Structure of $1f_{7/2}$ and $1g_{9/2}$ nuclei

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In order to investigate some approximations involved in the generalized semimicroscopic model (GSMM), a cluster-phonon model is presented in which a hole or two holes and one particle are coupled between them through Q-Q plus pairing interactions, and to the quadrupole vibrations of a doubly even core having a closed shell. Anharmonicities are also included phenomenologically. Matrix elements are computed using graphical methods and numerical calculations are performed for ^{57}Co and ^{115}In, selected as well known examples of odd-mass nuclei in the $1f_{7/2}$ and $1g_{9/2}$ shells, respectively. A good fit to energy levels is achieved through a reasonable variation of the model parameters, and the predictions of the GSMM about the nature of the low-lying levels in both nuclei are confirmed. The calculated electromagnetic properties and spectroscopic factors compare favorably with experiment except for some discrepancies in ^{115}In.

I. INTRODUCTION

The classical semimicroscopic model (SMM),[1] in which a nucleon cluster of particles and/or holes interacts with the quadrupole vibrations of a core, has been successful in describing some properties of the excited states in nuclei lying near closed shells. However, the SMM cannot explain the $\frac{1}{2}$ and $\frac{3}{2}$ low-lying energy levels observed in nuclei with a hole in the closed $1f_{7/2}$ and $1g_{9/2}$ major shells (strictly speaking the latter is a subshell of the shell ending at proton number $Z = 50$). In the SMM it is assumed that the excited states of such a nucleus with mass number A arise from the coupling of the hole state to the vibrations of the $(A + 1)$ single-closed core (hole-like states). That limitation has been recently overcome with the generalized semimicroscopic model (GSMM)[2] which differs from the SMM in the addition of two-hole one-particle seniority-zero and seniority-three cluster states in an approximate way. The main approximation was to replace the matrix elements of the model Hamiltonian between those additional excitations by matrix elements of a modified Hamiltonian between states arising from the coupling of a nucleon to the vibrations of the $(A - 1)$ core with two holes in the closed shell (particlelike states). A further approximation, required for the calculation of cross matrix elements, was to replace that part of the particlelike states corresponding to the vibrations of the $(A - 1)$ core, by states of the $(A + 1)$ vibrating core coupled to a major-shell two-hole states with zero angular momentum. In this way, only

the interaction of a particle or a hole with surface vibrations of a core was required to treat nuclei with a hole in a proton or neutron closed shell.

Within the framework of the GSMM a detailed study of the low-lying states of odd-mass indium isotopes,[2,3] and of ^{49}Ti, ^{51}Cr, and odd-mass cobalt isotopes[4,5] was performed with a restricted variation of the particle-vibration interaction strength, and taking the single-particle and phonon energies as given.

We felt that the following points deserved further investigation: the approximate treatment of 2h-1p excitations (which defines the GSMM), the assumption that the core vibrations are harmonic, and the influence of all the involved parameters on the calculated properties. The latter point has some importance in the case of single-particle energies, because in the GSMM calculations on indium isotopes[2] the assumption of model independence was made, and in those on ^{49}Ti, ^{51}Cr, and cobalt isotopes[4,5] the mass and charge dependence were neglected. Of course, single-particle energies must be model independent, but there is no unique way of extracting them from experimental data without recourse to a model.

With the above aims in mind, in the present work we start by treating the matrix elements involving 2h-1p excitations in an exact manner, in the sense that we first assume that an appropriate base is that composed by the states of a hole and those of two holes plus one particle coupled to an $(A + 1)$ doubly even single-closed quadrupole harmonic vibrating core, and then diagonalize numerically the resulting matrix of

the model Hamiltonian. This approach requires the explicit inclusion of residual interactions between the members of the cluster, which, as stated above, were not needed in the GSMM for the nuclei we are interested in. These interactions are taken to be the long-range quadrupole-quadrupole (Q-Q) interaction plus the short-range pairing one. The influence of single-particle energies and interaction strengths on energy levels is studied through the search of the best set of these parameters by least-squares fits to experimental energy levels. Finally, anharmonic effects are included phenomenologically by allowing for a nonzero quadrupole moment of the first excited 2+ core state.

We devote Sec. II to the description of the formalism used, listing the matrix elements involved. In Sec. III we give some details of the numerical calculations together with a critical comparison with the GSMM results. Finally, some general conclusions are presented in Sec. IV.

II. FORMULATION

A. Model Hamiltonian, energy matrix, and spectroscopic factors

Our starting Hamiltonian has the usual form in cluster-phonon models (cf. Ref. 2):

$$H = H_0 + H_{sp} + H_{int} + H_{res}, \tag{1}$$

where the Hamiltonians are, respectively, associated with: (i) the harmonic quadrupole vibrations of the $(A+1)$ doubly even core; (ii) the motion of the extracore nucleons and holes in an effective spherical potential; (iii) the interaction of the cluster with the $(A+1)$ vibrating core, and (iv) the residual two-body interaction within the cluster.

The operators H_0 and H_{sp} have the usual form; H_{int} is taken to be

$$H_{int} = \sum_\mu Q^k_{2\mu} \, \alpha^\dagger_{2\mu} \equiv Q^k_2 \cdot \alpha_2, \tag{2}$$

where

$$Q^k_{2\mu} = -\sum_i k(r_i) \, Y_{2\mu}(\Omega_i) \tag{3}$$

and the coupling strength is

$$k(r) = \rho r \frac{\partial V}{\partial r}, \tag{4}$$

$V(r)$ being the effective spherical potential and ρ the renormalization parameter introduced in Ref. 4. The Hamiltonian H_{res} has been taken to be a sum of a pairing interaction H_P and a quadrupole-quadrupole (Q-Q) interaction H_Q:

$$H_{res} = H_P + H_Q. \tag{5}$$

The pairing interaction only affects the two-hole states coupled to zero angular momentum. The Q-Q interaction is

$$H_Q = -\chi \sum_{i>j} \sum_\mu Q^r_{2\mu}(i) Q^{r*}_{2\mu}(j), \tag{6}$$

where

$$Q^r_{2\mu} = (4\pi/5)^{1/2} r^2 Y_{2\mu}(\Omega) \tag{7}$$

and χ is the Q-Q strength.

According to our hypothesis above, we choose the following base vectors

$$|h^{-1}, NR; IM\rangle = [|h^{-1}m_h\rangle \otimes |NRM_R\rangle]_{IM}, \tag{8}$$

$$|[(h^{-2})J_h, \, p]\, J, \, NR; \, IM\rangle = [\, |JM_J\rangle \otimes |NRM_R\rangle]_{IM}. \tag{9}$$

where the symbols mean: $|h^{-1}m_h\rangle$ and $|JM_J\rangle$, the state vectors of the hole and the 2h-1p states, respectively, with h and J representing the set of quantum numbers necessary to completely describe them; $|NRM_R\rangle$, the state vectors of the $(A+1)$ core, assumed to be a harmonic vibrator, with N phonons coupled to angular momentum R; and I and M, the total angular momentum quantum numbers of the odd-A nucleus.

