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Isospin mixing in deformed nuclei*

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The effect of nuclear deformation on the isospin mixing matrix elements has been investigated for deformed configurations in ^{18}F. An enhancement of the mixing with deformation is found.

[NUCLEAR STRUCTURE Isospin mixing.]

The mixing of isospin states through the Coulomb interaction has been studied in detail some time ago by MacDonald.[1] In this early work, part of the isospin mixing in light nuclei was considered to result from the interaction of a core, consisting of completely filled shells, with the "valence" nucleons outside of this core. MacDonald treated the core as a uniform, spherical, charge distribution of radius R, providing a Coulomb potential

$$V(r)=\frac{3Ze^2}{2R}\left(1-\frac{r^2}{3R^2}\right),\quad r\leqslant R$$
$$=\frac{Ze^2}{r},\quad r\geqslant R\ . \tag{1}$$

Using this potential and harmonic oscillator wave functions, he calculated the Coulomb mixing matrix element $\langle T=1|V(r)|T=0\rangle$ for two and three nucleons outside closed shells.

Following the general approach of MacDonald, we have investigated the effects of nuclear deformation on isospin mixing in ^{18}F. We feel the results have greater generality, however. The present study was motivated by the suggestion (see Friedman, Ref. 2) that the excitation function for the isospin forbidden reaction ^{16}O(d, α_1)-^{14}N* might be understood in terms of pairs of simple configurations called "bridge states," for which isospin mixing is especially strong. It was further suggested in Ref. 2 that these pairs of bridge states in ^{18}F might be composed of a proton and neutron, coupled either to $T=0$ or $T=1$, outside of a ^{16}O* core treated as a ^{12}C$+\alpha$ cluster configuration.

As a further simplification to facilitate the calculation of isospin mixing matrix elements, we now employ the Nilsson model[3] and consider the valence proton and neutron to be strongly coupled to a spheroidal ^{16}O core via the potential

$$V(\vec{r},\vec{l},\vec{s})=\tfrac{1}{2}m[\omega^2(x_1^{\,2}+x_2^{\,2})+\omega_3^{\,2}x_3^{\,2}]+C\vec{l}\cdot\vec{s}+D\vec{l}^{\,2}\ . \tag{2}$$

The ω's are parametrized by

$$\omega^2=\omega_0^{\,2}(1+\tfrac{2}{3}\delta)\ , \tag{3a}$$

$$\omega_3^{\,2}=\omega_0^{\,2}(1-\tfrac{4}{3}\delta)\ , \tag{3b}$$

where δ is a measure of the deformation and ω_0 can be related to the rms radius through $\hbar/m\omega_0=\frac{4}{9}\langle r^2\rangle$ in the case of ^{16}O core. We make the conventional assumption that the oscillator deformation parameter δ is related to the core deformation parameter β by

$$\delta=\frac{3}{2}\left(\frac{5}{4\pi}\right)^{1/2}\beta\cong 0.946\beta\ . \tag{4}$$

To calculate the isospin mixing, we couple the valence proton and neutron to states with $T=0$ and $T=1$ and take the wave function for the whole system to be

$$|\alpha\Omega_a,\beta\Omega_b,IMK,T\rangle=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\frac{1}{(1+\delta_{\alpha\beta}\delta_{\Omega_a\Omega_b})^{1/2}}$$
$$\times\sum_{j_a j_b}C^{\alpha}_{j_a\Omega_a}C^{\beta}_{j_b\Omega_b}[\psi(j_a\Omega_a j_b\Omega_b T)D^{I*}_{MK}+(-1)^{I-j_a-j_b}\psi(j_a-\Omega_a j_b-\Omega_b T)D^{I*}_{M\bar{K}}]\ , \tag{5a}$$

where

$$\psi(j_a\Omega_a j_b\Omega_b T)=\frac{1}{\sqrt{2}}[\phi_{j_a\Omega_a}(1)\phi_{j_b\Omega_b}(2)+(-1)^T\phi_{j_b\Omega_b}(1)\phi_{j_a\Omega_a}(2)]\frac{1}{\sqrt{2}}[\epsilon_1(\tfrac{1}{2})\epsilon_2(-\tfrac{1}{2})-(-1)^T\epsilon_1(-\tfrac{1}{2})\epsilon_2(\tfrac{1}{2})]\ . \tag{5b}$$

In Eq. (5) we have used the expansion of the Nilsson states $\chi_{\alpha\Omega_a}$ in terms of spherical harmonic-oscillator states $\phi_{j_a\Omega_a}$ with coefficients $C^{\alpha}_{j_a\Omega_a}$. The coefficients are taken from Ref. 4.

We represent the potential for the Coulomb interaction of each valence particle with the core by the potential V_C arising from a uniformly charged prolate spheroid of semimajor axis b and semiminor axis a, with $a^2b = R^3 = (3.42\text{ fm})^3$. The value of R, the radius of the equivalent uniform distribution ($R^2 = \frac{5}{3}\langle r^2\rangle$), is taken from electron scattering from the ground state of ^{16}O.[5] The precise form of the Coulomb potential can be found in Ref. 6. In order to relate the core deformation parameter β to a and b, we use the self-consistency argument that the density distribution has the same shape as the Nilsson potential. Hence,[7] $a^2/b^2 = \omega_3{}^2/\omega^2 = (1-1.2616\beta)/(1+0.6308\beta)$.

