# Estimation of total reaction cross sections from elastic scattering data*

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A study is made of the interaction distance parameters from sharp and smooth cutoff analysis of elastic scattering data. These distance parameters are compared to interaction barrier parameters from analysis of total-reaction-cross-section data. A simple method is proposed for calculation of total reaction cross sections from the empirical parameters.

[NUCLEAR REACTIONS Systematics are given for the distance parameters from sharp and smooth cutoff analysis of elastic scattering data. A simple method is proposed for calculation of total reaction cross sections.]

## I. INTRODUCTION

There are three major kinds of experimental data from which various workers have extracted interaction barriers between complex nuclei[1-3]: (1) elastic scattering, (2) total reaction cross sections, and (3) inelastic scattering and simple transfer reactions. Early studies of elastic scattering were often characterized by one parameter, the so-called "strong interaction radius" in the sharp cutoff model.[4] Then for many years data were fitted to six-parameter optical potentials, simplified by arbitrary assumptions relating the form factors for the real and imaginary potentials.[5]

In recent years the weight of evidence has pointed toward very small absorptive potentials in the nuclear surface.[6,7] If this situation obtains, then the shape of the real potential dominates both the elastic scattering and the total reaction cross sections. With this assumption Wong[8] has developed a simple expression for the analysis of total reaction cross sections, and Vaz and Alexander[9] have systematized the parameters extracted from experimental data. Similarly, Berry,[10] Broglia and Winther,[11] and Da Silviera and Leclercq-Willian[12] have developed certain equations to describe elastic scattering for absorption confined to the nuclear interior. A systematic search of both elastic scattering and reaction cross sections for consistent potential parameters is now in progress.[13] This effort is difficult and can be expected to be quite time consuming.

In this work we develop purely empirical relationships between the "strong interaction radii" from elastic scattering and barrier heights from total reaction cross sections. Elastic scattering measurements are generally much easier to make than those for total reaction cross sections. Therefore the body of data from this source is more extensive. In order to estimate reaction cross sections for new systems it seems to us that, at present, semiempirical correlations of experimental data provide the best guides.

## II. RADIUS PARAMETERS FROM THE SHARP AND SMOOTH CUTOFF ANALYSES

The elastic scattering amplitude $f(\theta)$ depends on the phase shifts $\delta_l$ through the expression

$$f(\theta)=(\lambda/2i)\sum_{l=0}^{\infty}(2l+1)(e^{2i\delta_l}-1)P_l(\cos\theta), \quad (1)$$

with partial wave amplitudes $S_l$

$$S_l=e^{2i\delta_l}. \quad (2)$$

The differential elastic scattering cross section $\sigma_{el}$ is, of course, $|f(\theta)|^2$.

The sharp cutoff parametrization of the amplitudes $S_l$ is as follows:

$$S_l=0 \quad \text{for } l<l_{\max}, \quad (3a)$$

$$S_l=e^{2i\sigma_l} \quad \text{for } l>l_{\max}, \quad (3b)$$

where $\sigma_l$ is the phase shift for Coulomb scattering by point charges. Blair and Frahn have shown the special significance of the angle $(\theta_{1/4})$ and energy $(E_{1/4})$, where the observed elastic scattering is $\frac{1}{4}$ of that for pure Rutherford scattering.[4] In this parametrization the strong interaction distance $D_{1/4}$ (and $l_{\max}$) can be obtained from the classical equations for the turning point:

$$D_{1/4}=(Zze^2/2E_{1/4})[1+\csc(\tfrac{1}{2}\theta_{1/4})], \quad (4)$$

$$l_{\max}=\eta\cot(\tfrac{1}{2}\theta_{1/4}), \quad (5)$$

where $\eta$ is the Sommerfeld parameter $Zze^2/\hbar v$, and $Z$ and $z$ are atomic numbers of the collision partners.

