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Pion elastic scattering from aligned targets*

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The differences in the interactions of π^+ and π^- particles with neutrons and protons can be used to investigate relative neutron and proton densities in nuclei. The use of aligned targets with π^+ and π^- beams presents considerable advantages for the extraction of neutron density information. As a framework for estimating pion scattering from aligned targets we use the Kisslinger optical potential including a deformed nucleon density distribution. The difference between aligned and unaligned differential cross sections (deformation effect) is found to be linear in β and allows for the extraction of neutron density shape information.

[NUCLEAR REACTIONS Scattering theory, $\pi^{\pm}$ elastic scattering from aligned targets. ^{165}Ho(π,π), $T_\pi = 100$–200 MeV; calculated $\sigma_{\text{aligned}}(\theta)$, $\sigma_{\text{unaligned}}(\theta)$.]

I. INTRODUCTION

It was suggested[1] some time ago that differences in the interactions of π^+ and π^- particles with neutrons and protons could be used to investigate relative neutron and proton densities in nuclei. A recent experimental measurement[2] using pion total-cross-section data obtained the experimental result that the neutron and proton rms radii are quite similar. This result agrees with data from other experimental techniques.[3] Very little information exists on the higher moments of the neutron density distribution. Analyses[4] of charge exchange inelastic scattering indicate a rather substantial difference in the neutron and proton deformation parameters $\beta(n)$ and $\beta(p)$, but the results are rather sensitive to the uncertain reaction analysis employed.

Cross-section measurements from aligned targets using conventional nuclear projectiles have been helpful in determining certain parameters of the optical potential. Fisher, Tabor, and Watson[5] have investigated proton scattering from an aligned ^{165}Ho target in order to obtain an estimate of the nuclear deformation parameter β. Their results were in reasonable agreement with the value $\beta = 0.33$ derived from Coulomb excitation measurements. In a subsequent experiment,[6] using α particles, a previous ambiguity which existed in α scattering from rare earth nuclei was resolved, thus establishing unique values for parameters of the optical potential.

The use of aligned targets in elastic scattering with π^+ and π^- beams presents considerable advantages for the extraction of neutron density information. The quadrupole interaction of the pion with the deformed target yields a scattering amplitude, dependent upon the angular momentum projection, which interferes with the usual spherical target scattering amplitude. Thus the aligned elastic cross sections contain a term *linear in* β which can be measured by comparing aligned and unaligned cross sections. Further comparison of π^+ and π^- data allows for the differentiation, in principle, of proton and neutron deformation effects. Present state-of-the-art techniques promise[7] a temperature of 0.1 K with an alignment $B_2/B_2(\max) = -0.48$ (compared with -0.5 for 0 K and 0 for an unaligned system) and perhaps 0.4% accuracy in relative cross sections. Such data should provide stringent tests on theories of pion-nucleus scattering and perhaps allow for some measurement of neutron density shapes.

The use of pions with aligned targets has some advantages[7] from an experimental standpoint in terms of target heating and beam handling. The ability to employ both π^+ and π^- beams is especially useful in the 100–200 MeV "resonance region" where a factor of almost 3 appears between the resonating π^+p (or π^-n) interactions compared with the π^-p (or π^+n) interactions. Thus a microscopic theory of π^+ scattering from a neutron rich nuclear target will yield a significant difference between the different charged pions. The effect is further enhanced by the Coulomb field which increases π^- cross sections relative to π^+ and provides for an additional Coulomb-nuclear interference region. These features cause substantial density-dependent effects which differ appreciably from the 4% black nucleus estimate.[8]

As a framework for estimating pion scattering from aligned targets we use a derived optical potential including a deformed nuclear density distribution.[9] The quadrupole effects are treated using the distorted-wave Born approximation (DWBA) which is found to be very accurate even for the large deformation ($\beta\sim 0.33$) appropriate for ^{165}Ho. Section II gives an outline of the formalism and Sec. III presents the results for $\pi^{\pm}$ elastic scattering from the ground state of ^{165}Ho. The reliability of the calculations is discussed in Sec. IV. Conclusions concerning the possibility of an experimental measurement of neutron density shapes are presented in Sec. V.