Using Eq. (5) and the deformed Coulomb potential, we find the Coulomb mixing matrix element,

$$M = \langle \alpha\Omega_a, \beta\Omega_b, T=1 | \sum_{i=1}^{2} \tfrac{1}{2} V_C(x_i)(1+\tau_z^{(i)}) | \alpha\Omega_a, \beta\Omega_b, T=0\rangle \tag{6}$$

reduces to

$$M = \tfrac{1}{2}[\langle \chi_{\alpha\Omega_a} | V_C | \chi_{\alpha\Omega_a}\rangle - \langle \chi_{\beta\Omega_b} | V_C | \chi_{\beta\Omega_b}\rangle] . \tag{7}$$

Note that M is the difference between the Coulomb energies of the two configurations occupied by the valence particles. It follows that no mixing can occur when the two particles are placed in the same Nilsson level. This is analogous to the conclusion reached by MacDonald in the spherical case, i.e., no mixing of different isospin states of $(1P_j)^2$ can occur.

In Fig. 1 we present the single particle Coulomb energy

$$\langle \chi_{\alpha\Omega_a} | V_C | \chi_{\alpha\Omega_a}\rangle$$

for the $N=2$ Nilsson levels. Substitution of these energies into Eq. (7) yields the Coulomb mixing matrix elements, which are tabulated in Table I, for all possible pairs of $N=2$ levels and for $\beta = 0.0$, 0.1, 0.2, and 0.3.

Note that the deformation allows for an increase in the Coulomb mixing over what is possible with a spherical nucleus. For example, the maximum Coulomb mixing matrix element for the spherical

TABLE I. Coulomb mixing matrix element M (in keV) between $T=0$ and $T=1$ Nilsson configurations for various values of deformation parameter β.

Nilsson configuration	$\beta=0.0$	$\beta=0.1$	$\beta=0.2$	$\beta=0.3$
$\frac{1}{2}[220]\,\frac{3}{2}[211]$	0	0.1	−7.6	−19.0
$\frac{1}{2}[220]\,\frac{5}{2}[202]$	0	15.8	25.5	33.9
$\frac{1}{2}[220]\,\frac{1}{2}[211]$	−25.8	−17.0	−14.5	−21.4
$\frac{1}{2}[220]\,\frac{1}{2}[200]$	0	−15.3	−47.2	−78.2
$\frac{1}{2}[220]\,\frac{3}{2}[202]$	0	13.9	22.9	31.3
$\frac{3}{2}[211]\,\frac{5}{2}[202]$	0	15.7	33.1	52.8
$\frac{3}{2}[211]\,\frac{1}{2}[211]$	−25.8	−17.1	−6.9	−2.4
$\frac{3}{2}[211]\,\frac{1}{2}[200]$	0	−15.4	−39.6	−59.2
$\frac{3}{2}[211]\,\frac{3}{2}[202]$	0	13.8	30.5	50.3
$\frac{5}{2}[200]\,\frac{1}{2}[211]$	−25.8	−32.8	−40.0	−55.2
$\frac{5}{2}[202]\,\frac{1}{2}[200]$	0	−31.1	−72.7	−112.0
$\frac{5}{2}[202]\,\frac{3}{2}[202]$	0	−2.0	−2.7	−2.6
$\frac{1}{2}[211]\,\frac{1}{2}[200]$	25.8	1.7	−32.8	−56.8
$\frac{1}{2}[211]\,\frac{3}{2}[202]$	25.8	30.8	37.3	52.7
$\frac{1}{2}[200]\,\frac{3}{2}[202]$	0	29.1	70.1	109.5

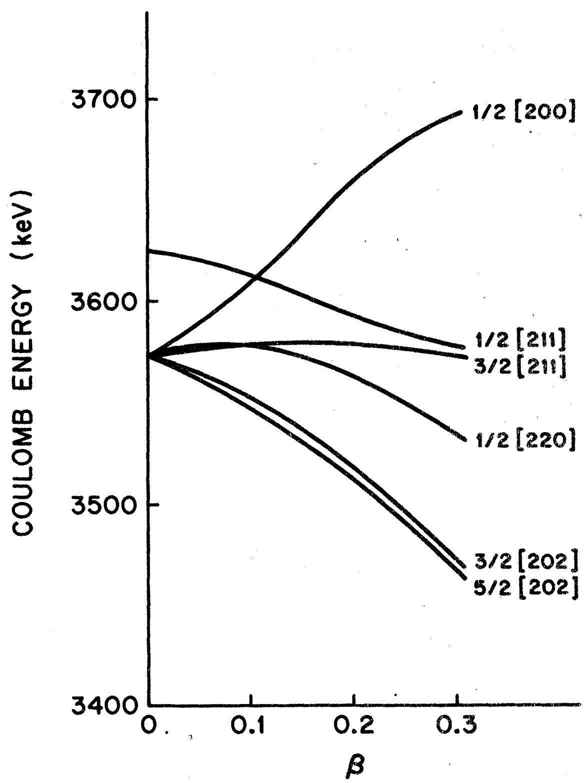

FIG. 1. Variation of the single-particle Coulomb energies for $N=2$ Nilsson levels with deformation parameter β ($R = 3.42$ fm).

case is 26 keV, while the maximum for the deformed nucleus (with $\beta = 0.3$) is 112 keV, an increase of over a factor of 4. Also important is the fact that the increase in Coulomb mixing through deformation is achieved without an appreciable increase in the sum of the nuclear energies ϵ_i of the valence particles. For example, the spherical configuration involving $(1d_{5/2})^2$ has $M = 0$ and $\epsilon_1 + \epsilon_2 = 6.8\hbar\omega_0$, while the deformed configuration ($\beta = 0.3$) involving orbits $\frac{1}{2}[220]$ and $\frac{1}{2}[200]$ has $M = 78$ keV and $\epsilon_1 + \epsilon_2 = 6.8\hbar\omega_0$ (the exact agreement of $\epsilon_1 + \epsilon_2$ is coincidental).

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