Differences of deformation parameter β for different transition mechanisms; comparison with data*

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(Received 19 June 1975)

The differences between the isovector and isoscalar parts of the collective deformation parameter β are obtained from the results of microscopic effective-charge theory. For single-closed-shell nuclei, differences, due to the isovector parts, of up to about 20% between $\beta_{nn'}$ and $\beta_{pp'}$ are predicted, $\beta_{nn'}$ being much closer to β_{em} than to $\beta_{pp'}$. Data on single-closed-shell nuclei are tabulated and their general trend supports our predictions.

[NUCLEAR REACTIONS $0^+ \to 2^+$ transitions; estimate of β_2 deformation parameter—differences for different transition mechanisms; comparison with data.]

I. INTRODUCTION

In the vibrational model the transition amplitude for the transition from the 0^+ ground state to the 2^+ first-excited state is proportional to the deformation parameter β, which is an intrinsic property of the nucleus and independent of how it is excited. As in elastic scattering, the transition operator contains an isoscalar part and an isovector part, the latter coming from the Lane potential. The differences between (p, p') and (n, n') in elastic scattering are due to the neutron excess, and this feature is carried over when the collective model is used in inelastic scattering. Yet in inelastic scattering there should be shell effects present which invalidate that procedure, particularly for the $0^+ \to 2^+$ transitions. The simplest example is that of a neutron-closed-shell nucleus. The 2^+ states consist primarily of $\Delta N=0$ transitions from the ground state, and these are not available for the neutrons. Thus in the extreme shell model the vibration consists solely of proton motion. Because the spin-independent two-body forces between unlike nucleons are much greater than those between like nucleons, the transition is expected to be much stronger for (n, n') than (p, p'). This is contrary to the collective-model result where differences are attributed to the excess neutrons.

In actual nuclei there are strong core-polarization contributions due to mixing of high-lying collective $\Delta N=2$ particle-hole excitations, the giant states, which mix with the valence wave function. Due to the signs of the V_0 and $V_1\, \vec{\tau}\cdot\vec{\tau}$ nuclear forces, the isoscalar giant collective state is constructive and the isovector state destructive to the valence components. The difference between β parameters, being due to the isovector transition strength, is thus reduced, and the collective-model result is, to a large extent, regained; nevertheless there should be some residue of the shell-structure effect. It is the aim of this paper to estimate on the basis of effective-charge ideas how great such effects are expected to be.

In another paper[1] we have presented a formulation of the consequences for inelastic scattering processes of core polarization including the effect of neutron excess on the purity in isospin of the giant $\Delta N=2$ excitations. In Sec. II of this paper we apply these formulas to the problem of obtaining estimates of the ratios of the β parameter for (p, p'), (n, n'), (α, α'), and electromagnetic transitions. Comparison with empirical β values is given in Sec. III for single-closed-shell nuclei, and the results are discussed in Sec. IV. A preliminary report of these results has been presented earlier.[2]

A single-closed-shell nucleus is referred to as a neutron-vibration nucleus if the closure is in protons and a proton-vibration nucleus if the closure is in neutrons.

II. CONNECTION TO CORE-POLARIZATION THEORY

A. Core-polarization parameters

Typically in a microscopic treatment of inelastic scattering, shell-model wave functions are used to describe the initial and final nuclear states in a perturbation treatment of the scattering. Because of inadequacies of the shell model, core polarization must somehow be taken into account. In electromagnetic transitions this is done through effective charges. Core-polarization corrections in inelastic scattering can be related to effective charges, and this has been done by several authors.[1,3] The result from Ref. 1 is that the isoscalar and isovector strength parameters a_0 and a_1 for a particular external field [electromagnetic, (p, p'), etc., see Table I] are replaced by effective parameters $a_0^{\rm eff}$ and $a_1^{\rm eff}$ when

TABLE I. Microscopic strength parameters of the external field for various transitions.

External field type	a_0	a_1
Electromagnetic	$\frac{1}{2}$	$-\frac{1}{2}$
(α, α')	V_0	0
(p, p')	V_0	$-V_1$
(n, n')	V_0	V_1

approximate shell-model wave functions are used. The connection between the strength parameters is

$$a_0^{\text{eff}} + a_1^{\text{eff}}\tau_z' = (\epsilon_{00}a_0 + \epsilon_{01}a_1) + (\epsilon_{10}a_0 + \epsilon_{11}a_1)\tau_z' , \quad (1)$$

where $\epsilon_{\tau'\tau}$ is an element of a 2×2 core-polarization matrix. If exact nuclear wave functions are used, $\epsilon_{00}=\epsilon_{11}=1$, $\epsilon_{10}=\epsilon_{01}=0$ so that the over-all strength for neutrons or protons would be $a_0 + a_1\tau_z$; otherwise the core-polarization matrix $\underline{\epsilon}$ is substituted to account for the inadequacies in the shell-model wave functions. The need for off-diagonal elements of $\underline{\epsilon}$ is due to the lack of purity in the giant quadrupole states which is attributable to the neutron excess.