The particular smooth cutoff parametrization proposed by McIntyre, Wang, and Becker[14] takes $f(\theta)$ as a sum

$$f(\theta)=f_c(\theta)+(\chi/2i)\sum_{l=0}^{\infty}(2l+1)e^{2i\sigma_l}(1-A_l e^{2i\delta_l}) \times P_l(\cos\theta), \quad (6)$$

with $f_c(\theta)$ for pure Coulomb scattering. The amplitude and real phase shifts are $A_l$ and $\delta_l$:

$$A_l=\{1+\exp[-(l-l_A)/\Delta l_A]\}^{-1}, \quad (7)$$

$$\delta_l=\delta_0\{1+\exp[(l-l_\delta)/\Delta l_\delta]\}^{-1}. \quad (8)$$

In this parametrization the critical angular momentum $l_A$ corresponds to that particular (interpolated) partial wave amplitude with a value of $\frac{1}{2}$. The interaction distance $D$ is then given by the semiclassical equation

$$\chi l_A(l_A+1)=D[(D/\chi)-2\eta]. \quad (9)$$

From the interaction distances $D_{1/4}$ (Eq. 4) or $D$ (Eq. 9) one may define radius parameters $r_{0B}$ or $r_{0M}$

$$D_{1/4}=r_{0B}(A_t^{1/3}+A_p^{1/3}), \quad (10)$$

$$D=r_{0M}(A_t^{1/3}+A_p^{1/3}), \quad (11)$$

where subscripts $t$ and $p$ denote target and projectile. These parameters are obtained by fitting either Eqs. 1–3 or 6–8 to measured elastic scattering data. Equations 4, 5, and 9 do not take account of deflections due to the nuclear potential. Therefore, the parameters $r_{0B}$, $r_{0M}$, $l_{\max}$ and $l_A$ can only be considered as empirical benchmarks for the observed elastic scattering. [Note that Eqs. 4 and 5 are purely classical while Eq. 9 has adopted $l(l+1)$ for the classical $l^2$. As most workers have used these relationships, we do also, even though some small inconsistencies result.]

## III. SYSTEMATICS OF THE RADIUS PARAMETERS AND A PRESCRIPTION FOR ESTIMATION OF THE TOTAL REACTION CROSS SECTION

The purpose of this study is to develop a simple means of estimating the total reaction cross section $\sigma_R$ from experimentally determined parameters. Let us review the equations developed for this purpose by Wong[8] and the systematic behavior of the interaction barrier parameters.[9] The interaction barrier is taken as an inverted parabola with height $E_0$, curvature $\hbar\omega_0$, and radial extent $R_0$. A uniform spectrum of barrier heights from $\overline{E}_0-\Delta$ to $\overline{E}_0+\Delta$ was added to cover the possible effects of static or dynamic deformations and/or vibrations.[9,15] Best-fit values of $\overline{E}_0$, $R_0$, $\hbar\omega_0$, and $\Delta$ were determined from the experimental total reaction cross sections.[9] For the energy region $\overline{E}_0 \lesssim E \lesssim 2\overline{E}_0$ the most sensitive parameter is the average barrier height $\overline{E}_0$. The quantity $\overline{E}_0$ was cast in terms of the distances $R_e$ or $r_e$, where

$$\overline{E}_0=Zze^2/R_e \quad (12)$$

and

$$R_e=r_e(A_t^{1/3}+A_p^{1/3}). \quad (13)$$

In Fig. 1 the values of $r_e$ are shown as determined in Ref. 9. The straight line was obtained by a least squares fit:

$$r_e=1.951-0.164\log_{10}(Zz). \quad (14)$$

In order to compare the pattern in Fig. 1 with studies of elastic scattering we have collected the reported values of $r_{0B}$ and $r_{0M}$ for systems with large $\eta$. The values are given in Tables I and II along with other pertinent experimental

