invariant. Physically, the ambiguity arises because pion emission (or absorption) can occur only if at least one of the particles is not on its mass shell—i.e., is not free.

Recently, the notion of GI for a theory of nonrelativistic nucleons interacting with relativistic (or nonrelativistic) pions has been questioned. Since the interaction (1)–(4) allows the creation and destruction of single pions, we are not dealing with a normal nonrelativistic one-particle quantum theory. Moreover, as first pointed out by Bolsterli *et al.*,[6] the NR reduction of Eq. (1) depends on whether the potential in which the nucleon is assumed to move is a relativistic scalar or the fourth component of a relativistic four-vector. This feature can be brought out also by rewriting Eq. (4) as

$$H_{\pi N}=-\frac{g}{2M}\sum_i\int d^3x\,\psi^\dagger\vec{\sigma}_D\cdot(\vec{\nabla}_\pi\phi_i+\tfrac{1}{3}\vec{\alpha}\dot{\phi}_i)\tau_i\psi\,. \tag{5}$$

The second term clearly indicates that the Galilean correction is proportional to the relativistic velocity operator $\vec{\alpha}$. The nonrelativistic reduction of this term is known to depend on the off-mass shell behavior of the nucleon.[7]

As already argued by Eisenberg, Noble, and Weber,[8] it is not the nonrelativistic pion-nucleon interaction Hamiltonian which must satisfy GI, but only the physical transition matrix element, since it is the latter which is the NR limit of a measurable relativistic quantity. We will demonstrate this feature explicitly, but in doing so we shall not restrict ourselves to a nucleon moving in a potential; indeed, the nucleon may be free. For a nucleon moving in a potential, there exists an obviously preferred frame and, as argued by Friar,[9] the GI correction term in the NR matrix element is actually of order v^2/c^2 in this case, where v is a nucleon velocity. By contrast, if the pion emission occurs in the presence of the field of another meson or photon, the GI term is of order v/c, but the nonrelativistic effective pion-nucleon interaction generally is not well defined. In particular, there is also a Barnhill-type ambiguity in the NR effective pion-nucleon interaction. We shall demonstrate these features in the next section. Throughout the remainder of this paper,

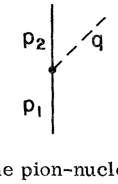

FIG. 1. The pion-nucleon vertex.

we shall consistently keep corrections of order v/c to the dominant terms, but generally discard relativistic corrections, i.e., those of order v^2/c^2.

II. EFFECTIVE π-N INTERACTION

We make use of perturbation theory to define an effective nonrelativistic pion-nucleon interaction. We will show that the form of the interaction to order v/c depends on the off-mass shell behavior of the meson or nucleon; however, the matrix element for the physical process remains Galilean invariant independent of this arbitrariness.

The pion-nucleon interaction is responsible for pion emission (absorption) as illustrated in Fig. 1. One of the particles shown must be off-mass shell since it is not possible to conserve both energy and momentum in the emission (absorption) process.

Consistent with the usual definitions of nonrelativistic potentials, we define an effective nonrelativistic interaction as one that in perturbation theory gives the same matrix element as the nonrelativistic limit of the relativistic scattering matrix. Consider a pion-nucleon system together with one further field (e.g., electromagnetic, nuclear, etc.) represented by H'; the Hamiltonian for such a system can be written as

$$H=H_0+H_{\pi N}^{\text{eff}}+H'\,, \tag{6}$$

where H_0 is the free nucleon and pion Hamiltonian, $H_{\pi N}^{\text{eff}}\equiv h$ is the effective interaction at the vertex of Fig. 1, and H' is the additional interaction with the other field. We stress that H' need not be an external potential; this interaction serves to bring the nucleon or pion back onto its mass shell. To first order in h and H', the nonrelativistic matrix for a reaction involving both of these operators is

$$\mathfrak{M}_{fi}^{\text{NR}}=\left\langle f\left|H'\frac{1}{E-H_0}h+h\frac{1}{E-H_0}H'\right|i\right\rangle\,. \tag{7}$$