Equation (1) shows that there is an ambiguity[1,4] in the terms *isoscalar* and *isovector*. These are clearly separated in Eq. (1), where a_τ refers to the strength of the external field and therefore to the isospin actually transferred to the nucleus. On the other hand $a_{\tau'}^{\text{eff}}$ refers to the strength of the effective operator acting on the nucleons in the shell-model wave functions; therefore, for electromagnetic transitions, for example, a_0^{eff}/a_0 and a_1^{eff}/a_1 are just the isoscalar and isovector effective charges. In the limit of pure isoscalar and isovector core excitations the matrix $\underline{\epsilon}$ is diagonal and the ambiguity disappears. The distinction between the reference of the adjectives *isoscalar* and *isovector* to the nucleus as a whole and to the model space is crucial to the application made in this paper.

B. Collective model

For electromagnetic transitions the collective-model multipole operator can be obtained crudely as follows. The microscopic operator is

$$Q_{\lambda\mu} = \sum_i Q_{\lambda\mu}(i)\tfrac{1}{2}[1-\tau_z(i)] \quad (2a)$$

$$\approx \sum_i Q_{\lambda\mu}(i)\tfrac{1}{2}\left[1-\frac{2T_z}{A}\right]. \quad (2b)$$

A slight extension of the usual connection[5] between microscopic and macroscopic operators gives us the form

$$Q_{\lambda\mu} = \tfrac{1}{2}AR^2\alpha_{\lambda\mu}\left(1-\frac{\beta_1}{\beta_0}\frac{N-Z}{A}\right). \quad (3)$$

The collective operator $\alpha_{\lambda\mu}$ is purely isoscalar. Equation (2b) attributes all isovector effects to the neutron excess, contrary to the shell-model picture of the transition described in Sec. I, whereas Eq. (3) allows more freedom by permitting a different nuclear-structure factor for the isoscalar and isovector parts of the interaction. If $\beta_1=\beta_0$, Eq. (3) becomes the usual collective-model electromagnetic operator $ZR^2\alpha_{\lambda\mu}$. The $0\to\lambda$ electromagnetic transition amplitude calculated from Eq. (3) is

$$\langle\lambda|\tfrac{1}{2}AR^2\alpha_{\lambda\mu}\left(1-\frac{\beta_1}{\beta_0}\frac{N-Z}{A}\right)|0\rangle = \tfrac{1}{2}AR^2\left(1-\frac{\beta_1}{\beta_0}\frac{N-Z}{A}\right)\frac{\beta_0}{(2\lambda+1)^{1/2}} \equiv ZR^2\frac{\beta_{\text{em}}}{(2\lambda+1)^{1/2}} . \quad (4)$$

The parameter β_{em} defined by Eq. (4) is then

$$\beta_{\text{em}} = \frac{A}{2Z}\left(1-\frac{\beta_1}{\beta_0}\frac{N-Z}{A}\right)\beta_0 \approx \beta_0\left[1-\frac{N-Z}{A}\left(\frac{\beta_1}{\beta_0}-1\right)\right] . \quad (5)$$

For inelastic scattering we can use the Lane model with deformation.[6] The collective interaction has a strength parameter

$$\hat{V}\beta = \hat{V}_0\beta_0 \pm \hat{V}_1\beta_1\frac{N-Z}{4A} \quad \begin{cases}(n,n')\\(p,p')\end{cases}, \quad (6)$$

where $\hat{V}_0$ and $\hat{V}_1$ are the optical potentials. The deformation parameter is defined as Eq. (6) divided by the optical potential (see Appendix):

$$\beta_{\text{inel}} = \frac{\hat{V}_0\beta_0 \pm \hat{V}_1\beta_1(N-Z)/4A}{\hat{V}_0 \pm \hat{V}_1(N-Z)/4A} \approx \beta_0\left[1\pm\frac{\hat{V}_1}{\hat{V}_0}\frac{N-Z}{4A}\left(\frac{\beta_1}{\beta_0}-1\right)\right] \quad \begin{cases}(n,n')\\(p,p')\end{cases}. \quad (7)$$

For α scattering for which $\hat{V}_1=0$, it follows that $\beta=\beta_0$. When $\hat{V}_1\neq 0$, there is still a common deformation parameter for all transitions if $\beta_1=\beta_0$, but otherwise differences are expected. Various cases are presented in Table II. It should be pointed out that the dependence of the isovector terms, both in Eq. (3) and in Eq. (7), on $\xi=(N-Z)/A$ is due to the assumption that the differences between neutron and proton effects are due to the neutron excess. The results which we shall obtain do *not* depend on this assumption, however, and we sim-

TABLE II. Ratios of deformation parameters.

