Abrasion-ablation in reactions between relativistic heavy ions

J. Hüfner, K. Schäfer, and B. Schürmann*
Institut für Theoretische Physik der Universität and Max-Planck-Institut für Kernphysik Heidelberg, Federal Republic of Germany
(Received 19 May 1975)

The abrasion-ablation model for reactions between relativistic heavy ions is derived from Glauber's multiple scattering theory. Simple expressions are found for the abrasion cross section and for the excitation energy of the projectile after abrasion. Both quantities depend on nuclear densities and on the nucleon-nucleon forward scattering amplitude. They do not contain any adjustable parameters. With a very simple model for the ablation process, fragmentation cross sections for the production of individual isotopes are calculated. The agreement with experiment is good for most cases. Systematic discrepancies are observed and are analyzed in terms of an extension of the abrasion-ablation model.

[NUCLEAR REACTIONS Relativistic heavy ions, calculated fragmentation cross sections.]

I. INTRODUCTION

The recent experiments with relativistic heavy ions (typical energies 2 GeV/nucleon) penetrate the no-man's land of nuclear research. Many bewildering phenomena have been observed (Heckman *et al.*,[1] Lindstrom *et al.*,[2] Greiner *et al.*,[3] and Crawford *et al.*[4]), and many more are to be expected. No theoretical tools are yet ready to interpret these experiments. In a situation like that one may try to borrow successful concepts from other fields of physics[5]: Regge theories from particle physics, multiple scattering approaches *à la* Glauber, and descriptions from classical physics like shock waves[6] or abrasion of matter. Whether these concepts successfully describe aspects of heavy ion scattering can only be decided after detailed calculations and their comparison with experiment. The present work is one step in this direction. We derive the abrasion-ablation model within the Glauber theory and apply it to reactions like

$$^{16}\mathrm{O} + {}^{9}\mathrm{Be} \to {}^{A}\mathrm{C} + X\ , \tag{1}$$

where fast oxygen ions hit a beryllium target and one looks for fast carbon isotopes.

To our knowledge the abrasion-ablation model was introduced by Bowman, Swiatecki, and Tsang.[7] It is based on the following very simple idea: When two relativistic heavy ions pass each other so closely that part of their volumes overlap, the overlapping volume elements are sheared away or scraped off ("abrasion"). The remaining chunk of projectile matter continues its path essentially undisturbed, i.e., with the same velocity. Of course, after abrasion, the projectile is in an excited state and looses energy by emitting one or several particles (ablation). The two steps, abrasion and ablation, determine the proton and neutron numbers of the isotope which enters the detector. Bowman *et al.*[7] have calculated cross sections within the liquid drop model for the nucleus: The number of abraded nucleons is proportional to the volume of the overlap region. The average excitation energy after abrasion is estimated from the additional surface energy. Total cross sections like $^{16}\mathrm{O} + {}^{9}\mathrm{B} \to \mathrm{C} + X$ are calculated and agree with experiment better than a factor of 2. We are not aware of any further developments of the abrasion-ablation concept.

The present work has the following aims: (i) To establish a connection between the classical and macroscopic treatment of the abrasion-ablation process and a quantummechanical and microscopic multiple scattering theory; (ii) thereby to derive formulas which are independent of any specific nuclear model; and (iii) to calculate cross sections for the production of specific isotopes. They have been measured and provide another test of the abrasion-ablation concept. We derive the abrasion cross section within the formalism of Glauber's multiple scattering theory.[8] This formalism has proven very successful in describing the scattering of fast elementary particles by nuclei. There have been already a number of attempts to extend Glauber theory to the scattering of nuclei by nuclei: Czyż and Maximon,[9] Kofoed-Hansen,[10] Formánek,[11] Fäldt,[12] Têkou,[13] Fäldt, Pilkuhn, and Schloile,[14] Fishbane and Trefil,[15] and Franco.[16] However, most of the quoted papers concentrate on elastic scattering and total cross sections. Only few of the papers, among them Ref.

14, treat true inelastic processes (e.g., removal of nucleons). Abrasion processes are true inelastic reactions. We show how Glauber theory is also a very successful starting point for the true inelastic processes. Simple expressions for the abrasion cross section can be derived with relatively few approximations. The second step, ablation, is more difficult to describe, since it is a low-energy phenomenonon. Using a sum rule we succeed to calculate the average excitation energy of the projectile after ablation. Typically, the energies range between 5 and 15 MeV. We have no precise idea about the nature of the excitation and the decay. Is a compound nucleus formed which evaporates nucleons or do particles decay directly? As long as only one particle is "ablated," the relative intensities of neutrons, protons, and α particles may not depend too strongly on the ablation mechanism and the ablation part may be rather sound. Thus it does not seem so surprising that the salient features of the data are always reproduced; in many cases agreement within 20–30% is even achieved. Yet the data show systematic discrepancies which go beyond the abrasion-ablation model. We discuss one extension of the model which seems to remove the discrepancies quite successfully.

Before closing the introduction we draw attention to yet another approach to describing reactions of the type of Eq. (1). The picture is essentially one of compound nucleus formation (Feshbach and Huang,[17] Bhaduri,[18] and Goldhaber[19]): The collision between the heavy ions only excites the projectile but leaves it undamaged otherwise. The excitation energy is distributed over the whole projectile nucleus and is "evaporated" then. This model successfully describes the momentum distribution of the final products in Eq. (1) and the factorization property. It has not yet been applied to the calculation of absolute fragmentation cross sections.

II. DERIVATION OF THE ABRASION CROSS SECTION

The following derivation is based on Glauber's multiple scattering theory[8] in a form which allows both projectile and target to be composite systems.[9–16] The starting point is the scattering amplitude

$$F_{O_P O_T \to M_P M_T}(\vec{q}) = \frac{k}{2\pi i}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\left\langle M_P M_T\right| \prod_{\substack{j\in P\\ l\in T}} e^{i\chi_{lj}} - 1 \left| O_P O_T\right\rangle \tag{2}$$

for the scattering of a projectile nucleus (P) by a target nucleus (T). Initially, the two ions are in their respective ground states, $|O_P\rangle$, $|O_T\rangle$; after the reaction they are excited into states $|M_P\rangle$ and $|M_T\rangle$, respectively. While projectile and target states are properly antisymmetrized, we can neglect antisymmetrization between the ions, because the fast nucleons in the projectile can well be distinguished from the slow ones in the target. The transition operator in Eq. (2) between initial and final states contains the phase shift functions $\chi_{jl} = \chi_{jl}(\vec{s}_j + \vec{b} - \vec{s}_l)$ for the scattering of nucleon j from the projectile by nucleon l of the target. We assume the projectile to move in the z direction. Then the position vector $\vec{x}_j$ of particle j (with respect to the projectile center of mass) can be decomposed into its z component and a component $\vec{s}_j$ in the impact parameter plane, $\vec{x}_j = (\vec{s}_j, z_j)$. Similarly, $\vec{s}_l$ refers to the target center of mass. The impact parameter $\vec{b}$ denotes the relative positions of target and projectile in the plane perpendicular to the z direction. The elementary phase shift function $\chi_{jl}(\vec{b})$ is related to the profile function $\Gamma_{jl}(\vec{b})$ and to the scattering amplitude $f_{jl}(\vec{q})$ for nucleon-nucleon scattering by

$$\begin{aligned}\Gamma_{jl}(\vec{b}) &= 1 - e^{i\chi_{jl}(\vec{b})}\,,\\ f_{jl}(\vec{q}) &= \frac{k}{2\pi i}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}(e^{i\chi_{jl}} - 1)\,.\end{aligned} \tag{3}$$

In this section, the phase shift function χ_{jl} is assumed to be real, i.e., the nucleons only scatter elastically. The generalization to a more realistic nucleon-nucleon amplitude is given in the Appendix.