Our definition of $H_{\pi N}^{\text{eff}}$ does not imply that it or H' is weak or that Eq. (6) is to be solved in perturbation theory; indeed one advantage of a NR treatment is the availability of distorted-wave Born-approximation (DWBA) methods. $H_{\pi N}^{\text{eff}}=h$ represents, rather, an expansion in powers of M^{-1} of the relativistic Hamiltonian which gives identical answers to that obtained with Eq. (6) *if* perturbation techniques are used.

In the comparison of the relativistic and nonrelativistic matrices, it must be realized that there also occur recoil corrections of order v/c.

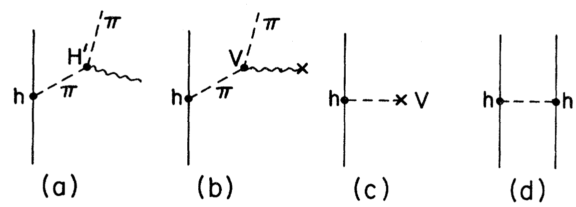

FIG. 2. Typical Feynman diagrams for physical processes which involve the effective pion-nucleon interaction with a virtual pion. (a) Scattering by an external light particle, e.g., photon. (b) Scattering by a potential. (c) Pion absorption by a nucleus (potential scattering). (d) Nucleon-nucleon scattering.

It is the propagator $(E-H_0)^{-1}$ in Eq. (7) which contains recoil corrections.

In order to discuss GI we shall consider, separately, the case of a pion off-mass shell and a nucleon off-mass shell. We further differentiate between a nucleon moving in a potential and in the field of another meson or photon.

A. Pion off-mass shell

The question of GI for the case of nucleons on mass, but the pion off-mass shell in Fig. 1, has not been considered by previous authors. Examples of physical processes where such considerations apply are shown in Fig. 2. It is straightforward to show that the effective interaction for all reactions illustrated in Fig. 2 is

$$h=-\frac{g}{2M}\sum_i\int d^3x\psi^\dagger\vec{\sigma}\cdot\vec{\nabla}_\pi\tau_i\psi\phi_i\,. \tag{8}$$

There is no GI term when the pion is off-mass shell but the nucleons are free. The matrix element for a physical process is nonetheless GI. This is readily appreciated for nucleon-nucleon scattering due to one-pion exchange illustrated in Fig. 2(d).

The lack of a GI term in h is brought out explicitly by Eq. (4) or (5) and shows the disadvantage of Eq. (3). There $\dot{\phi}=\pm iq_0\phi$, where q_0 is the time component of the pion four-momentum, has been approximated by $\pm i\mu\phi$. However, whereas q_0 (the energy of the virtual pion) vanishes to order M^{-1} for all situations of physical interest [except that represented by Fig. 2(b)], μ does not do so. Corrections to Eq. (8) are only of order v^2/c^2 and are thus relativistic ones. For the exception [Fig. 2(b)] where the pion interacts with a potential, the GI correction is also of order v^2/c^2 as shown in Sec. IIB, below [see Eq. (12) and beyond].

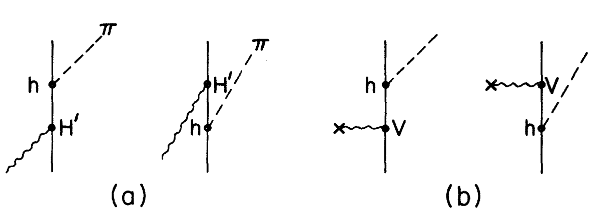

FIG. 3. Typical Feynman diagrams for physical processes which involve the effective pion-nucleon interaction with a virtual nucleon. (a) Pion production by an incident light particle (e.g., σ meson, photon, etc.). (b) Pion production in a "potential" (e.g., nucleus).

