Unified weak-coupling model of mass 13 nuclei, including exchange effects

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Structure calculations on ^{13}C and ^{13}N are reported which use the weak-coupling model as a starting point. The same model is used for both positive- and negative-parity states. Exchange effects, which are especially important in the negative-parity states, are included by perturbation theory. ^{12}C core states used are 0^+, 2^+, 0^{+*}, 3^-, 4^+, and single-particle shell model states up through the s-d shell (plus the $1f_{7/2}$, in one case) are included. Energy levels, moments, and transition rates are calculated and compared, where possible, with experiment. Generally, the results are excellent, except for $E1$ rates. Effective charges are used in the electromagnetic rate results.

[NUCLEAR STRUCTURE ^{13}C, ^{13}N. Calculated levels, moments, transition rates, log(ft); negative and positive parity, exchange effects.]

I. INTRODUCTION

The mass 13 nuclei have been the subject of many model calculations, as a look at the list of references given by Ajzenberg-Selove[1] will attest. Very briefly, these calculations can be grouped under the headings: intermediate coupling in the negative-parity states, intermediate coupling in the positive-parity states, strong-coupling models, and weak-coupling models. We now review these in more detail.

Generally, normal (negative) parity states in ^{13}C and ^{13}N are considered to arise from various recouplings of nine p shell particles. The earliest calculations were performed by Inglis,[2] Lane,[3] and Kurath,[4] with an emphasis on the intermediate-coupling aspects of the states involved, for nuclei throughout the entire set of p shell nuclei. More recently, extensive investigations within the same assumed configuration of particles have been conducted by Amit and Katz,[5] Cohen and Kurath,[6] and Goldhammer *et al.*[7] The major emphasis of these investigations was to obtain a consistent set of effective two body interaction matrix elements for p shell nuclei. These calculations are very useful and each of them has been a major source of p shell wave functions for transition rates and spectroscopic factors since their publication.

The energy spectrum for normal parity states throughout the p shell obtained by Halbert, Kim, and Kuo[8] compares impressively with experimental results in most cases. They took the viewpoint of using a realistic nucleon-nucleon interaction as the starting point to calculate the effective interaction matrix elements instead of using them as adjustable parameters to be obtained from experiment. Similarly good results using a realistic interaction have been obtained throughout the p shell by Hauge and Maripuu.[9]

Turning our attention to the positive-parity states in the mass 13 nuclei, we note that generally use is made of the weak coupling which exists between a particle in the s-d shell and the ^{12}C core. Moreover, only the lowest two core states are usually included in the model. This weak-coupling aspect will be discussed further below.

Broadly speaking, the calculations of Lane,[3] Barker,[10] Sebe,[11] Hsieh and Horie,[12] and Jäger, Kissener, and Eramghian[13] are standard intermediate-coupling studies. The choice of basis functions is limited, however, to those which might reasonably be expected to describe a single s-d shell particle coupled to the lowest two or three ^{12}C states.

Strong-coupling calculations for the positive-parity states in the mass 13 nuclei were done by Kurath and Lawson.[14] Also, all of the odd mass nuclei in the p shell were studied in the strong-coupling rotational model by El-Batanoni and Kresnin.[15] In this latter case, energies of both positive- and negative-parity rotational bands were obtained. In both cases, it was found to be sufficient to treat the ^{12}C case, within its own intrinsic system, as an inert entity for the mass 13 spectra, at least for energies below 10 MeV or so.

The idea behind the weak coupling for the positive-parity states of ^{13}C and ^{13}N, referred to above, was discussed somewhat qualitatively from various viewpoints by Lane,[3] Reich, Phillips, and Russell,[16] Baz,[17] and Phillips and Tombrello.[18] The model was discussed in detail in Lane's review article[19] and extensive calculations were carried out by Kurath and Lawson.[14] The concept also entered into intermediate-coupling calculations of positive-parity states as discussed above. In one form or another, the model has been used in discussions of mass 13 transition rates,[20] reaction theory,[21] analysis of scattering nucleons[22] by ^{12}C, and electron scattering[23] by ^{13}C, to mention a few of the applications.

The weak-coupling model of ^{13}C and ^{13}N is simple and elegant. It forms the basis of this paper and is discussed in detail in Sec. II. Our major purpose is to treat both positive- and negative-parity states within the same framework. In its simplest form, the model assumes that antisymmetry effects between the valance particle and those particles in the ^{12}C core can be ignored. This assumption works well for positive-parity states but is not expected to work at all for negative-parity states.[14] The simple model is extended in this paper to include antisymmetry effects by using the techniques of perturbation theory. This paper is part of a continuing program of studies of light nuclei using the weak-coupling model.[24]

The basic model and its extension are discussed in Sec. II. The results of the calculations are presented in Sec. III and our conclusions are presented in Sec. IV.

II. THEORY

The purpose of this paper will be to present a unified picture of ^{13}C and ^{13}N within the framework of the weak-coupling model. The weak-coupling model is based on the assumption that a reasonable picture of an odd A nucleus is to see its low-lying states as being formed by coupling the states of a particle (or hole) to the lowest few states of the appropriate even mass core. Explicitly, this gives the wave function of the odd A nucleus as

$$|J_c, j; IM\rangle = \sum_m (J_c M_c jm|IM)|J_c M_c\rangle|jm\rangle , \qquad (1)$$

where $|J_c M_c\rangle$ is an eigenfunction of the Hamiltonian describing the collective motion of the $A-1$ nucleons we have decided are core nucleons and $|jm\rangle$ is an eigenfunction of the single-particle Hamiltonian describing the motion of the odd particle. Antisymmetry between the particles in the core and the odd particle is ignored in this picture; that is, one assumes that there is some method by which we can differentiate between the particles labeled core particles and the odd particle.

