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Pion-nucleus elastic scattering at intermediate energies

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Dynamical modifications of the pion-nucleus optical potential due to binding effects on the target nucleons are calculated for pions of energies in the range 120–600 MeV. Calculations are performed using an independent particle model for the nucleus; however, a prescription for incorporation of the Pauli principle is given and its effects are studied. Resulting corrections to the potential are of the order of 50–100% near the (3, 3) resonance and fall to negligible values at energies less than 120 MeV and greater than 450 MeV. Our correction is such as to increase the nuclear opacity. Pion-nucleus total cross sections and angular distributions are modified by binding effects by 10–20%. Comparison with π-^{12}C total cross section data shows that our theory reproduces well the energy dependence of the correction required to reconcile theory and experiment, and produces about 50% of the necessary increase in the magnitude of the theoretical cross section.

[NUCLEAR REACTIONS $^{12}C(\pi,\pi)$; calculated $\sigma_T(E)$, $E=120$–600 MeV; binding corrections.]

I. INTRODUCTION

Over the past few years, very accurate total cross section and elastic scattering measurements for the interaction of intermediate energy pions with complex nuclei have been made.[1-7] These data provide the best testing ground available for our understanding of the physics underlying the construct known as the "optical potential," in terms of which such reactions are generally described. Such an understanding is essential if particle-nucleus reactions are to provide us with otherwise unattainable information about either nuclear structure[8] or the interactions with nucleons of short-lived hadronic systems.[9,10]

The construction of the optical potential has received an enormous amount of theoretical attention.[11] The ancient impulse approximation,[13,14] in which the nucleons on which the beam particle scatters are treated as free particles, is a basic ingredient of almost all such calculations. The purpose of this paper will be to examine the "binding corrections" to the optical potential which arise when this approximation is not made. It is intuitively obvious that for a beam particle (say a pion) of energy large compared with nuclear binding energies, the place where the impulse approximation is most likely to break down is at a pion-nucleon resonance, where the relatively long πN interaction time enhances the probability of interaction of the πN system with the surrounding nuclear medium. In fact, it has been known for a long time that the impulse approximation for π-nucleus scattering at energies near the (3, 3) resonance cannot *a priori* be justified.[15] Quantitative studies of the problem have been restricted almost entirely to the least complicated case of a deuterium target.[16-21] Of the two calculations done in the resonance region, Julius[20] finds a correction to the elastic πd cross section of up to 40% at backward angles, and Myhrer and Koltun[21] obtain a correction to the πd P-wave *amplitude* of 14% near the (3, 3) resonance.

For heavier nuclei, the problem of binding corrections has been examined recently by Révai[22] who constructs an alternative formalism to that of Goldberger and Watson,[23] but no results are taken past the formal stage. Attempts at inclusion of binding effects appear also in the work of Schmit[38] and Kujawski and Aitkin.[11] Here the influence of a potential binding the target nucleon is simulated by *shifting* (by a constant amount) the energy argument of πN scattering operator relation to the value for two free particles. This is the "quasiclassical" approximation in which binding effects reduce to a replacement of the πN energy E by $E-U$, where U is the binding potential of the target nucleon treated as *constant*. It is difficult to assess the goodness of this approximation, especially for light nuclei. We observe, however, that, as will be discussed below, the largest effects due to binding corrections are expected to occur on the high energy tail of the (3, 3) resonance. As can be seen in Fig. 1, this is just where the agreement of experiment with various theories uncorrected for binding effects is noticeably poor. One argument against the use of the classical approximation is that binding corrections added in this way can only *worsen* this agreement. We shall examine this point more closely in Sec. III. Outside the framework of the classical approximation, the present calculations represent the first quantitative evaluation of binding corrections to the intermediate energy optical potential.[24]

We shall see below that modifications of the optical potential due to interactions of the target nucleons are of the order of 50–100% near the (3,3) resonance and tend to increase the nuclear opacity. However, since the theory uncorrected for binding effects already predicts a very black nucleus at the resonance, the final effect of binding corrections on π-nuclear cross sections is reduced to 15–20%. It is highly instructive to examine this point more closely with the help of an extremely simple picture of π-nucleus scattering. Here we take the simplest conceivable optical potential, given by the product of the forward πN amplitude and the nuclear density (see Sec. II). We then neglect the real part of the πN amplitude, take the nucleus to be a uniform sphere of radius R and solve the Klein-Gordon equation in the eikonal approximation.[25] The total πA cross section then takes the very simple form

$$\sigma_{\pi A}=2\pi R^2\left[1-\frac{2}{x^2}+\frac{2e^{-x}}{x}\left(1+\frac{1}{x}\right)\right],$$

where $x=3A\sigma_{\pi N}/4\pi R^2$ and $\sigma_{\pi N}$ is the isospin averaged πN total cross section. We can now easily evaluate the sensitivity of the πA cross section to the two parameters R and $\sigma_{\pi N}$, say, $d\sigma_{\pi A}/dR^2$ and $d\sigma_{\pi A}/d\sigma_{\pi N}$, respectively. These two quantities, respectively, will indicate where one might expect the greatest sensitivity to (i) nuclear structure and (ii) dynamical modifications of the optical potential in a more realistic description of the problem. The two quantities $d\sigma_{\pi A}/dR^2$ and $d\sigma_{\pi A}/d\sigma_{\pi N}$ are shown in Fig. 2 as functions of pion kinetic energy. For orientation purposes, the averaged cross section $\frac{1}{2}(\sigma_{\pi^+p}+\sigma_{\pi^-p})$ is plotted as well. We see that sensitivity to R^2 (nuclear structure effects) is maximal at the resonance peak, but that sensitivity to $\sigma_{\pi N}$ (changes in the optical potential) is *minimal* at the resonance peak. In fact, doubling the πN cross section increases $\sigma_{\pi A}$ by less than 10% at the resonance energy. Thus if one wants to test one's dynamical theory of the optical potential, the peak of the resonance is the *worst* place to look.