Starting from Eq. (2), we calculate the angle integrated cross section for abrasion. Abrasion means: (i) the final state $|M_T\rangle$ of the target remains unobserved; (ii) out of the A projectile nucleons, n nucleons are abraded, i.e., they are kicked into high momentum states, rapidly leave the nucleus, and are not observed. The projectile fragment with $A-n$ nucleons is left in a low excited state (which later ablades). These physical requirements are translated into the formalism: The total formation rate is obtained by an angular integration in the standard approximation[8] $d\Omega = d^2q/k^2$:

$$\begin{aligned}\sigma^{\rm tot}_{O\to M} &= \int d\Omega |F_{O\to M}(\vec{q})|^2 \equiv \int d^2b\, \sigma_{O\to M}(\vec{b})\,,\\ \sigma_{O\to M}(\vec{b}) &= |\langle M | \prod e^{i\chi_{jl}} - 1 | O\rangle|^2,\end{aligned} \tag{4}$$

where we use the abbreviations $|O\rangle = |O_P, O_T\rangle$ and $|M\rangle = |M_P, M_T\rangle$. Since the final state of the target remains unobserved [point (i)], closure can be performed over M_T:

$$\sum_{M_T} \sigma(\vec{b})_{O_P O_T \to M_P M_T} = \left\langle O_T \left| \left(\left| \left\langle M_P \right| \prod_{j,l} e^{i\chi_{jl}} - 1 \left| O_P \right\rangle \right|^2 \right) \right| O_T \right\rangle . \tag{5}$$

In order to translate requirement (ii), the final projectile states $|M_P\rangle$ are labelled appropriately

$$|M_P\rangle = |\mu_1, \mu_2, \ldots, \mu_n, m\rangle ; \quad \mu_i > \mu_0, \; m < m_0 , \tag{6}$$

where μ_i denote single particle states of the ejected nucleons and m labels the states of the residual fragment with $A-n$ nucleons. The requirement that the μ_i describe highly excited orbits is symbolically written as $\mu_i > \mu_0$. For instance, if the μ_i stand for momenta, we demand $k_i > k_0$. Similarly, $m < m_0$ limits the $(A-n)$ system to low excited states. The nature of the states $|\mu_0\rangle$ and $|m_0\rangle$ is discussed later. We do not antisymmetrize among the states $|\mu_i\rangle$ and $|m\rangle$, but keep the residual states of the projectile appropriately antisymmetrized. The cross section $\sigma_n(\vec{b})$ for the abrasion of n $(n>0)$ nucleons is obtained from Eq. (5) by summing over the appropriate projectile states, Eq. (6):

$$\sigma_n(\vec{b}) = \binom{A}{n} \left\langle O_T \left| \left(\sum_{\substack{\mu_i > \mu_0 \\ m < m_0}} \left| \left\langle \mu_1 \cdots \mu_n, m \right| \prod_{\substack{j \in P \\ l \in T}} e^{i\chi_{jl}} \left| O_P \right\rangle \right|^2 \right) \right| O_T \right\rangle . \tag{7}$$

The factor $\binom{A}{n}$ takes into account that it is unimportant which n nucleons out of the A projectile nucleons are abraded. The 1 in the transition operator Eq. (5) has been dropped because of the orthogonality $\langle M_p | O_P \rangle = 0$. We introduce the operators Q_j,

$$Q_j = \prod_{l \in T} e^{i\chi_{jl}} . \tag{8}$$

They describe the interaction of nucleon j $(\in P)$ with all nucleons of the target. Equation (7) is rewritten identically as

$$\sigma_n(\vec{b}) = \binom{A}{n} \left\langle O_P O_T \left| \left(\prod_{j=1}^{n} Q_j^\dagger \sum_{\mu_j > \mu_0} |\mu_j\rangle \langle \mu_j | Q_j \right) \left(Q_A^\dagger \cdots Q_{n+1}^\dagger \sum_{m < m_0} |m\rangle\langle m| Q_{n+1} \cdots Q_A \right) \right| O_P O_T \right\rangle . \tag{9}$$

Closure over the states μ_j plus the property $|Q_j| = 1$ (real phase shift functions) lead to

$$\sigma_n(\vec{b}) = \binom{A}{n} \left\langle O_P O_T \left| \left[\prod_{j=1}^{n} \left(1 - Q_j^\dagger \sum_{\mu_j \in \mu_0} |\mu_j\rangle \langle \mu_j | Q_j \right) \right] \left(Q_A^\dagger \cdots Q_{n+1}^\dagger \sum_{m < m_0} |m\rangle \langle m| Q_{n+1} \cdots Q_A \right) \right| O_P O_T \right\rangle . \tag{10}$$

At this stage the average is taken over the target coordinates which appear in the operators Q_j. We are unable to perform the average of Eq. (10) exactly and have to introduce the approximation

$$\langle O_T O_P | F(Q_j) | O_T O_P \rangle = \langle O_P | F(\langle O_T | Q_j | O_T \rangle) | O_P \rangle . \tag{11}$$

We call it the coherent approximation in analogy to a similar procedure in standard multiple scattering theory. Corrections to Eq. (11) are discussed in the Appendix. We recall from the work by Glauber[8] that

$$\langle O_T | Q_j | O_T \rangle = \left\langle O_T \left| \prod_{l \in T} e^{i\chi_{lj}(\vec{s}_j + \vec{b} - \vec{s}_l)} \right| O_T \right\rangle = e^{i\chi_j^{\text{opt}}(\vec{s}_j + \vec{b})} \equiv Q_j^{\text{opt}}(\vec{s}_j + \vec{b}) , \tag{12}$$

where the optical phase shift χ_j^{opt} is calculated from the optical potential for the scattering of nucleon j by the target nucleus:

$$\chi_j^{\text{opt}}(\vec{b}) = -\frac{m}{k} \int_{-\infty}^{+\infty} U_T^{\text{opt}}(\vec{b}, z)\, dz . \tag{13}$$

