Quadrupole moments of 2^+ states of even tin isotopes*

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In high precision Coulomb excitation experiments α particles of 10.0, 10.5, and 10.6 MeV and ^{16}O ions of 42.0 and 46.0 MeV were used to excite the first 2^+ states in ^{112,116,118,120,122,124}Sn. Static quadrupole moments $Q(2^+)$ and reduced quadrupole transition probabilities $B(E2;0^+\to 2^+)$ were extracted from intensities of particles elastically and inelastically scattered at angles near 173° into an annular detector. The values of $Q(2^+)$ are very small and consistent with 0.0 b. These results are in agreement with theoretical predictions based on pairing plus quadrupole forces.

[NUCLEAR REACTIONS ^{112,116,118,120,122,124}Sn(α,α'), E=10.0, 10.5, 10.6 MeV, ^{112,116,118,120,122,124}Sn(^{16}O, ^{16}O$'$), E=42.0, 46.0 MeV, measured $d\sigma/d\Omega$ for θ near 173°, deduced $Q(2^+)$, $B(E2;0^+\to 2^+)$. Enriched targets.]

I. INTRODUCTION

The discovery by de Boer *et al.*[1] of an unexpectedly large static quadrupole moment $Q(2^+)$ of the first $J^\pi=2^+$ level in ^{114}Cd stimulated extensive experimental efforts to measure quadrupole moments of excited states in other nuclei. Measurement of the nuclear charge distribution can provide basic information about nuclear shapes and data for testing the validity of nuclear models. The static quadrupole moments of the first 2^+ levels (~1.2 MeV) of the stable even-mass Sn isotopes are especially interesting because the wide range of neutron number N from 62 to 74 permits investigation of the variation in $Q(2^+)$ with change in N. These neutron numbers lie approximately between ^{114}Cd ($N=66$) and the shell closure at $N=82$ where $Q(2^+)$ is expected to be small. With their closed proton shell of $Z=50$ the Sn isotopes are expected to be nearly spherical and should have particularly small quadrupole moments.

Earlier experiments to determine $Q(2^+)$ values for ^{116}Sn and ^{124}Sn by means of Coulomb excitation and the reorientation effect were carried out by Kleinfeld *et al.*,[2] who measured γ rays in coincidence with backscattered ^{4}He, ^{16}O, and ^{32}S ions. The major experimental uncertainties arose in the determination of the efficiency of the γ ray detector and in the determination of the effective bombarding energies of the heavy ion beams in the 100–500 μg/cm^2 targets. Stelson *et al.*[3] obtained relative values of $Q(2^+)$ for all the stable even Sn isotopes, except the very rare ^{114}Sn, from γ rays emitted during bombardments with ^{4}He and ^{16}O ions. They then normalized the relative values to their absolute determination of $Q(2^+)$ for ^{120}Sn from inelastic scattering and particle-γ coincidence measurements. A fundamental difficulty in these measurements was the determination of the relative efficiency of the γ ray detector as a function of γ ray energy. Therefore, measurement of $Q(2^+)$ for the even Sn isotopes by another technique, subject to different experimental problems, is of considerable value.

Here we report new absolute determinations[4] of $Q(2^+)$ values and of the reduced electric quadrupole transition probabilities $B(E2;\ 0^+\to 2^+)$ for the six stable even-mass Sn isotopes 112, 116, 118, 120, 122, and 124. The results were obtained by precision measurements of inelastic scattering of α particles and ^{16}O ions. The apparatus, experimental procedures, and data are described in Sec. II below, and the extraction of $Q(2^+)$ values is discussed in Sec. III. Our results are compared with the earlier experimental data and with theoretical computations in Sec. IV.