Now, whereas the uncorrected optical potential is linear in the πN scattering amplitude, our binding correction will be quadratic in this quantity. Figure 2 then shows that if we contemplate moving in energy from the (3, 3) resonance upward, the point at which binding effects will be biggest will be determined by a compromise as the size of the effect decreases, but sensitivity to changes in the optical potential as a whole increases.

We thus expect binding effects to be minimal at the resonance position, to increase to a maximum somewhere beyond the right hand wing, and to fall to zero at high energies. This is indeed what we shall find. What is far more interesting is that the discrepancy between the measured πA total cross sections and *all* of the theories compiled by Clough *et al.*[7] to describe them show *the same* energy dependence. This is clearly seen in Fig. 1, which we have reproduced from Ref. 7. It is difficult to see how this energy dependence could be produced by some purely structural effect, such as nucleon-nucleon correlations. "Off-shell effects" in the lowest order theory could be a viable alter-

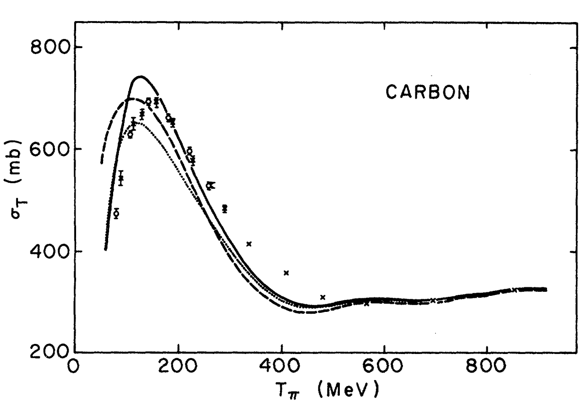

FIG. 1. Comparison of measured charge averaged pion-carbon total cross sections as a function of energy with those calculated using the following: ——, Laplacian model; ---, Kisslinger model; and •••, simple density proportional model (from Ref. 7).

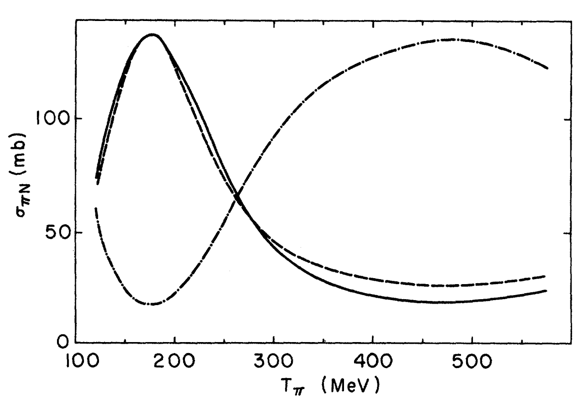

FIG. 2. The "sensitivity" functions $d\sigma_{\pi A}/dR^2$ (solid curve) and $d\sigma_{\pi A}/d\sigma_{\pi N}$ (dash-dot-curve) in arbitrary units as functions of pion kinetic energy. The dashed curve is the average total cross section $\frac{1}{2}(\sigma_{\pi p}+\sigma_{\pi n})$.

native source of the discrepancy. These have, however, been convincingly shown to be small; in particular, they appear to affect the π-^{12}C total cross sections above the resonance by only about 4% at most,[11] whereas the discrepancy mentioned above is 10–20%. $1/A$ effects are excluded at the outset by the data.[7]

These observations encourage the view that binding corrections to the optical potential are, in fact, a relevant object of study. We turn in Sec. II to our evaluation of these effects. Our results and conclusions are contained in Sec. III.

II. OPTICAL POTENTIAL

We shall construct a theory in which the optical potential reduces in the limit of zero binding correction to simply the product of the free πN amplitude and the nuclear single particle density distribution [see Eq. (8) below]. We refer to this ancient result as the "lowest order theory." It is well known that gradient and Laplacian terms represent important modifications of this theory at the (3, 3) resonance energy. However, for pion kinetic energies above 200 MeV, we see from Fig. 1 that these modifications are small compared with the discrepancy between the lowest order theory and experiment. In this region we expect binding corrections to provide the dominant correction to the lowest order theory, and shall neglect gradient and Laplacian terms.

A. Lowest order theory

We wish to construct an optical potential V_q for the interaction of a projectile, which we take for definiteness to be a pion, with a target nucleus. Here q is the incoming pion momentum. We consider first the quantity

$$\langle \vec{k}' | V_q^{(0)} | \vec{k} \rangle \equiv \sum_\alpha \langle \vec{k}', \Omega | \mathcal{T}_\alpha | \vec{k}, \Omega \rangle , \quad (1)$$

where $|\vec{k}, \Omega\rangle$ represents the ground state of the target nucleus and a noninteracting projectile of momentum $\vec{k}$. $\mathcal{T}_\alpha$ is the scattering operator for the pion with the αth target nucleon, in which the latter is treated as a *free* particle. For the reaction $\pi(\vec{k}) + N_\alpha(\vec{p}) \to \pi(\vec{k}') + N_\alpha(\vec{p}')$, we have

$$\langle \vec{k}', \vec{p}' | \mathcal{T}_\alpha | \vec{k}, \vec{p} \rangle = \delta^3(\vec{p}' + \vec{k}' - \vec{p} - \vec{k}) \times \langle \vec{k}', \vec{p}' | T_\alpha | \vec{k}, \vec{p} \rangle \quad (2)$$

and the normalization condition

$$d\sigma = (2\pi)^4 \left| \frac{\vec{k}}{\epsilon_\pi(k)} - \frac{\vec{p}}{\epsilon_N(p)} \right|^{-1} \int d^3k' d^3p' \delta(\epsilon_\pi(k') + \epsilon_N(p') - \epsilon_\pi(k) - \epsilon_N(p)) \, \delta^{(3)}(\vec{k}' + \vec{p}' - \vec{k} - \vec{p}) |\langle \vec{k}', \vec{p}' | T_\alpha | \vec{k}, \vec{p} \rangle|^2 , \quad (3)$$

where $\epsilon_i(p) = (p^2 + m_i^2)^{1/2}$.