Transition	β/β_0
(α, α')	1
(n,n')	$1+\frac{\hat{V}_1}{\hat{V}_0}\frac{N-Z}{4A}\left(\frac{\beta_1}{\beta_0}-1\right)$
(p,p')	$1-\frac{\hat{V}_1}{\hat{V}_0}\frac{N-Z}{4A}\left(\frac{\beta_1}{\beta_0}-1\right)$
Electromagnetic	$1-\frac{N-Z}{A}\left(\frac{\beta_1}{\beta_0}-1\right)$

ply treat $\xi\beta_1/\beta_0$ as a single parameter. The important physical point in the connection between Eqs. (3) and (7) is that the ratio of the isovector to the isoscalar term in β is proportional to the ratio of the isovector to isoscalar transition-operator strengths.

C. Deformation-parameter ratios for single-closed-shell nuclei

When the model space consists only of protons or neutrons as in the case of a single-closed-shell nucleus, an over-all strength factor $a^{\text{eff}}=a_0^{\text{eff}}+a_1^{\text{eff}}\tau_z'$ applies since τ_z' then takes on a single definite value. From Eq. (1) the factor is

$$a^{\text{eff}}=(\epsilon_{00}\pm\epsilon_{10})a_0\pm(\epsilon_{11}\pm\epsilon_{01})a_1 \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}, \tag{8}$$

where the terms of Eq. (1) have been regrouped according to the isospin actually transferred to the nucleus by the external field. Now, the connection with the collective deformation parameters can be made by taking the ratio of the isovector to isoscalar terms of Eqs. (3) and (6) equal to the corresponding ratio in Eq. (8). For example, in electromagnetic transitions, for which according to Table I $a_0=\frac{1}{2}$, $a_1=-\frac{1}{2}$, we write

$$\frac{\beta_1}{\beta_0}\frac{N-Z}{A}=\pm\frac{\epsilon_{11}\pm\epsilon_{01}}{\epsilon_{00}\pm\epsilon_{10}} \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}. \tag{9}$$

Therefore, from Eq. (5) we have

$$\beta_{\text{em}}=\beta_0\left(1\mp\frac{\epsilon_{11}\pm\epsilon_{01}}{\epsilon_{00}\pm\epsilon_{10}}+\frac{N-Z}{A}\right) \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}. \tag{10}$$

Similarly, for inelastic scattering we take the ratio of the isovector and isoscalar terms in Eq. (6) to be proportional to the corresponding terms of Eq. (8) with $a_0=V_0$, $a_1=V_1$ for (n,n'), and $-V_1$ for (p,p'). If the appropriate connection $(\hat{V}_1/4\hat{V}_0)=V_1/V_0$ is made, Eq. (9) again results; Eq. (7) then yields

$$\beta_{\text{inel}}=\beta_0\left[1+\frac{\hat{V}_1}{4\hat{V}_0}\left(\pm\frac{\epsilon_{11}\pm\epsilon_{01}}{\epsilon_{00}\pm\epsilon_{10}}-\frac{N-Z}{A}\right)\times\tau_z(\text{projectile})\right] \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}. \tag{11}$$

For the pure isoscalar projectile scattering (α,α') or (d,d') the second term vanishes. Note that if $\hat{V}_1=-4\hat{V}_0$, then $\beta_{nn'}=\beta_{\text{em}}$. Since this is not far from the case in actual nuclear forces, where $\hat{V}_1\approx-2\hat{V}_0$, $\beta_{nn'}$ is expected to be much closer to β_{em} that it is to $\beta_{pp'}$.

D. Evaluation of the polarization parameters $\epsilon_{\tau'\tau}$

In Ref. 1 there are four alternative methods used to evaluate $\epsilon_{\tau'\tau}$. Perturbation theory was used to obtain expressions for the polarization parameters in terms of matrix elements for core-collective states. The matrix elements are then evaluated both by use of a generalization of the Brown schematic model and by comparison with the electromagnetic effective-charge formulation of Ref. 4. For each of these no-parameter theories it is anticipated that the isoscalar enhancement will be inadequate due to fractionation of the isoscalar strength. Accordingly, a one-parameter formula is devised for each which allows ϵ_{00} to be determined from any single piece of empirical information on effective charge; ϵ_{11} is taken from the theory and ϵ_{10} and ϵ_{01} are determined in terms of ϵ_{00} and ϵ_{11}. The proposed formulations are given in Table III. The parameters in the table are isovector to isoscalar two-nucleon strength V_1/V_0, isoscalar and isovector collective-coupling constants χ_0 and χ_1, shell-model spacing $\hbar\omega$, and giant resonance energies E_0 and E_1.