The interaction between the particle and the core can be written, following the de-Shalit model,[25] as the scaler product of spherical tensor operators which operate separately on the core and particle coordinates as shown in Eq. (2).

$$H_{\text{int}} = \sum_{km} \chi_k T_{km}(c)^\dagger T_{km}(p) . \qquad (2)$$

The operator $T_{km}(c)$ $[T_{km}(p)]$ is a spherical tensor operator of order k and projection m operating on the core (particle) coordinates and χ_k is the strength of the interaction. If this strength is sufficiently large to justify intermediate coupling, the wave functions of Eq. (1) become a basis and the states of the odd A nucleus are a superposition of these basis states.

The advantage of the weak- or intermediate-coupling approach is that, while one cannot write down the wave function of the $A-1$ core particles, one does know a great deal about the matrix elements of many operators between the core states. These matrix elements can be taken from experiment if we assume that the states available to the core particles are precisely those of the nuclide with the same number of nucleons. In applying our version of the intermediate-coupling model to ^{13}C and ^{13}N we will make use of the ^{12}C experimental energies and transition rates.

For the moment let us concentrate on the particular problem of interest to us here: the properties of the lowest $\frac{1}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states in ^{13}C and ^{13}N. In the weak-coupling picture the $\frac{1}{2}^-$ state would be the $p_{1/2}$ single-particle state and the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states would come from the coupling of the $p_{1/2}$ state to the 4.43 MeV 2^+ state in ^{12}C. Other workers have assumed that this is not a reasonable picture in that it cannot be correct to ignore antisymmetry between the odd particle and the ^{12}C core.[14] This point of view is supported by the calculations on the nature of the ^{12}C 4.43 MeV 2^+ state by Goswami and Pal[26] which indicate that the $(p_{1/2}p_{3/2}^{-1})_2$ configuration is very important in the ^{12}C 4.43 MeV state. This would certainly lead one to assume that the effects of blocking and exchange would have to be taken into account in any acceptable approach to describing the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states.

We have been able to retain the simple weak-coupling interpretation of both negative- and positive-parity states of ^{13}C and ^{13}N, yet approximate the effects of antisymmetry between the core and odd particles by making a few simple reasonable assumptions concerning the core excitation mechanism in ^{12}C. These assumptions are as follows: (1) The interaction in Eq. (2) has been assumed to be quadrupole-quadrupole and octupole-octupole in nature, where the operator T_{km} is assumed to be $r^k Y_{km}$, and (2) we assume that in the zeroth order approximation the 0^+ ground state and 2^+ 4.43 MeV state are members of the ground state rotational band.

The $k=2$ term of H_{int} in this picture of ^{12}C can connect collective rotational states where the change in angular momentum is 0 or 2, annihilating or creating a particle-hole pair. In effect, we are assuming that we can approximate the contribution to the positive-parity core state of all configurations other than 1p-1h as a collective rotation.

Returning now to the ^{13}C and ^{13}N case one now must include 2p-1h configurations in the basis. The possible particle states include the $1p_{1/2}$, $2s_{1/2}$,

$1d_{3/2}$, and $1d_{5/2}$ states, and the hole states considered will be $1s_{1/2}$ and $1p_{3/2}$. At first glance it would seem that by including this vast new array of basis states we have lost the simplicity which we initially said was the virtue of the weak- or intermediate-coupling approach. In order to preserve this simplicity we make the assumption that the effect of increasing the size of the basis to include the 2p-1h configurations can be approximated by applying the first order perturbation theory correction to the wave functions shown in Eq. (1) so as to include the 2p-1h states and then correcting all matrix elements in the original basis for the additional components of the wave functions. In effect, the basis remains unchanged, but all matrix elements between basis states are corrected to take into account the possibility of 2p-1h excitations. Our strategy, as we mentioned above, is to take as much information as we can from the available data on ^{12}C. In line with this approach, we calculate only the exchange correction to the mass 13 matrix elements and only to second order. The direct contributions due to the addition of 2p-1h configurations have presumably already been included in the experimental ^{12}C transition rates and energies. A complete discussion of these ideas and a derivation of the following equations may be found in Ref. 27.

The exchange contributions to the energies in the basis of Eq. (1) are given in Eqs. (3) and (4).

$$\delta E(0, j_p; j_p) = \hat{\chi}_2^2 \sum_{j_h} |\langle j_h \| T_2 \| j_p \rangle|^2 [5(2j_p+1)(E_2+\epsilon_{ph})]^{-1}, \qquad (3)$$

$$\delta E(2, j_p; I) = -\hat{\chi}_2^2 \sum_{j_h} |\langle j_h \| T_2 \| j_p \rangle|^2 \left[\frac{1}{5} \begin{Bmatrix} I & j_p & 2 \\ j_h & j_p & 2 \end{Bmatrix} \frac{1}{\epsilon_{ph} - E_2} - \frac{1.44}{\epsilon_{ph}} \begin{Bmatrix} 2 & I & j_p \\ 2 & j_p & j_h \\ 2 & 2 & 2 \end{Bmatrix} - \frac{2.56}{\epsilon_{ph} + 2.33 E_2} \begin{Bmatrix} 2 & I & j_p \\ 2 & j_p & j_h \\ 4 & 2 & 2 \end{Bmatrix} \right]. \qquad (4)$$

The variable $\hat{\chi}_2$ in Eqs. (3) and (4) is the product of the strength of the quadrupole interaction χ_2 and the reduced matrix element of the core quadrupole operator $\langle 0^+ \| T_2(c) \| 2^+ \rangle$. The numerical factors 1.44 and 2.56 found in Eq. (4) are the squares of the matrix elements $\langle 2^+ \| T_2(c) \| 2^+ \rangle$ and $\langle 2^+ \| T_2(c) \| 4^+ \rangle$ in units of the matrix element $\langle 0^+ \| T_2(c) \| 2^+ \rangle$, assuming that these states are purely rotational in nature.[28] T_2 is the particle quadrupole operator of Eq. (2). The index j_h runs over all the hole states. The two energies ϵ_{ph} and E_2 are the energy required to excite a particle from the state h to the state p and the energy of the first excited state in the core rotational band, respectively. In this calculation, we actually calculate only the shifts in basis states of the form $|2, j_h; I\rangle$ since the shift indicated by Eq. (3) has, we assume, already been included in the measured single-particle energies. We take this effect into account by using the difference between Eq. (4) and Eq. (3) as the shift would be measured, in the energy of the $|2, j_p; I\rangle$ state.