We treat the nucleus in an independent particle model, in which case Eq. (1) reads[12]

$$\langle \vec{k}' | V_q^{(0)} | \vec{k} \rangle = \sum_\alpha \int d^3p \langle \vec{k}', \vec{k} + \vec{p} - \vec{k}' | T_\alpha | \vec{k}, \vec{p} \rangle \times \phi_\alpha(\vec{k} + \vec{p} - \vec{k}') \phi_\alpha(\vec{p}) , \quad (4)$$

where ϕ_α is the momentum space wave function of the αth nucleon. We neglect the "Fermi motion" in the matrix element by setting $\vec{p} = 0$ there, and assume that the matrix element is slowly varying in $|\vec{k} - \vec{k}'|$ compared with ϕ_α ("large nucleus approximation") so that Eq. (4) reduces to

$$\langle \vec{k}' | V_q^{(0)} | \vec{k} \rangle = \sum_\alpha \langle \vec{k}, 0 | T_\alpha | \vec{k}, 0 \rangle F_\alpha(\vec{k} - \vec{k}') , \quad (5)$$

where F_α is the single particle form factor. Transforming to coordinate space and writing F_α as the Fourier transform of the single particle probability distribution n_α gives

$$\langle \vec{r}' | V_q^{(0)} | \vec{r} \rangle = (2\pi)^{-3} \sum_\alpha \int d^3k' d^3k d^3r'' \langle \vec{k}, 0 | T_\alpha | \vec{k}, 0 \rangle n_\alpha(\vec{r}'') \, e^{i\vec{k}' \cdot \vec{r}'} e^{-i\vec{k} \cdot \vec{r}} e^{i(\vec{k} - \vec{k}') \cdot \vec{r}''} . \quad (6)$$

In the "high energy approximation" we assume that the optical potential is small compared with the asymptotic pion energy. In this case the pion momentum is modified only slightly from its asymptotic value as the pion traverses the nuclear medium, and we replace $\langle \vec{k}, 0 | T_\alpha | \vec{k}, 0 \rangle$ by $\langle \vec{q}, 0 | T_\alpha | \vec{q}, 0 \rangle$ in Eq. (6), giving

$$\langle \vec{r}' | V_q^{(0)} | \vec{r} \rangle = \delta^{(3)}(\vec{r} - \vec{r}')(2\pi)^3 \times \sum_\alpha \langle \vec{q}, 0 | T_\alpha | \vec{q}, 0 \rangle n_\alpha(\vec{r}) . \quad (7)$$

We define $V_q^{(0)}(r)$ as the coefficient of the δ function in Eq. (7). If we average the forward scattering

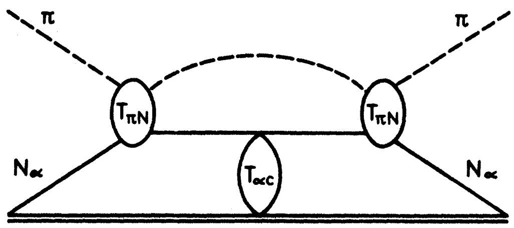

FIG. 3. Binding correction to the πN amplitude used in constructing the optical potential. The double line represents the inert source of the binding potential for the struck nucleon.

amplitude over the isospin of the target nucleons, and approximate n_α by an effective distribution $A^{-1}\rho(r)$ independent of α, we arrive at the standard expression[26]

$$V_q^{(0)}(r) = (2\pi)^3 \langle q, 0 | T | q, 0\rangle \rho(r)\,, \tag{8}$$

where $\rho(r)$ is the nuclear density distribution normalized to A, the target nucleon number.

B. Binding corrections

We wish now to modify the result (8) so as to include the effects of binding of the target nucleons. To do this we consider a struck nucleon α as interacting with an inert core via a potential U_α. The relationship between the operator t_α for scattering from a bound nucleon α and $\mathcal{T}_\alpha$ is[23]

$$t_\alpha = \mathcal{T}_\alpha + \mathcal{T}_\alpha (E - K_0 + i\eta)^{-1} \times U_\alpha (E - K_0 - U_\alpha + i\eta)^{-1} t_\alpha\,, \tag{9}$$

where E is the energy of the πN system and K_0 is the Hamiltonian for a free pion and a free nucleon. For a pion of incident momentum q we have $E = \epsilon_\pi(q) + M_N - B$, where B is the binding energy of the nucleon. We replace $\mathcal{T}_\alpha$ by the first iteration of Eq. (9) in calculating the optical potential from Eq. (1), giving a result which includes to second order in the πN amplitudes corrections due to nucleon binding:

$$\langle \vec{r}' | V_q | \vec{r} \rangle = \delta^{(3)}(\vec{r} - \vec{r}') V_q^{(0)}(r) + \frac{1}{2(2\pi)^3} \sum_{\alpha} \sum_{m, m', m''} \int d^3q' \langle \vec{q}, 0; m | T_\alpha | \vec{q}', \vec{q} - \vec{q}'; m' \rangle F^{(\alpha)}_{m' m''}(\vec{q}'; \vec{r}, \vec{r}') \times \langle \vec{q}', \vec{q} - \vec{q}'; m'' | T_\alpha | \vec{q}, 0; m \rangle\,. \tag{10}$$

The quantum numbers m, m', m'' in Eq. (10) are nucleon spin projections. This result includes an average over spin orientations m of the struck nucleon, and the πN amplitudes have again been treated in the high energy approximation with Fermi motion neglected (see Sec. II A). The quantity F is given by

$$F^{(\alpha)}_{m', m''}(\vec{q}'; \vec{r}, \vec{r}') = \int d^3k\, d^3k'\, e^{i \vec{k}' \cdot \vec{r}'} e^{-i \vec{k} \cdot \vec{r}} \int d^3p\, d^3p'\, \phi_\alpha(\vec{p}' + \vec{k}' - \vec{q}') [E - \epsilon_\pi(q') - \epsilon_N(p') + i\eta]^{-1} \times \langle \vec{p}', m' | \mathcal{T}_{\alpha c}[E - \epsilon_\pi(q')] | \vec{p}, m'' \rangle [E - \epsilon_\pi(q') - \epsilon_N(p) + i\eta]^{-1} \phi_\alpha(\vec{p} + \vec{k} - \vec{q}')\,. \tag{11}$$

Here we have used the equality

$$U_\alpha \frac{1}{E - \epsilon_\pi(q') - K_\alpha - U_\alpha + i\eta} = \mathcal{T}_{\alpha c}[E - \epsilon_\pi(q')] \frac{1}{E - \epsilon_\pi(q') - K_\alpha + i\eta}\,,$$

where K_α is the kinetic energy operator for the αth nucleon, and $\mathcal{T}_{\alpha c}$ is the full scattering operator for the αth nucleon against the "inert" core.[39]

Our binding correction to the free πN amplitude used in constructing the optical potential is shown pictorially in Fig. 3.