B. Nucleon off-mass shell

The second case of physical interest is when the nucleon is off-mass shell, but the pion is a physical one; we consider first cases illustrated in Fig. 3 in which either the initial *or* final nucleon is off-mass shell. In Fig. 3(a) the particle of momentum $\vec{k}$ is assumed to be a meson or photon. In Fig. 3(b) the nucleon moves in a potential $H'=V$. In both of these cases, and for an arbitrary interaction H', there is a 1:1 correspondence between the nonrelativistic limit of the relativistic matrix element and Eqs. (4) or (5), used in conjunction with a nonrelativistic propagator, but *only if* the second (Galilean) term of the last two equations is included. It is the nonrelativistic reduction of the *matrix* for the physical process which must be evaluated, rather than for the pion-nucleon vertex alone. Furthermore, if the interaction H' is a pseudoscalar (e.g., pion-nucleon interaction), it is not sufficient to retain terms of order g in the NR reduction of Eq. (1); terms of order g^2 must be kept.[5,10]

It is necessary to specify the interaction H' to obtain the effective pion-nucleon interaction h. The result depends not only on whether $[h,H']=0$, but also on whether H' represents a coupling to a light meson as in Fig. 3(a) or to a potential as in Fig. 3(b). To bring out the differences of these situations we consider three separate examples: (1) H' is an isoscalar and spatial scalar, (2) H' is the third component of an isovector and we consider $\pi^\pm$ emission, (3) H' is an isoscalar but of the form $\vec{\tau}\cdot\vec{T}$, where T is the isospin of the nucleus or of the external meson. In each of these instances we shall differentiate between a potential and the field of an external meson or photon.

1. H' = isoscalar and space scalar

We assume that the vertex function can simply be represented by the function $f(k)$. The NR reduction of the relativistic matrix represented by the two diagrams of Fig. 3(a) is

$$\mathfrak{M}_i = gf(k)\bar{u}(p')\tau_i[\gamma^5(\not{p}' + \not{q} - M)^{-1} + (\not{p} - \not{q} - M)^{-1}\gamma^5]u(p)$$

$$\approx gf(k)\chi^\dagger\tau_i\left[\frac{\vec{\sigma}\cdot\vec{q} - (\omega/2M)\vec{\sigma}\cdot(\vec{p}+\vec{p}')}{2M\omega - 2\vec{p}'\cdot\vec{q} + \mu^2} - \frac{\vec{\sigma}\cdot\vec{q} - (\omega/2M)\vec{\sigma}\cdot(\vec{p}+\vec{p}')}{2M\omega - 2\vec{p}\cdot\vec{q} - \mu^2}\right]\chi, \tag{9}$$

where χ is a two-component spinor. In order to obtain the same matrix for the same diagrams with Eq. (7) we must choose h as

$$h = -\frac{g}{2M}\sum_i \int d^3x\,\psi^\dagger\vec{\sigma}\cdot\vec{\nabla}_\pi\phi_i\left(1 - \frac{\omega}{2M}\right)\tau_i\psi. \tag{10}$$

This equation is analogous to that obtained by Bolsterli *et al.*[6] and is not "Galilean invariant," but h nevertheless gives rise to the GI matrix element Eq. (9). The reasons for the difference of h and Eqs. (4) and (5) or Eq. (1) is that with these equations the GI terms appear for each of the diagrams of Fig. 3(a), whereas the *sum* of these diagrams is required to produce the GI of the matrix with the use of Eqs. (7) or (10).