E. Evaluation of β

The use of Table III makes it possible to evaluate the deformation parameter β. As can be seen from actual calculations, the no-parameter and one-parameter models give roughly the same ratios of β for different transitions. We have, therefore, chosen to present a formula[7] for Eq. (9) based on the no-parameter schematic model for polarization:

$$\frac{\beta_1}{\beta_0}\xi=\pm\frac{\epsilon_{11}\pm\epsilon_{01}}{\epsilon_{00}\pm\epsilon_{10}}\approx\pm0.212+0.388\xi \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}. \tag{12}$$

Then we may write, using Eqs. (10) and (11):

$$\frac{\beta_{pp'}}{\beta_{\text{em}}}=\frac{1\pm0.106-0.306\xi}{1\mp0.212+0.612\xi} \quad \begin{cases}\text{target } n\\ \text{target } p\end{cases}, \tag{13}$$

TABLE III. Four different evaluations of the polarization parameters: the no-parameter Bohr-Mottelson model (NPBM), the one-parameter Bohr-Mottelson model (OPBM), the no-parameter schematic model (NPSM), and the one-parameter schematic model (OPSM). Parameters used (text and Ref. 1) in the evaluation are $V_1/V_0 \approx -0.5$ for the schematic model and -0.65 for the Bohr-Mottelson model, $\chi_0 = 1$, $\chi_1 = -0.64$, $\hbar\omega = 41A^{-1/3}$, $E_0 \approx 60A^{-1/3}$, and $E_1 \approx 120A^{-1/3}$, and $\xi \equiv (N-Z)/A$.

Case	ϵ_{01}	ϵ_{10}	ϵ_{00}	ϵ_{11}
NPBM	$\xi(\epsilon_{00}-\epsilon_{11})$	$\frac{V_1}{V_0}\epsilon_{01}$	$1+\chi_0=2$	$1+\chi_1(1+\frac{V_1}{V_0}\xi)^{-1}$ $\approx 0.36+0.42\,\xi$
OPBM	$\xi(\epsilon_{00}-\epsilon_{11})$	$\frac{V_1}{V_0}\epsilon_{01}$	free	$1+\chi_1(1+\frac{V_1}{V_0}\xi)^{-1}$ $\approx 0.36+0.42\,\xi$
NPSM	$\frac{2}{3}\frac{V_0}{V_0-V_1}\xi(\epsilon_{00}-\epsilon_{11})$	$\frac{V_1}{V_0}\epsilon_{01}$	$\frac{4\hbar\omega}{E_0}-1\approx 1.74$	$\frac{4\hbar\omega}{E_1}-1\approx 0.366$
OPSM	$\frac{2}{3}\frac{V_0}{V_0-V_1}\xi(\epsilon_{00}-\epsilon_{11})$	$\frac{V_1}{V_0}\epsilon_{01}$	free	$\frac{4\hbar\omega}{E_1}-1\approx 0.366$

$$\frac{\beta_{nn'}}{\beta_{\text{em}}} = \frac{1 \mp 0.106 + 0.306\xi}{1 \mp 0.212 + 0.612\xi} \quad \begin{cases}\text{target } n \\ \text{target } p\end{cases} . \qquad (14)$$

From Eqs. (13) and (14) we see that differences between $\beta_{pp'}$ and β_{em} of the order of 20% are expected for single-closed-shell nuclei. As a typical case ^{120}Sn has $\xi = \frac{1}{6}$ and $\beta_{pp'}/\beta_{\text{em}} = 1.19$, $\beta_{nn'}/\beta_{\text{em}} = 1.06$ and $\beta_{pp'}/\beta_{nn'} = 1.12$. By contrast, if there were no polarization effects, then $\epsilon_{\tau'\tau} = \delta_{\tau'\tau}$ and $(\beta_1/\beta_0)\xi = 1$; so, according to Eq. (5), β_{em} would be zero for a pure neutron vibration, as it must. In this limit Eq. (7) gives $\beta_{pp'}/\beta_{nn'} = (3-\xi)/(1+\xi) = 2.4$ for ^{120}Sn. The change from these extreme values of ratios of β in a shell-model description to the values near unity of Eqs. (13) and (14) is the effect both of isoscalar enhancement and isovector retardation due to core-polarization.

For nuclei other than single-closed-shell nuclei both neutrons and protons are excited even in the extreme shell model. We therefore expect the differences in β due to shell effects represented by Eqs. (13) and (14) will tend to be upper limits to the differences in other nuclei.