While $|Q_j| = 1$, the optical potential U_T^{opt} is non-Hermitian in general and therefore $|Q_j^{\text{opt}}| \leq 1$. The abrasion cross section takes the form

$$\sigma_n(\vec{b}) = \binom{A}{n} \left\langle O_P \left| \left[\prod_{j=1}^{n} \left(1 - Q_j^{\text{opt}\,\dagger} \sum_{\mu_j < \mu_0} |\mu_j\rangle \langle \mu_j | Q_j^{\text{opt}} \right) \right] \left[(Q_{n+1}^{\text{opt}} \cdots Q_A^{\text{opt}})^\dagger \sum_{m < m_0} |m\rangle \langle m| (Q_{n+1}^{\text{opt}} \cdots Q_A^{\text{opt}}) \right] \right| O_P \right\rangle . \tag{14}$$

At this stage we specify μ_0 and m_0, i.e., we define what is high energy of the ejected nucleons or low energy of the remaining fragments. The optical profile function $Q_j^{\text{opt}}(\vec{s}_j+\vec{b})-1$ is a slowly varying function of $\vec{s}_j$; characteristic distances are the target radius and the target surface thickness. Therefore the operator $Q_j^{\text{opt}}(\vec{s}_j+\vec{b})$ does not induce high Fourier components when acting on $|O_P\rangle$. This is verified later in the calculation of the average excitation energy. We choose the limits μ_0 and m_0 in Eq. (14) so that they include *all* Fourier components generated by Q_j^{opt}. Then the limits μ_0 and m_0 are unnecessary and closure can be performed (with $\mathcal{P}(\vec{s}_j+\vec{b}) = |Q_j^{\text{opt}}(\vec{s}_j+\vec{b})|^2$,

$$\sigma_n(\vec{b}) = \binom{A}{n}\left\langle O_P\left|\prod_{j=1}^{n}\left[1-\mathcal{P}(\vec{s}_j+\vec{b})\right]\right.\right. \times \left.\left.\prod_{l=n+1}^{A}\mathcal{P}(\vec{s}_l+\vec{b})\right|O_P\right\rangle. \qquad (15)$$

Equation (15) is readily interpreted: $\mathcal{P}(\vec{s}_j+\vec{b})$ is the probability that particle j stays in the projectile; $1-\mathcal{P}$, the probability that it is ejected from the projectile.

Finally we average over the projectile coordinates, again in the coherent approximation, Eq. (11). This leads to the expression for the cross section for the abrasion of n nucleons

$$\sigma_n(\vec{b}) = \binom{A}{n}\left[1-P(\vec{b})\right]^n P(\vec{b})^{A-n} \qquad (16)$$

in terms of the probability function $P(\vec{b})$,

$$P(\vec{b}) = \langle O_P|\,|Q_j^{\text{opt}}|^2|O_P\rangle = \int d^2s\, dz\, \rho_P(\vec{s},z)\left|e^{i\chi^{\text{opt}}(\vec{s}+\vec{b})}\right|^2 . \qquad (17)$$

No free parameter, nor any specific nuclear model, enters the derivation of Eq. (16). $\sigma_n(\vec{b})$ depends only on the single particle density $\rho_P(\vec{x})$ of the projectile and on the nucleon-nucleus optical potential. The optical potential can be extracted from elastic nucleon-nucleus scattering or can be calculated rather reliably from multiple scattering theory (e.g., Feshbach, Gal, and Hüfner[20]). The physics of Eqs. (16) and (17) can be made even more transparent if one neglects all correlations in the calculation of χ^{opt}. Then

$$P(\vec{b}) = \int d^2s\, dz\, \rho_P(\vec{s},z) \times\exp\left[-A_T\sigma_{\text{tot}}^{NN}\int_{-\infty}^{+\infty}\rho_T(\vec{s}+\vec{b},z')dz'\right] \approx 1-A_T\sigma_{\text{tot}}^{NN}\int d^2s\, dz\, \rho_P(\vec{s},z)\rho_T(\vec{s}+\vec{b},z')dz' . \qquad (18)$$

The integral is essentially the overlap of projectile and target densities when they pass at a distance $\vec{b}$ from each other. σ_{tot}^{NN} is the total nucleon-nucleon cross section. Thus, $P(\vec{b})-1$ is proportional to the volume element sheared off in the abrasion process. Thus the geometric model of Bowman *et al.*[7] is derived.

III. AVERAGE EXCITATION ENERGY AFTER ABRASION

In order to calculate the average excitation energy of the fragment after abrasion, we use the following generalized Thomas-Reiche-Kuhn sum rule:

$$(\bar{E}-E_0) = \frac{1}{W}\sum_{n\neq 0}(E_n-E_0)|\langle n|K|O\rangle|^2 \qquad (19\text{a})$$

$$= \frac{1}{2}\frac{1}{W}\langle O|[K^\dagger,[H,K]]|O\rangle \qquad (19\text{b})$$

$$= \frac{1}{W}\frac{1}{2M}\left\langle O\left|\sum_{i=1}^{A}|\vec{\nabla}_i K|^2\right|O\right\rangle; \qquad (19\text{c})$$

$$W = \sum_{n\neq 0}|\langle n|K|O\rangle|^2 . \qquad (19\text{d})$$

Equality (19b) holds for *any* A-body operator K between eigenstates $|O\rangle$ and $|n\rangle$ of the many-body system (associated energies: $(H-E_n)|n\rangle=0$). Equation (19c) only follows if K commutes with the potential energy terms in H. For nuclei this condition is met if K depends only on the spatial coordinates (not on spin and isospin), and if the forces do not contain velocities.

Starting from Eq. (14) we *define* the average excitation energy of the projectile after abrasion of n nucleons by

$$(\bar{E}-E_0)(\vec{b}) = \frac{1}{\sigma_n(\vec{b})}\binom{A}{n}\left\langle O_P\left|\left[\prod_{j=1}^{n}(1-|Q_j^{\text{opt}}|^2)\right]\left[(Q_{n+1}^{\text{opt}}\cdots Q_A^{\text{opt}})^\dagger\sum_m|m\rangle(E_m-E_0)\langle m|\,(Q_{n+1}^{\text{opt}}\cdots Q_A^{\text{opt}})\right]\right|O_P\right\rangle. \qquad (20)$$

After the factorization approximation (related to the coherent approximation) $|O_P\rangle = |\alpha_0\rangle\cdot|O_F\rangle$, where $|O_F\rangle$ is the ground state of the abraded fragment with $(A-n)$ nucleons, the coordinates $\vec{x}_1,\ldots,\vec{x}_n$ can be factored out. Then the sum rule Eq. (19) leads to

$$(\bar{E}-E_0)(\vec{b})=\frac{1}{2M}\frac{\left\langle O_F\left|\sum_{i=n+1}^{A}|\vec{\nabla}_i Q^{\mathrm{opt}}_{n+1}\cdots Q^{\mathrm{opt}}_{A}|^2\right|O_F\right\rangle}{\langle O_F||Q^{\mathrm{opt}}_{n+1}\cdots Q^{\mathrm{opt}}_{A}|^2|O_F\rangle}\,. \tag{21}$$