The correction to the transition matrix elements of an operator O_L between an initial state, which is a member of the core excited multiplet formed from the coupling of the 2^+ core state to the single-particle state j_p giving a total angular momentum I, and a final state, which is the single-particle state j_0, is given by Eq. (5).

$$\delta\langle j_0 \| O_L \| 2, j_p; I \rangle = -\frac{\hat{\chi}_2}{\sqrt{5}} \sum_{j_h} \Big[\delta_{j_h I} \langle j_p \| T_2 \| j_h \rangle \langle j_0 \| O_L \| j_h \rangle / (E_2 + \epsilon_{ph}) + (2I+1)^{1/2} (-1)^L \langle j_p \| O_L \| j_h \rangle \langle j_0 \| T_2 \| j_h \rangle \begin{Bmatrix} j_h & j_p & L \\ I & j_0 & 2 \end{Bmatrix} \Big/ (E_2 - \epsilon_{0h}) \Big]. \qquad (5)$$

The energy ϵ_{ih} is the energy required to promote a particle from the state j_h to the state j_i where i is p or 0. Only the exchange contribution to the change in the reduced matrix element is shown in Eq. (5). The direct part is not needed since we assume it has already been included in the experimental core transition rates.

The corrections obtained from Eqs. (3), (4), and (5) depend on the interaction strength $\hat{\chi}_2$ and the moment of inertia parameter of the core $K=0$ rotational band, which we have included as the energy of the 2^+ state E_2. We consider only one of these parameters as independent in that, given a value for the parameter $\hat{\chi}_2$, we require that E_2 fulfill the condition that the perturbed energy of the rotational 2^+ state must, to second order, reproduce the energy difference between the 2^+ state and the ground state in ^{12}C. The dependence of this separation on these parameters, ignoring the small ground state shift, is shown in Eq. (6).

$$4.43\text{ MeV} = E_2 + \hat{\chi}_2{}^2 \sum_{j_p j_h} |\langle j_p \| T_2(p) \| j_h \rangle|^2 \left[\frac{1}{E_2 - \epsilon_{ph}} - \frac{1.44}{\epsilon_{ph}} - \frac{2.56}{2.33E_2 + \epsilon_{ph}} \right] \Big/ 25 . \tag{6}$$

The calculation was carried out by varying the two parameters $\chi_2\langle 0^+\|T_2(c)\|2^+\rangle$ and $\chi_3\langle 0^+\|T_3(c)\|3^-\rangle$ to achieve the best fit to the available experimental data. The energies of the single-particle states were assumed initially, as is commonly and perhaps rather unrealistically done, to correspond to the energies of states in ^{13}C and ^{13}N which are thought to be roughly single particle in character. It was found that the positive-parity states were too low in energy and so an additional parameter was needed. This parameter was a simple constant added to each of the positive-parity single-particle energies while the spacing between these states was assumed unchanged. The single-particle energies cover a region of about 10 MeV when this parameter is included, so it was felt that all the ^{12}C states within this same region of excitation should be included in the calculation. Thus, the 0^+ ground, 4.43 MeV 2^+, 7.66 MeV 0^+, and 9.64 MeV 3^- states were included. The core matrix elements needed in Eq. (2) were taken from experiment in that they were assumed to be proportional to the reduced matrix elements of the electric quadrupole and octupole operators.

The three parameters were varied until the best agreement with experiment was obtained. The same values were used in both the ^{13}C and ^{13}N calculations and, as we shall see in the succeeding section, produce remarkably good results.

III. RESULTS AND DISCUSSION

A. Parameters

As was stated in the previous section, the parameters of the calculation which were allowed to vary freely were the product of the quadrupole interaction strength and the core reduced matrix element $\chi_2\langle 0^+\|T_2(c)\|2^+\rangle$, the product of the octupole interaction strength and the core reduced matrix element $\chi_3\langle 0^+\|T_3(c)\|3^-\rangle$, and a constant energy which was added to each positive-parity single-particle state. In addition there were parameters which were taken from experimental information available on the ^{12}C core and from assumptions concerning the nature of the core states.

We assumed the 0^+ ground state, the 4.43 MeV 2^+ state, and the 14.1 MeV 4^+ states of ^{12}C are to zeroth order members of the $K=0$ rotational band and that the quadrupole reduced matrix elements between the states are those of a rigid rotor.[28] The reduced quadrupole matrix element between the core 7.66 MeV 0^+ state and the 4.43 MeV 2^+ state was taken from the experimental quadrupole transition rate between these two states[29] and is found to be 0.46 in units of the core reduced matrix element between the ground state and first excited state $\langle 0^+\|T_2(c)\|2^+\rangle$. The octupole reduced matrix elements were assumed to be zero except for the matrix element connecting the 9.64 MeV 3^- state to the ground state.