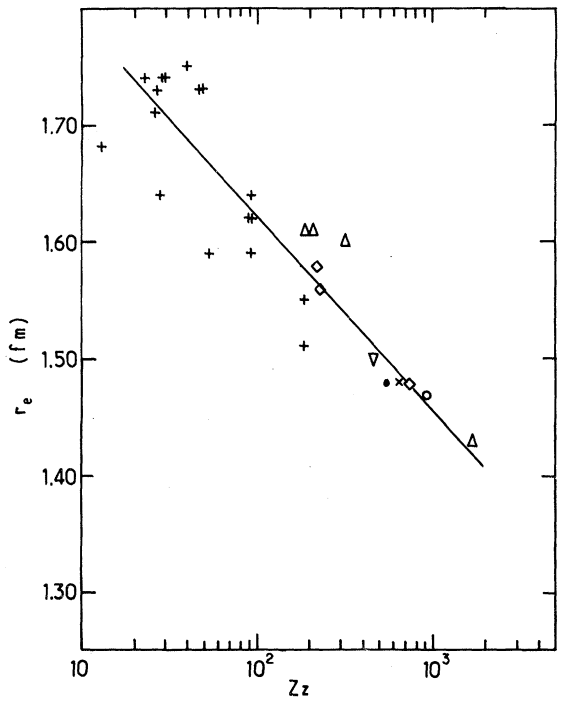


FIG. 1. Interaction barrier parameter $r_e$ from total reaction cross sections vs $Zz$ product [after Ref. 9, with additional points from J. C. Wells, R. L. Robinson, H. J. Kim, and J. L. C. Ford (unpublished)]. Symbols denote the projectiles as follows: +, $Z \leq 6$; ▽, $^{10,11}$B; ●, $^{12,13}$C; ×, $^{14}$N; ◇, $^{16,18}$O; o, $^{20,22}$Ne; △, $^{32}$S, $^{40}$Ar; ▼, $^{56}$Fe; □, $^{84}$Kr. The line is a least squares fit, Eq. 14.

TABLE I. Interaction distance parameters $r_{0B}$ from sharp cutoff analysis. Values of $E_{1/4}$ and $\theta_{1/4}$ were taken from the original references, and Eqs. 4 and 10 were used to obtain $r_{0B}$. Only those reactions with $\eta \geq 10$ are included.