The matrix elements of the total Hamiltonian (1) between the hole states (8) are the same as those calculated in Ref. 2. The remaining ones are most readily calculated by means of the graphical methods of El-Baz, Lafourcrière, and Castel,[6] together with the symbol shown in Fig. 1(b) for the hole states. The graphical calculation has some advantages: (i) the determination of phases (rather cumbersome if done analytically) is elementary, (ii) it yields directly reduced matrix elements between *particle* states because the holes are automatically transformed into particles in the course of the calculation, and (iii) certain simplifications which are obvious in the diagrams may be rather difficult (if not practically impossible) to see in analytical expressions. These
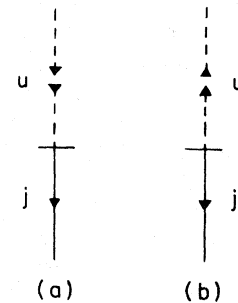

FIG. 1. Graphical symbols for (a) particle states $\langle u | \, j\, m\rangle$ and (b) hole states $\langle u | \, j^{-1} m\rangle$.

advantages are particularly important for the complex matrix elements involving many-particle many-hole configurations.

The final expressions for the matrix elements of the model Hamiltonian (1) without anharmonicities are given below. The following notation is used: $\epsilon(j)$, single-particle energy; $\hbar\omega$, phonon

energy; $\{abc\}$, 3-j symbol indicating the allowed values of the angular momenta involved; $\begin{Bmatrix} a & b & e \\ c & d & f \end{Bmatrix}$, 6-$j$ symbol; $\hat{J} \equiv (2J+1)^{1/2}$; $\langle \; || \; || \; \rangle$, reduced matrix elements; $\delta_{p'p} = \delta_{n_{p'} n_p} \delta_{j_{p'} j_p} \delta_{l_{p'} l_p}$, where n is the number of oscillator quanta and l the orbital angular momentum of the particle; and G is the pairing strength constant.

$$\langle [(h^{-2})J'_h, p']J', N'R'; IM | H | [(h^{-2})J_h, p]J, NR; IM \rangle$$

$$= \delta_{J'_h J_h} \delta_{p'p} \delta_{J'J} \delta_{N'N} \delta_{R'R} [\epsilon(p) + N\hbar\omega]$$

$$+ (-)^{R'-p+I} \hat{J}'\hat{J} \begin{Bmatrix} J' & J & 2 \\ R & R' & I \end{Bmatrix} \langle N'R' || \alpha_2 || NR \rangle$$

$$\times \left[2\hat{J}'_h\hat{J}_h \delta_{p'p} \begin{Bmatrix} h & h & 2 \\ J_h & J'_h & h \end{Bmatrix} \begin{Bmatrix} J'_h & J_h & 2 \\ J & J' & p \end{Bmatrix} \langle h || Q_2^k || h \rangle + (-)^{J'-J} \delta_{J'_h J_h} \{hhJ_h\} \begin{Bmatrix} p' & p & 2 \\ J & J' & J_h \end{Bmatrix} \langle p' || Q_2^k || p \rangle \right]$$

$$+ \delta_{N'N} \delta_{R'R} \delta_{J'J} \{JRI\} \left\{ \delta_{p'p} \delta_{J'_h J_h} \{J_h pJ\} \left[\chi \begin{Bmatrix} h & h & 2 \\ h & h & J_h \end{Bmatrix} \langle h || Q_2^r || h \rangle^2 - \delta_{J_h 0} G(h+\tfrac{1}{2}) \right] \right.$$

$$\left. - (-)^{p'+J} 2\chi \hat{J}'_h \hat{J}_h \begin{Bmatrix} J'_h & J_h & 2 \\ h & h & h \end{Bmatrix} \begin{Bmatrix} J'_h & J_h & 2 \\ p & p' & J \end{Bmatrix} \langle h || Q_2^r || h \rangle \langle p' || Q_2^r || p \rangle \right\}, \quad (10)$$

$$\langle h^{-1}, N'R'; IM | H | [(h^{-2})J_h, p]J, NR; IM \rangle = (-)^{R'-J+I+p-h} \begin{Bmatrix} h & J & 2 \\ R & R' & I \end{Bmatrix} \begin{Bmatrix} h & p & 2 \\ J & h & J_h \end{Bmatrix} \langle N'R' || \alpha_2 || NR \rangle \langle h || Q_2^k || p \rangle$$

$$- \chi(-)^{p-h} \{hRI\} \delta_{N'N} \delta_{R'R} \delta_{J_h} (\hat{J}_h/\hat{h}) \begin{Bmatrix} h & p & 2 \\ h & h & J_h \end{Bmatrix} \langle h || Q_2^r || h \rangle \langle h || Q_2^r || p \rangle, \quad (11)$$

$$\langle h^{-1}, N'R'; IM | H | h^{-1}, NR; IM \rangle = \delta_{N'N} \delta_{R'R} [-\epsilon(h) + N\hbar\omega] + (-)^{R'-h+I} \begin{Bmatrix} h & h & 2 \\ R & R' & I \end{Bmatrix} \langle N'R' || \alpha_2 || NR \rangle \langle h || Q_2^k || h \rangle. \quad (12)$$

As stated in the Introduction, anharmonic effects are included phenomenologically by allowing for a nonzero static quadrupole moment $Q(^12)$ of the $^12+$ state of the core. The corresponding matrix elements are obtained from those shown above just by adding a term proportional to

$$\delta_{N1} \delta_{NN'} Q(^12)$$

to the reduced matrix elements of the collective operator α_2. This treatment differs from others, such as that of Stewart, Castel, and Singh,[8] in the off-diagonal core matrix elements, which in the present case are still given by those of α_2. In this way, no extra parameters are needed in principle, since $Q(^12)$ may be taken from experiment.

The eigenstates of the model Hamiltonian, with total angular momentum I, are linear combinations of the base vectors (8) and (9):

$$|\tau IM\rangle = \sum_{NR} \eta(h, NR; \tau I) |h^{-1}, NR; IM\rangle$$

$$+ \sum_{\substack{J_h p \\ JNR}} \eta(J_h, p, J, NR; \tau I)$$

$$\times |[(h^{-2})J_h, p]J, NR; IM\rangle, \quad (13)$$

where the amplitudes η are obtained by diagonalizing the energy matrices, and the superindex $\tau = 1, 2, \ldots$ distinguishes between states of the same total angular momentum in order of increasing energy.

The spectroscopic factors for nucleon transfer reactions are calculated from the appropriate amplitudes η as indicated in Ref. 2.

B. Electromagnetic properties

The electromagnetic transition operators are a sum of a particle contribution and a collective one.

They are defined as usual (cf. Ref. 2) by the following.

Electromagnetic quadrupole operator:

$$T_{2\mu}^{s} = (5/4\pi)^{1/2} e^{\text{eff}} \sum_i Q_{2\mu}^r(i)$$

$$+ (3R_0^2/4\pi\alpha_0) e_v^{\text{eff}} \alpha_{2\mu} \quad \text{(in } e\text{ fm}^2) . \qquad (14)$$

where $Q_{2\mu}^r$ is given by Eq. (7), the summation over i corresponds to the components of the cluster, e^{eff} is the particle effective charge, $R_0 = 1.2 A^{1/3}$ fm the nuclear radius, $\alpha_0 = 5^{-1/2}\beta$ (with β the quadrupole deformation of the core) is the amplitude of the vibrations, and

$$e_v^{\text{eff}} = Ze\alpha_0 \qquad (15)$$

is the vibrator effective charge.