In the coherent approximation (ρ_F refers to the single particle density of the residual fragment):

$$(\bar{E}-E_0)(\vec{b}) =(A-n)\frac{1}{2M}\frac{\int d^2s\,dz\,\rho_F(\vec{s},z)|\vec{\nabla}_s Q^{\mathrm{opt}}(\vec{s}+\vec{b})|^2}{\int d^2s\,dz\,\rho_F(\vec{s},z)|Q^{\mathrm{opt}}(\vec{s}+\vec{b})|^2}\,. \tag{22}$$

The physics is again rather transparent. The operator Q^{opt}_j is the "knife" with which parts of the projectile are cut away. The "sharper" the knife, i.e., the larger $|\vec{\nabla}Q^{\mathrm{opt}}|$, the higher the excitation energy. We note that all nucleons of the projectile have their tail cut off. Thus, in a sense, abrasion is a collective phenomenon, and thermalization of the excitation energy before nucleon emission (ablation) seems the more likely process. The factorization assumption $|O_P\rangle=|\alpha_0\rangle|O_F\rangle$, where $|O_P\rangle$ and $|O_F\rangle$ are the ground states of projectile and fragment, respectively, seems plausible within an independent particle model, except for a shift in the center of mass.[22] Since matter is sheared off one side of the projectile only, the center of mass of the fragment is displaced from the one of the projectile by

$$\vec{a}(\vec{b})=\frac{\left\langle O_P\left|[(1-Q^{\mathrm{opt}}_1)\cdots(1-Q^{\mathrm{opt}}_n)Q^{\mathrm{opt}}_{n+1}\cdots Q^{\mathrm{opt}}_A]\frac{1}{A-n}\sum_{i=n-1}^{A}\vec{x}_i[(1-Q^{\mathrm{opt}}_1)\cdots(1-Q^{\mathrm{opt}}_n)Q^{\mathrm{opt}}_{n+1}\cdots Q^{\mathrm{opt}}_A]^{\dagger}\right|O_P\right\rangle}{\langle O_P||(1-Q^{\mathrm{opt}}_1)\cdots(1-Q^{\mathrm{opt}}_n)Q^{\mathrm{opt}}_{n+1}\cdots Q^{\mathrm{opt}}_A|^2|O_P\rangle}\,, \tag{22a}$$

where $\vec{a}$ lies in the impact parameter plane. The excitation energy of the final fragment has to be evaluated with respect to a ground state $|O'_F\rangle$ which equals the state $|O_F\rangle$, only displaced by $\vec{a}$. Using an independent particle model and a harmonic oscillator potential, the center-of-mass shift reduces the excitation energy Eq. (21) by the amount

$$\Delta E(\vec{b})=-\tfrac{1}{2}(A-n)M\omega^2\vec{a}^2(\vec{b})\,. \tag{22b}$$

The sign can be understood easily: The fragment with one side sheared off fits better into a potential well, if the c.m. of fragment and well coincide. The correction Eq. (22b) is significant, as can be seen on Fig. 2. Equations (22) and (22b) are our final result for the excitation energy. The expressions do not contain any free parameter, nor do they refer to a specific nuclear model.

IV. NUMERICAL RESULTS FOR THE ABRASION-ABLATION MODEL

The cross section for the abrasion of n out of N neutrons and z out of Z protons from the projectile nucleus is calculated from the expression

$$\sigma_{nz}(\vec{b})=\binom{N}{n}\binom{Z}{z}[1-P(\vec{b})]^{n+z}P(\vec{b})^{A-n-z}\,, \tag{23}$$

where $P(\vec{b})$ is defined in Eq. (18). The densities $\rho_P(\vec{x})$ and $\rho_T(\vec{x})$ in Eq. (18) are built from harmonic oscillator wave functions (s and p orbits); the oscillator constant is chosen so to reproduce the experimental rms radii for the target ^{9}Be and the projectile ^{16}O. The total nucleon-nucleon cross section $\sigma^{NN}_{\mathrm{tot}}=45$ mb is a value averaged over proton-proton and proton-neutron data at 2 GeV.[21] The total abrasion cross section $\sigma^{\mathrm{tot}}_{nz}$ is simply an integral over $\sigma_{nz}(\vec{b})$ [Eq. (23)] in the impact parameter plane. Figure 1 shows various abrasion cross sections for the production of carbon isotopes as a function of the impact parameter b. As expected, the cross sections peak at large impact parameters when only few nucleons are removed ($b_{\max}=4.1$ fm for ^{14}C), while the removal of several particles occurs largely at smaller impact parameters ($b_{\max}=2.3$ fm for ^{10}C). Then target and projectile ions overlap considerably. The shapes of the distributions do not depend dramatically on the number of removed particles.

The average excitation energy per nucleon is calculated

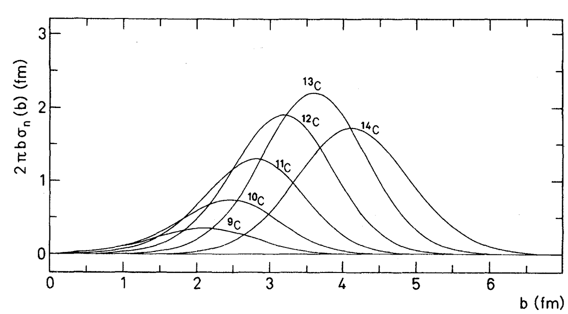

FIG. 1. The abrasion cross section $\sigma_n(\vec{b})$ as a function of the impact parameter b for the production of carbon isotopes in the reaction ^{16}O + ^{9}Be → C + X.

$$\frac{(\bar{E}-E_0)(\vec{b})}{A-n}=\frac{1}{2M}\frac{\int d^2s\,dz\,\rho_P(\vec{s}-\vec{b},z)\left|\vec{\nabla}\exp[-\frac{1}{2}A_T\sigma_{\rm tot}^{NN}(1-i\alpha)\int_{-\infty}^{+\infty}\rho_T(\vec{s},z')dz']\right|^2}{\int d^2s\,dz\,\rho_P(\vec{s}-\vec{b},z)\exp[-A_T\sigma_{\rm tot}^{NN}\int_{-\infty}^{+\infty}\rho_T(\vec{s},z')dz']} . \tag{24}$$

plus correction Eq. (22b). Eq. (24) agrees with Eq. (22), except that ρ_F is replaced by the projectile density (which should be a good approximation as long as only a few nucleons are removed). The parameter $\alpha=\mathrm{Re}f^{NN}(O)/\mathrm{Im}f^{NN}(O)$ is a measure of the real part of the nucleon-nucleon forward scattering amplitude $f^{NN}(O)$. Unfortunately, α is ill determined[21] at 2 GeV. However, the nucleon-nucleon data are compatible with $|\alpha|<0.3$. Since only the absolute value $|1-i\alpha|^2=1+\alpha^2$ enters Eq. (24), the uncertainty in α does not seem crucial. The dependence of the excitation energy per nucleon as a function of b is displayed in Fig. 2. As expected, the excitation energy goes to zero