II. APPARATUS, PROCEDURES, AND DATA

A. Apparatus and experimental procedures

For the inelastic scattering measurements beams of doubly charged α particles at energies of 10.0, 10.5, and 10.6 MeV were obtained from the University of Pittsburgh three-stage Van de Graaff accelerator. Oxygen ions stripped of six electrons were accelerated to 42.0 and 46.0 MeV. Inside a 1 m diameter scattering chamber the projectile beam passed through two beam collimators and an annular detector before passing through a thin target and into a Faraday cup beam stop about 4.5 m behind the target. Typical target currents were 150 nA for the α beams and 400 nA for the oxygen beams. The collimated beam was about 2 mm in diameter at the target.

Spot targets about 4 mm in diameter were prepared by vacuum evaporation of isotopically enriched[5] tin oxide powders except for the targets of ^{118}Sn which were prepared from metallic Sn enriched to 99.98%. Target thicknesses ranged from 5 to 40 $\mu g/cm^2$ on 15 $\mu g/cm^2$ carbon backings. Some target thicknesses were measured in a small scattering chamber with 11.0 MeV α particles from the Pennsylvania State University CN Van de Graaff accelerator. The beam charge collected in a Faraday cup was integrated during Rutherford scattering. Similarly, some target thicknesses were measured during data collection at Pittsburgh with α or ^{16}O beams. Thicknesses were also estimated from the amount of target material evaporated, the target-crucible distance, and the target mask size. Finally, the observed resolution or full width at half the maximum amplitude (FWHM) of ^{16}O spectral lines was used to set an upper limit on target thicknesses. The energy loss rate in Sn is about 3.4 keV/$\mu g\,cm^{-2}$ for the incident 42 MeV ^{16}O ions and about 5 keV/$\mu g\,cm^{-2}$ for the backscattered 25 MeV ions. For SnO_2 these loss rates must be increased by a factor of 1.2. Thus, ^{16}O ions scattered from the deepest layer of a 40 $\mu g/cm^2$ SnO_2 target would lose about 400 keV. Because the observed FWHM was typically 150 keV, and the detector itself contributed much of that observed energy spread, the targets were thinner than 40 $\mu g/cm^2$.

Ions elastically and inelastically scattered from the targets were detected in an annular 150 μm thick Si surface-barrier detector. The scattering angle was typically 173° with an angular acceptance of 3° and a solid angle of 32 msr. The detector was maintained at $-30°$C by a freon refrigerant and was operated with a bias voltage 50% above the manufacturer's specification. A liquid nitrogen trap in the chamber helped to minimize carbon buildup on the target. A magnet located between target and detector prevented electrons ejected from the target from reaching the detector. Detector pulses were initially accumulated in a 4096 channel analyzer and then transferred to magnetic tape for subsequent spectrum analysis. In later experiments pulses were digitized and stored in an online PDP-15 computer. A pulse-pair rejector was essential to minimize pileup effects in the α spectra. Additional details about beam optics, chamber geometry, and electronics are available elsewhere.[6]

TABLE I. Coulomb excitation parameters for Sn.

Projectile	E (MeV)	a (fm)	$\lambda\!\!\!^{-}$ (fm)	η	ξ
^{4}He	10.0	7.5	0.75	10	0.667
^{16}O	42.0	7.8	0.20	39	0.634

The beam bombarding energies were chosen to optimize the Coulomb excitation measurements of $Q(2^+)$ while minimizing interference from nuclear reactions. Parameters relevant for the description of Coulomb excitation[7] include a, one-half the distance of closest approach in 180° scattering, and $\lambda\!\!\!^{-}$, the reduced de Broglie wavelength of the projectile. The ratio $\eta \equiv a/\lambda\!\!\!^{-}$ should be much greater than 1 to ensure the validity of the semiclassical computations of Coulomb excitation used here. Typical values of a, $\lambda\!\!\!^{-}$, η, and the adiabaticity parameter ξ, are collected in Table I. Our choice of a bombarding energy low enough to avoid nuclear interference effects was guided by Saladin, Glenn, and Pryor,[8] who detected a 1% deviation from Rutherford scattering at back angles of 11 MeV α particles incident on Cd ($Z=48$). Thus, 10.6 MeV α particles scattered from Sn ($Z=50$) should not yield a larger deviation.