Magnetic dipole operator:

$$T_{1\mu}^{m} = (3/4\pi)^{1/2}[g_R(I_\mu - J_\mu) + g_l(J_\mu - S_\mu) + g_s^{\text{eff}} S_\mu]$$

$$\text{(in } \mu_N) , \qquad (16)$$

where $\vec{I}$, $\vec{J}$, and $\vec{S}$ are the (vector) operators of the nuclear angular momentum, total angular momentum, and spin of the cluster, respectively; the numbers g_R, g_l, and g_s^{eff} are the collective, orbital, and spin gyromagnetic ratios.

The reduced $E2$ and $M1$ transition probabilities, and the electric quadrupole Q and magnetic dipole μ moments of the states are then, respectively, given by

$$B(E2; I' \to I) = \hat{I}^2(Ae^{\text{eff}} + Be_v^{\text{eff}})^2 \quad \text{(in } e^2\text{ fm}^4); \qquad (17)$$

$$B(M1; I' \to I) = (3/4\pi)\hat{I}^2[(C - D)g_R + (D - E)g_l + Eg_s^{\text{eff}}]^2$$

$$\text{(in } \mu_N^2); \qquad (18)$$

$$Q = \delta_{I'I}\delta_{\tau'\tau}\left[\frac{16\pi}{5}\frac{I(2I+1)(2I-1)}{(I+1)(2I+3)}\right]^{1/2}(Ae_{\text{eff}} + Be_v^{\text{eff}})$$

$$\text{(in } e\text{ fm}^2) ; \qquad (19)$$

$$\mu = \delta_{I'I}\delta_{\tau'\tau}\left[\frac{I(2I+1)}{I+1}\right]^{1/2}[(C - D)g_R + (D - E)g_l + Eg_s^{\text{eff}}]$$

$$\text{(in } \mu_N) . \qquad (20)$$

The quantities A, B, C, D, and E are of the form

$$F = \sum_{\gamma'\gamma} \eta(\gamma')\eta(\gamma)F^* , \qquad (21)$$

where the amplitudes η are those of Eq. (19), with γ representing the corresponding quantum numbers, and the F^*'s are the matrix elements between the base states. Their expressions are

$$A^* = \delta_{N'N}\delta_{R'R}(-)^{J'+I+R}(5/4\pi)^{1/2}\begin{Bmatrix} J' & J & 2 \\ I & I' & R \end{Bmatrix}$$

$$\times \langle J' || Q_2^r || J \rangle ; \qquad (22)$$

$$B^* = \delta_{p'p}\delta_{J_h' J_h}\delta_{J'J}(-)^{J+I'+R}\frac{3}{4\pi\alpha_0}R_0^2\begin{Bmatrix} R' & R & 2 \\ I & I' & J \end{Bmatrix}$$

$$\times \langle N'R' || \alpha_2 || NR \rangle ; \qquad (23)$$

$$C^* = \delta_{p'p}\delta_{J_h' J_h}\delta_{J'J}\delta_{N'N}\delta_{R'R}\delta_{I'I}\delta_{\tau'\tau}\left[\frac{I(I+1)}{2I+1}\right]^{1/2} ; \qquad (24)$$

$$D^* = \delta_{p'p}\delta_{J_h' J_h}\delta_{J'J}\delta_{N'N}\delta_{R'R}(-)^{J'+I+R+1}$$

$$\times [J(J+1)(2J+1)]^{1/2}\begin{Bmatrix} J' & J & 1 \\ I & I' & R \end{Bmatrix} ; \qquad (25)$$

$$E^* = \delta_{N'N}\delta_{R'R}(-)^{J'+I+R+1}\begin{Bmatrix} J' & J & 1 \\ I & I' & R \end{Bmatrix}\langle J' || \vec{S} || J \rangle . \qquad (26)$$

In these equations, J' and J stand for the angular momentum of the cluster (either a hole or two holes and one particle), and the rest of the symbols have the same meaning as in Eqs. (10)–(12).

III. RESULTS AND DISCUSSION

In this section we give some details of the calculations, together with a critical comparison with the results of the GSMM.[2-5]

The space spanned by the base vectors (8) and (9) must be truncated for practical reasons. Where to truncate it, is mainly determined by physical considerations. In the first place, only the single-particle orbits immediately above the major closed shell are considered, since higher ones are expected to contribute negligibly. Of course, only single-particle orbits belonging to the same number of oscillator quanta are involved because of parity conservation. As regards the collective part, it must be noted that in cluster-phonon models, the phonon accounts for collective effects of the appropriate mixing of many-particle shell-model configurations. Which ones, it cannot be ascertained *a priori*, but in any case it is to be expected that an increase in the number of phonons brings about a mixing of higher particle configurations. Consequently, in order to be systematic, 2h-1p states must be coupled to a smaller number of phonons than the 1h state. We considered the hole state in Eq. (8) coupled to up to two phonons, and the particle states in Eq. (9), up to one phonon. Calculations were also performed for one more phonon in both states and it was found that, although the structure of the nuclear states was not substantially altered, their energy was exceedingly lowered. This may be interpreted as indicating that from a certain number of phonons upwards, inadequate high-lying particle configurations are

mixed. It is to be noted that even with the above restriction on the number of phonons, the allowed configurations for each spin more than doubled those appearing in the GSMM.

The influence of single-particle energies and of the parameters ρ, χ, and G was studied by adjusting two of them at a time in least-squares fits to the excitation energies of the lowest unambiguously determined levels. In a first series of fits, only the single-particle energies were varied, since they are not expected to affect the levels identified in the SMM and GSMM as belonging to the one-phonon multiplet. The first fit of this series included the variation of the hole energy, which is the most uncertain and is supposed to fix the relative position of the multiplet and the rest of the levels which, according to the GSMM, should be particlelike. In a second series of fits, at least one interaction-strength constant was included. At the end of each fit the first three levels of each spin up to the highest observed spin were calculated. The intermediate steps of the process of χ^2 minimization in each fit showed how the levels changed as functions of the two parameters being adjusted. A final simultaneous variation of all parameters in order to obtain the best fit was precluded because of the available computational facilities. Therefore, taking advantage of the information provided by the intermediate steps of each two-parameter fit, new ones were performed and the set of parameters yielding the lowest χ^2 was chosen.

The phonon energy was considered as the observed energy of the $^1 2+$ state of the $(A+1)$ core; also, the deformation parameter β, appearing in the reduced matrix elements of the collective operator α_2, was taken from experiment. The input values of the parameter ρ and of the single-particle energies were those of our previous works,[2-5] except that for the hole energy an estimate was obtained from binding-energy considerations. We chose as origin the energy of the first single-particle orbit above the major closed shell. For the strength constants G and χ of the residual interactions, we took as input values those commonly quoted in the literature for the respective mass regions (cf. Ref. 7).