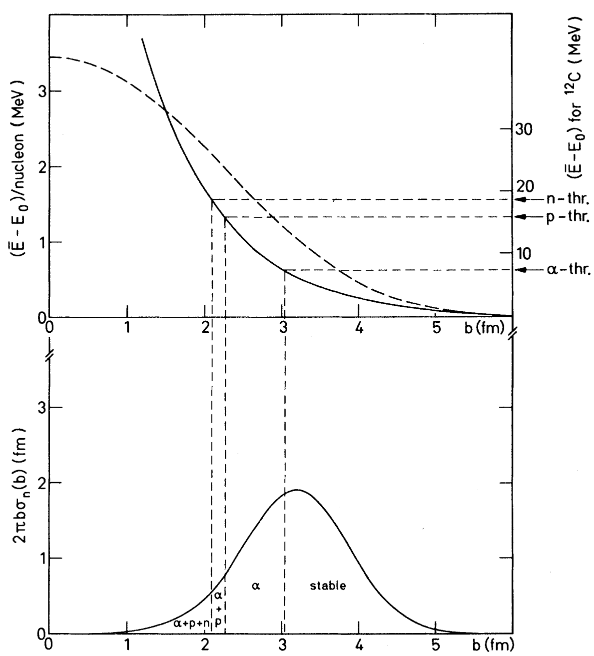

FIG. 2. (a): Average excitation energy per nucleon after abrasion as a function of the impact parameter. [dashed curve, Eq. (24); solid curve with c.m. correction, Eq. (22b)]. The ordinate on the right-hand side is multiplied by 12 to give the excitation energy for ^{12}C. The thresholds for α, proton, and neutron decays of ^{12}C are indicated. (b): Abrasion cross section for ^{12}C as in Fig. 1. The lines indicate how we calculate the intensities in the ablation process: The area of $\sigma_n(\vec{b})$ for impact parameters which correspond to excitation energies below the first threshold is the amount of stable ^{12}C. The next area of $\sigma_n(\vec{b})$ corresponding to energies above α and below proton thresholds is the amount of decay $^{12}C\to{}^8Be+\alpha$, etc.

for large impact parameters. Bowman *et al.*[7] calculate the average excitation energy from the excess surface energy after abrasion (in the liquid drop model the surface energy is about 1 MeV/fm^2). Although we cannot make the conceptual bridge between their model and our expression (24), the numerical results are rather similar. However, there is one important difference: Since the peaks in Fig. 1 are rather wide, one cannot associate *one* excitation energy to each abrasion cross section, but one has a distribution of energies. This fact is displayed in Fig. 2. For ^{12}C, the resulting fragment has an excitation energy between 3 and 22 MeV depending on the impact parameter. As various thresholds open in this energy interval ($^{12}C\to{}^8Be+\alpha$ at 7.9 MeV, $\to{}^{11}B+p$ at 15.6 MeV, etc.), the resulting fragment $^{12}C^*$ after abrasion may be particle stable or may decay by α emission, or α and proton decay may compete, etc. The probability for ^{12}C to be particle stable is proportional to the area with $b>b_1$, where b_1 is that impact parameter for which $(\bar{E}-E_0)(b_1)=7.9$ MeV. The probability for α decay is measured by the area between b_1 and b_2, where $(\bar{E}-E_0)(b_2)=15.6$ MeV, as indicated in Fig. 2. When two and more channels compete like α's, protons, and neutrons, we assume the relative probabilities W to be proportional to the penetrabilities

$$W_\alpha(b):W_p(b):W_n(b)$$
$$=T_\alpha(\bar{E}(b)-E_\alpha{}^{\rm th}):T_p(\bar{E}(b)-E_p{}^{\rm th}):T_n(\bar{E}(b)-E_n{}^{\rm th}) , \tag{25}$$

where for the neutron

$$T_n(E)=kR|j_0(kR)|^2, \quad k^2=2mE . \tag{26}$$

T_p and T_α involve the Coulomb functions instead of the spherical Bessel function j_0. R is the radius of the decaying nucleus and $E^{\rm th}$ is the threshold energy. We assume $l=0$ only. The approximation Eq. (25) is certainly a crude one. For instance, it neglects all nuclear structure effects. An inclusion of these effects, however, seems premature since we do not even know whether the excited fragment after abrasion decays "immediately" i.e., by direct decay, or whether it thermalizes the energy and then evaporates particles. In both cases, the decay probabilities must be proportional to the penetrabilities T multiplied by factors appropriate to the specific model. Thus

TABLE I. The results of an abrasion-ablation calculation for the reaction $^{16}O+{}^{9}Be \rightarrow {}^{A}Z+X$ compared with experimental values of Lindstrom *et al.* (Ref. 2). Columns 3 and 4 give the details of how the abraded projectile decays (partial decay cross sections in brackets) and from which sources the final observed cross section arises.

