Coulomb displacement energies of excited states*

R. Sherr
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

G. Bertsch
Michigan State University, East Lansing, Michigan 48823
(Received 11 June 1975)

The Bansal-French-Zamick model is quite successful in accounting for the Coulomb displacement energies of excited particle-hole states in a variety of light nuclei. Level shifts are typically reproduced to within 50 keV. However, the model fails for certain excited 0^+ states, and this remains a puzzle.

[NUCLEAR STRUCTURE Calculated Coulomb energies of particle-hole states $A=13$ to 45.]

The Coulomb displacement energies of the ground states in mirror nuclei exhibit the well-known odd-even effect.[1] This "pairing" is an enhanced displacement energy when $Z_>$ of the isospin doublet is even so that this member of the doublet has an additional $J=0$ proton pair. This is shown with the solid line for mirror nuclei from $A=13$ to 39 in Fig. 1. The same systematics for the one-particle–m-hole excited states is shown by the dashed line in the figure. Note that the $\frac{7}{2}^-$ and $\frac{5}{2}^+$ energies are lower than the sd and $1p$ shell energies in odd $Z_>$ mirror pairs. This merely reflects the larger rms radius of the particle shell relative to the hole shell. Note also that the "pairing" effect is much more pronounced for the $\frac{7}{2}^-$ and $\frac{5}{2}^+$ excited states than for the ground state. This enhancement is rather surprising since, for example, the $\frac{7}{2}^-$ states in ^{35}Cl and ^{35}Ar arise from a single $f_{7/2}$ nucleon coupled to $(d_{3/2})^2$, $J=0$, which precludes $f_{7/2}$ pairing. However, as we shall see, two thirds of the time, ^{35}Ar $(\frac{7}{2}^-)$ is an $f_{7/2}$ *neutron* coupled to a pair of $d_{3/2}$ *protons*, and vice versa for ^{35}Cl $(\frac{7}{2}^-)$. The large "pairing" effect must arise from the $d_{3/2}$ nucleons. In the present note we describe a simple method to explain this enhanced pairing quantitatively.

In a recent paper[2] the systematics of the binding energies of particle-hole states were found to be consistent with predictions of the Bansal-French-Zamick model[3,4] (BFZ) for a wide range of nuclei in the $2s$-$1d$ and $1f_{7/2}$ shells. We can extend this method to Coulomb displacement energies by finding the energies of a particle-hole state and of its analog and subtracting the two. Under the usual assumption that analog states have the same nuclear energies, all differences in binding energies of analogs are ascribed to Coulomb energy differences. Equivalently, we can in the BFZ model replace the (total) energies of the nuclei involved by their Coulomb energies and retain only the Coulomb term of the particle-hole interaction.

If one or more particles are excited into a higher orbital, let us designate the particle configuration as P and the residue of the original configuration as C. Schematically, we describe the particle-hole state as $P\otimes C$ and take these to be the corresponding real nuclei. Thus, for example, $^{33}\mathrm{S}(\frac{7}{2}^-, \frac{1}{2}) = {}^{41}\mathrm{Ca}(\frac{7}{2}^-, \frac{1}{2}) \otimes {}^{32}\mathrm{S}(0^+, 0)$, and its analog is $^{41}\mathrm{Sc}(\frac{7}{2}^-, \frac{1}{2}) \otimes {}^{32}\mathrm{S}(0^+, 0)$.

The Coulomb energy of a state with isospin (T, T_z) is given in this method by

$$E = \sum_i (T^{C_i} T_Z^{C_i} T^{P_i} T_z^{P_i} \mid TT_z)^2 \times [E(C_i) + E(P_i) - \nu_i \mu_i c], \tag{1}$$

where the index i designates the various constituent states needed for a state of good isospin. The ν_i and μ_i are the numbers of proton particles and proton holes and c is the Coulomb interaction between two protons in different shells. To obtain ΔE_c we write down the corresponding expression for the analog state and take the difference.

Let us first see what happens for the states in mirror nuclei described by an excitation of one particle from the valence shell to a higher shell. It will be necessary to distinguish between nuclei whose particle number is $4N+1$ or $4N+3$. In the former case, as for example ^{33}Cl, the excitation is described by

$$|A^*\rangle = \{(1\mathrm{p}) \otimes (A-1)^{T=0}\}^{T=1/2}. \tag{2}$$

We apply this to Eq. (1), with $T_Z = +\frac{1}{2}$ and $-\frac{1}{2}$, and subtract. The result is, trivially,

$$\Delta E_c(A^*) = \Delta E_c(1\mathrm{p}) - \mu_{A-1}c. \tag{3}$$

For the example of ^{33}Cl, this is

$$\Delta E_c(^{33}\mathrm{Cl}, \tfrac{7}{2}^-) = \Delta E_c(^{41}\mathrm{Sc}, \tfrac{7}{2}^-) - 4c.$$