In order to investigate the effect of anharmonicities (included phenomenologically as indicated in Sec. II), instead of taking the experimental values of the quadrupole moments $Q(^1 2)$ of the first excited state of the cores, which are reported with large relative errors, we have used $Q(^1 2)$, together with ρ, as free parameters in least-squares fits to energy levels, keeping the remaining parameters fixed at the values resulting from the fits without anharmonicities. The parameter ρ was chosen be-

cause it is the one affecting more strongly all levels. Since no substantial difference with the calculations without anharmonicities was obtained (see below), except in the splitting within the multiplet, we thought it unnecessary to perform further fits to adjust the rest of the parameters (which, in any case, do not affect the multiplet appreciably; see below).

Once the best set of parameters was chosen, the first three levels of each spin up to the highest observed spin were calculated; then those below a certain energy (3 MeV for ^{57}Co and 2 MeV for ^{115}In) were selected; and finally the state vectors were used to calculate spectroscopic factors, electromagnetic properties, branching ratios, and lifetimes, all in one program. We did not look for the best fit of the latter through variation of their specific parameters (effective charges and gyromagnetic ratios), although calculations were performed with the eight combinations of the sets of values indicated in Ref. 4.

In Sec. III A the results for ^{57}Co are described, and in Sec. III B, those for ^{115}In. Both nuclei were chosen as typical and well known representatives of $1f_{7/2}$ and $1g_{9/2}$ ones, respectively.

A. ^{57}Co nucleus

We first mention the following general features of the behavior of energy levels as functions of the parameters, without the inclusion of anharmonicities. The criterion used to distinguish between particlelike and phononlike levels is their fast or very slow change with single-particle energies. This criterion was confirmed *a posteriori* by the structure of the state vectors.
(i) The $\frac{11}{2}$ state, and the $\frac{3}{2}$ and $\frac{5}{2}$ ones not belonging to the one-phonon multiplet increase strongly with the decrease of either $\epsilon(\frac{7}{2})$, ρ, or χ.
(ii) Mainly as a consequence of this, the values of the parameters yielding reasonable fits appear to be rather critical. The best values of the energy $\epsilon(\frac{7}{2})$ of the hole state are of the order of the estimated one; the other single-particle energies are practically the same as those used in Ref. 4, i. e. as the observed level energies of ^{57}Ni. The parameter ρ must be reduced by 30% with respect to the value used in Ref. 4, whereas the strength χ of the Q-Q interaction must be half the value commonly quoted in the literature (cf. Ref. 7). This result was to be expected, since in the present model the total interaction of nucleons outside closed shells is represented by particle-vibration plus two-particle interactions, whereas in the GSMM only the former are considered and in shell-model-like calculations only the latter appear.
(iii) Within ranges wider than the critical one, the

levels of the one-phonon multiplet (which correspond to the observed $1\frac{9}{2}$, $1\frac{11}{2}$, $2\frac{3}{2}$, $2\frac{7}{2}$, $1\frac{5}{2}$) are not significantly affected by any of the parameters, which implies that the model cannot provide, without anharmonicities, the observed splitting; in particular, the relative position of the $\frac{3}{2}$ member is incorrect.

(iv) The main components of the state vectors are but slightly affected by the variation of the single-particle energies. The only dramatic exceptions are the first two $\frac{3}{2}$ states, one of them belonging to the multiplet, which cross each other at about $\epsilon(\frac{7}{2}) = -4$ Mev: both become more mixed in nature either as $\epsilon(\frac{7}{2})$ is shifted up or as $\epsilon(\frac{1}{2})$ is shifted down.

The resulting energy sequence of negative-parity states, as well as the corresponding spectroscopic factors for proton stripping reactions, are compared in Fig. 1 with the experimental ones[9] and with those predicted by the GSMM.[4] We comment first on the results without anharmonicities. The main components of the state vectors are listed in

Table I and the calculated electromagnetic properties, branching ratios, and lifetimes are compared in Table II with experimental data. It can be seen from Fig. 1 that the fit to energy levels and spectroscopic factors is better than in the GSMM; in particular, the correct order of the first two $\frac{3}{2}$ states, namely, the lower particlelike and the other collective, has been achieved (see also Table I), although the latter still lies too low. On the other hand, the total splitting of the one-phonon multiplet is narrow as compared with the experimental one and with that given by the GSMM. It appears then that the correlations (anharmonicities excluded) used in the present model cannot provide a satisfactory description of the multiplet. The predictions about electromagnetic properties, branching ratios, and lifetimes (Table II) are similar to those of GSMM,[4] except: (i) the $2\frac{3}{2}$ is now correctly described, and (ii) the quadrupole moments of the low-spin ($\frac{3}{2}$, $\frac{5}{2}$) members of the multiplet are substantially increased, whereas those of the high-spin ones and of the ground state

TABLE I. Components $\times 10^3$ of some low-lying negative-parity states in ^{57}Co calculated without anharmonicities. Each state is indicated by its ordering number and spin. The base vectors are defined in the text. Only components contributing more than 4% are listed.

$1\frac{7}{2}$		$1\frac{3}{2}$		$1\frac{9}{2}$	
$\lvert[0,f_{7/2}]\frac{7}{2},00\rangle$	910	$\lvert[0,p_{3/2}]\frac{3}{2},00\rangle$	−719	$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	899
$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	350	$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	444	$\lvert[0,f_{7/2}]\frac{7}{2},24\rangle$	331
		$\lvert[0,p_{3/2}]\frac{3}{2},12\rangle$	−231	$\lvert[0,f_{7/2}]\frac{7}{2},22\rangle$	−223
		$\lvert[2,p_{3/2}]\frac{7}{2},12\rangle$	203		

$1\frac{1}{2}$		$1\frac{11}{2}$		$2\frac{3}{2}$	
$\lvert[0,p_{1/2}]\frac{1}{2},00\rangle$	−605	$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	918	$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	790
$\lvert[2,p_{3/2}]\frac{1}{2},00\rangle$	454	$\lvert[0,f_{7/2}]\frac{7}{2},24\rangle$	332	$\lvert[0,f_{7/2}]\frac{7}{2},22\rangle$	328
$\lvert[0,p_{3/2}]\frac{3}{2},12\rangle$	358			$\lvert[0,p_{3/2}]\frac{3}{2},00\rangle$	301
$\lvert[2,f_{5/2}]\frac{1}{2},00\rangle$	−298			$\lvert[2,p_{3/2}]\frac{3}{2},00\rangle$	−288
$\lvert[0,f_{5/2}]\frac{5}{2},12\rangle$	268				
$\lvert[2,p_{1/2}]\frac{5}{2},12\rangle$	217				

$2\frac{7}{2}$		$1\frac{5}{2}$		$2\frac{5}{2}$	
$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	827	$\lvert[0,f_{7/2}]\frac{7}{2},12\rangle$	951	$\lvert[0,f_{5/2}]\frac{5}{2},00\rangle$	673
$\lvert[0,f_{7/2}]\frac{7}{2},00\rangle$	−333	$\lvert[0,f_{7/2}]\frac{7}{2},24\rangle$	224	$\lvert[2,p_{1/2}]\frac{5}{2},00\rangle$	303
$\lvert[0,f_{7/2}]\frac{7}{2},24\rangle$	285			$\lvert[0,p_{1/2}]\frac{1}{2},12\rangle$	−214
				$\lvert[2,p_{3/2}]\frac{5}{2},00\rangle$	240
				$\lvert[0,f_{5/2}]\frac{5}{2},12\rangle$	247
				$\lvert[2,f_{5/2}]\frac{9}{2},12\rangle$	−221
				$\lvert[2,f_{5/2}]\frac{5}{2},00\rangle$	−269

are decreased by about 30% with respect to the values predicted by the GSMM; only for the ground state is an experimental value available and it is correctly reproduced by the GSMM but not by the present treatment.