B. α particle spectra

A spectrum of α particles scattered at 172.0° from ^{112}Sn is shown on Fig. 1. The FWHM is about 30 keV. It is primarily because of detector resolution with small contributions due to variations of beam energy loss in the target, variations in energy of the incident beam particles, kinematic energy variation across the face of the detector, and possibly electronic drift. The quality of the spectrum can be characterized by the ratio of the amplitude of the elastic peak to the amplitude of the background at energies just above the 2^+ peak. This elastic-peak-to-valley ratio is about 2×10^4 on Fig. 1. Additional details are given in the figure caption.

The yields for the elastic and inelastic peaks were determined manually by summing the appropriate channels, subtracting an estimated background, and correcting for yield due to other Sn isotopes and impurities. The net yield for the 2^+ peak shown on Fig. 1 is about 2800 counts, while the total net yield accumulated for the 2^+ peak in several exposures was 8400 counts. The inelastic differential cross section is about 92 μb/sr. However, the experimental quantity relevant for determining $Q(2^+)$ and $B(E2)$ values is the ratio of the inelastic to elastic cross section. For ^{112}Sn this ratio $R_{exp} = 7.99 \times 10^{-4}$. An uncertainty of 2% stems primarily from uncertainty in subtraction of background under the inelastic peak and from the statistical fluctuation in the inelastic yield.

A correction of 4.5% was made for excitation of the 2^+ levels of ^{116}Sn and ^{114}Sn which were not resolved from the ^{112}Sn 2^+ peak. The correction was computed from the isotopic abundances,[5] our measured $B(E2)$ for ^{116}Sn, and an estimated $B(E2) = 0.200\ e^2 b^2$ for ^{114}Sn. No significant uncertainty is expected in this correction. A correction of 0.16% for an unresolved peak due to Coulomb excitation of a 1.354 MeV level in ^{119}Sn is based on the isotopic abundance[5] of ^{119}Sn in the target material and on an experimental $B(E2)$ value.[9] Another unresolved contribution to the ^{112}Sn 2^+ peak could arise from Coulomb excitation of the 1.280 MeV level in ^{115}Sn for which the $B(E2)$ is unknown. The level structure of ^{115}Sn is similar to that of ^{119}Sn and this level has the same $J^\pi = \frac{5}{2}^+$ as the 1.354 MeV level in ^{119}Sn. If the $B(E2)$ values are similar, the correction would be only 0.02% because of the very low abundance of ^{115}Sn in the target. Correction of the observed yield for elastic scattering from ^{57}Fe, which also is unresolved from the 2^+ peak, should be negligible. The natural abundance ratio of $^{57}Fe/^{56}Fe$ is 0.024, and the ^{56}Fe peak on Fig. 1 is very weak.

The choice of the bombarding energy of 10.6 MeV was, however, dictated by the presence of ^{56}Fe in the target. For $E_\alpha = 10.0$ MeV the elastic ^{56}Fe and the inelastic ^{112}Sn peaks coincided and gave rise to a rather large apparent $B(E2)$ value.[4] For $E_\alpha = 9.5$ MeV a prominent ^{56}Fe peak was well resolved, but the ^{112}Sn 2^+ peak was unresolved from a cluster of unidentified peaks and did not yield useful data.

The only other Sn isotope for which there were potential difficulties due to impurities was ^{124}Sn. For a bombarding energy of 10.0 MeV the α particles inelastically scattered from ^{124}Sn have nearly the same energy as those scattered elastically from Cu and Zn. Consistent data for ^{124}Sn were obtained with both 10.0 and 10.5 MeV α particles from a target which appeared to be free of Cu and Zn contamination.