For nuclei of the type $4N+3$, the BFZ model predicts that the lowest excited state has the form

$$|A^*\rangle = \{(1\mathrm{p}) \otimes (A-1)^{T=1}\}^{T=1/2}. \tag{4}$$

We apply this to Eq. (1) with $T_Z = +\frac{1}{2}$ and $-\frac{1}{2}$, and subtract. The resulting Coulomb displacement energy is

$$\Delta E_c(A^*) = \tfrac{2}{3}[\Delta E_c(A-1)^{T=1}_{T_Z=-1} + \Delta E_c(A-1)^{T=1}_{T_Z=0}] - \tfrac{1}{3}\Delta E_c(1\mathrm{p}) + \tfrac{1}{3}(\mu_0+2)c. \tag{5}$$

Here μ_0 is the number of proton holes in the $(A-1)^{T=1}_{T_Z=0}$ nucleus. An example of a mirror pair of the type (4) are the $\frac{7}{2}^-$ states of ^{35}Ar and ^{35}Cl, for which the Coulomb displacement energy is

$$\Delta E_c(\tfrac{7}{2}^-) = \tfrac{2}{3}[\Delta E_c(^{34}\mathrm{Ar}) + \Delta E_c(^{34}\mathrm{Cl})] - \tfrac{1}{3}\Delta E_c(^{41}\mathrm{Sc}) + \tfrac{5}{3}c.$$

We now insert experimental data on the right-hand side of Eqs. (3) and (5). The parameter c is chosen to fit one displacement energy (indicated by "fit" in Table I) in each shell, except for $c(d_{5/2}p_{1/2})$, which was chosen to give a reasonable over-all fit to the relevant data. Throughout the paper, we use the experimental Coulomb displacement energies for the *lowest* pair of states with the appropriate spin, parity, and isospin. The results are given in Table I for nuclei for which sufficient input data exist. For the $2s_{1/2}$ and $1d_{5/2}$ shells the particle nuclei were taken to be the $\frac{7}{2}^-$ states of $A=33$ and the $\frac{7}{2}^-$ states of $A=29$, respectively. The experimental data were taken from the compilations of Endt and Van Der Leun[5] and Ajzenberg-Selove.[6] Our value of 289 keV for $c(f_{7/2}d_{3/2})$ is considerably smaller than the 400 keV used by Bansal and French[3] and our value of 355 keV for $c(d_{5/2}p_{1/2})$ is considerably smaller than the 500 keV used by Zamick.[4]

The results are in excellent agreement with experiment for the $d_{3/2}$ shell. The pairing effect in the excited state is several times as large as in the ground state, and is predicted to within 15%. Agreement is also good for the $2s_{1/2}$ shell. The large deviations in the very light nuclei may be due to the fact that the orbits are unbound and