It is interesting to point out that the electromagnetic properties of the $1\frac{3}{2}$ level (and hence those of the $1\frac{1}{2}$ one) show a manifest discrepancy with experimental results, both in the GSMM and in the present treatment; also, the value of $B(E2; 1\frac{11}{2} \rightarrow 1\frac{9}{2})$ (and hence that of the corresponding branching ratio) predicted in both models is too low. Taking into account that 2h-1p configurations are treated in a

different way in both approaches, it may be concluded that additional configurations would be needed to get the correct nature of the $1\frac{3}{2}$ level and, to a lesser extent, of the $1\frac{9}{2}$ one.

The levels calculated with anharmonicities are shown in Fig. 2, for a value of the quadrupole moment of the $1{2+}$ state of ^{58}Ni compatible with the experimental one, $Q(1{2}) = (-12 \pm 13)\,e\,\text{fm}^2$ (Ref. 10). The only changes which deserve being mentioned are the downward shift of the $1\frac{9}{2}$ state and the opposite one of the $2\frac{3}{2}$ state, which implies a good prediction of the splitting within the one-phonon multiplet. It appears then that in the present treat-

TABLE II. Experimental and theoretical electromagnetic properties of some negative-parity states of ^{57}Co calculated without anharmonicities. Weisskopf units [A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I, p. 387.] are indicated W.u. The values of the proton effective charge and of the gyromagnetic ratios are $e^{\text{eff}} = e$; $g_R = 0$; $g_s^{\text{eff}} = 0.6 g_s^{\text{free}}$.

τ_I	Q (e b) Experiment[a]	Theory	μ (μ_N) Experiment[a]	Theory	τ (fs) Experiment[b]	Theory
$1\frac{7}{2}$	0.52 ± 0.09	0.289	4.733 ± 0.017	4.746		
$1\frac{3}{2}$		0.164	3.0 ± 0.6	2.896	$(19 \pm 4) \times 10^3$ [a]	3.4×10^3
$1\frac{9}{2}$		0.164		4.215	84 ± 18	174
$1\frac{1}{2}$				-0.064	$(866 \pm 72) \times 10^3$ [c]	5.2×10^3
$2\frac{3}{2}$		0.124		3.460	360 ± 70	663
$1\frac{11}{2}$		0.258		4.796	260^{+120}_{-90}	429
$2\frac{7}{2}$		0.081		3.906	120 ± 30	289
$1\frac{5}{2}$		0.111		3.508	30 ± 20	52

τ_{I_i}	τ'_{I_f}	$B(E2)$ (W.u.) Experiment[b]	Theory	$B(M1)$ (W.u.) Experiment[b]	Theory	δ Experiment[b,d]	Theory	b (%) Experiment[d]	Theory
$1\frac{3}{2}$	$1\frac{7}{2}$	0.45 ± 0.08 [e]	4.3					100	100
$1\frac{9}{2}$	$1\frac{7}{2}$	18 ± 4	17	0.19 ± 0.04	0.03	-0.27 ± 0.01	-0.68	100	100
$1\frac{1}{2}$	$1\frac{3}{2}$		5.6	0.02	1.21		0.01	100	100
$1\frac{11}{2}$	$1\frac{9}{2}$	39 ± 2	2.4	0.5 ± 0.2	0.15	-0.09 ± 0.01	-0.01	56 ± 3	1
$1\frac{11}{2}$	$1\frac{7}{2}$	10 ± 4	13					44 ± 3	99
$2\frac{3}{2}$	$1\frac{7}{2}$	10 ± 2	9.8					98	99.8
$2\frac{7}{2}$	$1\frac{9}{2}$	1 ± 1	2.4	0.6 ± 0.2	0.45	-0.02 ± 0.01	-0.01	60 ± 7	6.4
$2\frac{7}{2}$	$1\frac{7}{2}$		10	0.02 ± 0.01	0.006		1.61	40 ± 7	93.6
$2\frac{5}{2}$	$1\frac{3}{2}$		0.69		0.0007	$0.34^{+0.20}_{-0.11}$	0.40	14 ± 5	0.3
$2\frac{5}{2}$	$1\frac{7}{2}$		0.03		0.01	0.06 ± 0.07	-0.08	83 ± 4	99.4

[a] V. S. Shirley and C. H. Lederer, Lawrence Berkeley Laboratory, Report No. LBL-3450, 1974 (unpublished); Proceedings of the International Conference on Hyperfine Interactions Studied in Nuclear Reactions and Decay, Uppsala, Sweden, 1974 (unpublished).

[b] K. S. Burton and L. C. McIntyre, Jr., Phys. Rev. C **3**, 621 (1971), unless otherwise stated.

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[e] Calculated from experimental data assuming negligible total conversion coefficient.

ment of 2h-1p excitations, unlike that in the GSMM, anharmonicities are indispensable for the description of the low-energy spectrum of ^{57}Co. As regards the state-vector components, very slight changes are obtained except in the case of the two $\frac{3}{2}$ states: both become purer in nature through a 30% reduction of the component corresponding to the opposite nature of the respective state. Also, there is practically no change in the electromagnetic properties, branching ratios, and lifetimes, except for some improvement in the value of $B(E2; \frac{1}{2}\frac{3}{2} \rightarrow \text{g.s.})$, which does not remedy the manifest disagreement with the experimental one. Because of the above results of the calculations with anharmonicities, we thought it unnecessary to show the numerical values on state-vector components and electromagnetic properties.

B. ^{115}In nucleus

We first mention the following features of the behavior of energy levels as functions of the parameters, without anharmonicities. (The distinction **between** particlelike and phononlike levels is based on the same criterion as that used for ^{57}Co.)
(i) For a wide range of all the model parameters, the pair $\frac{1}{2}\frac{7}{2} - \frac{1}{2}\frac{5}{2}$ lies below the pair $\frac{1}{2}\frac{3}{2} - \frac{1}{2}\frac{11}{2}$, which is the reverse of the experimental order. The final values of the single-particle energies appear to be rather critical in this respect.
(ii) The levels of the one-phonon multiplet $(\frac{1}{2}\frac{11}{2}, \frac{2}{2}\frac{5}{2}, \frac{2}{2}\frac{9}{2}, \frac{1}{2}\frac{13}{2}, \text{ and } \frac{2}{2}\frac{7}{2})$ are not affected by the variation of

G and χ, whereas the particlelike levels $\frac{1}{2}\frac{3}{2}, \frac{1}{2}\frac{11}{2}, \frac{1}{2}\frac{7}{2}$, and $\frac{1}{2}\frac{15}{2}$ decrease when both parameters increase.
(iii) The particle levels decrease with increasing ρ, whereas the reverse holds true for those of the multiplet.
(iv) The members of the multiplet lie at a somewhat high energy whichever the choice of parameters.
(v) The main components of the state vectors are practically not affected by the variation of single-particle energies.