The single-particle energies were also taken from experiment. One should not take the word experiment too seriously here since we have assumed that the ground state in both ^{13}C and ^{13}N is a pure $1p_{1/2}$ state and that the lowest energy states of each spin both ^{13}C and ^{13}N were pure single-particle states with the exception of the $\frac{3}{2}^+$ states. Following Reynolds *et al.*[22] we have assumed that the second $\frac{3}{2}^+$ state in each nuclide is the $1d_{3/2}$ state. Shell model calculations[12,13] indicate that at best the above assumption is only approximately true. In view of this, we have assumed that these energies give a reasonable estimate of the relative energies of the positive-parity states and have only parametrized their separation from the $1p_{1/2}$ state. The $1s_{1/2}$-$1d_{3/2}$ energy separation was taken to be 33.63 MeV and the $1p_{3/2}{}^{-1}$-$1p_{1/2}$ energy to be 13.77 MeV. All other particle-hole excitation energies were calculated using these energies and the relative single-particle energies.

The final values of the three parameters were $\chi_2\langle 0^+\|T_2(c)\|2^+\rangle = -3.3$ MeV/fm^2, $\chi_3\langle 0^+\|T_3(c)\|3^-\rangle = -1.5$ MeV/fm^3, and an energy of 1.5 MeV added to each of the positive-parity single-particle states. The same three parameters were used in both the ^{13}C and the ^{13}N calculations.

B. Negative-parity states

In Sec. II the additions necessary to the weak-coupling model to enable us to include, at least approximately, antisymmetry effects between the "core" particles and the "odd" particle were discussed. It was pointed out that one would expect these effects to be most dramatic in those states with a large parentage of $1p$ particles. In our model one would be talking primarily about the negative-parity states and so we have elected to discuss the results for the negative-parity states first.

We found that the ground state was a relatively pure (90%) $1p_{1/2}$ single-particle state, the remaining 10% coming from the coupling of a $d_{5/2}$ particle to the 3^- core state. We can compare this wave function with those obtained using the (6–16)2BME interaction of Cohen and Kurath[6] and the Sussex interaction as used by Hauge and Maripuu.[9] A somewhat lower percentage of the ground state is found to be single particle in character in these shell model calculations, 61% and 64%, respectively.[30] Spectroscopic factors found experimentally vary considerably, due to a strong dependence on the parameters used to extract them from the data. An average value[9] indicates the ground state wave function is approximately 85% single particle in character, clearly favoring our calculation over shell model results.

Both the lowest $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states are formed in our calculation by coupling the $p_{1/2}$ single-particle state to the core first excited state. Again we can compare this result with shell model calculations. The overlap between the $|2^+, p_{1/2}; \frac{3}{2}^-\rangle$ basis state and the Cohen and Kurath[6] $\frac{3}{2}^-$ wave function calculated using the (6–16)2BME interaction is 84% and 88% in the case of the Hauge and Maripuu calculation.[30]

Finally, we find that the second $\frac{1}{2}^-$ state is formed from the coupling of the second core 0^+ state to the $1p_{1/2}$ single-particle state.

The experimental and calculated energies of the four lowest energy negative-parity states in ^{13}C are shown in Fig. 1. The results of two calculations are shown, one with and one without the energy corrections of Eqs. (3) and (4). Only the ^{13}C results are shown since the ^{13}N results are essentially the same.

In a weak- or intermediate-coupling model calculations using the basis and interaction we have used but not including the exchange effects due to 2p-1h configurations, one would expect that the $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states would be degenerate as in calculation A shown in Fig. 1. The reason for this degeneracy in the absence of 2p-1h configurations is that the shift in energy of both states is proportional to the diagonal matrix element of the mass quadrupole operator in the $1p_{1/2}$ single-particle state. This matrix element is, of course, zero. Within our basis, there are no off diagonal matrix elements and so the splitting between these two states is entirely due to exchange effects between particle involved in the core excitation and the odd particle. While the splitting we obtain is not precisely equal to the experimentally observed value, certainly the agreement with experiment is reasonable considering we are using only the lowest nonzero correction.

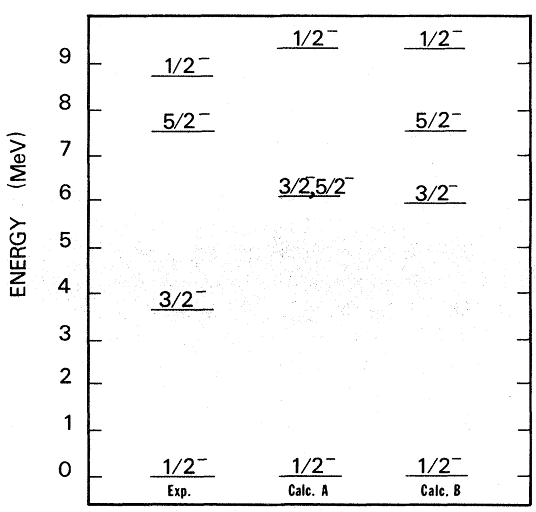

FIG. 1. Lowest negative-parity states in ^{13}C. Calculation B includes exchange effects, A does not.

In Table I we present our calculated magnetic dipole moments for the states shown in Fig. 1 and the log(ft) value for the β decay between the ground states of ^{13}N and ^{13}C.

Due to the simplicity of the ground state wave function, one can easily see the reasons for the remarkable agreement between calculation and experiment for the magnetic dipole moments. The extreme single-particle model would predict ground state moments of $0.64\mu_N$ for ^{13}C and $-0.26\mu_N$ for ^{13}N. The deviation of our calculated value from the Schmidt limits is due to the presence of the $|3^-, d_{5/2}; \frac{1}{2}^-\rangle$ component in the ground state wave function. The intensity of this component is only 10%; however, it produces a relatively large increase in the magnitude of the magnetic dipole moment over the Schmidt limit, particularly in the ^{13}N case.

If the ground state wave function were purely

TABLE I. Dipole moments of negative-parity states and ground state log(ft).