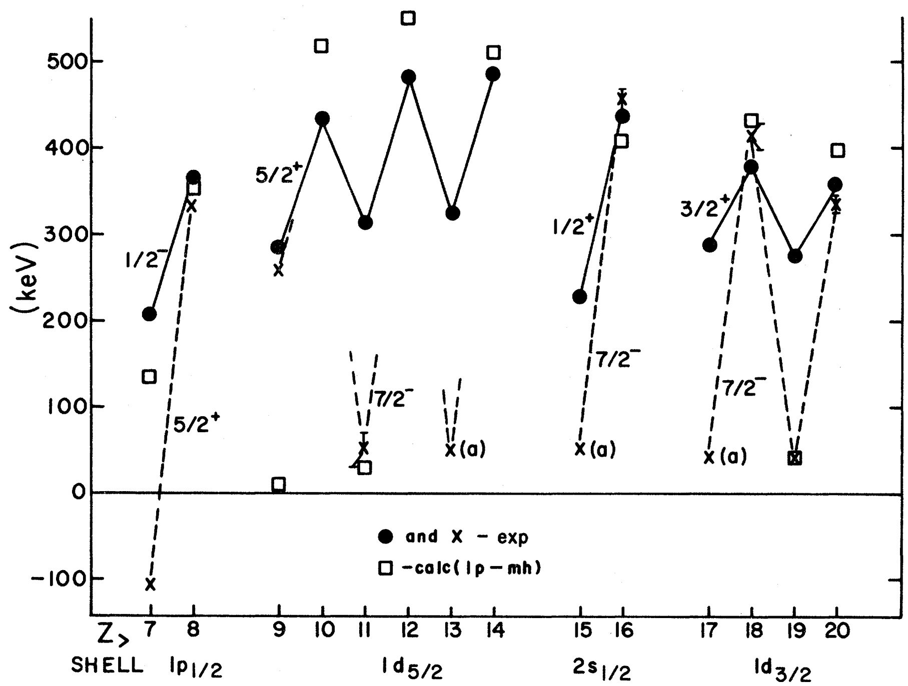

FIG. 1. Coulomb displacement energies of 1-particle–m-hole states for $T=\frac{1}{2}$ nuclei (Table I). $Z_>$ is the higher Z of the mirror pair. Arbitrary expressions linear in $Z_>$ have been subtracted from ΔE_c in order to emphasize the pairing effect: in the $1p_{1/2}$ shell, 2800 +380 ($Z_> - 7$); in the $1d_{5/2}$ shell, 3260 +370 ($Z_> - 9$); in the $2s_{1/2}$ and $1d_{3/2}$, 5500 +290 ($Z_> - 15$), in keV. Ground-state points in the $1p_{1/2}$, $2s_{1/2}$, and $1d_{3/2}$ shells are designated by ●, as are $\frac{5}{2}^+$ states in the $1d_{5/2}$ shell. One-particle–m-hole states ($J=\frac{7}{2}^-$ in the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ shells, and $\frac{5}{2}^+$ in the $1p_{1/2}$ shell) are indicated by X. Computed values are indicated by □. Points marked with (a) were used to determine c.

have substantial widths, as noted in the table. The Thomas-Ehrman shift has the correct sign to explain the discrepancy in mass 13. The magnitude and sign of the deviation for $A=17$ suggest that the lowest $\frac{7}{2}^-$ states used here are not single-particle states.

The above treatment of one-particle–m-hole states also applies to n-particle–one-hole states. In this case the BFZ model has the structure

$$|A^*\rangle=\{(A+1)^{T=1}\otimes(1\mathrm{h})\}^{T=1/2}\quad A=4N+1\,, \tag{6}$$

$$|A^*\rangle=\{(A+1)^{T=0}\otimes(1\mathrm{h})\}^{T=1/2}\quad A=4N+3\,. \tag{7}$$

This model can be applied to $\frac{3}{2}^+$ states in $f_{7/2}$ shell mirror nuclei, and to $\frac{1}{2}^-$ states in $d_{5/2}$ shell mirror nuclei. As in the case of one-particle excitations, an enhanced pairing effect is predicted. Comparison with the observed energies is shown in Table II and in Fig. 2. The same values of c are used as in Table I. The enhanced pairing in the $d_{5/2}$ shell nuclei is excellently reproduced with the exception of $Z_>=10$. However, the calculated pairing effect in the $f_{7/2}$ shell is too large by a factor of 1.4, with the ^{41}Sc displacement energy particularly out of line. This suggests that Eq. (6) may not be an appropriate model for that $\frac{3}{2}^+$ state. The structure $4\mathrm{p}\otimes3\mathrm{h}$ is estimated to be less than 2 MeV above the $2\mathrm{p}\otimes1\mathrm{h}$ state, and has a predicted displacement energy much closer to the observed.

We note an interesting difference between Fig. 1 and Fig. 2. In the former, the energy for the particle-hole state is very much lower than that of the ground state for $Z_>$ odd, while in Fig. 2 the energy of the particle-hole state is much greater than the ground state for $Z_>$ even. From Eqs. (2) and (6) we see that the isospin structure of these two sets of particle-hole states is $\{T=\frac{1}{2}\otimes T=0\}^{T=1/2}$. The Coulomb energy is therefore just that of the odd particle or odd hole.

TABLE I. Coulomb displacement energies of lowest one-particle–m-hole states $J=j_\mathrm{p}$, $T=\frac{1}{2}$ in mirror nuclei. The configurations are assumed to be $[P(j_\mathrm{p},\frac{1}{2})\otimes C(j_\mathrm{h}^{-m},0^+,T_m)]$. The third and fourth columns show the experimental ΔE_c for the ground and particle-hole states. The fifth column lists the computed values for the particle-hole states and the last column, Δ, is the difference between computed and experimental data. The value of $c(j_\mathrm{p}j_\mathrm{h})$ used in the computations is given in the subcaptions. All energies are in keV. Errors assigned to the computed values are based solely on uncertainties in the experimental Coulomb energies needed in the computations. References given in this and the subsequent tables are to publications in addition to Refs. 5 and 6 containing new data needed for determining either experimental or calculated ΔE_c.