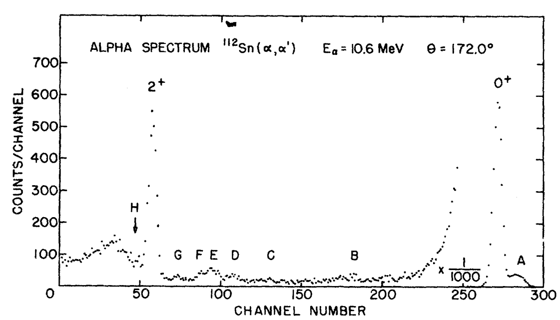

FIG. 1. Spectrum of α particles backscattered from ^{112}Sn. The target material was isotopically enriched to 87.5% in ^{112}Sn. The elastic peak, labeled 0^+, has been reduced by a factor of 1000. The elastically scattered α particles have a laboratory energy of 9.2 MeV. The excitation energy of the 2^+ level is 1.257 MeV. Adjacent channels in a raw data spectrum were added in pairs for this illustration which has a dispersion of 5.5 keV/channel. The FWHM is about 30 keV and the inelastic-peak-to-valley ratio is about 11. Lettered peaks correspond to α particles scattered from: *A*, ground states of all stable Sn isotopes heavier than ^{112}Sn; *B*, ^{115}Sn (0.498 MeV); *C*, ^{119}Sn (0.921 MeV); *D*, ^{65}Cu (0.0 MeV); *E*, ^{63}Cu (0.0 MeV); *F*, ^{120}Sn (1.171 MeV); *G*, ^{118}Sn (1.230 MeV); *H*, ^{56}Fe (0.0 MeV). Also see Sec. II B.

C. Oxygen spectra

A spectrum of ^{16}O scattered from ^{120}Sn at a mean laboratory scattering angle of 172.0° is shown on Fig. 2. The FWHM of 129 keV was the best resolution obtained in our oxygen spectra. A FWHM of 150 keV was typical. The resolution is primarily due to the detector and secondarily to the variable energy loss in the target, depending on the thickness traversed by the bombarding ion. The total net inelastic yield obtained from all oxygen bombardments of ^{120}Sn was 9100 counts. The inelastic differential cross section is about 1 mb/sr. The ratio $R_{exp} = 9.13 \times 10^{-3}$ with a standard deviation of 1.5%.

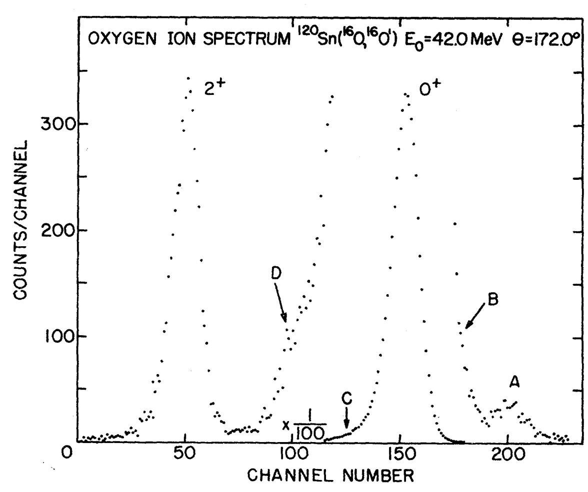

FIG. 2. Spectrum of ^{16}O backscattered from ^{120}Sn. The elastically scattered ions have a laboratory energy of 23.7 MeV. The dispersion is about 8.7 keV/channel and the FWHM is 129 keV. The elastic-peak-to-valley ratio is about 3.3×10^4. Letters *A*, *B*, *C*, and *D* indicate elastic scattering from ^{124}Sn, ^{122}Sn, ^{118}Sn, and ^{116}Sn, respectively. See Sec. II C for additional details.

The results presented here for ^{120}Sn are typical of all the ^{16}O data except for the spectra obtained from targets of ^{124}Sn. The peaks due to elastic scattering from several lower-mass Sn isotopes, which had abundances of about 1% in the sample of ^{124}Sn, were comparable in intensity to the 2^+ peak. Together these peaks formed a broad and flat tail below the elastic ^{124}Sn peak and could not be analyzed. With increasing bombarding energy the inelastic yield increases more rapidly than the elastic. Useful data were obtained with ^{16}O of 46.0 MeV. This energy still seemed sufficiently low to avoid interference of nuclear effects.