		Magnetic moment (μ_N)			
I	Present work	Exp [a]	B-K [b]	V-G [c]	C-K [d]
		^{13}C			
$\frac{1}{2}$	0.700	0.702	0.742	0.644	0.701
$\frac{3}{2}$	0.517				
$\frac{5}{2}$	0.638				
$\frac{1}{2}$*	0.638				
		^{13}N			
$\frac{1}{2}$	−0.327	−0.322	−0.368	−0.315	⋯
$\frac{3}{2}$	1.06				
$\frac{5}{2}$	0.736				
$\frac{1}{2}$*	−0.264				
		log(ft) ^{13}N$\rightarrow$^{13}C			
	3.62	3.66	3.65	3.78	

[a] Reference 1.
[b] Reference 15.
[c] Reference 7.
[d] Reference 6.

that of a single-particle state the log(ft) value for the decay of ^{13}N into ^{13}C by positron emission would be 3.62 which is somewhat below the experimental value of 3.66. In order to obtain the experimental result the Gamow-Teller (GT) transition rate would have to be reduced by about 30%. We had hoped that the $|3^-, d_{5/2}; \frac{1}{2}^-\rangle$ component of the ground state wave function would have just this effect; however, the contribution to the GT transition rate by this component is precisely that of the $p_{1/2}$ component. As a result, the calculated decay rate between the ^{13}N ground state and the ^{13}C ground state is the $p_{1/2}$ single-particle rate.

Another possible source of the difference between calculation and experiment which was considered was that some exchange process between the particles involved in the 3^- core excitation and the odd particle was affecting the GT decay rate. In order to test this hypothesis we assumed that the 3^- state could be approximated as an octupole vibration. The only possible exchange process within our basis is shown graphically in Fig. 2. When we evaluate the expression resulting from this diagram we find that its contribution is, unfortunately, exactly zero.

We are left with a calculated value which could have been found from simply applying the extreme single-particle model. One must remember, however, that this result is in very good agreement with experiment.

Perhaps the most striking example of an exchange effect is seen in the reduced $M1$ transition rates presented in Table II. Again we include only those states shown in Fig. 1. Without the introduction of the 2p-1h states the $M1$ transition between the $\frac{3}{2}^-$ state and the ground state would be completely forbidden. In our model this transition can occur only by exchange, yet we find quite good agreement between our calculation and experiment[31,32] for both ^{13}C and ^{13}N.

The exchange effects are small in the calculated reduced $E2$ rates. The rate between the $\frac{5}{2}^-$ and the ground state and the $\frac{3}{2}^-$ and the ground state is close to the rate between the ^{12}C 2^+ state and the ground state,[29] which is 8.43 $e^2\,\mathrm{fm}^4$. The reduction of these rates toward the experimental values is mainly a function of the single-particle strength of the ground state. The uncertainty in the measured $B(E2)$ is unfortunately extremely large, on the order of 30%, in each of the values presented in Table II. All our calculated values fall within this uncertainty and are quite close to the most probable values.

C. Positive-parity states

The same parameters were used in calculating the wave functions of the positive-parity states as were used in calculating the negative-parity wave functions discussed in the previous section. We find that the lowest $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states contain most of the $2s_{1/2}$ (79%) and $1d_{5/2}$ (69%) single-particle strength, in agreement with shell model calculations.[12,13] In contrast with the relatively pure configurations found in the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states, the $1d_{3/2}$ single-particle strength is distributed over a large number of states. The principle strength (32%) is in the ^{13}C 8.25 MeV state and 36% in the ^{13}N 7.90 MeV state. This distribution is due in large measure to the fact that the unperturbed energy of the $|2^+, d_{5/2}; \frac{3}{2}^+\rangle$ state is almost exactly equal to the assumed $d_{3/2}$ single-particle energy for both ^{13}C and ^{13}N. The lowest $\frac{3}{2}^+$ state is largely $|2^+, s_{1/2}; \frac{3}{2}^+\rangle$ as is the second $\frac{5}{2}^+$ state, although this state has in addition a large $|2^+, d_{5/2}; \frac{5}{2}^+\rangle$ component. The $\frac{7}{2}^+$ state in ^{13}N at 7.17 MeV is about 95% $|2^+, d_{5/2}; \frac{7}{2}^+\rangle$, in agreement with both shell model calculations[12,13] and the weak-coupling model.[22]

It is interesting to compare our wave functions with those found in a shell model calculation. Jager *et al.*[13] in a calculation using a variety of forces were able to reproduce the energy spectrum ^{13}N quite well. They also have recalculated the wave functions of the ^{12}C ground state and first excited state using the Cohen and Kurath (8-16)2BME interaction[6] and using these wave functions they write their ^{13}N shell model wave functions in terms of the ^{12}C ground state and first excited state coupled to the $2s_{1/2}$, $1d_{3/2}$, and $1d_{5/2}$ single-particle states. The overlap between our wave functions and theirs, and the major amplitudes in our wave functions are given in Table III. In all cases, we find the overlap to be almost exactly one.[33]

In Fig. 3 we show the calculated and experimental energies of the positive-parity states. The results are obviously reasonably good. Any discrepancies could be easily corrected by small changes in the single-particle energies, certainly a reasonable procedure in view of the lack of experimental information on these energies. Such a procedure was

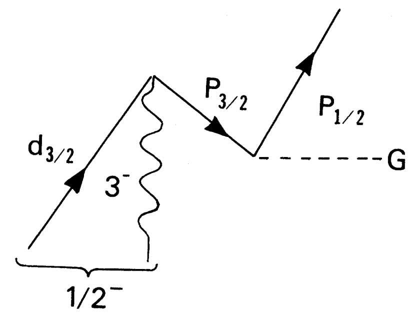

FIG. 2. Second order correction to the Gamow-Teller β decay matrix element.