Mirror pair	m, T_m	ΔE_c (ground) (exp)	$\Delta E_c\,(J^\pi)$ (exp)	$\Delta E_c\,(J^\pi)$ (computed)	Δ (comp–exp)
		(a) $J^\pi=\frac{7}{2}^-$ states in the $1d_{3/2}$ shell: $c(f_{7/2}d_{3/2})=289$ keV			
^{33}Cl–^{33}S	8, 0	6367 (2)	6121 (5)	fit	0 (7)
^{35}Ar–^{35}Cl	6, 1	6747 (2)	6784 (14)	6800 (5)	+16 (16)
^{37}K–^{37}Ar	4, 0	6932 (2)	6700 (3)	6700 (7)	0 (8)
^{39}Ca–^{39}K	2, 1	7305 (5)	7284 (8)	7347 (6) [a]	+63 (10)
		(b) $J^\pi=\frac{7}{2}^-$ states in the $2s_{1/2}$ shell: $c(f_{7/2}s_{1/2})=286$ keV			
^{29}P–^{29}Si	4, 0	5726 (3)	5549 (4)	fit	0 (7)
^{31}S–^{31}P	2, 1	6224 (11)	6245 (13)	6194 (6)	−51 (14)
		(c) $J^\pi=\frac{7}{2}^-$ states in the $1d_{5/2}$ shell: $c(f_{7/2}d_{5/2})=380$ keV			
^{17}F–^{17}O [b]	12, 0	3542 (1)	3518 (10)	3269 (4)	−249 (11)
^{19}Ne–^{19}F	10, 1	4062 (1) [c]	···	4147 (4)	···
^{21}Na–^{21}Ne	8, 0	4311 (2) [c]	[4050 (20)] [d]	4029 (4)	−21 (20)
^{23}Mg–^{23}Na	6, 1	4850 (2) [c]	···	4909 (2)	···
^{25}Al–^{25}Mg	4, 0	5062 (2)	4790 (5)	fit	0 (7)
^{27}Si–^{27}Al	2, 1	5593 (1)	···	5619 (4)	···
		(d) $J^\pi=\frac{5}{2}^+$ states in the $1p_{1/2}$ shell: $c(d_{5/2}p_{1/2})=355$ keV			
^{13}N–^{13}C [e]	4, 0	3003 (1)	2696 (6)	2832 (1)	+136 (6)
^{15}O–^{15}N	2, 1	3542 (1)	3513 (1)	3542 (2)	+29 (2)

[a] References 10 and 11.
[b] $\Gamma(^{17}\mathrm{F}\ 5.67\ \mathrm{MeV},\frac{7}{2}^-)=40$ keV, $\Gamma(^{17}\mathrm{O}\ 5.70\ \mathrm{MeV},\frac{7}{2}^-)=3.2$ keV.
[c] $\frac{5}{2}^+$ states rather than ground state.
[d] Using $E_x(^{21}\mathrm{Ne}\ \frac{7}{2}^-)=5.33$ MeV (Ref. 12) and $E_x(^{21}\mathrm{Na}\ \frac{7}{2}^-)=5.05$ MeV (Ref. 13).
[e] $\Gamma(^{13}\mathrm{N}\ 3.55\ \mathrm{MeV},\frac{5}{2}^+)=74$ keV.

TABLE II. Coulomb displacement energies for excited n-particle–one-hole mirror states. The n-particle configuration has $J^\pi = 0^+$.

Mirror pair	(J^π, T)	n, T_n	ΔE_c (ground) (exp)	ΔE_c (J^π) (exp)	ΔE_c (J^π) (computed)	Δ
			(a) $J^\pi = \frac{3}{2}^+$ states in $1f_{7/2}$ shell			
^{41}Sc-^{41}Ca	$\frac{3}{2}^+, \frac{1}{2}$	2, 1	7278 (5)	7364 (9)	7264 (5)	−100 (11)
^{43}Ti-^{43}Sc	$\frac{3}{2}^+, \frac{1}{2}$	4, 0	7634 (11) [a]	7802 (20) [b]	7883 (5)	+81 (20)
			(b) $J^\pi = \frac{1}{2}^-$ states in $1d_{5/2}$ shell			
^{17}F-^{17}O	$\frac{1}{2}^-, \frac{1}{2}$	2, 1	3542 (1)	3592 (8)	3576 (4)	−16 (10)
^{19}Ne-^{19}F	$\frac{1}{2}^-, \frac{1}{2}$	4, 0	4062 (1) [c]	4186 (2)	4252 (2)	+66 (3)
^{21}Na-^{21}Ne	$\frac{1}{2}^-, \frac{1}{2}$	6, 1	4313 (2) [c]	4345 (5)	4354 (3)	+9 (6)
^{23}Mg-^{23}Na	$\frac{1}{2}^-, \frac{1}{2}$	8, 0	4849 (4) [c]	4970 (4)	4962 (2)	−8 (5)
^{25}Al-^{25}Mg	$\frac{1}{2}^-, \frac{1}{2}$	10, 1	5062 (2)	···	5080 (3)	···
^{27}Si-^{27}Al	$\frac{1}{2}^-, \frac{1}{2}$	12, 0	5593 (1)	5674 (7)	5672 (2)	−2 (7)

[a] Reference 14.
[b] Reference 15.
[c] $\frac{5}{2}^+$ state rather than ground state.

Since the Coulomb energy of a higher orbital is less than that of a lower orbital, the odd particle (Fig. 1) gives a reduced energy relative to the ground state while the odd hole (Fig. 2) gives an enhanced energy.