In the analysis of the ^{122}Sn + ^{16}O spectrum there is an ambiguity due to the possible existence of a ^{115}Sn elastic peak at nearly the same energy as the ^{122}Sn inelastic peak. Spectrographic analysis[5] indicated an upper limit of 0.05% for the isotopic abundance of ^{115}Sn which could contribute a maximum of 3.6% of the observed intensity. In Table II, the summary of experimental results, we also list an alternative determination of $B(E2)$ and $Q(2^+)$, based on the assumption that there was 0.05% ^{115}Sn in the target.

III. DETERMINATION OF $Q(2^+)$ AND $B(E2)$

The extraction of static quadrupole moments $Q(2^+)$ and reduced electric quadrupole transition probabilities $B(E2;\ 0^+ \to 2^+)$ from measurements of the reorientation effect has been described in detail by de Boer and Eichler[10] and illustrated for Sn isotopes by Kleinfeld *et al.*[2] Because the cross section ratio $R \equiv d\sigma(2^+)/d\sigma(0^+)$ is 0.01 or less in this experiment, R can be written in terms of a perturbation expansion as

$$R \simeq AB(E2;\ 0^+ \to 2^+)[1.0 + \rho Q(2^+)]\,, \tag{1}$$

where A and ρ depend on the nuclear charge and mass of projectile and target, on the bombarding energy and scattering angle, and on the excitation energy of the 2^+ level. A computer program,[11] which numerically solves the time-dependent Schrödinger equation in a semiclassical approximation, was used to determine $B(E2)$ and $Q(2^+)$ values. The input data required for the program included the parameters on which A and ρ depend and electric quadrupole matrix elements M_{ik}. These matrix elements are related to $B(E2)$ and $Q(2^+)$ by[10]

$$(M_{ik})^2 = B(E2;\ I_i \to I_k)(2I_i + 1) \tag{2}$$

and

$$Q(2^+) = -0.758 M_{22}\,. \tag{3}$$

The program output included differential cross sections and the ratio R.

By holding $Q(2^+) = 0$ ($M_{22} = 0$) and varying the input matrix element M_{12} around values which yielded computed ratios of R close to $R_{\exp}$, it was possible to determine A from Eq. (1). The computed values of A for α and ^{16}O bombardment of ^{120}Sn are 2.901×10^{-3} and 4.718×10^{-2} $(e^2\,\mathrm{b}^2)^{-1}$, respectively. The parameter ρ was determined by varying both M_{12} and M_{22} and using Eqs. (1), (2), and (3). Our value of ρ is 0.0735 (b)$^{-1}$ for α on ^{120}Sn and 0.2952 (b)$^{-1}$ for ^{16}O on ^{120}Sn. On Fig. 3 the quantity $R_{\exp}/AB(E2) = 1.0 + \rho Q$ is plotted as a function of ρ with M_{12} as a parameter. A "best-fit" computation with M_{12} adjusted so that the straight line has an intercept of 1.0 at $\rho = 0$ is also shown. The $B(E2)$ value is determined from the best-fit value of M_{12} and Eq. (2). The slope of the best-fit line is $Q(2^+)$.

The standard deviations in $R_{\exp}$ are indicated with error bars on the best-fit line on Fig. 3. The largest and smallest slopes consistent with the limits of the error bars define extreme upper and lower limits of $Q(2^+)$. For the standard deviations in $Q(2^+)$ we have adopted approximately $\frac{2}{3}$ of the range defined by these extreme limits. For the uncertainty in $B(E2)$ we have simply used the uncertainty in $R_{\exp}$ from the α data. A similar analysis for ^{124}Sn is also shown on Fig. 3. The results for the six Sn isotopes are collected in Table II.