III. NUMERICAL RESULTS

A. Data from single-closed-shell nuclei

According to Secs. I and II there should be a systematic effect such that for closed-neutron-shell nuclei $\beta_{\text{em}} > \beta_{pp'}$, but for closed-proton-shell nuclei the reverse should hold. Table IV shows that this kind of behavior does seem to occur, although in many cases the β's could be equal within the limits of experimental error. The table includes all single-closed-shell nuclei with $A > 40$ for which we were able to find β_{em} in the compilation of Stelson and Grodzins[8] and $\beta_{pp'}$ in the literature. No systematic averaging has been done for $\beta_{pp'}$ as has been done for β_{em} in Ref. 8. The neutron inelastic scattering deformation parameter $\beta_{nn'}$ should lie between β_{em} and $\beta_{pp'}$ and be nearly equal to β_{em}. The few data available are consistent with this expectation, but the errors are fairly large, so nothing really definite can be said. The average difference $\langle\beta_{\text{em}} - \beta_{pp'}\rangle$ from Table IV is -0.015 for neutron-vibration nuclei with a standard deviation of 0.012, and for proton-vibration nuclei it is $+0.028$ with a standard deviation of 0.020. The t test of significance[9] gives $t = 3.8$ and 3.1 for the departure of these means from zero which could occur by chance with probabilities $P = 0.004$ and 0.03 for neutron-vibration and proton-vibration nuclei, respectively. The difference between the two means gives $t = 5.1$ which could occur by chance with probability $P = 10^{-4}$. It is possible, however, that there could be systematic errors due to common approximations used to determine β. (See Sec. IV for some discussion.)

B. Comparison with theory

Table V shows the ratio $\beta_{pp'}/\beta_{\text{em}}$ and a comparison with calculations using various procedures of Table III. Effective charges used in the one-parameter models were taken from the literature as noted in the table. The trends in the data seem to be followed by the calculations, the one-parameter schematic model being especially close. There are only two cases among the data which disagree with the theoretical prediction of whether $\beta_{pp'}/\beta_{\text{em}}$ should be greater than or less than unity,

TABLE IV. Parameters for single-closed-shell nuclei.

Nucleus	Type [a]	β_{em} [b]	$\beta_{nn'}$ [c]	$\beta_{pp'}$
^{44}Ca	n	0.22 ± 0.02		0.25 [d]
^{50}Ti	p	0.175 ± 0.02		0.14, [e] 0.15 [f]
^{52}Cr	p	0.23 ± 0.01	0.21 ± 0.02	0.17 [f]
^{54}Fe	p	0.18 ± 0.01		0.14, [g] 0.17 [h]
^{58}Ni	n	0.187 ± 0.009	0.19 ± 0.02	0.20 [h]
^{60}Ni	n	0.211 ± 0.009		0.23 [h]
^{62}Ni	n	0.193 ± 0.009		0.229 [i]
^{64}Ni	n	0.192 ± 0.009		0.200 [i]
^{88}Sr	p	0.14 ± 0.02		0.10, [j] 0.11 [j]
^{90}Zr	p	0.074 ± 0.026		0.07 [k]
^{92}Mo	p	0.116 ± 0.009		0.105 [l]
^{112}Sn	n	0.130 ± 0.014		0.152 [m]
^{116}Sn	n	0.118 ± 0.007	0.12 ± 0.01	0.133 [m]
^{118}Sn	n	0.116 ± 0.006		0.134 [m]
^{120}Sn	n	0.112 ± 0.006		0.119 [m]
^{122}Sn	n	0.118 ± 0.006		0.112 [m]
^{124}Sn	n	0.108 ± 0.006		0.108 [m]
^{138}Ba	p	0.120 ± 0.017		0.069 [n]

[a] n means "neutron vibration" or proton-closed-shell nucleus and vice versa for p.

[b] The electromagnetic parameters are all taken from Ref. 8.

[c] Measurements done on natural targets, reported by P. H. Stelson, R. L. Robinson, H. J. Kim, J. Rapaport, and G. R. Satchler, Nucl. Phys. 68, 97 (1965).

[d] R. J. Peterson and D. M. Perlman, Nucl. Phys. A108, 185 (1968).

[e] H. F. Lutz, W. Bartolini, T. H. Curtis, and G. M. Klody, Phys. Rev. 187, 1479 (1969).

[f] H. O. Funsten, N. R. Roberson, and E. Rost, Phys. Rev. 134, B117 (1964).

[g] S. F. Eccles, H. F. Lutz, and V. A. Madsen, Phys. Rev. 141, 1067 (1966).