Conversely, the particle-hole energies are about the same as that of ground states for $Z_>$ even in Fig. 1 and $Z_>$ odd in Fig. 2. These particle-hole states have the isospin structure from Eqs. (4) and (7), $\{T=\frac{1}{2}\otimes T=1\}^{T=1/2}$. In these cases the proton-rich nucleus has an odd-neutron particle (or proton hole) two-thirds of the time, as noted in the introductory paragraph. The resulting effect on the Coulomb energy can be seen qualitatively from the classical coupling of isospin vectors: the vectors of length $\frac{1}{2}$ and 1 must be antiparallel to couple to $\frac{1}{2}$. Thus, the $\frac{1}{2}$ component is antiparallel to the total vector. Under isospin rotation on the average one neutron is changed into a proton. However, rotation of the antiparallel ($\frac{1}{2}$) component changes a proton into a neutron and this must be compensated for by changing two neutrons of the $T=1$ component in protons. Thus, in the states of Fig. 1, for $Z_>$ even, we gain, roughly speaking, twice the energy of a valence proton while losing the energy of the excited proton. The net gain on this picture would therefore be greater than that for the ground state mirrors, in qualitative agreement with the calculations. For the states in Fig. 2, this vector picture leads to a similar result, namely that for $Z_>$ odd, the particle hole energy should be slightly less than the ground state energy. In summary, the apparent "pairing" effect for the particle-hole states arises primarily from the alternating role of the odd particle rather than from the pairing of protons as in the ground states. This reversal from normal of the charge of the odd particle (or hole) would also be reflected in the magnetic moments of the $(4N+3)$ one-parti-

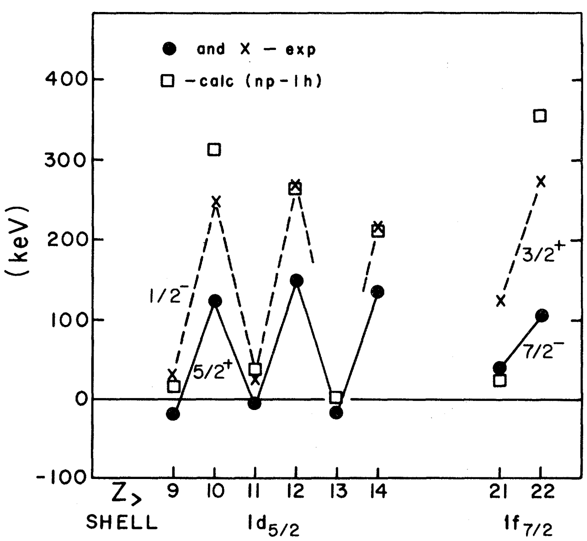

FIG. 2. Coulomb displacement energies of n-particle–1-hole states for $T=\frac{1}{2}$ nuclei in the $1d_{5/2}$ and $1f_{7/2}$ shells (Table II). The n-particle–one-hole states (X) are $\frac{1}{2}^-$ in the $1d_{5/2}$ shell and $\frac{3}{2}^+$ in the $1f_{7/2}$ shell. The ordinates are ΔE_c minus $3560+380$ $(Z_>-9)$ in the $1d_{5/2}$ shell, and $7240+290$ $(Z_>-21)$, in the $1f_{7/2}$ shell, in keV.

cle–m-hole states and the $(4N+1)$ n-particle–one-hole states.[7]

Let us now turn to excitations in non-mirror nuclei. In Table III we collect that data on odd-A nuclei, in Table IV for even-A nuclei with $J\neq 0$, and in Table V for even-A nuclei with $J=0$.

The agreement between computed and experimental values in Table III(a) and III(b) is quite good, the maximum significant deviation being less than 50 keV with the exception of ^{41}K-^{41}Ar. The agreement in Table III(c) is very poor. For the ^{15}N-^{15}O pair, the 80 keV width of the ^{15}N state could be responsible for the +125 keV deviation. However, the width of the $(\frac{1}{2}^-, \frac{3}{2})$ state of ^{17}O is reported to be <20 keV and it is therefore unlikely that a Thomas-Ehrmann shift would be large enough to account for the +80 keV deviation.

The agreement in Table IV is very good for all the cases listed. The analog states in ^{38}Ar, ^{40}Ca, and ^{42}Ca are particle-unbound but their widths are probably too small to be shifted significantly. It should be remarked that our method is in principle valid only for the centroid of the $(J_P\otimes J_C)^J$ multiplets. For ^{40}Ca-^{40}K, the energies of the 4^- states relative to the respective centroids are the same within 4 keV, and for the 5^- states of ^{38}Ar-^{38}Cl within an estimated 20 keV. For ^{16}O-^{16}N, the 2^- and the 2^--3^- centroids have the same ΔE_c. For ^{40}Sc-^{40}Ca and ^{42}Ca-^{42}K we can only compare the ground states, there being insufficient information on other states of the multiplets.

We consider 0^+ states separately in Table V because deviations are generally large. Several calculations are at least qualitatively successful and we discuss these first. The Coulomb displacement energies of the $(f_{7/2}{}^2 d_{3/2}{}^{-4})$ states in ^{38}K and ^{38}Ar

TABLE III. Coulomb displacement energies for n-particle–m-hole states with $T\geq\frac{3}{2}$ for odd-A nuclei.