| Target | $E_{1/4}$ (lab) (MeV) | $\theta_{1/4}$ (deg) | $r_{0B}$ (fm) | Ref. |
|---|---|---|---|---|
| | | $^{11}$B | | |
| $^{208}$Pb | 72.2 | 67.1 | 1.48 | 45 |
| | | $^{12}$C | | |
| $^{58}$Ni | 42.0 | 69.0 | 1.56 | 42 |
| $^{96}$Zr | 38.0 | 130.0 | 1.57 | 19 |
| $^{144}$Nd | 118.0 | 34.0 | 1.40 | 20 |
| $^{146}$Nd | 118.0 | 34.0 | 1.39 | 20 |
| $^{152}$Sm | 118.0 | 36.0 | 1.36 | 20 |
| $^{154}$Sm | 118.0 | 36.0 | 1.36 | 20 |
| $^{181}$Ta | 124.6 | 37.6 | 1.39 | 21 |
| $^{197}$Au | 124.0 | 40.0 | 1.41 | 22 |
| $^{206}$Pb | 124.2 | 40.0 | 1.45 | 23 |
| $^{207}$Pb | 123.7 | 40.0 | 1.45 | 23 |
| $^{208}$Pb | 116.4 | 42.0 | 1.49 | 52 |
| $^{208}$Pb | 122.8 | 40.0 | 1.46 | 23 |
| $^{208}$Pb | 118.0 | 43.0 | 1.44 | 20 |
| $^{208}$Pb | 125.0 | 38.0 | 1.49 | 21 |
| $^{209}$Bi | 118.0 | 45.0 | 1.41 | 20 |
| $^{209}$Bi | 121.4 | 41.6 | 1.45 | 22 |
| | | $^{13}$C | | |
| $^{40}$Ca | 40.0 | 54.5 | 1.58 | 7 |
| $^{40}$Ca | 48.0 | 41.0 | 1.59 | 7 |
| $^{94}$Mo | 51.0 | 75.5 | 1.55 | 24 |
| $^{96}$Mo | 54.5 | 67.0 | 1.53 | 24 |
| | | $^{14}$N | | |
| $^{48}$Ca | 50.0 | 43.7 | 1.59 | 25 |
| $^{103}$Rh | 81.0 | 54.0 | 1.44 | 26 |
| $^{103}$Rh | 121.0 | 29.0 | 1.42 | 26 |
| $^{197}$Au | 145.0 | 38.0 | 1.46 | 22 |
| $^{206}$Pb | 146.8 | 40.0 | 1.42 | 23 |
| $^{208}$Pb | 147.0 | 37.9 | 1.47 | 21 |
| $^{208}$Pb | 147.2 | 40.0 | 1.41 | 23 |
| $^{209}$Bi | 145.5 | 40.5 | 1.43 | 22 |
| | | $^{16}$O | | |
| $^{26}$Mg | 45.0 | 47.0 | 1.59 | 27 |
| $^{28}$Si | 33.0 | 98.0 | 1.61 | 28 |
| $^{28}$Si | 36.0 | 83.0 | 1.59 | 28 |
| $^{28}$Si | 38.0 | 75.0 | 1.59 | 28 |
| $^{40}$Ca | 33.6 | 177.0 | 1.62 | 29 |
| $^{40}$Ca | 40.0 | 93.0 | 1.62 | 29 |
| $^{40}$Ca | 60.0 | 48.0 | 1.57 | 19 |
| $^{40}$Ca | 34.7 | 144.1 | 1.60 | 39 |
| $^{40}$Ca | 36.7 | 123.2 | 1.58 | 39 |
| $^{40}$Ca | 38.0 | 116.7 | 1.55 | 39 |
| $^{40}$Ca | 40.0 | 94.4 | 1.60 | 39 |
| $^{40}$Ca | 41.4 | 90.0 | 1.58 | 39 |
| $^{40}$Ca | 42.0 | 85.6 | 1.60 | 39 |
| $^{48}$Ca | 60.0 | 42.0 | 1.58 | 19 |
| $^{48}$Ca | 31.5 | 177.0 | 1.59 | 29 |
| $^{48}$Ca | 40.0 | 80.0 | 1.60 | 29 |
| $^{48}$Ca | 32.0 | 147.0 | 1.59 | 30 |
| $^{48}$Ca | 33.1 | 128.0 | 1.60 | 30 |
| $^{48}$Ca | 35.1 | 109.0 | 1.59 | 30 |
| $^{48}$Ca | 38.0 | 88.0 | 1.60 | 30 |
| $^{48}$Ti | 42.0 | 90.0 | 1.58 | 31 |
| $^{48}$Ti | 48.0 | 73.0 | 1.53 | 31 |
| $^{50}$Ti | 60.0 | 47.0 | 1.58 | 19 |
| $^{52}$Cr | 35.1 | 177.3 | 1.65 | 32 |
| $^{52}$Cr | 60.0 | 52.0 | 1.58 | 19 |
| $^{54}$Fe | 46.0 | 96.3 | 1.57 | 42 |
| $^{54}$Fe | 48.0 | 87.9 | 1.57 | 42 |
| $^{54}$Fe | 52.0 | 74.5 | 1.57 | 42 |
| $^{54}$Fe | 60.0 | 57.0 | 1.59 | 19 |
| $^{58}$Ni | 41.1 | 176.0 | 1.57 | 40 |
| $^{58}$Ni | 44.0 | 126.0 | 1.55 | 40 |
| $^{58}$Ni | 50.0 | 90.0 | 1.56 | 40 |
| $^{58}$Ni | 55.4 | 74.0 | 1.55 | 40 |
| $^{58}$Ni | 60.0 | 63.0 | 1.56 | 40 |
| $^{58}$Ni | 60.0 | 62.0 | 1.58 | 19 |
| $^{60}$Ni | 60.0 | 62.0 | 1.56 | 19 |
| $^{62}$Ni | 40.0 | 172.6 | 1.57 | 32 |
| $^{62}$Ni | 40.4 | 157.5 | 1.57 | 32 |
| $^{62}$Ni | 60.0 | 60.0 | 1.57 | 19 |
| $^{86}$Sr | 60.0 | 84.0 | 1.56 | 19 |
| $^{88}$Sr | 48.2 | 176.0 | 1.54 | 33 |
| $^{88}$Sr | 52.0 | 119.0 | 1.54 | 33 |
| $^{88}$Sr | 60.0 | 82.0 | 1.56 | 19 |
| $^{88}$Sr | 48.0 | 176.0 | 1.55 | 40 |
| $^{88}$Sr | 52.0 | 120.0 | 1.54 | 40 |
| $^{88}$Sr | 55.0 | 100.3 | 1.55 | 40 |
| $^{88}$Sr | 59.5 | 85.1 | 1.55 | 40 |
| $^{88}$Sr | 60.0 | 83.0 | 1.55 | 40 |
| $^{92}$Zr | 60.0 | 88.0 | 1.56 | 19 |
| $^{96}$Zr | 49.0 | 160.0 | 1.56 | 19 |
| $^{96}$Zr | 60.0 | 85.0 | 1.57 | 19 |
| $^{92}$Mo | 60.0 | 98.0 | 1.56 | 19 |
| $^{116}$Sn | 66.0 | 102.0 | 1.54 | 34 |
| $^{120}$Sn | 65.7 | 100.0 | 1.54 | 34 |
| $^{142}$Nd | 65.5 | 176.0 | 1.36 | 40 |
| $^{144}$Nd | 130.0 | 46.0 | 1.36 | 20 |
| $^{146}$Nd | 130.0 | 42.0 | 1.44 | 20 |
| $^{148}$Nd | 130.0 | 41.0 | 1.46 | 20 |
| $^{152}$Sm | 130.0 | 46.0 | 1.38 | 20 |
| $^{154}$Sm | 130.0 | 43.0 | 1.44 | 20 |
| $^{197}$Au | 164.0 | 38.0 | 1.47 | 22 |
| $^{206}$Pb | 130.0 | 57.0 | 1.44 | 20 |
| $^{206}$Pb | 167.1 | 40.0 | 1.42 | 23 |
| $^{207}$Pb | 166.1 | 40.0 | 1.43 | 23 |
| $^{208}$Pb | 82.0 | 145.5 | 1.51 | 46 |
| $^{208}$Pb | 104.0 | 79.8 | 1.48 | 41 |
| $^{208}$Pb | 130.0 | 56.0 | 1.45 | 20 |
| $^{208}$Pb | 166.0 | 40.0 | 1.42 | 23 |
| $^{208}$Pb | 170.0 | 37.0 | 1.47 | 21 |
| $^{209}$Bi | 164.1 | 40.0 | 1.46 | 22 |
| $^{209}$Bi | 170.0 | 37.5 | 1.47 | 21 |
| | | $^{18}$O | | |
| $^{28}$Si | 36.0 | 86.0 | 1.60 | 28 |
| $^{52}$Cr | 37.1 | 177.4 | 1.58 | 32 |
| $^{58}$Ni | 63.4 | 58.5 | 1.56 | 35 |

