Momentum-cutoff sensitivity in Faddeev calculations of trinucleon properties

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By increasing the cutoff q_{max} from 1.7 fm^{-1} to a value $\gtrsim 2.8$ fm^{-1} in momentum-space Faddeev calculations, we eliminate several discrepancies previously existing between momentum- and coordinate-space Faddeev calculations of trinucleon properties.

[NUCLEAR STRUCTURE ^{3}He; calculated binding energy and charge form factor. Momentum space Faddeev calculation with several different momentum cutoffs.]

Several discrepancies involving the trinucleon binding energy and charge form factor (for $Q^2 \gtrsim 10$ fm^{-2}) currently exist among Faddeev and variational calculations based on essentially the same nuclear interaction.[1] In this note, we give evidence that these discrepancies may be substantially reduced if the cutoff q_{max} for the magnitude of

$$\vec{q} = [\tfrac{1}{2}(\vec{k}_2 + \vec{k}_3) - \vec{k}_1]/\sqrt{3}\,, \tag{1}$$

in momentum-space Faddeev calculations is increased appropriately.

Our discussion is based on the comparison of the results of three different calculations for the trinucleon bound state: the momentum-space Faddeev calculation of Harper, Kim, and Tubis, (HKT)[2] extended to higher values of q_{max}, the coordinate-space Faddeev calculation of Laverne and Gignoux (LG)[3] and the recent variational calculation of Strayer and Sauer (SS).[4] The Faddeev calculations[2,3] give complete solutions for the case of two-nucleon interactions (given by the Reid soft-core potential[5] in the $^1S_0$ and coupled $^3S_1$-$^3D_1$ states). The SS calculation is probably the most elaborate variational trinucleon calculation to date (with over 4000 linear variational parameters) for the Reid potential. It includes two-nucleon interactions in the $^1D_2$ and $^3D_2$ states, in addition to interactions in the $^1S_0$ and $^3S_1$-$^3D_1$ states. However, the effects of the $^1D_2$ and $^3D_2$ interactions should not be large enough to prevent a meaningful comparison of results.

The HKT calculation[2] was done using the value $q_{max} = 1.7$ fm^{-1}. We have repeated the calculation using $q_{max} = 2.4$, 2.8 and 3.1 fm^{-1}. In Table I, we list the bound-state energy and in Table II the probabilities of the components of the bound-state wave function for different values of q_{max}. As in Ref. 2, we use the cutoff $p_{max} = 13$ fm^{-1}. In Table II, we also give the results of Laverne and Gignoux.[3] The components are classified according to the $\mathcal{L}$-$\mathcal{S}$ basis $|pq(Ll)\mathcal{L}W\mathcal{S}\mathcal{J}\mathcal{J}_z\rangle$, where $\vec{p} = \frac{1}{2}(\vec{k}_2 - \vec{k}_3)$ and $\vec{q}$ is given by Eq. (1). $W\mathcal{S}$ is a spin-isospin state with total spin $\mathcal{S} = \frac{1}{2}$ or $\frac{3}{2}$ and total isospin $\mathcal{T}$ $|\mathcal{T}_z| = \frac{1}{2}$. $r = A$, S, t, and $-$ denote, respectively, complete antisymmetry, complete symmetry, mixed over-all symmetry with symmetry under 23 exchange, and mixed over-all symmetry with antisymmetry under 23 exchange.

TABLE I. The trinucleon binding energy for different values of q_{max}, with $p_{max} = 13$ fm^{-1}. LG denotes the Laverne-Gignoux (see Ref. 3) result.

q_{max} (fm^{-1})	Binding energy (MeV)
1.7	6.7
2.4	6.9
2.8	6.96
3.1	6.98
LG	7.0

We see that the binding energy and wave-function probabilities are stable for $q_{max} \gtrsim 2.8$ fm^{-1}, and are in very good agreement with the LG results. The total S, S', and D probabilities $[P(S), P(S'), P(D)] = (90.2\%, 1.7\%, 8.1\%)$ are also in satisfactory agreement with the SS values (89.8%, 1.4%, 8.8%) especially in view of the fact that the SS variational results are estimated to be accurate only to about 10%. The SS lower bound on the binding energy is 6.6 MeV (with an estimated extrapolated value of 7.2 ± 0.2 MeV). This is also in fair accord with our result.