Analog pair	J^π, T	m, J_m^π, T_m	n, J_n^π, T_n	ΔE_c (ground)	$\Delta E_c(J^\pi)$ (exp)	$\Delta E_c(J^\pi)$ (computed)	Δ (keV)
			(a) $d_{3/2}{}^{-1}f_{7/2}{}^n$ states				
^{41}Ca-^{41}K	$\frac{3}{2}^+, \frac{3}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$2, 0^+, 1$	7045 (15)	gnd	7051 (4)	+6 (16)
^{43}Ca-^{43}K	$\frac{3}{2}^+, \frac{5}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$4, 0^+, 2$	6952 (20)	gnd	7001 (4) [a]	+49 (20)
^{45}Ca-^{45}K	$\frac{3}{2}^+, \frac{7}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$6, 0^+, 3$	···	gnd	6947 (5) [a]	···
^{43}Sc-^{43}Ca	$\frac{3}{2}^+, \frac{3}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$4, 0^+, 2$	7238 (4)	···	7175 (6)	···
^{45}Ti-^{45}Sc	$\frac{3}{2}^+, \frac{3}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$6, 0^+, 1$	7571 (4) [b]	7641 (10) [b]	7660 (4) [a]	+19 (11)
^{47}Ti-^{47}Sc	$\frac{3}{2}^+, \frac{5}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$8, 0^+, 2$	7550 (20) [b]	7580 (20) [b]	7600 (5) [a]	+20 (21)
^{49}Ti-^{49}Sc	$\frac{3}{2}^+, \frac{7}{2}$	$1, \frac{3}{2}^+, \frac{1}{2}$	$10, 0^+, 3$	7533 (20) [b]	7536 (20) [b]	7562 (4) [a]	+26 (20)
			(b) $f_{7/2}{}^n d_{3/2}{}^{-m}$ states				
^{35}Cl-^{35}S	$\frac{7}{2}^-, \frac{3}{2}$	$6, 0^+, 1$	$1, \frac{7}{2}^-, \frac{1}{2}$	6266 (4)	6175 (5)	6223 (3)	+48 (6)
^{37}Cl-^{37}S	$\frac{7}{2}^-, \frac{5}{2}$	$4, 0^+, 2$	$1, \frac{7}{2}^-, \frac{1}{2}$	6150 (30)	gnd	6203 (8)	+53 (31)
^{37}Ar-^{37}Cl	$\frac{7}{2}^-, \frac{3}{2}$	$4, 0^+, 2$	$1, \frac{7}{2}^-, \frac{1}{2}$	6591 (6) [c]	···	6582 (36) [d]	···
^{39}K-^{39}Ar	$\frac{7}{2}^-, \frac{3}{2}$	$2, 0^+, 1$	$1, \frac{7}{2}^-, \frac{1}{2}$	6765 (16) [e]	gnd	6784 (3)	+19 (16)
^{41}K-^{41}Ar	$\frac{7}{2}^-, \frac{5}{2}$	$2, 0^+, 1$	$3, \frac{7}{2}^-, \frac{3}{2}$	6639 (10) [f]	gnd	6726 (4)	+87 (11)
^{41}Ca-^{41}K	$\frac{7}{2}^-, \frac{3}{2}$	$2, 0^+, 1$	$3, \frac{7}{2}^-, \frac{1}{2}$	7045 (15)	7082 (15)	7096 (5)	+14 (16)
^{43}Ca-^{43}K	$\frac{7}{2}^-, \frac{5}{2}$	$2, 0^+, 1$	$5, \frac{7}{2}^-, \frac{3}{2}$	6952 (20)	6987 (33)	7043 (4)	+56 (33)
^{45}Ca-^{45}K	$\frac{7}{2}^-, \frac{7}{2}$	$2, 0^+, 1$	$7, \frac{7}{2}^-, \frac{5}{2}$	···	···	7013 (20) [b]	···
			(c) $d_{5/2}{}^n p_{1/2}{}^{-m}$ states				
^{15}N-^{15}O	$\frac{5}{2}^+, \frac{3}{2}$	$2, 0^+, 1$	$1, \frac{5}{2}^+, \frac{1}{2}$	2779 (13) [g]	gnd	2904 (2)	+125 (13)
^{17}O-^{17}N	$\frac{1}{2}^-, \frac{3}{2}$	$1, \frac{1}{2}^-, \frac{1}{2}$	$2, 0^+, 1$	3185 (16)	gnd	3265 (2)	+80 (16)

[a] Reference 16.
[b] Reference 17.
[c] Reference 18.
[d] Reference 19.
[e] Reference 20.
[f] Reference 21.
[g] $\Gamma(^{15}\mathrm{N}, \frac{5}{2}^+, \frac{3}{2}) = 80$ keV.

TABLE IV. Coulomb displacement energies for particle-hole ground state analogs for even-A nuclei with $T>\frac{1}{2}$ and $J^\pi \neq 0^+$.