[h] T. Stovall and N. M. Hintz, Phys. Rev. 135, B330 (1964).

[i] A. L. McCarthy and G. M. Crawley, Phys. Rev. 150, 935 (1966).

[j] J. Picard, O. Beer, A. El Behay, P. Lopato, Y. Terrien, G. Vallois, and R. Schaeffer, Nucl. Phys. A128, 481 (1969).

[k] W. S. Grey, R. A. Kenefick, J. J. Kraushaar, and G. R. Satchler, Phys. Rev. 142, 735 (1966).

[l] H. F. Lutz, D. W. Heikkinen, and W. Bartolini, Phys. Rev. C 4, 934 (1971).

[m] W. Makofske, W. Savin, H. Ogata, and T. H. Kruse, Phys. Rev. 174, 1429 (1968).

[n] D. Larson, S. M. Austin, and B. H. Wildenthal, Phys. Rev. C 9, 1574 (1974).

these being ^{122}Sn and ^{124}Sn for which β_{em} does not appear to follow the trend of other isotopes. Although ^{138}Ba does follow the trend, the $\beta_{pp'}/\beta_{em}$ ratio is so large that is is not included in the calculation of the mean values.

Equations (5) and (7) may be used to calculate β for one kind of transition when any two others are known. For example, from β_{em} and $\beta_{pp'}$ one can calculate the parameter $\xi(\beta_1/\beta_0)-1$ and use it to calculate $\beta_{nn'}$ from the ratio of $\beta_{nn'}/\beta_{pp'}$. This procedure applied to ^{52}Cr, natural Ni, and natural Sn gives $\beta_{nn'}=0.21$, 0.20, and 0.12 compared to 0.21 ± 0.02, 0.19 ± 0.02, and 0.12 ± 0.01 of the data, respectively. This procedure, in contrast to the effective-charge method, applies whether or not the nucleus in question has a single-closed shell.

IV. SUMMARY AND DISCUSSION

The data available on even single-closed shell nuclei indicate a definite effect of the kind predicted in the Introduction. The empirical deformation parameters for the electromagnetic and inelastic $0^+ \to 2^+$ transitions satisfy the result $\langle\beta_{em}-\beta_{pp'}\rangle < 0$ for neutron vibrations (proton closed shell) and $\langle\beta_{em}-\beta_{pp'}\rangle > 0$ for proton vibrations (neutron closed shell). The magnitude of these differences is rather well accounted for by very general features of nuclear core polarization.[1]

It should be mentioned that there are other effects which should lead to differences between β_{em} and $\beta_{pp'}$. The deformed optical model normally assumes a zero-range interaction between the projectile and the deformed nuclear matter. A more realistic range for the interaction would have the effect of averaging[10,11] over a range of angular positions in the nucleus, leading to a smaller effective β parameter in (p,p') than in electromagnetic transitions. In addition, it has been shown[12,13] that improvements on the uniform-density approximation used in Ref. 8 can lead to significant changes in the determination of β_{em}. These effects would be in the same direction both for neutron-vibration and proton-vibration nuclei, whereas the isovector effect we have studied reverses from one type of nucleus to the other. The data presented in Table IV and in the text do show $\langle\beta_{em}-\beta_{pp'}\rangle > 0$ for both kinds of single-closed-shell nuclei (if both kinds are given equal weighting). This trend seems also to be present in the calculated ratios presented in Table V based on effective-charge considerations alone. However, the theory is probably not reliable enough to make definite conclusions about the finite-range effect.

It is apparent from these considerations that fairly accurate (~10%) (n,n') data on single-closed-shell nuclei would be of considerable value in confirming the isospin effect we have found on the parameter β. It is not clear from the few (n,n') data available whether $\beta_{nn'}$ fits in the scheme or not. Furthermore, the interaction-range effect discussed above would be present both in $\beta_{pp'}$ and $\beta_{nn'}$ obtained from analysis of the data leaving only the isospin effect to be determined from the

TABLE V. Comparison of $\beta_{pp'}/\beta_{em}$ with theory.