No matrix elements other than M_{12} and M_{22} were included in our computations. The determination of $Q(2^+)$ from our data could be affected by interference due to excitation of higher levels. This problem has been considered by Stelson *et al.*,[3] who carried out computations with estimated values for $B(E2;\ 0 \to 2')$ and $B(E2;\ 2' \to 2)$. Here $2'$ denotes the next 2^+ level expected near 2.2 MeV in the even Sn isotopes. The conclusion was that

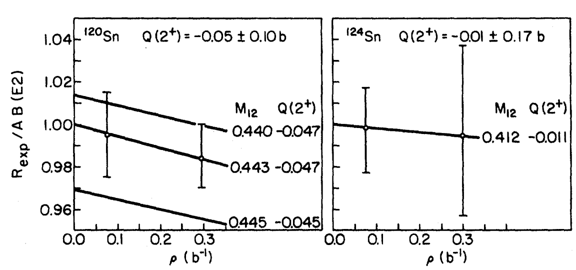

FIG. 3. Determination of $Q(2^+)$ for ^{120}Sn and ^{124}Sn. The procedure is discussed in Sec. III. For ^{120}Sn three lines corresponding to three different values of the matrix element M_{12} are shown. The quadrupole moment is given by the slope of the line which passes through (1, 0). Extreme limits on $Q(2^+)$ were determined from the slopes of lines (not shown) connecting opposite ends of the two error bars. For ^{120}Sn these limits are −0.20 b and +0.11 b. We adopted about $\frac{2}{3}$ of this range as the standard error.

TABLE II. Experimental results.

Isotope	Level [a] (MeV)	Projectile	E (MeV)	$R_{exp}\times 10^4$	$B(E2; 0^+\rightarrow 2^+)$ [b] (e^2b^2)	$Q(2^+)$ [b] (b)	$B(E2)$ [a] (e^2b^2)	$Q(2^+)$ [a] (b)	$B(E2)$ [c] (e^2b^2)	$Q(2^+)$ [c] (b)
112	1.257	α	10.6	8.19 ± 0.18	0.229 ± 0.005	−0.03 ± 0.11	0.256 ± 0.006	−0.15 ± 0.18		
		O	42.0	81.8 ± 1.3						
116	1.293	α	10.0	3.97 ± 0.15	0.195 ± 0.007	0.07 ± 0.10	0.216 ± 0.005	0.07 ± 0.16	0.22 ± 0.01	0.09 ± 0.13
		O	42.0	68.2 ± 1.0						
118	1.230	α	10.0	4.87 ± 0.15	0.199 ± 0.006	−0.05 ± 0.14	0.216 ± 0.005	−0.23 ± 0.16		
		O	42.0	79.5 ± 1.2						
120	1.171	α	10.0	5.67 ± 0.11	0.197 ± 0.004	−0.05 ± 0.10	0.203 ± 0.004	0.09 ± 0.10		
		O	42.0	91.3 ± 1.4						
122	1.140	α	10.0	6.05 ± 0.12	0.188 ± 0.004 [d]	0.02 ± 0.12 [d]	0.196 ± 0.004	−0.28 ± 0.17		
		O	42.0	98.6 ± 2.0						
124	1.131	α	10.0	5.66 ± 0.25	0.170 ± 0.004	−0.01 ± 0.17	0.161 ± 0.004	0.07 ± 0.17	0.19 ± 0.01	−0.24 ± 0.15
		α	10.5	8.08 ± 0.02						
		O	46.0	169 ± 6.8						

[a] From Ref. 3.
[b] Present work.
[c] From Ref. 2.
[d] If the ^{16}O yield for the 2^+ peak is reduced 3.6% for the maximum possible contribution from ^{115}Sn elastic scattering (see text, Sec. II C), the resulting $B(E2) = 0.189 \pm 0.004\ e^2b^2$ and $Q(2^+) = -0.16 \pm 0.12$ b.

$Q(2^+)$ values might be affected by as much as ±0.1 b. Kleinfeld *et al.*[2] made similar computations including the 2′ level and a 0′ level but found that the effect of the 0′ level was negligible. The possibility of such interference effects must be kept in mind in assessing the results listed in Table II.