TABLE II. Reduced transition rates between negative-parity states in ^{13}C and ^{13}N.

I_i	I_f	$B(M1)$ (μ_N^2) Present work	Exp	V-G [a]	C-K [b]	$B(E2)$ $(e^2\,fm^4)$ Present work	Exp
				^{13}C			
$\frac{3}{2}$	$\frac{1}{2}$	0.82	0.70 [c]	0.97	0.59	7.9	7.5 [d]
$\frac{5}{2}$	$\frac{1}{2}$					5.4	
$\frac{5}{2}$	$\frac{3}{2}$	0.06				2.4	
$\frac{1}{2}$*	$\frac{1}{2}$	0.0				0.0	
$\frac{1}{2}$*	$\frac{3}{2}$	0.0				3.6	
$\frac{1}{2}$*	$\frac{5}{2}$	0.0				5.4	
				^{13}N			
$\frac{3}{2}$	$\frac{1}{2}$	1.20	1.32 [c]	···	···	8.2	10 [c]
$\frac{5}{2}$	$\frac{1}{2}$		···	···	···	3.2	
$\frac{5}{2}$	$\frac{3}{2}$	0.10				2.4	
$\frac{1}{2}$*	$\frac{1}{2}$	0.0				0.0	
$\frac{1}{2}$*	$\frac{3}{2}$	0.0				3.6	
$\frac{1}{2}$*	$\frac{5}{2}$	0.0				5.4	

[a] Reference 7.
[b] Reference 6.
[c] Reference 32.
[d] Reference 31.

ruled out, however, since the agreement was more than satisfactory, making the introduction of more parameters unwarranted. As can be seen in Fig. 3 the results for ^{13}N appear to exhibit the Thomas-Ehrmann shift. The difference between the ^{13}C and ^{13}N is not produced by adding any Coulomb interaction in our ^{13}N calculation but by using different single-particle energies in this calculation as was pointed out above.

In Table IV we present the magnetic dipole moments for the positive-parity states. Unfortunately, we have only one experimental magnetic dipole moment with which to compare our calculation; however, we can compare our result with the results of other calculations for this case.

The measured spin factor for the 3.85 MeV $\frac{5}{2}^+$ state in ^{13}C is $|g_{5/2}| = 0.59 \pm 0.5$.[34] Our result is $g_{5/2} = -0.62$, in excellent agreement with the measured value. This result differs markedly from the single-particle value for a pure $d_{5/2}$ state ($g_{5/2} = -0.76$) due to substantial admixtures of $|2^+, d_{5/2}; \frac{5}{2}^+\rangle$ and $|3^-, p_{1/2}; \frac{5}{2}^+\rangle$ configurations in the wave function of this state.

Beene *et al.*[34] quote g factors calculated using the weak-coupling model wave functions of Kurath and Lawson[20] and of Sebe.[11] In the Kurath and Lawson calculation, for coupling strengths between −0.5 and −2.0 they find a g factor of −0.74 to −0.69. Using the Sebe wave functions they find $g_{5/2} = -0.72$. Beene *et al.*[34] also quote a result

TABLE III. Dominant amplitudes for lowest positive parity ^{13}N states and overlaps with the corresponding shell model amplitudes of Reference 13. See explanatory note, Reference 33.

$\|E_{th}$ (MeV), $I^\pi\rangle$	$\|0, 2s_{1/2}\rangle$	$\|0, 1d_{5/2}\rangle$	$\|0, 1d_{3/2}\rangle$	$\|2, 2s_{1/2}\rangle$	$\|2, 1d_{5/2}\rangle$	$\|2, 1d_{3/2}\rangle$	Overlap
2.37, $\frac{1}{2}^+$	0.8949	···	···	···	0.3688	−0.2201	0.970
3.02, $\frac{5}{2}^+$	···	0.8345	···	0.2468	0.3547	−0.0928	0.928
6.33, $\frac{5}{2}^+$	···	−0.0273 [a]	···	−0.7709	0.6166	−0.9180	0.958
6.50, $\frac{3}{2}^+$	···	···	−0.4680	−0.8454	0.0791	0.0740	0.926
7.63, $\frac{3}{2}^+$	···	···	0.6119	−0.2530	0.5614	0.4887	0.966
7.79, $\frac{7}{2}^+$	···	···	···	···	−0.9666	0.1175	0.944

[a] This amplitude, given as −0.4961 in Ref. 13, was taken to be −0.0496 based on normalization considerations.

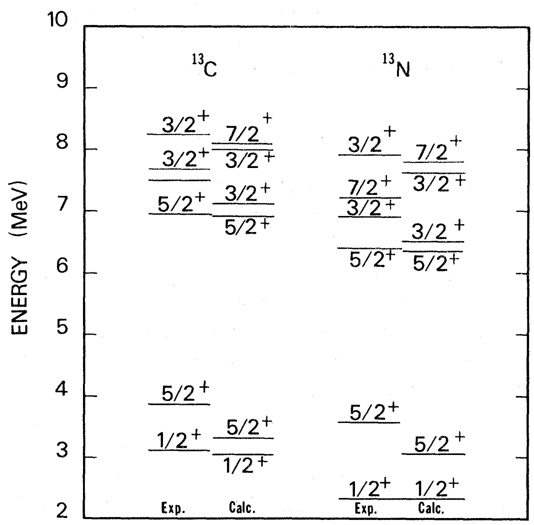

FIG. 3. Calculated and experimental energies of positive-parity states in ^{13}C and ^{13}N.

based on a recent and as yet unpublished SU(3) calculation giving $g_{5/2} = -0.64$. In each case, we find the calculated g factor is farther from the measured value than our result which is well within the uncertainty of the measurement.

The transition rates between the positive-parity states are presented in Table V. Again we find that experimental data are scarce. The one experimental result available is in good agreement with our calculated value.