II. THEORY OF PION SCATTERING FROM DEFORMED NUCLEI

In order to obtain quantitative estimates of the scattering of pions from aligned targets we employ the Kisslinger optical potential[10] which is derivable from π-nucleon phase shifts using the Watson multiple-scattering formalism.[11] This potential gave good fits[12] to π^--^{12}C elastic scattering data[13] and a collective-model generalization[9] yielded 2^+ and 3^- inelastic scattering cross sections in agreement with the data without adjustable parameters (the extracted deformation parameters were found to be energy-independent and in agreement with those obtained by other techniques). It should be noted that other theories of elastic and inelastic pion-^{12}C scattering also give reasonable fits to the Binon data.[13] For example, a WKB-Glauber approach,[14,15] a local (i.e., Laplacian) model,[16] a deformed black-nucleus model,[17] and a field theoretic Low-equation approach[18] all work quite adequately in the region where the data exist. The use of aligned targets may provide a means of discriminating between these theories. The generalized Kisslinger potential which we use is convenient for treating spin and Coulomb effects and should give reasonable estimates of what structure information might be deducible from experiments using aligned nuclear targets.

We begin with the Kisslinger optical potential which for a spherical target is given by[10]

$$U(r)=\frac{(\hbar c)^2A}{2E}\{-b_0k^2\rho(r)+b_1\vec{\nabla}\cdot[\rho(r)\vec{\nabla}]\}, \qquad (1)$$

where A is the mass of the target, k and E are the lab momentum and total lab energy of the incident pion, respectively, $\rho(r)$ is the nucleon density normalized to unity, and b_0 and b_1 are directly related to the pion-nucleon phase shifts as shown in Ref. 19.

For a deformed target, the density in Eq. (1) is generalized to be functionally dependent upon the body-fixed polar angle θ':

$$\rho(r,\theta')=\rho_0\left[1+\exp\left(\frac{r-R(\theta')}{a}\right)\right]^{-1},$$

$$R(\theta')=R_0[1+\beta_2Y_0^2(\theta')] \qquad (2)$$

for a quadrupole deformation which in turn gives rise to a deformed Kisslinger potential[9]

$$U(r)=U^{(0)}(r)+U^{(1)}(r,\theta'), \qquad (3)$$

where $U^{(0)}$ is the usual spherical term as in Eq. (1) and

$$U^{(1)}=\beta_2\frac{(\hbar c)^2A}{2E}\{-b_0k^2F(r)Y_0^2(\theta')+b_1\vec{\nabla}\cdot[F(r)Y_0^2(\theta')\vec{\nabla}]\}, \qquad (4)$$

where

$$F(r)=R_0\frac{\partial\rho}{\partial R}\bigg|_{R=R_0}.$$

If the neutron and proton density distributions are not the same, one can easily generalize Eq. (4) by making the replacement

$$\beta_2AF(r)\to\beta_2(n)NF_n(r)+\beta_2(p)ZF_p(r). \qquad (5)$$

The nuclear target is described in the rotational model by its total angular momentum J, projection M along a space fixed axis (z axis), and projection K along the nuclear symmetry axis (z' axis). Because Eq. (4) contains no dependence on the azimuthal angle ϕ', the value of K does not change in the scattering process.

The degree of orientation of an ensemble of nuclei may be specified by a density matrix $\rho_{MM'}$ or, equivalently, by a set of statistical tensors[8] B_{IQ} defined by

$$B_{IQ}=(2I+1)^{1/2}\sum_{MM'}\rho_{MM'}(JIMQ\,|JM'|), \qquad (6)$$

where $0\le I\le 2J$. If the spin system has an axis of symmetry and this is chosen as the z axis, then only $Q=0$ survives in Eq. (6) and one speaks of polarization (odd I) and/or alignment (even I). For definitiveness we will consider the simple $I=2$ alignment in which case Eq. (6) simplifies to

$$B_2=\sqrt{5}\sum_M\rho_{MM}(J2M0\,|JM). \qquad (7)$$

The ratio $B_2/B_2(\max)$ is often called the nuclear alignment where $B_2(\max)=\sqrt{5}\,(J2J0\,|JJ)$ is obtained from Eq. (7) when $M=\pm J$. For the unaligned case, B_2 is obtained using $\rho_{MM}=(2J+1)^{-1}$ in Eq. (7) resulting in $[B_2/B_2(\max)]_{\text{unaligned}}=0$.