C. Approximation scheme

To proceed with the evaluation of Eqs. (10) and (11) we note first that the k and k' integrations in Eq. (11) simply transform the nucleon wave functions to coordinate space. We then replace the energy denominators in Eq. (11) by their δ function parts and obtain

$$F^{(\alpha)}_{m' m''}(\vec{q}', \vec{r}, \vec{r}') = -8\pi^5 p^2 \epsilon_N^2(p) \psi_\alpha(\vec{r}') \psi_\alpha(\vec{r}) e^{i \vec{q}' \cdot (\vec{r}' - \vec{r})} \int d\Omega_{p'}\, d\Omega_p\, e^{-i \vec{p}' \cdot \vec{r}'} \langle \vec{p}', m' | \mathcal{T}_{\alpha c}[E - \epsilon_\pi(q')] | \vec{p}, m'' \rangle e^{i \vec{p} \cdot \vec{r}}\,, \tag{12}$$

where p and p' satisfy the energy conservation condition

$$\epsilon_N(p) = \epsilon_N(p') = E - \epsilon_\pi(q')\,. \tag{13}$$

To perform the angular integrations in Eq. (12), we introduce a partial wave expansion of the nucleon-core scattering amplitude[27] (the latter is treated as spinless):

$$\langle \vec{p}', m' | T_{\alpha c} | \vec{p}, m'' \rangle = \sum_{l, m_l, m_l'} Y_l^{m_l'}(\Omega_{p'}) Y_l^{m_l *}(\Omega_p) \{ \langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l_+} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l+}^{(\alpha)} \langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l_-} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l-}^{(\alpha)} \}, \tag{14}$$

where

$$\Lambda_{l_+} = \frac{l+1+\vec{\sigma}\cdot\vec{L}}{2l+1}, \tag{15a}$$

$$\Lambda_{l_-} = \frac{l-\vec{\sigma}\cdot\vec{L}}{2l+1}, \tag{15b}$$

$$T_{l_\pm}^{(\alpha)} = -\frac{1}{\pi p \epsilon_N(p)} \exp(i\delta_{l_\pm}^{(\alpha)}) \sin\delta_{l_\pm}^{(\alpha)}. \tag{15c}$$

Insertion of Eq. (14) into Eq. (12) gives

$$F_{m'm''}^{(\alpha)}(\vec{q}'; \vec{r}, \vec{r}') = -(2\pi)^7 p^2 \epsilon_N{}^2(p) \psi_\alpha(\vec{r}') \psi_\alpha(\vec{r}) e^{i\vec{q}'\cdot(\vec{r}'-\vec{r})} \times \sum_{l, m_l, m_l'} j_l(pr') j_l(pr) Y_l^{m_l'}(\Omega_{r'}) Y_l^{m_l *}(\Omega_r) \times \{ \langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l_+} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l+}^{(\alpha)} + \langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l_-} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l-}^{(\alpha)} \}. \tag{16}$$

Here $\psi_\alpha(\vec{r})$ is the coordinate space wave function of the αth nucleon.

To render the theory tractable, we first take the usual local approximation to the quantity F in Eq. (16):

$$F(\vec{q}'; \vec{r}, \vec{r}') \to F(\vec{q}', \vec{r}) = \int d^3r' F(\vec{q}'; \vec{r}, \vec{r}'). \tag{17}$$

Further, we replace $F(q', \vec{r})$ by its average over orientations of $\vec{r}$ and $\psi_\alpha(\vec{r})$ by a spherically symmetric effective single particle wave function $\psi(r) = [\rho(r)/A]^{1/2}$, independent of α. If we then write the optical potential (10) as

$$\langle \vec{r}' | V_q | \vec{r} \rangle = \delta^3(\vec{r}' - \vec{r}) [V_q^{(0)}(r) + V_q^{(1)}(r)], \tag{18}$$

our local, spherically symmetric binding correction $V_q^{(1)}(r)$ takes the form

$$V_q^{(1)}(r) = -(2\pi)^5 \psi(r) \sum_\alpha \sum_{m, m', m''} \int d^3q' \langle \vec{q}, 0; m | T_\alpha | \vec{q}', \vec{q} - \vec{q}'; m' \rangle \langle \vec{q}', \vec{q} - \vec{q}'; m'' | T_\alpha | \vec{q}, 0; m \rangle p^2 \epsilon_N{}^2(p) \times \sum_{l, m_l, m_l'} j_l(pr) j_l(q'r) y_l^{m_l'}(\Omega_{q'}) Y_l^{m_l *}(\Omega_{q'}) \left(\int_0^\infty x^2 dx\, j_l(px) j_l(q'x) \psi(x) \right) \times [\langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l+} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l+}^{(\alpha)}(p) + \langle l, m_l'; \tfrac{1}{2}, m' | \Lambda_{l-} | l, m_l; \tfrac{1}{2}, m'' \rangle T_{l-}^{(\alpha)}(p)]. \tag{19}$$

Finally, we restrict ourselves to an on shell theory in which the magnitude q' of the intermediate pion momentum is replaced in the πN T matrices by its on shell value q_f determined by q and the scattering angle $\cos^{-1}(\hat{q}\cdot\hat{q}')$.