The best value of the energy $\epsilon(\frac{9}{2})$ of the hole state is very stable and of the order of magnitude of the estimation from binding energies. In order to reproduce the observed sequence of the levels $\frac{1}{2}\frac{3}{2}, \frac{1}{2}\frac{11}{2}, \frac{1}{2}\frac{7}{2}$, and $\frac{1}{2}\frac{15}{2}$, all the other single-particle energies had to be lowered with respect to the values used in Ref. 2, which implies that the main change is an upward shift of $\epsilon(\frac{7}{2})$; this was to be expected from the fact that the values of single-particle energies must be model-dependent in this case, owing to the large number of neutrons outside closed shells. Since only ρ affects the members of the multiplet noticeably, its value had to be halved with respect to that used in the GSMM^2 in order to bring them to the correct energy region. For the same reason, the phonon energy had to be taken slightly less than the observed energy of the $^{1}2+$ state of ^{116}Sn.

The resulting spectrum of positive-parity states below 1.5 MeV, as well as the corresponding spectroscopic factors, are compared in Fig. 2 with
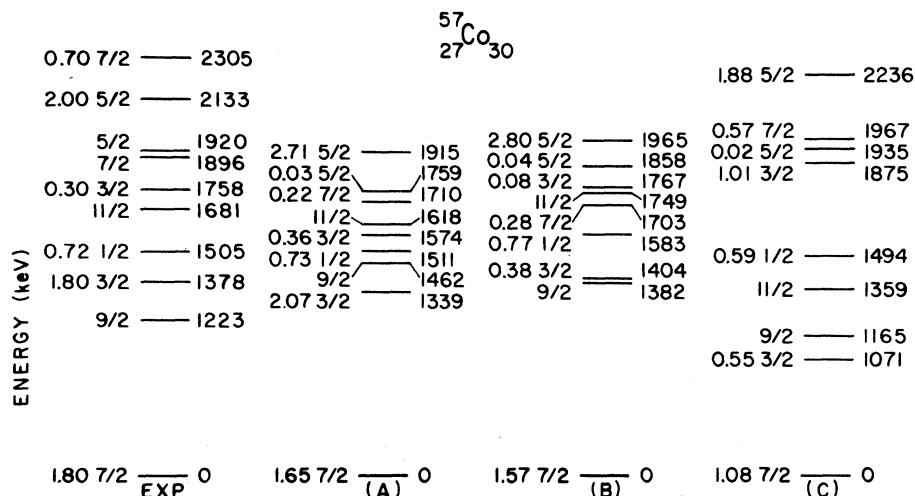
$$^{57}_{27}\text{Co}_{30}$$

FIG. 2. Experimental (Ref. 9) and calculated low-lying negative-parity states in ^{57}Co. The spectroscopic factors $(2I+1)S$ for the ^{56}Fe(^{3}He, d)^{57}Co reaction (Ref. 9) are indicated at the left of each level. (A): Present results without anharmonicities for $\hbar\omega(^{58}$Ni$) = 1.453$ MeV; $\beta(^{58}$Ni$) = 0.187$; $\rho = 0.5$; $G = 0.316$ MeV; $\chi = 0.007\,00$ MeV fm^{-4}; $\epsilon(f_{7/2}) = -3.86$ MeV; $\epsilon(p_{3/2}) = 0$; $\epsilon(f_{5/2}) = 0.75$ MeV; $\epsilon(p_{1/2}) = 1.15$ MeV. (B): Present results with anharmonicities for $\rho = 0.55$ and $Q(^12) = -17.5$ $e$fm^2; the other parameters are the same as in (A). (C): GSMM (Ref. 4) results.

experimental data[11] and with the predictions of the GSMM.[2] We comment first on the results without anharmonicities. The main components of the state vectors are listed in Table III and the calculated electromagnetic properties are compared with the observed ones in Table IV. (Owing to the lack of experimental data on branching ratios and lifetimes, the corresponding numerical results are omitted; they may be obtained on request from the authors.)

It may be seen that the states are purer than in the GSMM[3] in the sense that the multiplet is more one-phonon-like and the other levels are more particle-like. It appears that this is the reason for the exceedingly small splitting within the multiplet as compared with experiment and with the GSMM. On the other hand, the predictions about electromagnetic properties differ from those of the GSMM[2]:

(i) The calculated quadrupole moments of the ground and $^1\frac{3}{2}$ states are decreased, becoming definitely smaller than the observed ones. Also, the latter is predicted with the wrong sign (in fact, no negative quadrupole moments result from the present treatment)

(ii) The electric quadrupole transitions $^1\frac{1}{2} \rightarrow {}^1\frac{3}{2}$ and

TABLE III. Components $\times 10^3$ of some low-lying positive-parity states in ^{115}In calculated without anharmonicities. Each state is indicated by its ordering number and spin. The base vectors are defined in the text. Only amplitudes contributing more than 4% are listed.

$^1\frac{9}{2}$		$^1\frac{3}{2}$		$^1\frac{1}{2}$	
$\|[0,g_{9/2}]\frac{9}{2},00\rangle$	943	$\|[0,d_{3/2}]\frac{3}{2},00\rangle$	457	$\|[2,d_{5/2}]\frac{1}{2},00\rangle$	567
$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	260	$\|[2,g_{7/2}]\frac{3}{2},00\rangle$	433	$\|[0,s_{1/2}]\frac{1}{2},00\rangle$	538
		$\|[2,s_{1/2}]\frac{3}{2},00\rangle$	299	$\|[0,d_{5/2}]\frac{5}{2},12\rangle$	-325
		$\|[2,d_{5/2}]\frac{3}{2},00\rangle$	-290	$\|[2,d_{3/2}]\frac{1}{2},00\rangle$	-221
		$\|[2,d_{3/2}]\frac{3}{2},00\rangle$	-271		
		$\|[0,g_{7/2}]\frac{7}{2},12\rangle$	-254		
		$\|[4,d_{5/2}]\frac{3}{2},00\rangle$	210		

$^1\frac{7}{2}$		$^1\frac{5}{2}$		$^1\frac{11}{2}$	
$\|[0,g_{7/2}]\frac{7}{2},00\rangle$	667			$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	933
$\|[2,g_{7/2}]\frac{7}{2},00\rangle$	-369	$\|[0,d_{5/2}]\frac{5}{2},00\rangle$	663	$\|[0,g_{9/2}]\frac{9}{2},24\rangle$	269
$\|[2,d_{3/2}]\frac{7}{2},00\rangle$	315	$\|[2,d_{5/2}]\frac{5}{2},00\rangle$	-363		
$\|[0,g_{7/2}]\frac{7}{2},12\rangle$	239	$\|[2,s_{1/2}]\frac{5}{2},00\rangle$	333		
		$\|[0,d_{5/2}]\frac{5}{2},12\rangle$	235		
		$\|[2,d_{5/2}]\frac{9}{2},12\rangle$	-213		