IV. DISCUSSION

A. Comparison with other experiments

Experimental results obtained for $B(E2)$ and $Q(2^+)$ values from two γ ray studies[2,3] are also listed in Table II. Compared with the $B(E2)$ values of Stelson *et al*.[3] and their assigned errors of 2–3%, our $B(E2)$ values with uncertainties of 2–4% are slightly lower but almost overlap for five of the six isotopes studied. Our $B(E2)$ value of 0.197 ±0.004 e^2b^2 for ^{120}Sn is in very close agreement with the value of 0.199 ± 0.005 e^2b^2 determined by Stelson *et al.* from an inelastic scattering measurement. However, these results are lower than the 0.207 ± 0.005 e^2b^2 also determined by Stelson *et al.* from the measurement of γ rays in coincidence with projectiles backscattered from a thick target. Thus, the consistency between our results and those of Ref. 3 would be even better if their γ ray data were normalized only to their inelastic scattering data. For ^{124}Sn our $B(E2)$ value is intermediate between the results of the two γ ray experiments.[2,3]

For ^{112}Sn our $B(E2)$ value is almost 11% lower than that of Ref. 3. This difference corresponds to nearly 3 times the standard deviations of the two measurements. Our $B(E2)$ is largely determined by the α data. For example, if our 2^+ ^{16}O yield were decreased by 5%, our $B(E2)$ would increase only 2% to 0.234 e^2b^2, while $Q(2^+)$ would decrease 0.22 b to a value of −0.25 b. For ^{112}Sn the available isotopic enrichment of 87.51% was slightly lower than that typically available for the other enriched SnO_2 samples. The 4.5% correction to the observed inelastic α yield, discussed in Sec. II B, was unusually large because of the abundances of ^{116}Sn (3.11%) and ^{114}Sn (1.06%). Our data were not precise enough to provide an isotopic analysis. However, the intensity of the satellite peak labeled A on Fig. 1 is consistent with the isotopic analysis[5] provided with the target material. Even if we assumed that the observed inelastic α yield were *entirely* due to ^{112}Sn, our $B(E2)$ would increase only 4.5%. At present we have no explanation for the discrepancy.

The static quadrupole moments obtained from the three experiments are in very good agreement. Our values may be low because our computations with the Winther-de Boer code were carried out with the incident beam energy and not adjusted for energy lost in the target. For 10 MeV α particles the energy lost in penetrating a typical 15 $\mu g/cm^2$ SnO_2 target was less than 4 keV and did not affect the results of the computations. However, for incident 42 MeV ^{16}O the energy lost was about 60 keV and the mean effective bombarding energy was about 30 keV less than the incident energy. Carbon deposition on the target during ^{16}O bombardment also could have decreased the effective bombarding energy. This loss of energy could increase the value of $R_{exp}/AB(E2)$ for ^{16}O (see Fig. 3) by about 1% and increase $Q(2^+)$ by about 0.05 b. No correction was included because some target thicknesses were not measured, some targets were thinner than 15 $\mu g/cm^2$, and some of the values of R_{exp} in Table II are average results obtained from different targets. The standard errors listed in Table II may therefore be increased slightly. All our values of $Q(2^+)$ are consistent with zero. No significant variation with neutron number is apparent.

B. Theoretical computations

For the even Sn nuclei with a closed proton shell the first 2^+ levels are expected to be quadrupole core vibrations. The system of particles oscillates from a prolate shape through the spherical to an oblate shape and back again. If the oscillation is harmonic, the average shape is spherical and the static quadrupole moment is zero. Kisslinger and Sorensen[12] described the nuclear properties of medium and heavy mass nuclei with an interaction including short-range pairing and long-range quadrupole components. This interaction was diagonalized in a basis of quasiparticle and phonon states.