Nucleus	Type [a]	e_{eff} [b]	NPBM [c]	OPBM	NPSM	OPSM	Exp.
			$\beta_{pp'}/\beta_{em}$				
^{44}Ca	n	1.49 [d]	1.30	1.15	1.28	1.08	1.13
		1.95 [e]	1.30	1.12	1.28	1.05	
^{50}Ti	p	1.55 [f]	0.77	0.83	0.67	0.75	0.83
^{52}Cr	p	1.92 [f]	0.761	0.856	0.692	0.805	0.74
^{54}Fe	p	1.93 [f]	0.754	0.852	0.72	0.83	0.86
^{58}Ni	n	1.90 [g]	1.33	1.13	1.35	1.10	1.07
^{60}Ni	n	1.34 [g]	1.31	1.18	1.30	1.12	1.09
^{62}Ni	n	0.91 [g]	1.30	1.24	1.27	1.17	1.19
^{64}Ni	n	0.97 [g]	1.28	1.21	1.24	1.12	1.04
^{88}Sr	p	1.65 [h]	0.77	0.84	0.66	0.75	0.75
^{90}Zr	p	2.4 [i]	0.77	0.88	0.67	0.81	0.95
^{92}Mo	p	1.9 [j]	0.77	0.86	0.69	0.80	0.91
^{112}Sn	n	0.72 [k]	1.29	1.29	1.26	1.22	1.17
^{116}Sn	n	0.72 [k]	1.27	1.26	1.20	1.18	1.13
^{118}Sn	n	0.75 [k]	1.26	1.24	1.20	1.15	1.16
^{120}Sn	n	0.73 [k]	1.26	1.24	1.19	1.14	1.06
^{122}Sn	n	0.84 [k]	1.24	1.20	1.17	1.09	0.95
^{124}Sn	n	0.89 [k]	1.24	1.19	1.16	1.06	1.00
^{138}Ba	p	1.79 [l]	0.78	0.86	0.63	0.73	0.58

[a] n means "neutron vibration" or proton-closed-shell nucleus and vice versa for p.
[b] Effective charge of the valence nucleons.
[c] Abbreviations have been defined in Table III.
[d] B. A. Brown, D. B. Fossan, J. M. McDonald, and K. A. Snover, Phys. Rev. C 9, 1033 (1974), f,p shell neutrons.
[e] See Brown, Fossan, McDonald and Snover, footnote d, $f_{7/2}$ shell neutrons only.
[f] P. G. Bezzeti, J. Phys. Soc. Jpn. Suppl. 34, 338 (1973), $f_{7/2}$ shell only.
[g] Calculated from Table VI of L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).
[h] T. A. Hughes, Phys. Rev. 181, 1586 (1969).
[i] W. G. Love and G. R. Satchler, Nucl. Phys. A101, 424 (1967).
[j] Chosen arbitrarily as characteristic of proton nuclei in this mass region.
[k] Calculated from cloud-nucleon strength parameters and experimental $B(E2)$ of S. Yoshida, Nucl. Phys. 38, 380 (1962).
[l] D. Larson, S. M. Austin, and B. H. Wildenthal, Phys. Lett. 42B, 153 (1972); Phys. Rev. C 11, 1638 (1975).

comparison.

We realize that there is a considerable amount of (α, α') data available. There is in these data some tendency to fit the pattern which is predicted by Eqs. (10) and (11), but there is much more scatter. The large projectile size in this case makes the analysis and comparison less certain. We intend to pursue this question further, however.

Finally, we wish to state that the effective-charge theory used in our calculations of the β parameters is rather crude and the accuracy of the empirical determinations is somewhat uncertain, so the closeness of the agreement of the data with the schematic model results is probably fortuitous. Nevertheless, the estimate of 10–25% differences in $\beta_{pp'}$ and β_{em} from both the schematic-model and the Bohr-Mottelson parameters, borne out by the data, seems to be meaningful. We wish to reemphasize the point that although numerical results presented in this paper have been restricted to single-closed-shell nuclei, differences in β are expected in general for all nuclei. Although the magnitude of the differences in β derived from effective-charge theory may not be accurate, the expressions derived in Sec. II B are quite general and can be used to deduce the unknown β from measurements of the other two. For example, $\beta_{nn'}$ can be obtained from a comparison of $\beta_{pp'}$ and β_{em}.

We wish to thank Mr. G. M. Kingsley for helpful advice concerning the significance test used in Sec. III.

APPENDIX

We give here a more detailed derivation of Eq. (7) including imaginary parts of the optical potentials. The inelastic interaction is calculated as usual, expanding around the equilibrium spherical shape. We assume that the real terms in the optical potential have a common form factor f, and the imaginary parts have a common form factor g. The interaction is

$$\Delta V=-R_r(\beta_0\hat{V}_0\pm\beta_1\hat{V}_1\eta)\frac{\partial f}{\partial r}-iR_i(\beta_0\hat{W}_0\pm\beta_1\hat{W}_1\eta)\frac{\partial g}{\partial r}\,, \tag{A1}$$

where $\eta=(N-Z)/(4A)$. An over-all deformation parameter β is defined as minus the ratio of Eq. (A1) to the R-multiplied derivative of the optical potential:

$$\beta\equiv\Delta V\Big/\left[-R_r(\hat{V}_0\pm\hat{V}_1\eta)\frac{\partial f}{\partial r}-iR_i(\hat{W}_0\pm\hat{W}_1\eta)\frac{\partial g}{\partial r}\right]. \tag{A2}$$