The transition amplitude for elastic scattering from the state with angular momentum projection M to the state M' is

$$T_{MM'}=T^{(0)}\delta_{MM'}+T^{(1)}_{MM'},$$

where $T^{(0)}$ is the usual elastic scattering transition amplitude obtained from the outgoing wave boundary condition to the spherical optical model potential scattering solution. The M-dependent part is written in the DWBA as

$$T^{(1)}_{MM'}=\int d^3r\chi_f^{(-)*}(\vec{k}_f,\vec{r})\,\langle JM'K|U^{(1)}(r,\theta')+V^{(1)}_{\rm Coul}(r,\theta')|JMK\rangle\chi_i^{(+)}(\vec{k}_i,\vec{r})\,, \tag{8}$$

where the Coulomb excitation part $V^{(1)}_{\rm Coul}(r,\theta')$ is obtained in a manner similar to $U^{(1)}(r,\theta')$ of Eq. (4) and is therefore also proportional to the proton deformation parameter $\beta_2(p)$. The distorted waves, $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are solutions to the Klein-Gordon equation (including a Coulomb potential) with appropriate outgoing or incoming boundary conditions. It is seen that $T^{(1)}_{MM'}$ depends on the nuclear quantum numbers through the matrix element

$$\langle JM'K|Y_0^{\,2}(\theta')|JMK\rangle=(J2M\mu|JM')(J2K0|JK)Y_\mu^{\,2}(\hat{r})\,,$$

where $\mu=M'-M$. Here $\hat{r}$ refers to the space-fixed set of axes and we have used the properties of the rotation matrices[20] to evaluate the integral.

Expanding the distorted waves in multipoles and performing some angular momentum algebra yields the expression

$$T_{MM'}=T^{(0)}\delta_{MM'}+\sqrt{5}\,(J2M\mu|JM')(J2K0|JK)T_{fi}(2,\mu)\,, \tag{9}$$

where $T_{fi}(L,M)$ is independent of J or K and is written as[9]

$$T_{fi}(L,\mu)=\frac{(\hbar c)^2A}{2E}\beta_L\frac{(4\pi)^{3/2}}{k_fk_i}\sum_{ll'}(i)^{l-l'}Y_\mu^{l'}(\hat{k}_f)Y_0^{l*}(\hat{k}_i)\left(\frac{2l+1}{2l'+1}\right)^{1/2}(Ll00|l'0)(Ll\mu0|l'\mu)$$
$$\times[-b_0k^2G_{\rm I}+b_1G_{\rm II}+b_1G_{\rm III}\Delta^L_{l',l}]+\text{Coulomb}\,. \tag{10}$$

The radial integrals are

$$G_{\rm I}=\int dr\,\chi_{l'}^{(-)}F\chi_l^{(+)}\,,$$

$$G_{\rm II}=\int dr\left(\frac{d\chi_{l'}^{(-)}}{dr}-\frac{\chi_{l'}^{(-)}}{r}\right)F\left(\frac{d\chi_l^{(+)}}{dr}-\frac{\chi_l^{(+)}}{r}\right),$$

$$G_{\rm III}=\int dr\frac{\chi_{l'}^{(-)}}{r}F\frac{\chi_l^{(+)}}{r}\,,$$

where $\chi_l^{(+)}$ ($\chi_l^{(-)}$) are the radial wave functions for partial wave l in the incident (or final) channel. The quantity $\Delta^L_{l',l}$ arises from the $\vec{\nabla}$ operator in the Kisslinger potential and is given by[9]

$$\Delta^L_{l',l}=\tfrac{1}{2}[L(L+1)-l'(l'+1)-l(l+1)]\,.$$

Using Eq. (9) we write

$$|T_{MM'}|^2\approx\left(|T^{(0)}|^2+\sqrt{5}\,(J2M0|JM)(J2K0|JK)\times\{2{\rm Re}[T^{(0)}T^*_{fi}(2,0)]\}\right)\delta_{MM'}\,, \tag{11}$$

where terms of order $(U^{(1)})^2$ have been ignored. These are negligible except near deep minima in the elastic cross section when $|T^{(0)}|^2$ is anomalously small.