^{A}Z	Abrasion cross section (mb)	Abraded fragment ablates into cross section (mb)	Final fragment intensity arises from cross section (mb)	Abrasion-ablation cross section (mb)	σ_{exp} (mb)
^{15}O	131	$^{15}O(126)+(^{14}N+p)(2)+(^{14}O+n)(3)$	$^{15}O(126)$	126	43.0 ± 2.1
^{14}O	33	$^{14}O(27)+(^{13}N+p)(5)+(^{10}C+\alpha)(1)$	$^{14}O(27)+{}^{15}O(3)$	30	1.6 ± 0.1
^{13}O	10	$^{13}O(0.5)+(^{12}N+p)(10.5)$	$^{13}O(0.5)$	0.5	0.32 ± 0.04
^{12}O	3	?	...	...	
^{15}N	131	$^{15}N(128)+(^{14}N+n)(3)$	$^{15}N(128)$	128	54.1 ± 2.7
^{14}N	75	$^{14}N(70)+(^{13}N+p)(2)+(^{13}C+p)(3)$	$^{14}N(70)+{}^{15}N(3)+{}^{15}O(2)$	75	49.5 ± 4.0
^{13}N	40	$^{13}N(6)+(^{12}N+n)(2)+(^{12}C+p)(30)+(^{9}B+\alpha)(2)$	$^{13}N(6)+{}^{14}N(2)+{}^{14}O(5)$	13	8.0 ± 0.4
^{12}N	19	$(^{11}C+p)(16)+(^{8}B+\alpha)(3)$	$^{12}N(10.5)$	10.5	0.66 ± 0.06
^{11}N	8	?	...	...	
^{14}C	33	$^{14}C(31)+(^{13}C+n)(2)$	$^{14}C(31)$	31	5.2 ± 0.3
^{13}C	40	$^{13}C(24)+(^{12}C+n)(14)+(^{12}B+p)(1)+(^{9}Be+\alpha)(1)$	$^{13}C(24)+{}^{14}C(2)+{}^{14}N(3)$	29	28.6 ± 1.4
^{12}C	33	$^{12}C(25)+(^{11}C+n)(2)+(^{8}Be+\alpha)(6)$	$^{12}C(25)+{}^{13}C(14)+{}^{13}N(30)$	69	60.8 ± 4.9
^{11}C	21	$^{11}C(10)+(^{10}C+n)(3)+(^{10}B+p)(5)+(^{7}Be+\alpha)(3)$	$^{11}C(10)+{}^{12}C(2)+{}^{12}N(16)$	28	21.0 ± 1.0
^{13}B	10	$^{13}B(5)+(^{12}B+n)(5)$	$^{13}B(5)$	5	0.50 ± 0.04
^{12}B	19	$^{12}B(4)+(^{11}B+n)(12)+(^{8}Li+\alpha)(3)$	$^{12}B(4)+{}^{13}B(5)+{}^{13}C(1)$	10	2.8 ± 0.2
^{11}B	21	$^{11}B(12)+(^{10}B+n)(3)+(^{10}Be+p)(3)+(^{7}Li+\alpha)(3)$	$^{11}B(12)+{}^{12}B(12)$	24	27.5 ± 1.4

the requirement Eq. (25) is a minimal one. Although absolute values for lifetimes and decay widths differ by several orders of magnitude depending on the direct or compound mechanism, relative intensities are expected to be much less sensitive to the detailed models for ablation.

Table I contains our results for the reaction $^{16}O + {}^{9}B \to {}^{A_F}Z_F + X$. The calculation is performed according to the lines described above with *no adjustable parameter*. A first glance reveals good agreement between calculation and experiments. All trends are quantitatively reproduced: ^{15}O, ^{15}N, ^{12}C, and ^{11}B carry the maximal intensity in calculation and experiment. Particularly small values, for instance in ^{13}O (with the particle threshold at 1.5 MeV), are also understood. In most cases agreement within a factor of 2 is achieved. Nevertheless, the theoretical values show a systematic discrepancy to be too large in the oxygen and nitrogen isotopes and for ^{14}C. For all these nuclei abrasion dominates the final cross section. For instance, the calculated final intensity of 13 mb for ^{14}O is that part of the abrasion cross section (33 mb) which is below the first threshold. On the other hand, for the carbon and boron isotopes only a small fraction of the initial abrased intensity is stable. For ^{12}C, only 7 mb of the abrasion cross section are stable. The final intensity of 58 mb arises mainly from the particle decays of ^{13}C and ^{13}N. Since this is a rather complicated process, involving our assumption about the ablation mechanism, the agreement with experiment is surprising. Of course, the fact that ^{12}C carries the largest intensity of all C isotopes is a consequence of the stability of this nucleus, but this fact does not explain the nearly perfect matching between calculation and experiment.

In order to find the reason for the systematic discrepancies in the oxygen and nitrogen isotopes, we have investigated various approximations which enter the derivation of the abrasion-ablation model, especially the coherent approximation (cf. Appendix). We find that abrasion cross section and average excitation energies after abrasion are amazingly stable against all kinds of improvements. The only weak assumption of our calculation, namely, Eq. (25), cannot be the source for the discrepancies, since it becomes only important for the carbon and boron isotopes. Therefore we face the paradoxical situation: Although the over-all agreement between the calculations and experiment is good, systematic discrepancies appear where the model should be best. Bowman *et al.*[7] observed a similar trend. Their total cross sections for the production of oxygen and nitrogen isotopes are a factor of 2 too large, while their calculation is perfect for carbon.

V. BEYOND THE ABRASION-ABLATION MODEL

Since the abrasion-ablation model, defined by the Glauber expression for the scattering amplitude, is internally consistent and is carried through without major approximations, we have to analyze whether the physics of abrasion is sufficiently well reproduced by the model. Abrasion means that nucleon from the projectile ion is hit by a target nucleon and is "kicked out" of its bound orbit. More precisely, in the nucleon-nucleon scattering event, momentum $\vec{q}$ is transferred to the projectile nucleon. This momentum transfer is determined by the angular distribution of nucleon-nucleon scattering

$$\frac{d\sigma^{NN}}{dt} = \left(\frac{d\sigma^{NN}}{dt}\right)_{t=0} e^{at} , \tag{27}$$

where t is the four-momentum transfer. In the rest system of the projectile, $t = -q^2$. With the experimental value[21] $a = 6\ (\mathrm{GeV}/c)^{-2}$, we calculate an average momentum transfer to the sheared off nucleon

$$\langle q^2 \rangle^{1/2} = 1/\sqrt{a} \approx 400\ \mathrm{MeV}/c , \tag{28}$$

and an average kinetic energy of $\langle q^2 \rangle / 2M = 80$ MeV. The momentum transfer $\vec{q}$ is essentially *perpendicular* to the to the beam direction. In many cases, the struck nucleon has a strong final state interaction (FSI) with the remaining fragment. The abrasion-ablation model, developed in the previous sections, does not contain this effect. We consider the neglecting of FSI to be the reason why there are systematic discrepancies between calculation and experiment in Table I.

Before proposing a model, we discuss some general features of the FSI. In all cases considered in this paper, the abrasion process is located on the surface of the projectile. As mentioned above, the struck nucleon receives momentum perpendicular to the beam direction, i.e. the nucleon is either kicked immediately out of the nucleus (without FSI) or it has to cross a considerable amount of projectile matter. The first possibility happens with probability P_E, the second one with $1 - P_E$ (E stands for "escape"). Since the interaction happens at the surface, simple geometrical arguments yield $P_E \approx 0.5$. When the struck nucleon has to cross the fragment it experiences FSI. Since the mean free path $\lambda = (\sigma_{NN}^{\mathrm{tot}} \rho)^{-1} \approx 1 - 2$ fm, there will be several scatterings inside the nucleus. We are not aware of any experiment which could shed light on our FSI problem. Therefore we have to investigate models. We are guided by the results of a Monte Carlo calculation (Metropolis *et al.*[23]). The case in their paper which comes closest to our situation is the follow-

TABLE II. Results of fragmentation cross sections in an abrasion-ablation calculation which includes the final state interaction (FSI) of the struck nucleon according to assumptions (a), (b), and (c) of the text. For comparison we quote the abrasion, the abrasion-ablation, and the experimental cross sections from Table I.