We note that the amplitude $\langle \vec{q}, 0; m | T_\alpha | \vec{q}', \vec{q} - \vec{q}'; m' \rangle$ appearing in Eq. (19) can be expressed as a laboratory frame amplitude by the application of parity and time reversal invariance:

$$\langle \vec{q}, 0; m | T_\alpha | \vec{q}', \vec{q} - \vec{q}'; m' \rangle = (-1)^{m-m'} \langle \vec{q}', \vec{q} - \vec{q}'; -m' | T_\alpha | \vec{q}, 0; -m \rangle. \tag{20}$$

To perform the spin algebra, we introduce the decomposition

$$\langle \vec{q}', \vec{q} - \vec{q}'; m'' | T_\alpha | \vec{q}, 0; m \rangle = N \chi_{m''}^\dagger (f_\alpha + i g_\alpha \hat{n}\cdot\vec{\sigma}_\alpha) \chi_m, \tag{21}$$

where χ_m represents the Pauli spinor for spin projection m, and $\hat{n}$ is a unit vector in a direction normal to the scattering plane. We normalize f and g such that the laboratory frame cross section summed and averaged over polarizations is

$$\frac{d\sigma^{\text{lab}}}{d\Omega} = |f|^2 + |g|^2. \tag{22}$$

The kinematical factor N in Eq. (21) is then given by

$$N(q,\mu)=\frac{1}{(2\pi)^2}\left[\frac{q}{\epsilon_\pi(q)}\frac{1}{q_f^2}\left(\frac{q_f}{\epsilon_\pi(q_f)}+\frac{q_f-q\mu}{\epsilon_N(\Delta)}\right)\right]^{1/2}, \quad (23)$$

where $\mu=\cos(\hat{q}\cdot\hat{q}')$ and $\Delta=(q^2+q_f^2-2qq_f\mu)^{1/2}$.

Insertion of the decomposition (21) for the πN amplitudes into Eq. (19) permits the integration over azimuthal angles ϕ describing the orientation of q' to be performed trivially. We obtain the result

$$V_q^{(1)}(r)=-(2\pi)^5\psi(r)\sum_\alpha\int_{-1}^{1}d\mu N^2(q,\mu)[f_\alpha^2(q,\mu)+g_\alpha^2(q,\mu)]\int_0^\infty q'^2dq'p^2\epsilon_N^2(p)\sum_l j_l(pr)j_l(q'r)$$
$$\times\left(\int_0^\infty x^2dx j_l(px)j_l(q'x)\psi(x)\right)[(l+1)T_{l_+}^{(\alpha)}(p)+lT_{l_-}^{(\alpha)}(p)]. \quad (24)$$

The term f_α^2 in Eq. (24) corresponds to no spin flip of the struck nucleon in either of the πN interactions of Fig. 3, whereas the g^2 term corresponds to a double spin-flip.

D. Effects of the Pauli principle

We have attempted to delineate the effects of the Pauli principle by calculating in two extreme cases. In the first, Eq. (24) is used as it stands, with all effects of antisymmetrization ignored. In the second, we remove "by hand" from Eq. (24) those spin flip and charge exchange terms associated with the external lines which would lead after the first scattering of Fig. 3 to a double occupancy of shell model states. For this procedure we view the target nucleons α as being in the ground state shell model configuration prior to interaction with the incident pion, even though our earlier approximation $\psi_\alpha(\vec{r})\to[\rho(r)/A]^{1/2}$ deprived our description of the nucleus of all such structure. The quantum numbers of the struck nucleon following the first πN collision of Fig. 2 are partially dictated by the various elements of Eq. (24). For the term in Eq. (24) proportional to $f_\alpha^2 T_{l_-}^{(\alpha)}$, for example, the quantum numbers of the nucleon following the first collision of Fig. 2 are orbital angular momentum l, total angular momentum $j=l-\frac{1}{2}$, and spin projection unchanged from its value prior to collision. We now proceed to block the Pauli forbidden transitions under the assumption that the *principal* quantum number of the shell model wave function for the struck nucleon is *unchanged* by the first collision of Fig. 3; the effects of the Pauli principle are thereby *overestimated*. As we shall see below, this approximate inclusion of the Pauli principle affects a sizable reduction in the magnitude of $V_q^{(1)}$ but introduces little change in the scattering amplitudes calculated from the optical potential. Consequently, our approximate treatment at least as far as it concerns the binding correction should be entirely adequate.

III. RESULTS AND CONCLUSIONS

We have evaluated Eq. (24) numerically for a ^{12}C nucleus and a variety of pion energies. Here we simply approximate the nucleus by a uniform sphere of radius R. The πN amplitudes f and g were constructed from the CERN phase shifts[28]; both elastic scattering and double charge exchange were taken into account.

We have calculated the nucleon-"core" scattering amplitudes $T_{l_\pm}$ using a square-well isoscalar optical potential $U(r)$ including spin-orbit interaction:

$$U(r)=-\frac{1}{2M}[U_c\theta(R-r)+U_{so}R^{-1}\delta(R-r)\vec{\sigma}\cdot\vec{L}] \quad (25)$$

with[29] $U_c=(0.098+0.019i)$ GeV$^2-0.3p^2$ and $U_{so}=-0.62$. Here p is the nucleon momentum. This parametrization represents an average of neutron and proton potentials. The small imaginary component allows for the probability of virtual dissociation of the nuclear "core" in Fig. 3, otherwise the parameters here are in essential agreement with those of the effective independent particle potential used in the calculation of bound state properties.[30]

Our numerical results for the π-^{12}C optical potential are shown in Figs. 4 and 5. A value of 3.0 fm was used for the nuclear radius R. We see in Fig. 4 that below the resonance the imaginary part of the binding correction is small compared with the unembellished lowest order theory $V_q^{(0)}$, that it rises to the level of about 50% at the resonance peak, and has fallen to insignificance for a pion kinetic energy of 450 MeV. For all energies except the very lowest, the effect is to make the nucleus *more* absorptive. The real part of the binding correction in Fig. 5 shows a similar energy dependence, but modifies the real part of $V_q^{(0)}$ by up to about 100% within the resonance peak. Since the latter is only a small correction to the imaginary part of $V_q^{(0)}$ in this energy region, this result is significant only as far as concerns the very sensitive diffraction minimum in the π-nucleus differential cross section.