$^2\frac{5}{2}$		$^2\frac{9}{2}$		$^1\frac{13}{2}$	
$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	937	$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	891	$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	945
$\|[0,g_{9/2}]\frac{9}{2},22\rangle$	272	$\|[0,g_{9/2}]\frac{9}{2},24\rangle$	236	$\|[0,g_{9/2}]\frac{9}{2},24\rangle$	246
		$\|[0,g_{9/2}]\frac{9}{2},00\rangle$	-235		

$^2\frac{7}{2}$		$^3\frac{9}{2}$		$^2\frac{11}{2}$	
$\|[0,g_{9/2}]\frac{9}{2},12\rangle$	965	$\|[2,d_{5/2}]\frac{9}{2},00\rangle$	655	$\|[2,g_{7/2}]\frac{11}{2},00\rangle$	694
		$\|[0,d_{5/2}]\frac{5}{2},12\rangle$	-322	$\|[0,g_{7/2}]\frac{7}{2},12\rangle$	-350
		$\|[4,d_{5/2}]\frac{9}{2},00\rangle$	-286	$\|[4,g_{7/2}]\frac{11}{2},00\rangle$	-295
		$\|[4,s_{1/2}]\frac{9}{2},00\rangle$	280	$\|[2,g_{7/2}]\frac{11}{2},12\rangle$	238
		$\|[2,d_{5/2}]\frac{9}{2},12\rangle$	221	$\|[4,d_{3/2}]\frac{11}{2},00\rangle$	229
		$\|[2,s_{1/2}]\frac{5}{2},12\rangle$	-216	$\|[2,d_{3/2}]\frac{7}{2},12\rangle$	-214
				$\|[2,g_{7/2}]\frac{7}{2},12\rangle$	211

$^1\frac{7}{2} \to {}^1\frac{3}{2}$ are now hindered, whereas the experimental ones are unexpectedly enhanced.

(iii) The $B(E2)$ values for the ground-state transitions from the members of the multiplet are systematically higher than those predicted by the GSMM and than the experimental ones. These results seem to indicate that the phonons of the ^{116}Sn vibrating core do not take proper account of the collective effects of the many-particle configurations of its neutrons above the $N = 50$ shell. If this is taken for granted, the better results of the GSMM are to be attributed to the addition of ^{114}Cd phonons.

The levels calculated with anharmonicities are shown in Fig. 3 for a value of the quadrupole mo-
ment of the $^1 2+$ state of ^{116}Sn a bit higher than the experimental one, $Q(^1 2) = (10 \pm 11)\ e\ \text{fm}^2$ (Ref. 12). The only change worth mentioning is a downward shift of the one-phonon multiplet keeping the total splitting too narrow. As regards the state-vector components, very small changes are obtained indicating a slight increase in the collectivity of all states. The same is true for the calculated electromagnetic properties. In view of this, we thought it unnecessary to show the corresponding numerical results.

IV. CONCLUDING REMARKS

As mentioned in the Introduction, the GSMM predicts that some low-lying states in $1g_{9/2}$

TABLE IV. Experimental and theoretical electromagnetic properties of some positive-parity states of ^{115}In calculated without anharmonicities. Weisskopf units [A. Bohr and B. R. Mottelson, *Nuclear Structure* (W. A. Benjamin, New York, Inc., 1969), Vol. I, p. 387.] are indicated W.u. The values of the proton effective charge and of the gyromagnetic ratios are $e^{\text{eff}} = 2e$; $g_R = Z/A$; $g_s = 0.6 g_s^{\text{free}}$.

τ_I	Q (e b) Experiment[b]	Theory	μ (μ_N) Experiment[a]	Theory
$^1\frac{9}{2}$	0.861	0.547	5.5408±0.0002	5.834
$^1\frac{3}{2}$	−0.62±0.08	0.392	0.80 ±0.14	0.672

τ_{I_i}	τ'_{I_f}	B(E2) (W.u.) Experiment[b]	Theory	B(M1) (W.u.) Experiment	Theory	δ Experiment	Theory
$^1\frac{1}{2}$	$^1\frac{3}{2}$	88±20[c]	9.4	$0.0045^{+0.0009}_{-0.0007}$[d]	0.0015	$-0.16^{+0.04}_{-0.05}$[e]	−0.15
$^1\frac{7}{2}$	$^1\frac{3}{2}$	55[f]	2.8				
$^1\frac{7}{2}$	$^1\frac{9}{2}$		0.002		0.0012	−0.23±0.04 / −0.06±0.02 }[e]	−0.04
$^1\frac{5}{2}$	$^1\frac{9}{2}$	1.35±0.15	0.27				
$^1\frac{11}{2}$	$^1\frac{9}{2}$	25.1 ±2.8	19.9	0.139±0.030[b]	0.014	0.54±0.08 / −0.47±0.08 }[f]	−1.70
$^2\frac{5}{2}$	$^1\frac{9}{2}$	11.3 ±1.0	16.3				
$^2\frac{9}{2}$	$^1\frac{9}{2}$	3.73±0.60	15.1		0.0006		7.59
$^1\frac{13}{2}$	$^1\frac{11}{2}$	0.22±0.27 / 3.4 ±1.0 }[g]	2.5	0.28 ±0.06[f]	0.146	0.04±0.02 / 0.157±0.006 }[f]	−0.011
$^1\frac{13}{2}$	$^1\frac{9}{2}$	10.8 ±1.1	15.5				
$^2\frac{7}{2}$	$^1\frac{9}{2}$	4.67±1.20	12.0		0.027		1.05
$^3\frac{9}{2}$	$^1\frac{7}{2}$		0.02		0.111	0.04±0.02 / 0.28±0.04 }[e]	0.02
$^3\frac{9}{2}$	$^1\frac{9}{2}$	1.81±0.45	0.0001		0.0000		0.39

[a] V. S. Shirley and C. M. Lederer, Lawrence Berkeley Laboratory, Report No. LBL-3450, 1974 (unpublished); Proceedings of the International Conference on Hyperfine Interactions Studied in Nuclear Reactions and Decay, Uppsala, Sweden, 1974 (unpublished).

[b] F. S. Dietrich, B. Herskind, R. A. Naumann, R. G. Stokstad, and G. E. Walker, Nucl. Phys. A A155, 209 (1970), unless otherwise stated.

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[e] Reference 3.

[f] Reference 11.