The difference between Figs. 3(a) and 3(b) resides in energy considerations. For Fig. 3(a) we assume that the particle represented by the momentum $\vec{k}$ and the π meson both have masses much smaller than that of the nucleon. Energy conservation then gives $\omega \approx k_0$ and

$$\mathcal{O}\left(\frac{p}{M}\right) \sim \mathcal{O}\left(\frac{q}{M}\right) \sim \mathcal{O}\left(\frac{k}{M}\right) \sim \mathcal{O}\left(\frac{\omega}{M}\right). \tag{11}$$

By contrast, the potential in Fig. 3(b) can absorb (or give up) no energy but an arbitrary amount of momentum $\vec{k}$. In this case $k_0 = 0$ and

$$\frac{p^2 - p'^2}{2M} \approx \omega. \tag{12}$$

Thus, in this case we have ω/M of order $p^2/M^2 = v^2/c^2$. The Galilean term $\vec{\sigma}\cdot(\vec{p}+\vec{p}')\omega/2M$ is therefore of the order of a relativistic correction to the nonrelativistic interaction $\vec{\sigma}\cdot(\vec{p}+\vec{p}')$, even though it appears to be a correction of order v/c to the pion-nucleon interaction $\vec{\sigma}\cdot\vec{q}$. That is, the ratio of the Galilean correction term $\omega\vec{\sigma}\cdot(\vec{p}+\vec{p}')/2M$ to the primary interaction term $\vec{\sigma}\cdot\vec{q}$ is indeed of order p/M or v/c. It may, nevertheless, arise from relativistic corrections. If we only keep terms of order v/c and discard corrections of order v^2/c^2, then the GI correction term is ambiguous. This is brought out in an explicit example in the Appendix.

Thus, the correction term $\sim\omega/2M$ in Eq. (10) should not be retained if $H' = V$ since it is of order v^2/c^2 in that case or one should retain all corrections of this order. In particular the nonrelativistic reduction carried out, e.g., in Eq. (9), may not be unitary to this order.

2. $H' \propto \tau_3$ and space scalar

For case (2) it is straightforward to obtain the nonrelativistic limit of Eq. (1) as

$$\mathfrak{M} = gf(k)\chi^\dagger\left[\frac{\vec{\sigma}\cdot\vec{q} - (\omega/2M)\vec{\sigma}\cdot(\vec{p}+\vec{p}')}{2M\omega - 2\vec{p}'\cdot\vec{q} + \mu^2} + \frac{\vec{\sigma}\cdot\vec{q} - (\omega/2M)\vec{\sigma}\cdot(\vec{p}+\vec{p}')}{2M\omega - 2\vec{p}\cdot\vec{q} - \mu^2}\right]\chi \tag{13}$$

since $\{\tau_\pm, \tau_3\} = 0$. In this case, the effective nonrelativistic Hamiltonian which gives the same matrix element is

$$h = -\frac{g}{2M}\sum_i \int d^3x\,\psi^\dagger\tau_i\vec{\sigma}\cdot\left[\vec{\nabla}_\pi\phi_i - \frac{i}{2M}(\vec{\nabla}_N - \overleftarrow{\nabla}_N)\dot{\phi}_i\right]\psi \tag{14}$$

which has the appearance of a GI interaction.

If we consider the analogous case for potential scattering, then the Galilean correction term proportional to $\omega/2M$ in Eq. (14) should be dropped since it is a relativistic correction to an effective interaction proportional to $\psi^\dagger(\vec{\nabla}_N - \overleftarrow{\nabla}_N)\psi$. It is, however, also a correction of order v/c to the primary interaction Hamiltonian density proportional to $\vec{\nabla}_\pi\phi_i$. Note the ambiguity.

3. $H' \propto \Sigma_i\tau_i\Phi_i$ and space scalar

As a last example, we consider an interaction H' of the form $H' \propto \vec{\tau}\cdot\vec{\Phi}$, where $\vec{\Phi}$ is the field of a particle of isospin unity in Fig. 3(a); Φ is effectively replaced by the isospin $\vec{T}$ of the nucleus in Fig. 3(b). In this third case *no* solution to h [defined by Eq. (7)] can be found which reproduces the NR limit of the relativistic matrix element. The reason is that H' neither commutes nor anticommutes completely with h; it would be necessary to choose h of the form Eq. (10) for that part of H' which commutes with $\sum_i \tau_i\phi_i$ and h of the form Eq. (14) for that part of H' which anticommutes with $\sum_i \tau_i\phi_i$. There is, however, a more