We see in both Figs. 4 and 5 that the Pauli principle is very effective in suppressing the scattering of a nucleon which is initially deep in the nuclear interior, while the effect becomes negligible for nucleons which are highly peripheral. This is just what one would in fact expect in a more realistic A-body picture of the nucleus with a relatively dense central region and a less densely populated skin. In our simplified scheme the mechanism responsible can be seen directly from Eq. (24): Our treatment of the Pauli principle partially suppresses those low partial waves in nucleon-"core" scattering corresponding to occupied l levels in the shell model configuration. The behavior for small arguments of the Bessel functions in Eq. (24) implies that for small r, these low partial waves are the *only* ones of importance, whereas in the nuclear periphery the low l values suppressed by the Pauli principle make only a small contribution relative to those which remain unaffected. If we contemplate increasing the nucleon number A, the Pauli principle will partially suppress an increasing number of partial waves in Eq. (24). This is just as it should be, for the following reason. The low energy nucleon-nucleus optical potential is essentially A-independent, reflecting the saturation of nuclear forces; if the Pauli principle were *not* to play an increasingly important role in our theory with increasing A, then the size of our binding correction to the optical potential would certainly grow with A, in contradiction to this very saturation property.

To calculate a π-nuclear scattering amplitude, the Klein-Gordon (KG) equation must be solved with the potential V_q given by Eqs. (18), (8), and (24). In order to see approximately to what extent our results can reconcile theory and experiment, we have contented ourselves with a solution to the KG equation in the eikonal approximation which, as we shall see, is completely adequate for our purposes. The π-nuclear scattering amplitude corresponding to momentum transfer Δ is, in this ap-

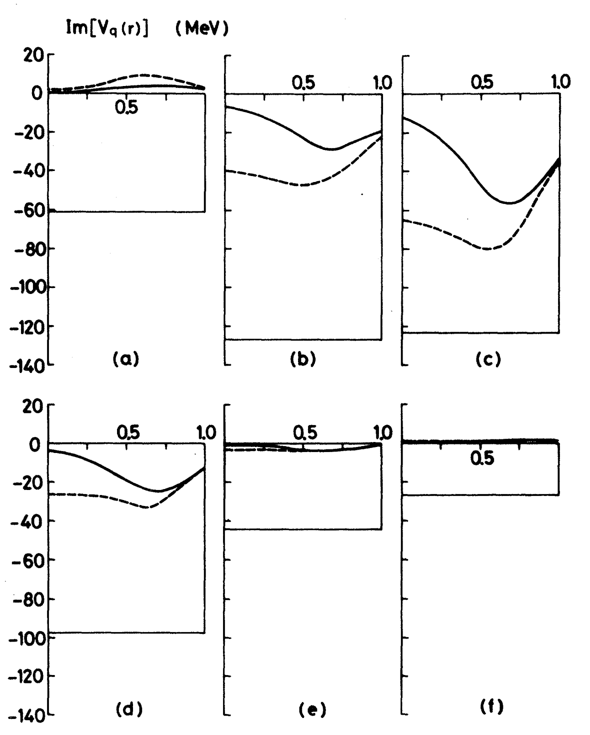

FIG. 4. Pion-^{12}C optical potential (imaginary part) as function of r/R. Horizontal line is the value for the theory without binding correction of a uniform spherical nucleus. Dashed curve: binding correction with Pauli priciple ignored. Solid curve: binding correction with Pauli principle included. Potentials are for pion kinetic energies of (a) 120 MeV, (b) 165 MeV, (c) 194 MeV, (d) 220 MeV, (e) 310, and (f) 450 MeV.

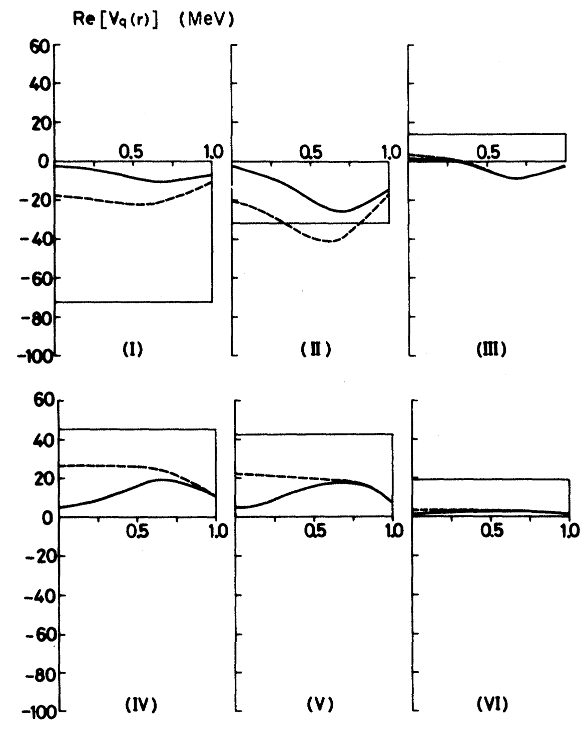

FIG. 5. Pion-^{12}C optical potential (real part) as a function of r/R. Horizontal line is the value for the theory without binding correction for a uniform spherical nucleus. Dashed curve: binding correction with Pauli principle ignored. Solid curve: binding correction with Pauli principle included. Potentials are for pion kinetic energies of (I) 120 MeV, (II) 165 MeV, (III) 194 MeV, (IV) 220 MeV, (V) 310 MeV, and (VI) 450 MeV.

proximation

$$\mathfrak{F}_q(\Delta) = iq \int_0^\infty b\,db\,J_0(b\Delta)\left(1 - \exp\left\{-i\frac{\epsilon_\pi(q)}{q}\int_{-\infty}^{\infty} dz\; V_q[(b^2+z^2)^{1/2}]\right\}\right). \tag{26}$$

For pion kinetic energies above 220 MeV we have used the form (8) as it stands for the uncorrected optical potential $V_q^{(0)}$, with the forward laboratory frame amplitude calculated directly from phase shifts.[28] For lower energies, we have included the modification due to Ericson and Hüfner[31]: $f_q(0) \to f_K(0)$ where K is the (complex) momentum of the pion *within* the nuclear medium. The forward πN amplitude (both on and off shell) is then given a resonance form, with the resulting lowest order optical potential[31]:

$$V_{q,\,\mathrm{res}}^{(0)}(r) = \frac{q^2\Delta}{2\epsilon_\pi(q)[\epsilon_\pi(q) - \epsilon_R + \Delta + i\Gamma/2]}, \tag{27}$$

where

$$\Delta = \frac{\Gamma\bar{\sigma}\rho(r)}{2q}. \tag{28}$$

Here ϵ_R and Γ are the position and width of the (3,3) resonance in πN scattering and $\bar{\sigma}$ is the average of πp and πn total cross sections at the resonance peak. We use numerical values[31,32] $\bar{\sigma} = 137$ mb, $\epsilon_R = 330.5$ MeV, and $\Gamma = 114$ MeV. The potential (27) has its peak shifted downward by about 50 MeV relative to the resonance peak in πN scattering.