TABLE I *(Continued)*

| Target | $E_{1/4}$ (lab) (MeV) | $\theta_{1/4}$ (deg) | $r_{0B}$ (fm) | Ref. | Target | $E_{1/4}$ (lab) (MeV) | $\theta_{1/4}$ (deg) | $r_{0B}$ (fm) | Ref. |
|---|---|---|---|---|---|---|---|---|---|
| | | $^{18}$O | | | | | $^{22}$S | | |
| $^{62}$Ni | 40.5 | 172.9 | 1.56 | 32 | $^{27}$Al | 85.0 | 84.2 | 1.55 | 36 |
| $^{62}$Ni | 40.9 | 158.4 | 1.56 | 32 | $^{27}$Al | 110.0 | 55.0 | 1.53 | 36 |
| $^{116}$Sn | 67.0 | 96.0 | 1.55 | 34 | $^{40}$Ca | 82.5 | 135.0 | 1.59 | 36 |
| $^{120}$Sn | 66.7 | 95.0 | 1.55 | 34 | $^{40}$Ca | 85.0 | 120.0 | 1.59 | 36 |
| $^{124}$Sn | 60.0 | 127.5 | 1.53 | 47 | $^{40}$Ca | 90.0 | 102.0 | 1.59 | 36 |
| | | $^{20}$Ne | | | | | $^{40}$Ar | | |
| $^{197}$Au | 209.6 | 39.0 | 1.40 | 22 | $^{nat}$Se | 146.0 | 89.0 | 1.42 | 26 |
| $^{206}$Pb | 207.2 | 40.0 | 1.42 | 23 | $^{nat}$Se | 201.0 | 50.0 | 1.43 | 26 |
| $^{207}$Pb | 208.1 | 40.0 | 1.42 | 23 | $^{209}$Bi | 288.0 | 61.0 | 1.41 | 51 |
| $^{208}$Pb | 206.2 | 40.0 | 1.43 | 23 | $^{238}$U | 208.0 | 167.5 | 1.40 | 49 |
| $^{209}$Bi | 207.6 | 40.0 | 1.43 | 22 | | | $^{56}$Fe | | |
| | | $^{22}$Ne | | | $^{116}$Sn | 219.0 | 164.0 | 1.46 | 50 |
| $^{88}$Sr | 61.8 | 176.2 | 1.53 | 48 | $^{124}$Sn | 220.0 | 163.0 | 1.41 | 50 |
| $^{88}$Sr | 65.4 | 127.7 | 1.53 | 48 | | | $^{84}$Kr | | |
| | | $^{22}$S | | | $^{181}$Ta | 457.0 | 112.0 | 1.33 | 37 |
| $^{24}$Mg | 75.0 | 110.0 | 1.58 | 36 | $^{197}$Au | 457.0 | 131.0 | 1.32 | 37 |
| $^{24}$Mg | 90.0 | 80.0 | 1.52 | 36 | $^{208}$Pb | 500.0 | 101.0 | 1.33 | 37 |
| $^{24}$Mg | 110.0 | 54.8 | 1.54 | 36 | $^{209}$Bi | 605.0 | 67.0 | 1.35 | 51 |
| $^{24}$Mg | 120.0 | 46.0 | 1.58 | 36 | $^{232}$Th | 500.0 | 125.0 | 1.29 | 37 |
| $^{27}$Al | 73.0 | 115.0 | 1.59 | 36 | $^{238}$U | 455.9 | 168.0 | 1.34 | 38 |