^{Z}A	σ_{abr} (mb)	$\frac{\sigma_{abr}}{2^n}$	$\sigma_{abr\text{-}abl\text{-}FSI}$ (mb) Assumptions (a)	(b)	(c)	σ_{exp} (mb)	$\sigma_{abr\text{-}abl}$ (mb)
^{15}O	131	65.5	63	63	63	43.0 ± 2.1	126
^{15}N	131	65.5	67	64	64	54.1 ± 2.7	128
^{14}O	33	8	9	8	9	1.6 ± 0.1	30
^{14}N	75	19	30	21	24	49.5 ± 4.0	75
^{14}C	33	8	12	10	12	5.2 ± 0.3	31
^{13}O	10	1	0.1	0.1	0.14	0.32 ± 0.04	0.5
^{13}N	40	5	8	8	13	8.0 ± 0.4	13
^{13}C	40	5	22	16	29	28.6 ± 1.4	29
^{13}B	10	1	2.5	2	4	0.50 ± 0.04	5
^{12}C	33	2	24	19	33	60.8 ± 4.9	69
^{12}B	19	1	5	3	8	2.8 ± 0.2	10

ing: A nucleon with 80 MeV strikes aluminium. In 60% of all cases it ejects *another* nucleon, while two nucleons (or no nucleon) are removed with about 20% probability each. The distribution of excitation energy after FSI extends from zero to 70 MeV.

Our model is constructed in the following way. A struck nucleon escapes (without FSI) with probability P_E. Therefore FSI is important in only 50% of all cases for the $A=15$ system. When two nucleons are sheared off ($A=14$), the escape probability for *both* nucleons is $P_E^2=0.25$. In a first very rough approximation, FSI amounts to the removal of at least one more nucleon. Therefore FSI reduces the abrasion cross section for the $A=15$ system by a factor of 2, and for $A=14$ by a factor of 4, etc. Thus the discrepancies for $A=15$ and 14 are understood qualitatively already. Of course, the part of the cross section which undergoes FSI must appear somewhere, i.e., for $A<14$. In order to see this we construct explicit models for the FSI.

(a) The struck nucleon, which crosses the nucleus, excites the fragment. The distribution of excitation energies E_x is constant in the interval $10 \leq E_x \leq 70$ MeV.

(b) On its way through the nucleus, the nucleon ejects one other nucleon *and* deposits energy as in model (a).

(c) If the struck nucleon ejects a 1s nucleon, this hole is refilled by an Auger transition ejecting another nucleon. Ejection of a p nucleon either directly or via Auger leads to a constant distribution of excitation energies in the interval $10 \leq E_x \leq 30$ MeV.

For each model the initial distribution intensities (as a function of mass number and E_x) are calculated. Then the decay cascade is computed assuming pure compound decay (Bondorf and Nörenberg[24]).

The results of the abrasion-ablation calculation with FSI are displayed in Table II. We observe the following features.

(i) The three assumptions (a)–(c) do not produce very different results. Other shapes of distributions in the excitation energy also do not alter the results dramatically.

(ii) For $A=14$ and 15, the calculated cross sections are considerably smaller than those of a pure abrasion-ablation calculation but are still somewhat larger than experiment.

(iii) The cascade calculation becomes important for the isotopes with $A=12$ and 13. The agreement with experiment is not too good; deviations in both directions are considerable.

In conclusion, we consider the abrasion-ablation-FSI model an adequate starting point to understand fragmentation cross sections in relativistic heavy ion reactions. While the abrasion part can be calculated reliably, FSI of the sheared off nucleons has to be understood better.

ACKNOWLEDGMENTS

We have profited from clarifying discussions with Dr. A. Abul-Magd, Dr. T. E. O. Ericson, Dr. H. Feshbach, Dr. A. S. Goldhaber, Dr. D. Julius, Dr. C. Mahaux, Dr. W. Nörenberg, Dr. A. Rinat, and Dr. H. A. Weidenmüller.

APPENDIX: CRITICAL DISCUSSION OF SOME APPROXIMATIONS

Two approximations have entered the derivation of the abrasion-ablation model in Sec. II: (i) the phase shift function χ_{jl} of the elementary nucleon-nucleon scattering event is real; (ii) the coherent approximation. While we can remove restriction (i) nearly completely without changing the final formulas, Eqs. (16) and (23), we can only estimate the accuracy of the coherent approximation. A real phase shift function implies that there is only elastic nucleon-nucleon scattering, because the associated S-matrix element

$$S_{00}(\vec{b}) = e^{i\chi(\vec{b})} \tag{A1}$$

exhausts unitarity; $|S_{00}| = 1$. Empirically, at kinetic energies of 2 GeV, elastic nucleon-nucleon scattering is only a fraction of total cross section. As an example, we quote values for proton-proton scattering at 2 GeV from the compilation by Benary, Price, and Alexander[21]

$$\sigma_{\text{tot}} = 45 \text{ mb}, \quad \sigma_{\text{el}} = 19 \text{ mb},$$
$$\sigma_{pp\to NN\pi} = 19 \text{ mb (of which } \sigma_{pp\to N\Delta} = 13 \text{ mb)} . \tag{A2}$$

Therefore, the S-matrix element S_{00} must be replaced by a matrix $S_{\alpha\beta}$, where α and β refer to the various channels. Unitarity requires

$$\sum_{\beta} S_{\alpha\beta}(\vec{b})S^*_{\gamma\beta}(\vec{b}) = \delta_{\alpha\beta} . \tag{A3}$$

The simple unitarity of Eq. (A1) has been used to derive Eq. (10) from Eq. (9). In the presence of inelastic nucleon channels, the sum over $\mu_j > \mu_0$ should also include unobserved nucleon channels, i.e., one has to replace

$$\sum_{\mu_j>\mu_0} Q_j^\dagger |\mu_j\rangle\langle\mu_j| Q_j \to \sum_{\substack{\mu_j>\mu_0\\ \alpha_l}} \left\{\prod_{l\in T} S^{jl}_{0\alpha_l}\right\}^\dagger |\mu_j\rangle\langle\mu_j| \left\{\prod_{l\in T} S^{jl}_{\alpha_l 0}\right\} \tag{A4a}$$

$$= 1 - \sum_{\substack{\mu_j<\mu_0\\ \alpha_l}} \left\{\prod_{l\in T} S^{jl}_{0\alpha_l}\right\}^+ |\mu_j\rangle\langle\mu_j| \left\{\prod_{l\in T} S^{jl}_{\alpha_l 0}\right\} . \tag{A4b}$$

The "1" again follows from unitarity. Even if the inelastic nucleon channels appear in Eq. (A4b), the average over target states again leads to the nucleon-nucleus optical potential. The presence of the inelastic channels manifests itself in the fact that the imaginary part of the optical potential is proportional to the total nucleon-nucleon cross section and not to the total elastic one:

$$Q_j^{\text{opt}} = |e^{i\chi^{\text{opt}}_{\text{tot}}}|^2$$
$$= \exp\left[-\sigma^{NN}_{\text{tot}} \int_{-\infty}^{+\infty} \rho(\vec{b}, z)dz\right]. \tag{A5}$$