In Table III, we give values for Q_{min}^2, the position of the minimum of $|F_{ch}^{^3He}(Q^2)|$ and R, the secondary maximum ratio

$$R = \frac{|F_{ch}^{^3He}(Q^2 = 20\ \text{fm}^{-2})|_{exp}}{|F_{ch}^{^3He}(Q^2 = 20\ \text{fm}^{-2})|_{calc}}$$

for different values of q_{max}. We also list the LG and SS values. Calculated form factors are plotted in Fig. 1. The agreement between our results,

TABLE II. Probabilities for $\mathfrak{L}$-$\mathfrak{S}$ wave-function components, for different values of q_{max} (fm^{-1}), with $p_{max}=13$ fm^{-1}. Values of q_{max} (fm^{-1}), with $p_{max}=13$ fm^{-1}. LG denotes the Laverne-Gignoux results (see Ref. 3).

L	l	$\mathfrak{L}$	$\mathfrak{S}$		Probability (%) q_{max} = 1.7	2.4	2.8	3.1	LG
0	0	0	$\frac{1}{2}$	A	88.3	89.0	89.0	88.9	87.8
2	2	0	$\frac{1}{2}$	A	1.4	1.3	1.3	1.3	1.4
0	0	0	$\frac{1}{2}$	−	0.8	0.8	0.8	0.8	0.8
1	1	0	$\frac{1}{2}$	+	0.8	0.9	0.9	0.9	0.8
2	0	2	$\frac{3}{2}$	−	3.1	3.1	3.1	3.1	3.3
0	2	2	$\frac{3}{2}$	−	1.0	1.0	1.0	1.0	1.1
1	1	2	$\frac{3}{2}$	+	2.5	2.5	2.5	2.5	2.7
3	1	2	$\frac{3}{2}$	+	0.3	0.4	0.4	0.4	0.4
1	3	2	$\frac{3}{2}$	+	1.0	1.1	1.1	1.1	1.2

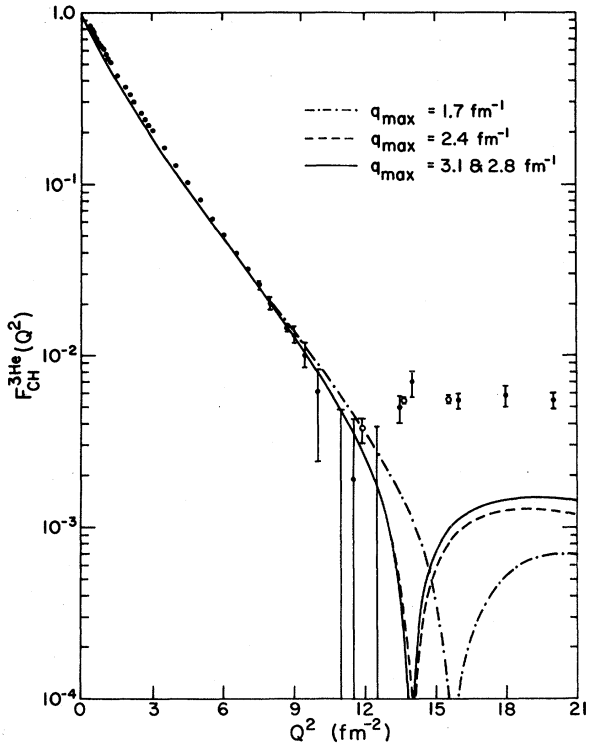

FIG. 1. Plots of $|F^{3He}_{ch}(Q^2)|$, calculated using $p_{max}=13$ fm^{-1} and $q_{max}=1.7$, 2.4, 2.8, and 3.1 fm^{-1}. The results for 2.8 and 3.1 fm^{-1} are essentially identical. The solid-circle data points are taken from Ref. 6 and the open-circle points from Ref. 7.

TABLE III. Parameters which characterize $|F^{3He}_{ch}(Q^2 \gtrsim 10\ \text{fm}^{-2})|$ for different values of q_{max}, with $p_{max}=13$ fm^{-1}. LG denotes the Laverne-Gignoux results (see Ref. 3) and SS the Strayer-Sauer result (see Ref. 4).

q_{max} (fm^{-1})	$Q_{min}{}^2$ (fm^{-2})	R
1.7	15.5	6
2.4	14.0	3.9
2.8	13.9	3.6
3.1	13.9	3.5
LG	14.0	3
SS	13.3	2.85

and those of LG is excellent. The Faddeev calculations give higher values of $Q_{min}{}^2$ and R than the variational calculations. The differences in the values of $Q_{min}{}^2$ and R are, however, compatible with estimated calculational uncertainties of the results.

In summary, we have shown that three different techniques for calculating three-nucleon bound-state observables give fairly consistent results. The excellent agreement between the two different Faddeev calculations and the convergence difficulties of the variational method give some support to the Faddeev formalism as the standard technique for calculating three-nucleon observables for realistic nuclear interactions.

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