The interpretation of experimental results is most straightforward if the symmetry axis of the nuclear spin system (often[5-7] the c-axis of a single ferromagnetic crystal) is chosen to be coincident with the incident beam direction. Then the density matrix is diagonal in M and the differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\left(\frac{2\pi}{\hbar c}\right)^4E^2\sum_{MM'}\rho_{MM}|T_{MM'}|^2\,. \tag{12}$$

Substituting Eq. (11) and using Eq. (7) yields

$$\frac{d\sigma}{d\Omega}=\left(\frac{2\pi}{\hbar c}\right)^4E^2\left(|T^{(0)}|^2+(J2K0|JK)\frac{B_2}{B_2(\max)}\left[\frac{5J(2J-1)}{(J+1)(2J+3)}\right]^{1/2}\{2\,{\rm Re}[T^{(0)}T^*_{fi}(2,0)]\}\right) \tag{13}$$

for the cross section in terms of the alignment $B_2/B_2(\max)$.

Experiments with aligned targets often measure the ratio

$$\frac{\Delta\sigma(\theta)}{\sigma(\theta)}=\frac{d\sigma/d\Omega(\text{aligned})-d\sigma/d\Omega(\text{unaligned})}{d\sigma/d\Omega(\text{unaligned})} \tag{14}$$

since the unaligned measurements may be conveniently made by warming the target. The quantity $\Delta\sigma/\sigma$ is sometimes called[8] the deformation effect. It may be compared for different energies, different particles (π^+ or π^-), or different neutron and proton density distributions. Using Eq. (13) and Eq. (7), we have the deformation effect

$$\frac{\Delta\sigma(\theta)}{\sigma(\theta)}\propto\frac{B_2/B_2(\max)\,{\rm Re}[T^{(0)}T^*_{fi}(2,0)]}{\sigma(\theta)}, \tag{15}$$

where $\sigma(\theta)$ denotes the unaligned differential cross section. Since $T^*_{fi}(2,0)$ is proportional to β, the sign of the deformation effect is directly related to the sign of β.

III. RESULTS FOR ^{165}Ho

The calculations to be discussed considered a ^{165}Ho target ($K=J=\frac{7}{2}$) whose density distribution was taken to be of Woods-Saxon form as in Eq. (2) with parameters $R_0=1.04A^{1/3}$ fm $=5.7$ fm, $a=0.605$ fm, and a Coulomb radius (uniform density) of $1.404A^{1/3}$ fm. The nuclear alignment was taken to be that of the proposed LAMPF experiment[7] $B_2/B_2(\max)=-0.48$.

The first results presented in Fig. 1 study the dependence of the various cross sections on the incident pion energy. In these calculations the neutron deformation parameter was taken to be the same as the proton one, i.e., $\beta(n)=\beta(p)=0.33$. From the figures one can see that the deformation effect $\Delta\sigma(\theta)/\sigma(\theta)$ behaves closely like the derivative (with respect to scattering angle) of the unaligned differential cross section. This follows from the fact that the first order amplitude $T_{fi}(2,0)$ in Eq. (15) is roughly proportional to the derivative of the main elastic amplitude $T^{(0)}$, as is evident from Eq. (4) for $U^{(1)}$. The same behavior was found by Davies *et al.*[21] who calculated the deformation effect in the elastic scattering of 43 MeV α particles from ^{55}Mn.

The π^- mesons are quite sensitive to the attractive Coulomb potential which "brings in" the wave function. As the energy approaches the resonance (195 MeV) the π^- mesons are further affected by the strong absorption at the nuclear surface which tends to further enhance the deformation effect.

The effect of the neutron density shape was studied by varying the neutron deformation parameter $\beta(n)$ for 150 MeV incident π^+ and π^- particles. From Fig. 2 it is seen that the sensitivity to $\beta(n)$ is most evident at those angles where the deformation effect is large, i.e., near minima in the differential cross section. Both theory and the data are uncertain at these angles; however, at more