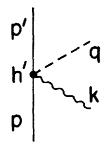

FIG. 4. Seagull or contact term required to obtain agreement between the NR matrix and the limit $v \ll c$ of the relativistic one.

desirable solution suggested by the requirement of gauge invariance in case the particle represented by $\vec{k}$ is a photon, and by the non-negligible contribution of intermediate nucleon pair states. This is the addition of a "seagull" diagram (Fig. 4) to those of Fig. 3. With the addition of this diagram it is a trivial matter to obtain a Galilean invariant matrix element as required by the NR reduction of the relativistic matrix which is

$$\mathfrak{M} = g f(k)\chi^\dagger \left[\left(\frac{\mathcal{O}}{2M\omega - 2\vec{p}\,'\cdot\vec{q} + \mu^2} - \frac{\mathcal{O}}{2M\omega - 2\vec{p}\cdot\vec{q} - \mu^2}\right)\vec{t}\cdot\vec{T} + \left(\frac{\mathcal{O}}{2M\omega - 2\vec{p}\,'\cdot\vec{q} + \mu^2} + \frac{\mathcal{O}}{2M\omega - 2\vec{p}\cdot\vec{q} - \mu^2}\right) i\vec{\tau}\cdot\vec{t}\times\vec{T}\right]\chi \tag{15}$$

with

$$\mathcal{O} = \vec{\sigma}\cdot\vec{q} - \frac{\omega}{2M}\vec{\sigma}\cdot(\vec{p} + \vec{p}\,')$$

and $\vec{t}$ is the isospin of the pion and $\vec{T}$ that of the field Φ. Although this matrix can readily be reproduced with the addition of Fig. 4, the operator h' is not unique. Three possible solutions are

$$h_a = \text{Eq. (8)},$$

$$h_a' = -\frac{g}{2M}\int \psi^\dagger \frac{\vec{\sigma}\cdot(\vec{\nabla}_N - \overleftarrow{\nabla}_N)}{M}\psi i\vec{\tau}\cdot\vec{\phi}\times\vec{\Phi}\, d^3x, \tag{16a}$$

$$h_b = \text{Eq. (14)}, \quad h_b' = +\frac{g}{2M}\int \psi^\dagger \frac{\vec{\sigma}\cdot\vec{\nabla}(\vec{\Phi}\cdot\vec{\phi})}{M}\psi\, d^3x, \tag{16b}$$

$$h = \alpha h_a + \beta h_b, \quad h' = \alpha h_a' + \beta h_b', \quad \alpha + \beta = 1. \tag{16c}$$

The solutions given in Eqs. (16) do not exhaust all possibilities. What these solutions indicate is that it is not always possible to define a nonrelativistic effective interaction through Eq. (7). In order to obtain a GI matrix for a physical process it may be necessary to add a contact term, represented by Fig. 4, with the field which serves to bring the nucleon back onto its mass shell. If this field represents a potential, as from a heavy nucleus, then this contact term is ambiguously a v/c correction to the dominant meson gradient term in the interaction and a v^2/c^2 correction to a nucleon gradient term. This ambiguity persists in the NR reduction of a relativistic matrix as illustrated in the Appendix.

All of the above considerations were carried out for a relativistic scalar interaction. They can be repeated for a four-vector interaction $H' = \int \bar{\psi}\gamma^\mu F_\mu \psi\, d^3x$, for the electromagnetic interaction, or for the time component of a relativistic four-vector.[6] The conclusions reached are identical to those obtained by our illustrative example. The nonrelativistic matrix element is always GI; this may be achieved by an effective interaction of the form of Eq. (8) or Eq. (14) or may require Eqs. (16). However, if $H' = V$, the GI term $\vec{\sigma}\cdot(\vec{p} + \vec{p}\,')/2M$ may be of order v^2/c^2, as pointed out above.