D. $E3$ and $E1$ transitions

The $E3$ transition rates shown in Table VI were calculated using an effective charge of magnitude 1 for both neutrons and protons. Exchange effects were found to be uniformly small in the $E3$ transitions. Unfortunately the only experimental $E3$ transition rate available[31] is the 3.85 MeV $\frac{5}{2}^+$ to the ground state in ^{13}C ($128 \pm 65\ e^2\,\mathrm{fm}^6$). Our calculated value is $90\ e^2\,\mathrm{fm}^6$, well within the limits of the uncertainty in the measured number. Almost exactly 50% of this transition amplitude is due to the large collective rate from the $|3^-, p_{1/2}; \frac{5}{2}^+\rangle$ component of the $\frac{5}{2}^+$ state to the $p_{1/2}$ single-particle component of the ground state although the amplitude of the $|3^-, p_{1/2}; \frac{5}{2}^+\rangle$ basis state is only -0.32. Again as in the ground state magnetic dipole moment calculation, we see the importance of including the 3^- core state.

The $E1$ rates calculated with this model do not in general agree with experiment. As has been found in previous weak-coupling calculations[14] and shell model calculation[12,13] the calculated transition rates are much lower than rates found experi-

TABLE IV. Magnetic dipole moments for positive-parity states in ^{13}C and ^{13}N.

I	Magnetic moment (μ_N) ^{13}C Present work	Exp [a]	^{13}N Present work	Exp
$\frac{1}{2}$	−1.92		2.80	
$\frac{5}{2}$	−1.55	±1.48	4.42	
$\frac{5}{2}$*	−1.03		3.83	
$\frac{3}{2}$	1.82		−0.76	
$\frac{3}{2}$*	1.25		0.08	
$\frac{7}{2}$			4.29	

[a] Reference 34.

TABLE V. Reduced transition rates between positive-parity states in ^{13}C and ^{13}N.

I_i	I_f	$B(M1)$ (μ_N^2)	$B(E2)$ ($e^2\,\mathrm{fm}^4$)
		^{13}C	
$\frac{5}{2}$	$\frac{1}{2}$	0.0	3.1 [a,b]
$\frac{5}{2}$*	$\frac{1}{2}$	0.0	7.2
$\frac{5}{2}$*	$\frac{5}{2}$	1.35×10^{-3}	4.7
$\frac{3}{2}$	$\frac{1}{2}$	2.07×10^{-5}	11.2
$\frac{3}{2}$	$\frac{5}{2}$	1.09	0.1
$\frac{3}{2}$	$\frac{5}{2}$*	1.43	3.7
$\frac{3}{2}$*	$\frac{1}{2}$	5.52×10^{-3}	0.3
$\frac{3}{2}$*	$\frac{5}{2}$	1.03	3.2
$\frac{3}{2}$*	$\frac{5}{2}$*	1.21	1.8
$\frac{3}{2}$*	$\frac{3}{2}$	1.86×10^{-2}	6.8
		^{13}N	
$\frac{5}{2}$	$\frac{1}{2}$	0.0	7.7
$\frac{5}{2}$*	$\frac{1}{2}$	0.0	7.2
$\frac{5}{2}$*	$\frac{5}{2}$	1.72×10^{-3}	4.3
$\frac{3}{2}$	$\frac{1}{2}$	9.82×10^{-5}	16.1
$\frac{3}{2}$	$\frac{5}{2}$	1.44	3.6
$\frac{3}{2}$	$\frac{5}{2}$*	2.32	6.8
$\frac{3}{2}$*	$\frac{1}{2}$	5.85×10^{-3}	0.3
$\frac{3}{2}$*	$\frac{5}{2}$	1.66	4.2
$\frac{3}{2}$*	$\frac{5}{2}$*	1.34	3.0
$\frac{3}{2}$*	$\frac{3}{2}$	8.88×10^{-3}	10.8

[a] Experimental value is 3.42, see Ref. 31 (this is the only experimental value available).

[b] The width for this transition (3.7×10^{-8} eV) has been calculated in Ref. 13; this yields $B(E2)=0.02$.

mentally.[1,29,31,32] The reduced $E1$ rate from the lowest $\frac{1}{2}^+$ state to the ground state is calculated to be 3×10^{-3} $e^2\,\text{fm}^2$ for ^{13}C and 4×10^{-3} $e^2\,\text{fm}^2$ in ^{13}N as compared with experimental values of 14×10^{-3} $e^2\,\text{fm}^2$ and 28×10^{-3} $e^2\,\text{fm}^2$. If one were to ignore exchange effects the results would be 28×10^{-3} $e^2\,\text{fm}^2$ and 37×10^{-3} $e^2\,\text{fm}^2$, respectively. Therefore, part of our problem is that the $E1$ rate is the difference between two relatively large numbers, the direct and exchange contributions. Thus, our difficulty could conceivably stem from an error in the estimate of the size of the exchange term which is, after all, calculated using perturbation theory.

In the calculation of the $E1$ rates quoted in the previous paragraph, we have used the effective charge due to the recoil effect: $|e_{\text{eff}}|=Z/A$ for neutrons and $e_{\text{eff}}=1-Z/A$ for protons.[35] The magnitude of the recoil charge is $e_{\text{eff}}=0.46$. Had we used $e_{\text{eff}}=1$, then our results for the $\frac{1}{2}^+$ to ground state transition would have been 14×10^{-3} $e^2\,\text{fm}^2$ for ^{13}C and 16×10^{-3} $e^2\,\text{fm}^2$ for ^{13}N. The need for an effective charge beyond that resulting from recoil suggests a need for expanding the basis. In a previous calculation,[22] it has been assumed that the $\frac{7}{2}^-$ state at 10.7 MeV in ^{13}C and 10.36 MeV in ^{13}N was the $1f_{7/2}$ state. When we include the effects of this single-particle state in the direct contribution, the $E1$ rates, using the same parameters as quoted above and an effective charge of 0.46, we find the reduced $E1$ rate for the $\frac{1}{2}^+$ to ground state transition is 15×10^{-3} $e^2\,\text{fm}^2$ for ^{13}C. One would infer from this result that the p-f shell has a significant effect on the $E1$ rates. Rather than complicating the problem further by including the p-f shell single-particle states, we have elected to approximate the effects of the p-f shell by using an effective charge $e_{\text{eff}}=1$ for both protons and neutrons. The results are presented in Table VII.