TABLE II. Interaction distance parameters $r_{0M}$ from smooth cutoff analysis. Values of $l_A$ were taken from the original references, and Eqs. 9 and 14 were used to obtain $r_{0M}$. Only those reactions with $\eta \geq 10$ are included.

| Target | $E$ (lab) (MeV) | $l_A$ | $r_{0M}$ (fm) | Ref. | Target | $E$ (lab) (MeV) | $l_A$ | $r_{0M}$ (fm) | Ref. |
|---|---|---|---|---|---|---|---|---|---|
| | | $^{12}$C | | | | | $^{14}$N | | |
| Fe | 124.5 | 51.0 | 1.42 | 43 | $^{208}$Pb | 147.2 | 78.0 | 1.45 | 23 |
| Ni | 124.5 | 56.0 | 1.51 | 43 | $^{209}$Bi | 145.5 | 77.0 | 1.45 | 22 |
| $^{107}$Ag | 124.5 | 60.0 | 1.44 | 43 | | | $^{16}$O | | |
| In | 124.5 | 61.0 | 1.44 | 43 | $^{144}$Nd | 130.0 | 62.4 | 1.35 | 20 |
| $^{144}$Nd | 118.0 | 56.9 | 1.37 | 20 | $^{146}$Nd | 130.0 | 62.7 | 1.35 | 20 |
| $^{146}$Nd | 118.0 | 54.8 | 1.33 | 20 | $^{148}$Nd | 130.0 | 64.3 | 1.37 | 20 |
| $^{152}$Sm | 118.0 | 53.3 | 1.31 | 20 | $^{154}$Sm | 130.0 | 65.3 | 1.39 | 20 |
| $^{154}$Sm | 118.0 | 56.5 | 1.34 | 20 | $^{197}$Au | 164.0 | 88.0 | 1.47 | 22 |
| $^{181}$Ta | 124.5 | 65.0 | 1.45 | 43 | $^{206}$Pb | 130.0 | 62.6 | 1.37 | 20 |
| $^{197}$Au | 124.0 | 64.0 | 1.45 | 22 | $^{206}$Pb | 167.1 | 89.0 | 1.45 | 23 |
| $^{206}$Pb | 124.2 | 64.0 | 1.44 | 23 | $^{207}$Pb | 166.1 | 87.5 | 1.44 | 23 |
| $^{207}$Pb | 123.7 | 65.5 | 1.46 | 23 | $^{208}$Pb | 130.0 | 63.2 | 1.41 | 20 |
| $^{208}$Pb | 118.0 | 61.9 | 1.43 | 20 | $^{208}$Pb | 166.0 | 88.5 | 1.45 | 23 |
| $^{208}$Pb | 122.8 | 66.0 | 1.47 | 23 | $^{208}$Pb | 170.1 | 90.9 | 1.44 | 21 |
| $^{209}$Bi | 118.0 | 60.9 | 1.45 | 20 | $^{209}$Bi | 164.0 | 89.5 | 1.49 | 22 |
| $^{209}$Bi | 121.4 | 64.5 | 1.45 | 22 | $^{209}$Bi | 170.1 | 91.7 | 1.45 | 21 |
| | | $^{14}$N | | | | | $^{20}$Ne | | |
| $^{197}$Au | 145.0 | 78.0 | 1.47 | 22 | $^{197}$Au | 207.6 | 112.0 | 1.47 | 22 |
| $^{206}$Pb | 146.8 | 78.0 | 1.47 | 23 | $^{206}$Pb | 207.2 | 111.5 | 1.46 | 23 |
| $^{208}$Pb | 146.7 | 79.0 | 1.45 | 21 | $^{207}$Pb | 208.1 | 112.0 | 1.45 | 23 |