Equation (A1) can be rewritten as

$$\begin{aligned}\Delta V&=-R_r[\beta_0(\hat{V}_0\pm\hat{V}_1\eta)\pm(\beta_1-\beta_0)\hat{V}_1\eta]\frac{\partial f}{\partial r}-iR_i[\beta_0(\hat{W}_0\pm i\hat{W}_1\eta)\pm(\beta_1-\beta_0)\hat{W}_1\eta]\frac{\partial g}{\partial r}\\&=\beta_0\left[-R_r(\hat{V}_0\pm\hat{V}_1\eta)\frac{\partial f}{\partial r}-iR_i(\hat{W}_0+\hat{W}_1\eta)\frac{\partial g}{\partial r}\right]\pm(\beta_1-\beta_0)\left[-R_r\hat{V}_1\frac{\partial f}{\partial r}-iR_i\hat{W}_1\frac{\partial g}{\partial r}\right]\eta\,.\end{aligned} \tag{A3}$$

Thus to first order in η, Eq. (A2) becomes

$$\beta=\beta_0+\frac{\pm(\beta_1-\beta_0)\left[-R_r\hat{V}_1\frac{\partial f}{\partial r}-iR_i\hat{W}_1\frac{\partial g}{\partial r}\right]\eta}{-R_r(\hat{V}_0\pm\hat{V}_1\eta)\frac{\partial f}{\partial r}-iR_i(\hat{W}_0+\hat{W}_1\eta)\frac{\partial g}{\partial r}}\approx\beta_0\pm(\beta_1-\beta_0)\eta\frac{\hat{V}_1}{\hat{V}_0}\pm\frac{-iR_i\hat{V}_1(\beta_1-\beta_0)\eta\left(\frac{\hat{W}_1}{\hat{V}_1}-\frac{\hat{W}_0}{\hat{V}_0}\right)\frac{\partial g}{\partial r}}{-R_r\hat{V}_0\frac{\partial f}{\partial r}-iR_i\hat{W}_0\frac{\partial g}{\partial r}}\,. \tag{A4}$$

If it happens that $\hat{W}_1/\hat{V}_1=\hat{W}_0/\hat{V}_0$, then only the first two terms contribute. For simplicity assume the typical values $f=g$, $\hat{W}_1/\hat{V}_1=0.5$, and $\hat{W}_0/\hat{V}_0=0.2$, which lead to a correction term $(\beta_1-\beta_0)\eta\hat{V}_1/\hat{V}_0(0.3i-0.06)$ to Eq. (7). The first term gives an amplitude incoherent with the main terms, so its contribution will be small. The second term is a 6% correction to the $(\beta_1-\beta_0)$ term, which is already a correction term in β.

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*Work performed under the auspices of the U. S. Energy Research and Development Administration.

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[1]V. R. Brown and V. A. Madsen, Phys. Rev. C 11, 1298 (1975).

[2]V. A. Madsen, V. R. Brown, and J. D. Anderson, Phys. Rev. Lett. 34, 1398 (1975).

[3]W. G. Love and G. R. Satchler, Nucl. Phys. A101, 424 (1967); J. Atkinson and V. A. Madsen, Phys. Rev. C 1, 1377 (1970); H. McManus, in *The Two-Body Force in Nuclei, Proceedings of a Symposium held at Gull Lake, Michigan, 1971,* edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972).

[4]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II, Chap. 6.

[5]A. Bohr and B. R. Mottelson, K. Dan. Vidensk. Selsk., Mat.—Fys. Medd. 27, No. 16 (1953).

[6]G. R. Satchler, R. M. Drisko, and R. H. Bassel, Phys. Rev. 136, B637 (1964).

[7]The apparent inconsistency in the ξ dependence of the left- and right-hand sides of Eq. (12) is due to the parametrization in terms of the neutron excess of the differences in scattering from neutrons and protons in the nucleus. See Sec. II B.

[8]P. H. Stelson and L. Grodzins, Nuc. Data A1, 21 (1965).

[9]G. W. Snedecar and W. G. Cochran, *Statistical Methods,* (Iowa State U. P., Ames, Iowa, 1967).

[10]D. L. Hendrie, Phys. Rev. Lett. 31, 478 (1973).

[11]B. J. Verhaar, Phys. Rev. Lett. 32, 307 (1974).

[12]L. W. Owen and G. R. Satchler, Nucl. Phys. 51, 155 (1964).

[13]C. R. Gruhn, B. M. Preedom, and K. Thompson, Phys. Rev. Lett. 23, 1175 (1969).