As indicated in the Introduction, one of the primary advantages of a nonrelativistic treatment is that it permits the use of a DWBA development. It is therefore of interest to examine the appropriate operator to be used in conjunction with a DWBA treatment. For illustrative purposes we use a potential V:

$$H' = V = V_0 + V_1\vec{\tau}\cdot\vec{T}, \tag{17}$$

where T is the isospin operator for the nuclear target. The second term allows analog states to be excited by the optical potential.

For this example it is not possible to find a unique form for h to order ω/M. First of all, the correction term of this order may be considered to be of relativistic origin as indicated earlier and shown in the Appendix. Secondly, in order to obtain agreement with the corrections of order ω/M, i.e., with apparent GI, it is necessary to include Fig. 4.

For definiteness, consider the reaction

$$N + A \to N + A + \pi,$$

where A is a nucleus. The decomposition of the DWBA matrix is

$$\mathfrak{M}^{\text{NR}} = \langle \Psi^{(-)}\Phi^{(-)} | h | \Psi^{(+)} \rangle, \tag{18}$$

where $\Psi^{(\pm)}$ are nucleon and $\Phi^{(-)}$ pion distorted waves. With

$$\Psi^{(\pm)} = \chi + \frac{1}{E - H_0 \pm i\epsilon} t_{\text{opt}}\chi \tag{19}$$

we obtain the decomposition of $\mathfrak{M}^{NR}$ as

$$\mathfrak{M}^{NR} = \langle \chi\Phi^{(-)}|h|\chi\rangle + \left\langle \chi\Phi^{(-)}\left| h\frac{1}{E-H_0+i\epsilon}t_{opt}\right|\chi\right\rangle + \left\langle \chi\Phi^{(-)}\left| t_{opt}\frac{1}{E-H_0+i\epsilon}h\right|\chi\right\rangle$$
$$+ \left\langle \chi\Phi^{(-)}\left| t_{opt}\frac{1}{E-H_0+i\epsilon}h\frac{1}{E-H_0+i\epsilon}t_{opt}\right|\chi\right\rangle . \quad (20)$$

The various terms of this decomposition are illustrated in Fig. 5. If we assume that t_{opt} is of the form

$$t_{opt} = t_0 + t_1\vec{\tau}\cdot\vec{T},$$

then agreement of $\mathfrak{M}^{NR}$ with the NR limit of the relativistic matrix element to order ω/M is not possible unless h' is included and terms of order v^2/c^2 are kept. The effective Hamiltonian is not definable by Eq. (7) alone.

We believe that the difference between the NR reduction of the relativistic matrix and the use of an effective NR interaction Hamiltonian h as defined by us through Eq. (7) accounts for the differences obtained by Bolsterli *et al.*,[6] Friar,[9] and by Eisenberg, Noble, and Weber.[8]

III. DETECTION OF GALILEAN INVARIANCE

In the previous section we have argued that any matrix element which involves the pion-nucleon vertex should be Galilean invariant in the nonrelativistic limit. When the pion emission (absorption) occurs from a nucleon moving in a potential, then the GI correction $\omega\vec{\sigma}\cdot(\vec{p}+\vec{p}')/2M$ may be relativistic in origin. Nevertheless, such a correction term occurs together with other terms of order v^2/c^2 for any physical matrix element. The same term will occur to order v/c in the NR reduction of the pseudovector coupling Eq. (2). In this section we discuss possible means of detecting the presence of this "GI" term and argue that pion production by nucleons may be particularly appropriate.