We turn now to a comparison with the experimental π-^{12}C total cross sections. We denote by σ_0 the total cross section calculated from Eq. (26) wherein V_q is replaced by $V_q^{(0)}$, i.e., σ_0 is the result of the theory without binding corrections. The corresponding quantity using the full optical potential will be denoted by σ. A further quantity of interest is the πA cross section calculated by Clough *et al.*[7] with the lowest order theory [Eq. (8)]. Here the KG equation has been solved *without* the eikonal approximation[40] and with a more realistic nuclear density distribution than the uniform one which we have chosen. Let us call this third theoretical cross section σ_c.

In order to minimize the errors in our calculated cross sections σ_0 and σ arising from the use of the eikonal approximation and from our very approximate description of the nuclear shape, we have compared the *ratio* (σ/σ_0) with the quantity $\sigma_{\mathrm{exp}}/\sigma_c$, where σ_{exp} is the experimental cross section. This procedure has the additional feature of demonstrating very clearly the energy dependence of the difference between lowest order theory and experiment. In Fig. 6 we plot the quantity $\Delta\sigma_{\mathrm{exp}} \equiv (\sigma_{\mathrm{exp}}/\sigma_c) - 1$ as data points and the quantity $(\sigma/\sigma_0) - 1$ calculated using our theory. For purposes of orientation, the π-^{12}C total cross section, obtained by drawing a smooth curve through the data of Clough *et al.*,[7] is also shown.

We note first that the effects of the Pauli principle are very small; even above the resonance, the zero momentum transfer amplitude clearly remains most sensitive to the optical potential at the nuclear surface. We observe next that our correction has the right sign, and, what is much more encouraging, reproduces quite well the energy dependence of the correction required by the data. It should be emphasized that it appears difficult to modify the lowest order theory in some way other than we have done, and still produce a correction with the right energy dependence. Both the data and our theory clearly show the compromise mentioned in the Introduction between (i) a modification of the optical potential which increases in magnitude with increasing strength of the πN interaction, and (ii) a sensitivity to changes in the optical potential which *decreases* with increasing πN interaction strength. The result is a shift of the maximal effect from the resonance peak to considerably higher energies: the peak effect in Fig. 5 is at $T_\pi = 310$ MeV. There is no reason for purely structural effects, such as deviations from the independent particle model of the nucleus, to exhibit such an energy dependence. Finally, as

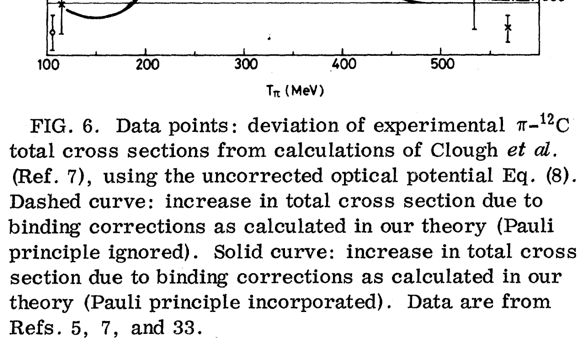

FIG. 6. Data points: deviation of experimental π-^{12}C total cross sections from calculations of Clough *et al.* (Ref. 7), using the uncorrected optical potential Eq. (8). Dashed curve: increase in total cross section due to binding corrections as calculated in our theory (Pauli principle ignored). Solid curve: increase in total cross section due to binding corrections as calculated in our theory (Pauli principle incorporated). Data are from Refs. 5, 7, and 33.

discussed in the Introduction, "off-shell effects" in the lowest order theory [the use of the πN T matrix as it stands in Eq. (6)] are simply too small to matter.[12]

Our theory reproduces on the average only about 50% of the required correction to the data. There are, of course, a number of possible sources of this discrepancy. First, we have neglected all off-shell effects in the propagation of the system between πN scatterings (see Fig. 3). There is no particular reason for these to be negligible at the energies in question. Second, we have used only the first iteration of Eq. (9). That is, we have neglected generalizations of the process shown in Fig. 3 which would involve three and more πN interactions. Of these two effects we expect the first to be the more important. In view of the encouraging results of Fig. 6 a calculation of these off-shell contributions would be a worthwhile undertaking. Such a calculation could be made feasible by the use of a separable πN interaction,[35-37] an approximation which should be fairly good near the (3,3) resonance. Finally, it is clear that in Fig. 3 other intermediate states than πN could play a role; ρN and $K\Sigma$ would be reasonable candidates. In fact, SU(3) predicts equal coupling constants (to within a sign) for $\Delta\pi N$ and $\Delta K\Sigma$.[34] Needless to say, the question of the size of the contribution made by such inelastic intermediate states is intimately connected with that of off-shell propagation, since in the on shell approximation made here, the inelastic intermediate states are forbidden energetically from contributing at all.