We come to approximation (ii): As is well studied in standard multiple scattering theory, the coherent approximation amounts to neglecting intermediate excited nuclear states. For instance, for two single particle operators B_1 and B_2,

$$\langle O|B_1B_2|O\rangle = \langle O|B_1|O\rangle\langle O|B_2|O\rangle + \sum_{n>0} \langle O|B_1|n\rangle\langle n|B_2|O\rangle . \tag{A6}$$

The second term is proportional to the two-body correlation function $C(\vec{x}, \vec{y})$,

$$\rho^{(2)}(\vec{x}, \vec{y}) - \rho(\vec{x})\rho(\vec{y}) = \rho(\vec{x})\rho(\vec{y})C(\vec{x}, \vec{y}) , \tag{A7}$$

with the nuclear two-body density $\rho^{(2)}(\vec{x}, \vec{y})$ and the one-body density $\rho(\vec{x})$. Because $\rho(\vec{x}) = \int d^3y\, \rho^{(2)}(\vec{x}, \vec{y})$ one has the important normalization

$$\int \rho(\vec{y})C(\vec{x}, \vec{y})d^3y = 0 . \tag{A8}$$

We introduce the correlation function into Eq. (A6):

$$\begin{aligned} D &= \langle O|B_1B_2|O\rangle - \langle O|B_1|O\rangle\langle O|B_2|O\rangle \\ &= \int d^3x\, d^3y\, \rho(\vec{x})\rho(\vec{y})C(\vec{x}, \vec{y})B_1(\vec{x})B_2(\vec{y}) \\ &= \frac{1}{2}\int d^3x\, d^3y\, \rho(\vec{x})\rho(\vec{y})C(\vec{x}, \vec{y}) \\ &\quad \times [B_1(\vec{x}) - B_1(\vec{y})][B_2(\vec{y}) - B_2(\vec{x})] , \end{aligned} \tag{A9}$$

where the last equality holds because of the normalization relation Eq. (A8). If the densities ρ and the operators B vary slowly over the correlation distance r_C ($C(\vec{x}, \vec{y}) \approx 0$ for $|\vec{x} - \vec{y}| > r_C$), one can expand $\vec{x}$ and $\vec{y}$ around $\vec{R} = \frac{1}{2}(\vec{x} + \vec{y})$:

$$D \approx -\tfrac{1}{2} \int d^3R\, \rho^2(\vec{R})(\vec{\nabla}B_1(\vec{R}))(\vec{\nabla}B_2(\vec{R}))\tfrac{1}{3}\epsilon ,$$
$$\epsilon = \int C(r)r^2 d^3r , \tag{A10}$$

where the correlation function is assumed to depend on the relative distance only. If the correlations are repulsive ($C(r)<0$) at small distances, there must be a tail with $C(r)>0$ because of the normalization Eq. (A8). Since the tail is weighted most in ϵ, we conclude that $\epsilon>0$ for repulsive correlations. The generalization to the case of A nucleons is straightforward if three-body and higher-order correlations are neglected. Then

$$\rho^{(A)}(\vec{x}_1, \ldots, \vec{x}_A) \approx \rho(\vec{x}_1) \ldots \rho(\vec{x}_A)\left[1+\sum_{i<j} C(\vec{x}_i, \vec{x}_j)\right], \tag{A11}$$

where the C's have to fulfill Eq. (A8). The correction $\Delta\sigma_n(\vec{b})$ to the result σ_n in the coherent approximation is then

$$\frac{\Delta\sigma_n(\vec{b})}{\sigma_n(\vec{b})} = -\frac{1}{12}\epsilon \frac{\int d^2s\, dz\, \rho^2(\vec{s}, z)[\nabla\mathcal{P}(\vec{s}+\vec{b})]^2}{[1-P(b)]^2 P(b)^2} \times\{[A(1-P)-n]^2-(A-n)(1-P)^2-nP^2\} . \tag{A12}$$

For short range repulsive correlations the correction is negative, but is small (a few percent) for the abrasion cross section.

---

*Work supported in part by a grant from the Federal Ministry for Research and Technology, Federal Republic of Germany.

[1] H. H. Heckman, D. E. Greiner, P. J. Lindstrom, and F. S. Bieser, Phys. Rev. Lett. 28, 926 (1972).

[2] P. J. Lindstrom, D. E. Greiner, H. H. Heckman, B. Cork, and F. S. Bieser, Report No. LBL-3650, Feb. 75 (unpublished).

[3] D. E. Greiner, P. J. Lindstrom, H. H. Heckman, B. Cork, and F. S. Bieser, Phys. Rev. Lett. 35, 152 (1975).

[4] H. J. Crawford, P. B. Price, J. Stevenson, and L. W. Wilson, Phys. Rev. Lett. 34, 329 (1975).

[5] Proceedings of the Second High Energy Heavy Ion Summer Study, Berkeley, July 1974 [Report No. LBL-3675 (unpublished)].

[6] W. Greiner (Ref. 5).

[7] J. D. Bowman, W. J. Swiatecki, and C. F. Tsang, (unpublished).

[8] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. 1.

[9] W. Czyż and L. C. Maximon, Ann. Phys. (N.Y.) 52, 59 (1969).

[10] O. Kofoed-Hansen, Nuovo Cimento A60, 621 (1969).

[11] J. Formánek, Nucl. Phys. B12, 441 (1969).

[12] G. Fäldt, Phys. Rev. D 2, 846 (1970).

[13] A. Tékou, Nucl. Phys. B46, 141,152 (1972).

[14] G. Fäldt, H. Pilkuhn, and H. G. Schlaile, Ann. Phys. (N.Y.) 82, 326 (1974).

[15] P. M. Fishbane and J. S. Trefil, Phys. Rev. D 10, 3005, 3128 (1974); Phys. Rev. Lett. 32, 396 (1974).

[16] V. Franco, Phys. Rev. Lett. 32, 911 (1974).

[17] H. Feshbach and K. Huang, Phys. Lett. 47B, 300 (1973).

[18] R. K. Bhaduri, Phys. Lett. 50B, 211 (1974).

[19] A. S. Goldhaber, Phys. Lett. 53B, 306 (1974).

[20] H. Feshbach, A. Gal, and J. Hüfner, Ann. Phys. (N.Y.) 66, 20 (1971).

[21] O. Benary, L. R. Price, and G. Alexander, *NN* and *ND* interactions (above 0.5 GeV/c), a compilation, Report No. UCRL-20 000 *NN* (unpublished).

[22] A. S. Goldhaber (private communication).

[23] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Phys. Rev. 110, 185 (1958).

[24] J. Bondorf and W. Nörenberg, Phys. Lett. 44B, 487 (1973).