TABLE II *(Continued)*

| Target | $E$ (lab) (MeV) | $l_A$ | $r_{0M}$ (fm) | Ref. | Target | $E$ (lab) (MeV) | $l_A$ | $r_{0M}$ (fm) | Ref. |
|---|---|---|---|---|---|---|---|---|---|
| | $^{20}$Ne | | | | | $^{32}$S | | | |
| $^{208}$Pb | 206.2 | 111.5 | 1.45 | 23 | $^{40}$Ca | 85.0 | 18.0 | 1.61 | 36 |
| $^{209}$Bi | 209.6 | 113.0 | 1.45 | 22 | $^{40}$Ca | 90.0 | 23.5 | 1.59 | 36 |
| | $^{32}$S | | | | | $^{40}$Ar | | | |
| $^{24}$Mg | 75.0 | 12.5 | 1.55 | 36 | $^{nat}$Se | 146.0 | 48.0 | 1.46 | 26 |
| $^{24}$Mg | 90.0 | 22.3 | 1.53 | 36 | $^{nat}$Se | 201.0 | 92.0 | 1.46 | 26 |
| $^{24}$Mg | 110.0 | 30.6 | 1.53 | 36 | $^{209}$Bi | 221 | | 1.37 | 37 |
| $^{24}$Mg | 120.0 | 36.0 | 1.57 | 36 | $^{238}$U | 302 | | 1.34 | 37 |
| $^{27}$Al | 73.0 | 15.5 | 1.64 | 36 | | $^{84}$Kr | | | |
| $^{27}$Al | 85.0 | 22.0 | 1.55 | 36 | | | | | |
| $^{27}$Al | 110.0 | 33.0 | 1.52 | 36 | $^{181}$Ta | 450 | | 1.30 | 37 |
| $^{40}$Ca | 82.5 | 14.0 | 1.59 | 36 | $^{232}$Th | 502 | | 1.27 | 37 |

quantities. Neither $r_{0B}$ nor $r_{0M}$ vary significantly with energy. Also, there are only very small differences between the values of $r_{0B}$ and $r_{0M}$ (see, for example, Ref. 36). The dependence of these quantities on the product $Zz$ is shown in Figs. 2 and 3. The trend is very similar to that shown in Fig. 1 for $r_e$. (It is interesting that $r_e$ gives the height of the barrier, not its radial extent. Thus, the quantities $D_{1/4}$ and $D$ probably reflect barrier height rather than radial distance, as one might infer from the term "strong interaction radius.")

Some of the scatter of the points may reflect real physical differences due to nuclear deforma-

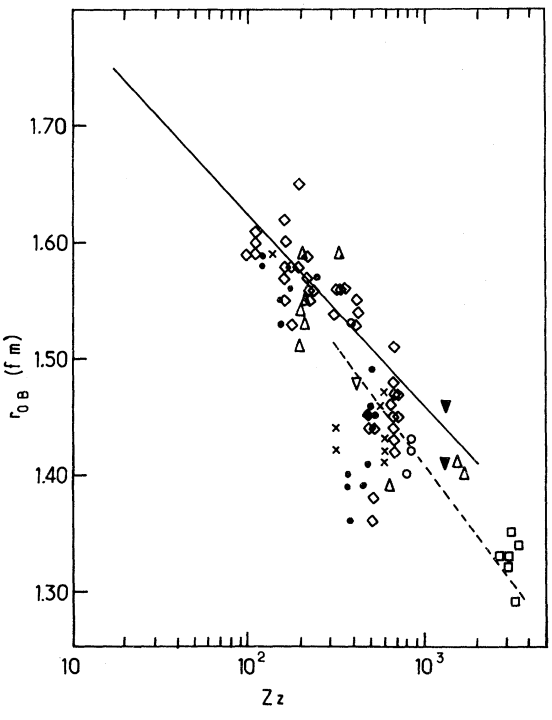


FIG. 2. Interaction distance parameter $r_{0B}$ from Blair's sharp cutoff analysis. Symbols and the solid line are as in Fig. 1. The dashed line is a least squares fit, Eq. 15.