Analog pair	(J^π, T)	m, J_m^π, T_m	n, J_n^π, T_n	ΔE_c (exp)	ΔE_c (computed)	Δ
			(a) $(f_{7/2})^n (d_{3/2})^{-m}$			
^{38}Ar-^{38}Cl	$5^-, 2$	$3, \frac{3}{2}^+, \frac{3}{2}$	$1, \frac{7}{2}^-, \frac{1}{2}$	6502 (7)	6546 (3)	+44 (8)
^{40}Ca-^{40}K	$4^-, 1$	$1, \frac{3}{2}^+, \frac{1}{2}$	$1, \frac{7}{2}^-, \frac{1}{2}$	7129 (2)	7146 (3)	+17 (4)
^{40}Sc-^{40}Ca	$4^-, 1$	$1, \frac{3}{2}^+, \frac{1}{2}$	$1, \frac{7}{2}^-, \frac{1}{2}$	7448 (3)	7435 (3)	−13 (4)
^{42}Ca-^{42}K	$2^-, 2$	$1, \frac{3}{2}^+, \frac{1}{2}$	$3, \frac{7}{2}^-, \frac{3}{2}$	7015 (11)	7038 (5)	+23 (12)
			(b) $(d_{5/2})^n (p_{1/2})^{-m}$			
^{16}O-^{16}N	$2^-, 1$	$1, \frac{1}{2}^-, \frac{1}{2}$	$1, \frac{5}{2}^+, \frac{1}{2}$	3327 (1)	3364 (1)	+37 (2)

TABLE V. Coulomb displacement energies for 0^+ particle-hole states with configuration $[(j_p^n, T_n, 0^+)\otimes(j_h^m, T_m, 0^+)]^{T,0^+}$.

Analog pair[h]	m, T_m	n, T_n	T	ΔE_c (ground) (exp)	ΔE_c (0^+p-h) (exp)	ΔE_c (0^+p-h) computed	Δ
				(a) $f_{7/2}{}^n d_{3/2}{}^{-m}$ states			
^{34}Cl-^{34}S	8, 0	2, 1	1	6273 (2)	···	6058 (3)	···
^{34}Ar-^{34}Cl	8, 0	2, 1	1	6842 (4)	···	6615 (7)	···
^{38}K-^{38}Ar	4, 0	2, 1	1	6826 (3)[a]	6693 (7)[c]	6636 (5)	−57 (9)
^{38}Ca-^{38}K	4, 0	2, 1	1	7399 (6)[b]	7215 (50)[c]	7193 (7)	−22 (50)
^{40}K-^{40}Ar	2, 1	2, 1	2	6665 (16)	gnd	6731 (3)	+66 (17)
^{40}Ca-^{40}K	2, 1	2, 1	2	7129 (2)	7062 (20)[d]	7113 (3)	+51 (20)
^{40}Ca-^{40}K	2, 1	2, 1	1	7129 (2)	7212 (13)[d]	7296 (5)	+84 (14)
^{42}Sc-^{42}Ca	2, 1	4, 0	1	7214 (3)	7265 (6)[e]	7404 (3)	+139 (7)
	2, 1	4, 2	1			7140[f]	−125
^{42}Ti-^{42}Sc	2, 1	4, 0	1	7771 (7)	7753 (30)	7977 (6)	+224 (31)
	2, 1	4, 2	1			7645[f]	−108
				(b) $(d_{5/2})^n (p_{1/2})^{-m}$			
^{14}N*-^{14}C	4, 0	2, 1	1	2938 (1)	2653 (5)	2770 (1)	+117 (6)
^{14}O*-^{14}N*	4, 0	2, 1	1	3614 (2)	3215 (13)	3477 (5)	+262 (14)
^{16}N*-^{16}C	2, 1	2, 1	2	2700 (18)	gnd	2854 (1)	+154 (18)
^{16}O*-^{16}N*	2, 1	2, 1	2	3327 (1)	3152 (8)	3321 (2)	+169 (8)
^{16}O-^{16}N	2, 1	2, 1	1	3327 (1)	···	3546 (3)	···
^{18}F-^{18}O	2, 1	4, 0	1	3480 (1)	3545 (6)	3648 (1)	+103 (6)
	2, 1	4, 2	1			3480 (34)[g]	−65 (34)
^{18}Ne-^{18}F*	2, 1	4, 0	1	4187 (5)	4066 (8)	4324 (1)	+258 (8)
	2, 1	4, 2	1			4280(110)[g]	+214(110)

[a] See Reference 10.
[b] References 10 and 11.
[c] $E_x(^{38}\mathrm{K}, 0_2^+) = 3376(6)$, Reference 22.
[d] Reference 23.
[e] Reference 24.
[f] $\Delta E_c(^{44}\mathrm{V}, 0^+, 2)$ and $\Delta E_c(^{44}\mathrm{Cr}, 0^+, 2)$ computed via IMME.
[g] Reference 25.
[h] Asterisk indicates unbound states.

are about 150 keV lower than those of the ground states and this feature is reproduced. In absolute values our predictions for the $A=40$ (2p-2h) states are consistently about 70 keV too high, perhaps due to 0^+ components. However, the calculations do reproduce the finding that in ^{40}Ca the splitting of the (2p-2h) $T=2$ and $T=1$ states is 150 keV less than in ^{40}K.