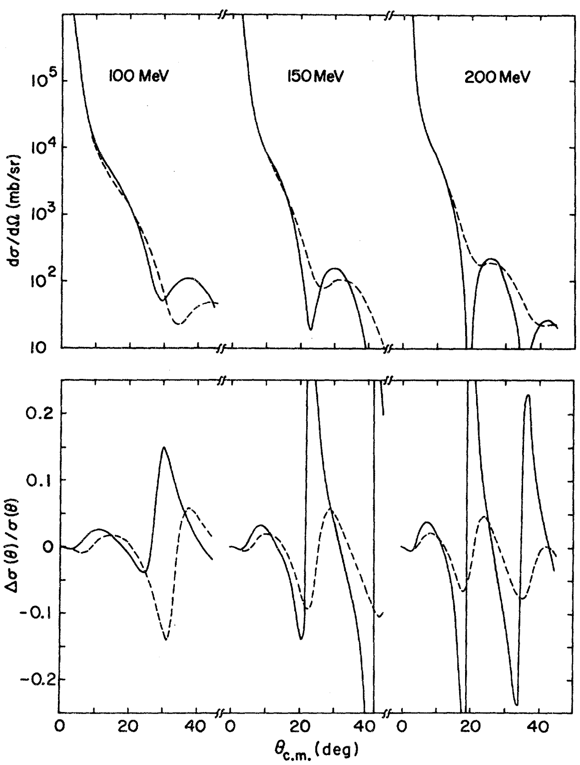

FIG. 1. Upper curve: The unaligned differential cross section for elastic scattering of pions from ^{165}Ho at incident pion energies 100, 150, and 200 MeV. The solid curve is for π^- mesons and the dashed curve for π^+. Lower curve: the corresponding deformation effect in the differential cross section as calculated from Eq. (15).

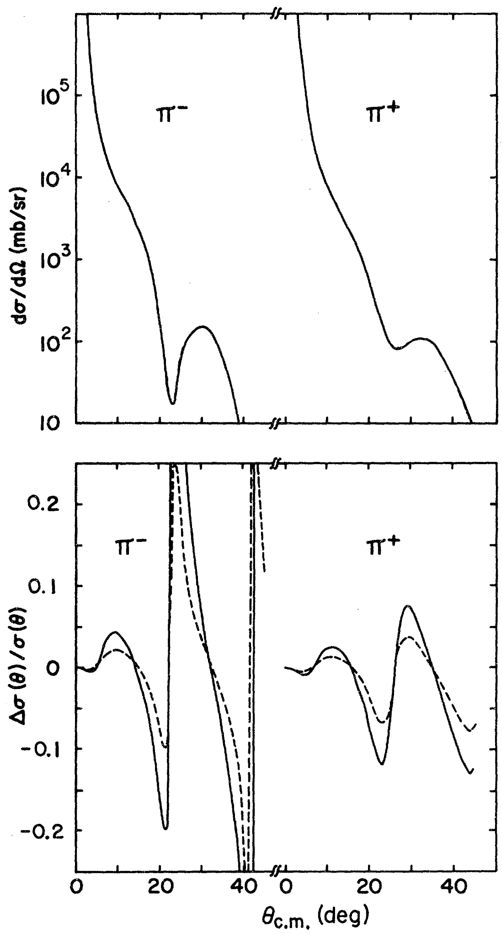

FIG. 2. Upper curve: the unaligned differential cross section for both π^- (on the left) and π^+ (on the right) particles. The incident energy was taken to be 150 MeV and the proton deformation parameter was $\beta_2(p)=0.33$. Lower curve: the corresponding deformation effect in the differential cross section (Eq. 15) for the cases $\beta_2(n)=0.51$ (solid curve) and $\beta_2(n)=0.15$ (dashed curve).

reasonable angles such as the first shoulder or second maximum (~15 and 30°), deformation effects of order 10% are predicted and these should be easily measurable.

The deformation effect in the integrated elastic cross section was calculated using a lower cutoff at 7° as appropriate for a typical experiment. (When the deformation effect in the total cross section is calculated as a function of the lower limit of scattering angle it is found to have a maximum for a lower cutoff angle near 7°. For angles much smaller than this, Coulomb scattering is so large that nuclear effects are completely invisible.) The results are shown in Fig. 3. A "black nucleus" calculation[8] estimated the deformation effect in the total cross section to be 4%, which is somewhat larger than our result. However, considerable information is available in the differential cross sections, as we have seen.

IV. RELIABILITY OF THE CALCULATIONS

A possible shortcoming of the formulation in Sec. II for pion scattering from aligned targets is the use of the DWBA in Eq. (8) to obtain the first-order M-dependent term. This is especially significant in that low energy neutron scattering shows[8] rather larger higher-order effects even for $\beta \approx 0.2$.