[g] Calculated from the experimental values of $B(M1)$ and δ.

nuclei[2,3] and $1f_{7/2}$ ones[4,5] are mainly due to 2h-1p excitations. Since these had been included in an approximate way, we have now treated them exactly within the framework of cluster-phonon models. This treatment required the explicit inclusion of residual two-body interactions between the members of the cluster, which have been taken to be of the Q-Q plus pairing type with adjustable strengths. A systematic study of the behavior of energy levels as functions of the parameters involved was performed, and the influence of anharmonicities in the core vibrations, which were not considered in the GSMM, was investigated phenomenologically by adding an interaction term yielding matrix elements diagonal in phonon number and proportional to the quadrupole moment of the $^1 2+$ state of the core. We think that this is the appropriate way of treating anharmonicities semimicroscopically in order to remain within the framework of a particle-*phonon* model. Other treatments, such as that of Stewart, Castel, and Singh,[8] are on the line of particle-*core* models which do not specify the nature of the core and thus do not allow a direct comparison with the GSMM, which is our purpose.

A good fit to energy levels has been achieved both with and without anharmonicities by adjusting single-particle energies and interaction strengths, with the following difference: Anharmonicities appear to be necessary to reproduce the observed one-phonon-multiplet splitting in ^{57}Co, whereas in ^{115}In they do not appreciably affect this splitting.

The predictions of the GSMM [2-5] about the origin of the lowest-lying levels in both nuclei have been confirmed: 2h-1p configurations are responsible for the appearance of the $^1\frac{3}{2}-$ and $^1\frac{1}{2}-$ states in ^{57}Co, and of the $^1\frac{3}{2}+$, $^1\frac{1}{2}+$, $^1\frac{7}{2}+$, and $^1\frac{5}{2}+$ states in ^{115}In, below the one-phonon multiplet arising from 1h configurations, which yield the lowest levels in the classical semimicroscopic model.[1] However, we point out that the failure of both the present model and the GSMM in predicting the precise nature of the $^1\frac{3}{2}$ state in ^{57}Co (as inferred from the calculated electromagnetic properties) seems to indicate that configurations other than 2h-1p ones are also needed for that nuclide.

As regards magnetic properties, it would be expected that by assuming an interaction of the form (2), they would not be correctly predicted because the collective motion is coupled only to the orbital motion of the particles. However, as in the GSMM, static magnetic dipole moments are properly reproduced and the predictions about magnetic dipole transitions are not worse than those about quadrupole electric ones; moreover, mixing ratios are predicted with the correct sign and in many cases with the correct magnitude too. It is to be noted that although the specific parameters of electromagnetic moments and transitions have not been varied in order to obtain the best fit, the calculations performed with different sets of them indicate that the discrepancies mentioned in Sec. III cannot be removed within the framework of the present model.
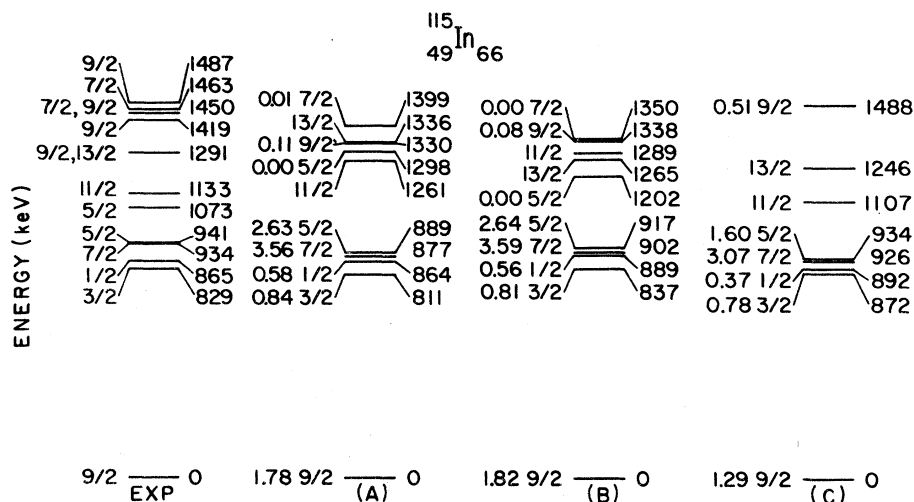

FIG. 3. Experimental (Ref. 11) and calculated low-lying positive-parity states in ^{115}In. The calculated spectroscopic factors $(2I+1)S$ for proton stripping reaction are indicated at the left of each level. (A): Present results without anharmonicities for $\hbar\omega$ (^{116}Sn) = 1.23 MeV; β(^{116}Sn) = 0.113; ρ = 0.52; G = 0.156 MeV; χ = 0.005 83 MeV fm^{-4}; ϵ ($g_{9/2}$) = −3.62 MeV; ϵ ($g_{7/2}$) = 0; ϵ ($d_{5/2}$) = 0.24 MeV; ϵ ($d_{3/2}$) = 1.63 MeV; ϵ ($s_{1/2}$) = 1.39 MeV. (B): Present results with anharmonicities for ρ = 0.46 and Q($^1 2$) = 25 $e$fm^2; the other parameters are the same as in (A). (C): GSMM (Ref. 2) results.

As was to be expected, the explicit introduction of residual interactions between the members of the cluster has the effect of reducing the strength of the particle-phonon interaction (as measured by the parameter p) with respect to that used in the GSMM. Therefore, since the latter interaction is the only one affecting all levels, there are differences in the predictions of both models about their energy position and their nature. Firstly, the quasirotational modes, which the GSMM suggests would coexist with the quasivibrational ones, do not appear in the present treatment; however, there are not enough experimental data to support either conclusion. Secondly, the GSMM somewhat exaggerates the splitting of the multiplet in ^{57}Co and is not able to yield the observed order of the first two $\frac{3}{2}$ states, which the present model does, providing also a better description of the nucleus. Finally, the GSMM appears to be more in accord with the physical situation in ^{115}In. The inclusion of anharmonicities, although strongly increasing the size of energy matrices, only improves the energy position of the members of the multiplet without affecting either the nature of all levels or their electromagnetic properties.

Finally, let us comment upon the remarkable difference in success of the present model when applied to ^{57}Co and ^{115}In, as compared with the similar success obtained with the GSMM for both and neighboring nuclei.[2-5] In the mass region around ^{57}Co, the core to which the nucleon cluster is coupled has only a few nucleons outside closed shells, whereas around ^{115}In the core has an approximately half-filled neutron shell. Therefore,

the collective effects of many particle configurations of the outer core nucleons are expected to be different in both mass regions. As stated previously, these collective effects are assumed in cluster-phonon models to be taken into account by the vibrations of the core. Now, the approximate treatment of 2h-1p excitations in the GSMM consists essentially in adding a second vibrator. It appears then that the two cores properly account for the relevant collective effects of many-particle configurations, including possible anharmonicities in the first vibrator, thus providing the GSMM with a more flexible adaptability to rather different situations. A less phenomenological treatment like the one we have described, would require, in the case of ^{115}In, the inclusion of other configurations, which does not seem feasible for computational reasons.

The important point to be emphasized is that the present more sophisticated approach demonstrates that the approximations involved in the GSMM are quite reasonable, rendering its results highly reliable. In order to elucidate these questions, a systematic study of different nuclei should be performed once enough experimental data for comparisons are available.

ACKNOWLEDGMENTS

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