The remainder of the cases in Table V show very poor agreement between calculations and data. The possibly large effects of configuration mixing can be seen in our following discussion of the $A=18$ and $A=42$ results.

The basic structure of the four-particle–two-hole states of ^{18}O and ^{42}Ca

$$[4\text{p}, T=0 \otimes 2\text{h}, T=1]^{T=1} \tag{8}$$

is reasonably assured on other theoretical grounds[8,9] besides the BFZ model. Nevertheless, it is worth considering admixtures of the type $[(T=2)\otimes(T=1)]^{T=1}$. Zamick[4] finds this configuration to have a low energy but not nearly as low as Ref. (8). Calculations of ΔE_c for these states in $A=42$ and $A=18$ are given in Table V. Agreement for ^{42}Sc would require a 50% component of this configuration, while for ^{42}Ti and ^{18}F a 65% component would be needed. (On the other hand, for ^{18}Ne this component appears to be of no help at all.) These admixtures are rather large and other components may be required for a full understanding of the Coulomb energies of these 0^+ states.

To summarize our results, we find that our method gives predictions, for particle-hole states with $J\neq 0$, which agree with experiment to within 20 keV for 45% of the 33 cases listed and within 100 keV for 94% of the cases. Predictions for the 0^+ states in the even-A nuclei listed in Table V are generally poor, with only 5 cases out of 13 agreeing with experiment within 100 keV. Thus, except for these 0^+ states our method has been generally successful in accounting for differences between Coulomb energies of particle-hole states and ground states which vary between −250 keV to +170 keV. Large deviations (>100 keV) from prediction can point up misidentification of states or Thomas-Ehrmann shifts, as discussed in several cases above.

The authors wish to acknowledge the hospitality and support of Los Alamos Scientific Laboratory extended to them as visiting staff members during the final drafting of this paper. We are indebted to Professor I. Talmi and Professor T. Fortune for helpful conversations.

---

*Supported in part by the National Science Foundation under Grant No. MPS71-03445.

[1] B. C. Carlson and I. Talmi, Phys. Rev. 96, 436 (1954).
[2] R. Sherr, R. Kouzes, and R. Del Vecchio, Phys. Lett. B 52B, 401 (1974).
[3] R. K. Bansal and J. B. French, Phys. Lett. 11, 145 (1964).
[4] L. Zamick, Phys. Lett. 19, 580 (1965).
[5] P. M. Endt and C. Van Der Leun, Nucl. Phys. A A214, 1 (1973).
[6] F. Ajzenberg-Selove, Nucl. Phys. A A152, 1 (1970); A166, 1 (1971); and A190, 1 (1972).
[7] This was pointed out to us by I. Talmi.
[8] G. E. Brown and A. M. Green, Nucl. Phys. 75, 401 (1966).
[9] B. H. Flowers and L. D. Skouras, Nucl. Phys. A A116, 529 (1968).
[10] B. H. Wildenthal, J. A. Rice, and B. M. Preedom, Phys. Rev. C 10, 2184 (1974).
[11] K. Seth *et al.*, Phys. Rev. Lett. 33, 233 (1974).
[12] C. A. Whitten (private communication).
[13] M. Lambert *et al.*, J. Phys. (Paris) 33, 155 (1972).
[14] D. Mueller, E. Kashy, W. Benenson, and H. Nann, Phys. Rev. C 12, 51 (1975).
[15] I. D. Proctor *et al.*, Phys. Rev. Lett. 29, 434 (1972).
[16] F. D. Becchetti, D. Dehnard, and T. G. Dzubay, Nucl. Phys. A A168, 151 (1971).
[17] M. B. Lewis and S. Raman, Nucl. Data B4, A=45, 47, 49 (1970).
[18] W. Benenson, E. Kashy, and I. D. Proctor, Phys. Rev. C 8, 210 (1973).
[19] J. Kroon, B. Hird, and G. C. Ball, Nucl. Phys. A A204, 609 (1973).
[20] K. T. Knöpfle, P. Doll, G. Mairle, and G. J. Wagner, Nucl. Phys. A A233, 317 (1974).
[21] L. R. Medsker, J. N. Bishop, S. C. Headley, and H. T. Fortune, Bull. Am. Phys. Soc. 19, 1033 (1974).
[22] H. T. Fortune (private communication).
[23] P. T. Debevec and G. T. Garvey, Phys. Rev. C 10, 680 (1974).
[24] R. Sherr, T. S. Bhatia, D. Cline, and J. J. Schwartz, Ann. Phys. (N.Y.) 66, 548 (1971).
[25] R. G. H. Robertson, S. Martin, W. R. Falk, D. Ingham, and A. Djalasis, Phys. Rev. Lett. 32, 1207 (1974).