Since, as shown in Sec. II, no GI correction term is expected to occur for reactions involving the production (absorption) of a virtual pion, we need only consider processes in which real mesons are present in the initial and/or final states. Possible reactions are

$$\gamma + A \to \pi + A', \quad (21a)$$

$$\pi + A \to \pi + A^*, \quad (21b)$$

$$p + A \to \pi + A'^*$$
$$\to p + \pi + A^*$$
$$\to n + \pi + A'^* . \quad (21c)$$

Whereas the dominant contribution to all of these reactions usually occurs for P-wave mesons, the "Galilean term" involves the creaction or destruction of S-wave mesons. We believe that nuclear, rather than nucleon, targets are appropriate because we anticipate that the excitation of certain states will enhance the effects of the GI term.

Reaction (21a) illustrated in Fig. 6 has the disadvantage that it is dominated by gauge invariance effects; in particular there is a seagull term [Fig. 6(a)] which produces S-wave pions and which has its main contribution from the minimal electromagnetic coupling required by the gradient coupling $\sim \vec{\nabla}_\pi\phi$. The GI contribution is thus but a small correction to this term due to $\sim\vec{\nabla}\psi$.

Reaction (21b) involves two pion-nucleon interactions and consequently has pair production (e.g., $\sim\int\psi^\dagger\phi^2\psi\, d^3x$) and other bilinear effects. It has nevertheless been proposed by Koltun and Nalcioglu[11] that excitation of the giant-dipole resonance in elastic pion scattering may be sensitive to the GI contribution.

Because reactions (21c) create single pions we believe that they are particularly appropriate as a test of GI. Whereas the dominant matrix element generally is that for P-wave production, the GI

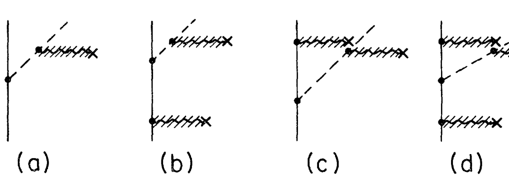

FIG. 5. Feynman diagrams for pion production in a DWBA theory. The nucleon optical scattering matrix t_{opt} is represented by a wavy line with slashes.

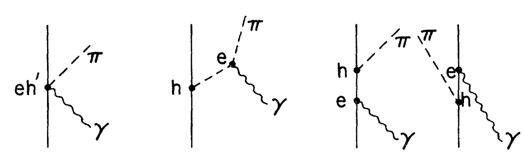

FIG. 6. Feynman diagrams for photopion production. Figure 6(a) is the seagull term.

contribution produces S-wave pions. Since the GI term is proportional to the nucleon momentum operator we expect it to preferentially excite the giant-dipole states. If we consider the production of low energy mesons at incident energies sufficiently small that the nonrelativistic approximation for nucleons is valid, then the GI term may be as large as the meson P-wave production contribution. The ratio of the GI to P-wave production terms in perturbation theory is of order $(p/M)\times(\omega/q)$; the ratio becomes large as $q \to 0$. In perturbation theory the effective interaction due to GI is proportional to $\vec{p}\tau_i$ and this operator is well known to have large matrix elements for excitation of the giant-dipole resonance. The additional proportionality to the spin operator $\vec{\sigma}$ means that the GI term not only excites 1^-, but also 2^- and 0^- states of even-even nuclei. Such states have been observed in inelastic electron scattering experiments.[12] Classically, these states correspond, for instance, to the vibrations of protons of spin up against neutrons of spin down. The 1^- state is a giant-dipole state, whereas the 2^- is a magnetic quadrupole state.

We thus propose that the excitation of giant-dipole and quadrupole resonant states in meson production at incident proton energies chosen so that the meson is produced close to threshold (in S states) should be particularly sensitive to the GI term of the production matrix element. We are presently investigating this proposal in detail with the inclusion of distortion effects.