We have used this model and the equations derived in Ref. 12 to evaluate $Q(2^+)$ for the even Sn isotopes. Because of typographical errors in Ref. 12 we restate here the equation for the quadrupole moment of a one phonon state:

$$Q = 8(5)^{-1/2} C^{2\,2\,2}_{2\,0\,2} N_\omega{}^2 \omega \sum_{jj'j''} \langle j' \| q \| j \rangle (U_j U_{j'} - V_j V_{j'}) W(2j'2j''; j2)(U_j V_{j''} + U_{j''} V_j)(U_{j'} V_{j''} + U_{j''} V_{j'}) \times \langle j \| q \| j'' \rangle \langle j' \| q \| j'' \rangle \frac{E_j + E_{j'} + 2E_{j''}}{[(E_j + E_{j''})^2 - \omega^2][(E_{j'} + E_{j''})^2 - \omega^2]}, \tag{4}$$

where the normalization factor

$$N_\omega{}^2 = \frac{4\pi}{8\omega} \left[\sum_{12} \frac{E_1 + E_2}{[(E_1 + E_2)^2 - \omega^2]^2} (U_1 V_2 + U_2 V_1)^2 (2j_1 + 1)(C^{2\,j_1\,j_2}_{0\,\frac{1}{2}\,\frac{1}{2}})^2 \langle 1 | r^2 | 2 \rangle^2 \right]^{-1}. \tag{5}$$

Here j (also j' and j'') denotes the five single neutron shell model states $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $h_{11/2}$, and $d_{3/2}$ which are filled by neutrons in the N=50–82 shell. The core of 50 protons and 50 neutrons is considered inert. The quasiparticle energies E_j and the corresponding occupation coefficients U_j and V_j depend on the energies ϵ_j of the single neutron states, the energy gap Δ, the Fermi energy λ, and the strength G of the pairing force for neutrons.

Values of λ and Δ were obtained by graphical interpolation from values given in Table IX of Ref. 12, while $G = 23/A$ in MeV where A is the mass number. Single particle energies ϵ_j for the Sn isotopes were interpolated from values in Table IX and equations in Appendix II of Ref. 12. In Eqs. (4) and (5) above ω is the phonon energy for which we have used the experimental excitation energies[3] of the 2^+ levels. The reduced single particle quadrupole matrix elements in Eq. (4) were expressed in terms of the matrix elements of r^2 evaluated with oscillator radial wave functions.[13] The resulting values of $Q(2^+)$, which are all very close to zero, are listed in Table III. Additional computations were also made for ^{116}Sn and ^{122}Sn with the values of λ and Δ changed by ±10% to check on the sensitivity of $Q(2^+)$ to the interpolated values of λ and Δ. The extreme values of $Q(2^+)$ obtained for ^{116}Sn were 0.007 b and −0.008 b, while for ^{112}Sn the values ranged from 0.024 to 0.030 b, well within the range of the uncertainties of the experimental values.

Also listed in Table III are results of computations[14,15] which take into account possible excitation of zero, two-, and four-quasiparticle mixtures in the excited Sn states. The range of values listed reflects various choices for the form of the residual interaction. While higher quasi-

TABLE III. Computed quadrupole moments.

Isotope	$Q(2^+)$ [a] (b)	$Q(2^+)$ [b] (b)
112	−0.024	
116	−0.001	0.15 to −0.08
118	0.013	
120	0.025	0.074 to 0.188
122	0.029	
124	0.027	0.012 to 0.156

[a] Computation procedure described in text, Sec. IV B.
[b] From Refs. 14 and 15.

particle admixtures can apparently generate larger quadrupole moments, most of the theoretical values still lie within the error range of the experimental values.

We conclude that the precision inelastic scattering measurements have provided independent absolute measurements of $B(E2)$ and $Q(2^+)$ values for six even Sn isotopes. Our experimental $B(E2)$ values tend to be slightly smaller than those obtained from earlier γ ray studies but are in satisfactory agreement except for ^{112}Sn. Our values of $Q(2^+)$ are consistent with the results of the γ ray experiments and with theoretical computations. No significant variation of $Q(2^+)$ with neutron number was found.

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