We return now to the treatment[11,38] of binding effects in the "classical approximation"[15] mentioned in Sec. I. Here the sole effect of the binding potential is taken to be a shift $E \to E'$ in the energy parameter of the πN scattering operator entering in the construction of the optical potential. The magnitude of the shift seems to be about 20 MeV, but is difficult to estimate accurately.[38] We note, however, that the *sign* of the shift is trivially determined: If the classical approximation were correct, the result would be $E' = E - U$ where U is the constant binding potential. Since U binds the nucleon we must have $E' > E$. Hence on the high energy wing of the (3,3) resonance, where the πN amplitude is falling in magnitude, binding corrections in the classical approximation can only *reduce* the nuclear opacity and thus *worsen* the agreement between theory and experiment (see Fig. 1). This effect is clearly seen in Fig. 7, where we plot the quantity $\sigma_0(E+\epsilon)/\sigma_0(E) - 1$ as a function of energy. As before, σ_0 is the total cross section calculated

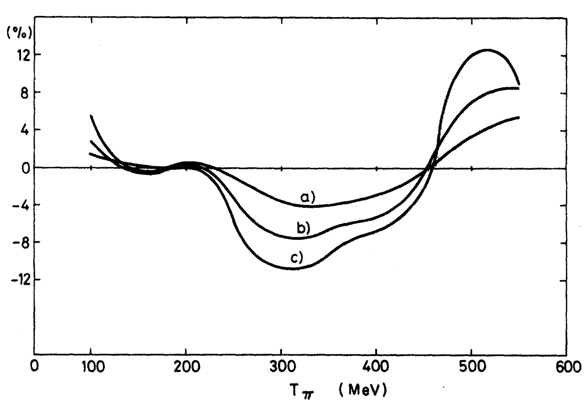

FIG. 7. The quantity $\sigma_0(E+\epsilon)/\sigma_0(E)-1$ as a function of energy E for different values of the "classical binding" shift ϵ: (a) $\epsilon = 20$ MeV, (b) $\epsilon = 40$ MeV, and (c) $\epsilon = 60$ MeV.

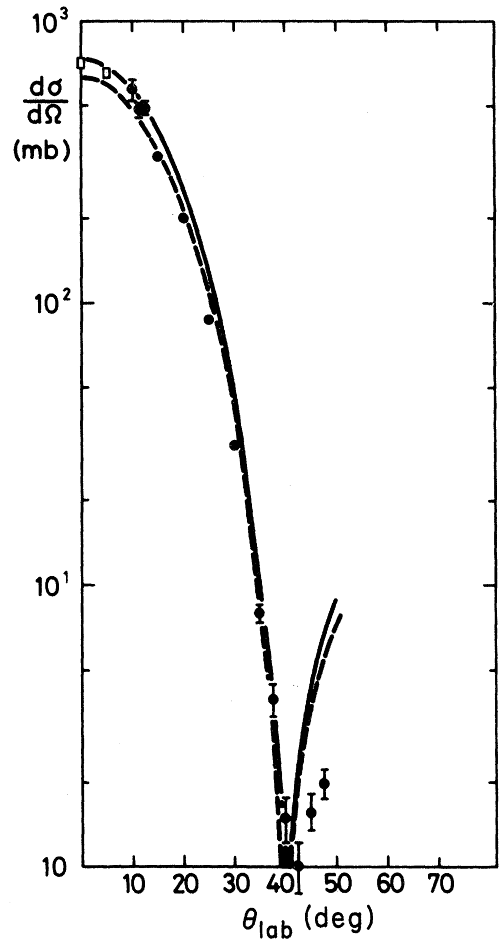

FIG. 8. Angular distribution for π^--^{12}C elastic scattering at 260 MeV. Dashed curve: calculation with optical potential uncorrected for binding effects. Solid curve: calculations with corrected optical potential (Pauli principle incorporated). Experimental points are from Ref. 1; open rectangles are computed from the data of Ref. 2 as described in the text.

from Eq. (26) with Vq replaced by the lowest order theory Vq. The quantity ϵ is the "classical binding" shift of the πN energy; we treat it as a free parameter. For all positive values of ϵ, we see that the treatment of binding corrections in the classical approximation *reduce* the theoretical cross section on the right hand wing of the (3, 3) resonance. Comparison with Fig. 1 shows that such a treatment is unsatisfactory.

We turn finally to the angular distributions for π-^{12}C elastic scattering. Because we have ignored all the finer features of the nuclear shape, agreement between our theory and experiment must at best be restricted to the diffraction peak. In Fig. 8 we show the π-^{12}C angular distribution at 260 MeV measured by Binon *et al.*,[1] as well as the eikonal solutions of the KG equation using both the uncorrected optical potential [Eq. (8)] and our improved version [Eqs. (18) and (24)]. The radius of the ^{12}C nucleus was taken for these calculations to be 3.2 fm, which is the equivalent charge radius.[31] The two open rectangles at extreme forward angles do not represent real data points, but rather the fit to the "pure nuclear" amplitude made in the Coulomb-interference experiment of Ref. 2. The binding correction to the optical potential affects an increase in the differential cross section of about 20% throughout the diffraction peak. Very near the forward direction, our modification of the lowest order optical potential represents a clear improvement. At wider angles, the data fall slightly below *both* curves, which probably reflects nothing more than the limitations of the eikonal approximation at these relatively low energies. The results of Fig. 8 do, however, assure us that the eikonal approximation is certainly *not* introducing any errors of a magnitude such as would cast doubt on the reliability of our earlier results in Fig. 6.

We conclude that binding effects represent an essential correction to the lowest order πA optical potential for pion kinetic energies between 120 and 420 MeV. Our theory for these correctly reproduces the distinctive energy dependence of the correction required to reconcile the lowest order theory with total cross section data. Comparison of the data with the various calculations as compiled by Clough *et al.*[7] and reproduced in our Fig. 1 shows that in this regard, the model described here would appear to be unique. This encourages us to believe that although our calculated effect is about a factor 2 too small in magnitude, the remaining discrepancy might very well be removed by the inclusion, as discussed above, of those portions of Fig. 3 corresponding to off-shell propagation of the intermediate πN system. A version of our model generalized in this way would then provide a completely satisfactory description of the optical potential at the energies in question.

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$$\frac{d\sigma}{d\Omega}(\vec{p}\rightarrow\vec{p}\,') = (2\pi)^4\epsilon_N{}^2(p)\,|\langle\vec{p}\,'|\mathcal{T}_{\alpha C}|\vec{p}\rangle|^2 .$$

[40]It is still not the full KG equation; one continues to discard terms quadratic in $V_q/\epsilon_\pi(q)$.