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APPENDIX

For simplicity, we consider the first Feynman diagram of Fig. (3b). If we neglect isospin labels we obtain

$$\mathfrak{M} = gf\bar{u}(p')\gamma^5 \frac{1}{\not{p}' + \not{q} - M} u(p) \tag{A1a}$$

$$= gf\bar{u}(p')\gamma^5 \frac{1}{\not{p} + \not{k} - M} u(p) . \tag{A1b}$$

Consider Eq. (A1a) which can be written as

$$D\mathfrak{M} = gf\bar{u}(p')\gamma^5(\not{p}' + \not{q} + M)u(p)$$

$$= gf\bar{u}(p')\gamma^5 \not{q} u(p) \tag{A2a}$$

with

$$D = (p' + q)^2 - M^2 = (p + k)^2 - M^2 . \tag{A3}$$

In the limit $v \ll c$, $D\mathfrak{M}$ becomes approximately

$$D\mathfrak{M} \approx gf\zeta^\dagger \left[\vec{\sigma}\cdot\vec{q} - \vec{\sigma}\cdot(\vec{p}+\vec{p}')\frac{\omega}{2M}\right]\zeta , \tag{A4a}$$

where only terms of (apparent) order v/c have been retained and ζ is a NR spinor.

If we use Eq. (A1b) we obtain

$$D\mathfrak{M} = gf\bar{u}(\not{p}')\gamma^5(2M + \not{k})u(p) \tag{A2b}$$

which for $v \ll c$ reduces to

$$D\mathfrak{M} \approx gf\zeta^\dagger \left[\vec{\sigma}\cdot\vec{q} - \frac{k_0}{2M}\vec{\sigma}\cdot(\vec{p}+\vec{p}')\right]\zeta . \tag{A4b}$$

If the particle represented by the momentum $\vec{k}$ in Fig. 3(b) were a real one as in Fig. 3(a), $k_0 \approx \omega$, and Eqs. (A4a) and (A4b) are consistent to order v/c. On the other hand, in Fig. 3(b) $\vec{k}$ is the momentum transferred by a potential for which $k_0 = 0$. Hence, to this order, Eqs. (A4a) and (A4b) do not agree. To obtain agreement it is necessary, for instance, to keep terms of order v^2/c^2 in the expression of Eq. (A2b). With $k_0 = 0$ this reduction becomes

$$D\mathfrak{M} = gf(E'+M)^{1/2}(E+M)^{1/2}\zeta^\dagger\left[\frac{\vec{\sigma}\circ\vec{p}}{E+M} - \frac{\vec{\sigma}\cdot\vec{p}'}{E'+M} + \frac{\vec{\sigma}\circ\vec{k}}{2M} + \frac{\vec{\sigma}\cdot\vec{p}'\vec{\sigma}\cdot\vec{k}\vec{\sigma}\circ\vec{p}}{(E'+M)(E+M)2M}\right]\zeta$$

$$\approx gf\zeta^\dagger\left[\vec{\sigma}\circ\vec{q} - \frac{\vec{\sigma}\cdot(\vec{p}+\vec{p}')\omega}{4M} + \frac{\vec{\sigma}\circ\vec{k}}{8M^2}(p^2+p'^2) + \frac{1}{8M^2}\left(\vec{\sigma}\cdot\vec{p}\,\vec{p}'\cdot\vec{k} - i\vec{p}\cdot\vec{p}'\times\vec{k} + \vec{\sigma}\cdot\vec{p}'\,\vec{p}\cdot\vec{k} - \vec{\sigma}\cdot\vec{k}\,\vec{p}\cdot\vec{p}'\right)\right]\zeta$$

$$= gf\zeta^\dagger\left[\vec{\sigma}\circ\vec{q} - \frac{\omega}{2M}\vec{\sigma}\cdot(\vec{p}+\vec{p}') + \text{other terms of order } \omega p/M\right]\zeta .$$

Agreement with the reduction of Eq. (A2a) requires that relativistic corrections be kept in both cases or in neither case. The second term in Eq. (A4a) is thus seen to be ambiguously of order v/c or of order v^2/c^2, depending on how it is obtained. This occurs because $\omega(p/M)$ is also of order $p^2/M\,(p/M)$.

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