It is certainly possible to obtain the scattering amplitude T_M in a full coupled-channel treatment although this was awkward and time consuming. A simpler and more instructive procedure was to compare the two parts of Eq. (9) separately using

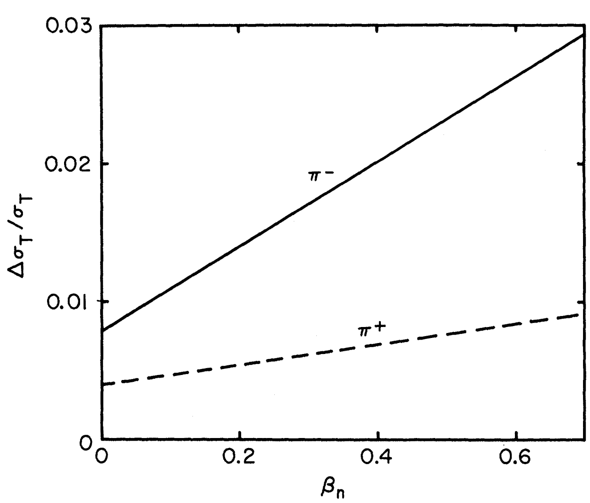

FIG. 3. The deformation effect in the total cross section as a function of $\beta_2(n)$ for π^- (solid curve) and π^+ (dashed curve) at 150 MeV. The total cross sections (aligned and unaligned) were calculated using a lower cutoff at 7°.

a coupled-channel calculation for a spin zero target with an artificially degenerate 2^+ excited state. The elastic scattering term is thus comparable to $|T^{(0)}|^2$ in Eq. (9) and was found to differ by $\lesssim 2\%$ from the spherical optical model approximation except for angles near deep minima. The artificial 2^+ inelastic scattering yields $\sum_M |T_{fi}(2,M)|^2$ which gives a measure of the accuracy of $T_{fi}(2,0)$ if one looks at several angles. These were found to agree with the DWBA approach [Eq. (10)] to about 8%. Adding quadratically as appropriate for $\Delta\sigma(\theta)/\sigma(\theta)$ in Eq. (15) gives an over-all error of about 5% due to the neglect of channel coupling. This is certainly small compared with nuclear projectile scattering and is traceable to the relative *weakness* of the pion-nucleus interaction as discussed elsewhere[22] and is a useful feature in analyzing such data.

The inelastic scattering calculations which we performed may also be used to estimate the effect of pions inelastically scattered by the target but unresolvable from the elastically scattered beam. The sum of the inelastic cross sections leading to states formed by coupling 2^+ and $\frac{7}{2}^-$ should be approximately that of a single inelastic $0^+ \to 2^+$ transition (i.e., the sum-rule result of a weak-coupling model). Furthermore, Coulomb excitation experiments[23] indicate that this multiplet dominates low-lying inelastic excitations and thus the $0^+ \to 2^+$ result should give a reasonable estimate of this source of "contamination." For the forward angular region $\theta \lesssim 20°$ we calculate about 10–30 mb/sr for maximum differential inelastic cross sections compared with the several b/sr from the elastic scattering. Thus for measurements of $\Delta\sigma_T/\sigma_T$ or forward angle measurements of $\Delta\sigma(\theta)/\sigma(\theta)$, the neglect of inelastic scattering is justified; at the more backward angles inelastic scattering may be a problem and should be included [at least in the denominator $\sigma(\theta)$ where it is a straightforward correction].

V. CONCLUSIONS

Higher moments of the neutron density distribution may be studied by looking at the deformation effect $\Delta\sigma/\sigma$ with aligned targets. Sample calculations for ^{165}Ho using rotational wave functions and a DWBA theory with the Kisslinger optical potential suggest a rather striking sensitivity to the model parameters which should allow for some differentiation of the many theories of pion-nucleus scattering. Our calculations give an angular dependence of the deformation effect which behaves roughly like the derivative of the unaligned cross section. Because of the relative weakness of the pion-nucleus interaction the effect of channel coupling is rather unimportant.

The π^- mesons are more sensitive to the neutron distribution because of both the pion-nucleon interaction and the Coulomb interaction. As a function of $\beta(n)$, the deformation effect in the π^- total cross section has a much steeper slope than the corresponding slope in the π^+ total cross section (Fig. 3). Thus the difference between π^+ and π^- deformation effects depends on the value of the neutron deformation parameter $\beta(n)$ and may provide a means of determining $\beta(n)$. The DWBA calculations also yield the result that the deformation effect is linearly dependent on β. This feature may be useful in some nuclides (not ^{165}Ho) where the sign of β is in question.

Finally, it is expected that with an experimental accuracy of a few percent in measuring the deformation effect in the differential cross sections, differences of the order of 20% in the neutron and proton deformations $\beta(n)$ and $\beta(p)$ should be readily discernible and significant tests of pion-nucleus scattering theories can be performed.

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