TABLE VI. Reduced $E3$ transition rates.

I_i^π	I_f^π	$B(E3)$ ($e^2\,\text{fm}^6$)	Exp.[a]
		^{13}C	
$\frac{5}{2}^+$	$\frac{1}{2}^-$	90.30	128
$\frac{5}{2}^+$	$\frac{3}{2}^-$	1.74	
$\frac{5}{2}^{+*}$	$\frac{1}{2}^-$	0.06	
$\frac{5}{2}^{+*}$	$\frac{3}{2}^-$	2.56	
		^{13}N	
$\frac{5}{2}^+$	$\frac{1}{2}^-$	88.10	
$\frac{5}{2}^+$	$\frac{3}{2}^-$	1.71	
$\frac{5}{2}^{+*}$	$\frac{1}{2}^-$	0.36	
$\frac{5}{2}^{+*}$	$\frac{3}{2}^-$	2.49	

[a] Reference 31.

IV. SUMMARY AND CONCLUSIONS

The purpose of this paper was to show that both the positive- and negative-parity states of ^{13}C and ^{13}N can be described within the framework of the weak-coupling model provided some mechanism is included for taking into account the effects of antisymmetry. Many authors have concluded that the weak-coupling formalism is adequate to describe the positive-parity states. On the other hand, it is assumed that, since the weak-coupling model in its usual form includes no provision for antisymmetrizing the wave functions, the negative-parity states cannot be described by it. The overlaps presented in Table III between our positive-parity wave functions and those obtained in an intermediate-coupling shell model emphasize the efficacy of the weak-coupling description of the positive-parity states. A detailed comparison between the amplitudes shown in Table III and those in Table VI of Ref. 12 shows that not only is the overlap almost perfect in each state, but that the individual amplitudes show a very strong correspondence. The negative-parity weak-coupling wave functions do not give this same correspondence with wave functions calculated using the intermediate-coupling shell model. However, we find that the properties of the negative-parity states calculated with our version of the weak-coupling model generally agree at least as well with experiment as do those calculated in the shell model.

The importance of exchange in determining the properties of the negative-parity states is particularly evident in the $M1$ rates between the lowest $\frac{3}{2}^-$ state and the ground state in both ^{13}C and ^{13}N.

TABLE VII. Reduced $E1$.

I_i^π	I_f^π	$B(E1)$ ($e^2\,\text{fm}^2$)	Exp.[a]
		^{13}C	
$\frac{1}{2}^+$	$\frac{1}{2}^-$	0.014	0.014
$\frac{3}{2}^-$	$\frac{1}{2}^+$	0.004	0.012
$\frac{5}{2}^+$	$\frac{3}{2}^-$	0.003	0.004
		^{13}N	
$\frac{1}{2}^+$	$\frac{1}{2}^-$	0.016	0.028
$\frac{3}{2}^-$	$\frac{1}{2}^+$	0.011	0.022
$\frac{5}{2}^+$	$\frac{3}{2}^-$	0.001	···

[a] These values are taken from Ref. 31 for ^{13}C and Refs. 29 and 32 for ^{13}N.

In our model this transition can proceed only through the exchange of a particle involved in the core excitation with the odd particle, yet we obtain excellent agreement between calculation and experiment. In addition, in our model the splitting between the lowest $\frac{3}{2}^-$ and $\frac{5}{2}^-$ is purely an exchange effect; again, emphasizing the importance of exchange in the negative-parity states.

We find on the whole, that the model does a very good job of calculating energies, electric quadrupole and octupole transition rates, and magnetic dipole transition rates, at least to the extent that comparison with experiment is possible. However, we encounter some difficulty in the electric dipole transition rates. It was found that the introduction of an $f_{7/2}$ single-particle state into the basis brought the $E1$ rate from the lowest-lying $\frac{1}{2}^+$ state to the ground state into exact agreement with experiment in ^{13}C and greatly improved agreement in ^{13}N. This suggests that the p-f shell is important in the $E1$ transition rates and so an effective $E1$ operator is needed. The use of an effective charge of 1 does improve the agreement of these rates with experiment but not to the extent one would like. A comparison with shell model calculations[12,13] again shows that we obtain agreement with experiment which, while not impressive, is generally at least as good as that obtained in the shell model approach.

The ordinary weak-coupling model was also extended to include 2p-1h states by Divadeenum and Beres[36] and Beres and Dorenbusch.[37] These calculations are complementary to ours in the sense that they take the 2p-1h states to be incoherent with the core-plus-particle states, whereas we assume coherence. Moreover, they investigated cases where antisymmetry effects could be ignored, while we emphasize the importance of those effects in some states.

One of the very intriguing aspects of the mass 13 system is the fact that the particle continuum lies very low in these nuclei: 1.94 MeV in ^{13}N and 4.95 MeV in ^{13}C. Therefore, most of the states discussed in this paper are resonances rather than bound states. Such processes as nuclear scattering and capture by ^{12}C and photonuclear processes on ^{13}C are additional phenomena which should be used in tests of nuclear models. Calculations related to these problems are currently underway. It will be of particular interest to see if exchange effects, as included in this paper, have a significant effect on these processes.

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