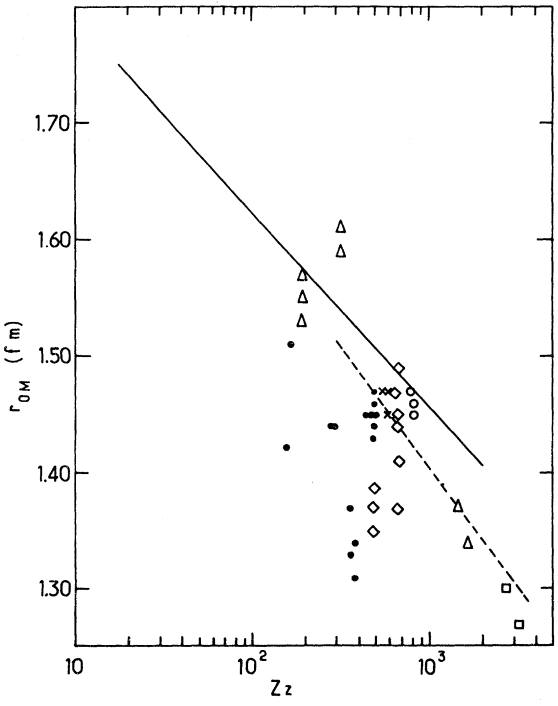


FIG. 3. Interaction distance parameter $r_{0M}$ from McIntyre's smooth cutoff analysis. Symbols and lines are as in Figs. 1 and 2.

tions or vibrations or velocity dependence.[15] However, the difficulties encountered in determining the size of such effects attest to their small magnitude.[16-18] Our feeling is that most of the scatter is due to random experimental uncertainties. If this is indeed the case, then smooth curve fits to the points provide a useful means of interpolation and extrapolation.[53] For $Zz$ products less than $\approx 300$, Eq. 14 seems to give a good representation for $r_{0B}$ as well as for $r_e$. For values of $Zz$ larger than $\approx 300$ the following equation provides a good fit for $r_{0B}$ and $r_{0M}$:

$$r_{0B} \text{ or } r_{0M} = 1.992 - 0.194 \log_{10}(Zz) \qquad \text{for } Zz > 300. \tag{15}$$

With these facts in mind we suggest the following prescription for estimating $\sigma_R(E)$ from elastic scattering data taken at one energy: (1) Obtain $r_{0B}$ from Blair's quarter point receipt (Eqs. 1-4 and 10) for the system in question. If reliable measurements are not available use Eq. 14 or 15.

(2) For $Zz$ products $\gtrsim 300$, estimate $r_e$ to be

$$r_e = r_{0B} - 0.041 + 0.030 \log_{10}(Zz). \tag{16}$$

(3) Use Wong's equation for $\sigma_R(E)$ and empirical values of $\hbar\omega_0$, $\Delta$, and $R_0$ from Ref. 9 (namely, $\hbar\omega_0 = 4.0$ MeV, $\Delta = 3.0$ MeV, and $r_0 = 1.41$ fm). Reference 9 gives a discussion of the precision that one can expect from this method; Ref. 17 describes the very strong sensitivity to $\overline{E}_0$ and $\Delta$ for low energies ($E \lesssim \overline{E}_0$).

As this prescription (and these parameters) is completely empirical, its precision depends on the precision of the experimental quantities employed. New data can be easily incorporated into the simple correlations shown in Figs. 1 and 2 and the uncertainties of extrapolation can be reduced correspondingly. A more sophisticated formulation of the interaction barrier, such as the folding potentials, is clearly desirable.[3,12,18,44] However, the precision of the parameters in any formulation will certainly depend on the quality of available experimental results. From Figs. 1-3 it is clear that more data are needed for $Zz \gtrsim 600$ and more precision is needed for all values of $Zz$.

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