Exchange effects in the coupled-equations nuclear-reaction formalism*

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Exchange symmetry is used to eliminate reference to physically indistinguishable channels in the coupled-equations nuclear-reaction formalism. The resulting equations are similar in structure to the original ones except that they are far fewer in number and the interactions contain exchange operators.

[NUCLEAR REACTIONS Coupled-equations nuclear-reaction formalism of Kouri, Levin, Hahn, and Tobocman is explicitly antisymmetrized.]

I. INTRODUCTION

It has always been difficult to make the consequences of exchange symmetry explicit in scattering formalisms. This is because the specification of the asymptotic behavior of a scattering-state wave function is in general not symmetric in the coordinates of identical particles. Usually, the best procedure is to solve the problem neglecting exchange symmetry, antisymmetrize the resulting wave function, and then sum the fluxes to physically indistinguishable channels.

In the last few years a new nuclear reaction formalism, which will be referred to as the coupled-equations formalism (CEF) has been developed.[1] This multipartition scattering formalism is an alternative to the Faddeev formalism which has the advantage that the structure of the equations is independent of the number of particles in the system. We will show that the CEF allows one to use exchange symmetry to eliminate reference to physically indistinguishable channels in the dynamical equations. This has the salutary effect of reducing considerably the number of equations to be solved, or equivalently, the number of transition amplitudes to be calculated. The price paid for this simplification is that the interaction potentials appearing in the dynamical equations now include exchange terms.

The approach we use to introduce the effects of exchange symmetry into the CEF is the same as the one used by Kouri and Levin[2] to analyze exchange effects for a particular case. We analyze the problem in a general way so that the results apply to all cases. A similar analysis was performed by Lovelace[7] for the Faddeev equations and by Bencze and Redish[8] for their N-particle connected-kernel formalism.

We will first review the integral form of the CEF in Sec. II. Then in Sec. III we will review the consequences of exchange symmetry for scattering. In Sec. IV we see how exchange symmetry can be incorporated into the dynamical equations of the integral form of the CEF. Section V is devoted to the derivation of an antisymmetrized version of the differential form of the CEF. A K matrix version of the antisymmetrized integral CEF is described in Sec. VI. Our results are summarized in Sec. VII.

II. COUPLED-INTEGRAL-EQUATIONS SCATTERING FORMALISM

We start by considering a system of N distinguishable fermions. For each partition α, β, ... of the fermions into two sets or clusters there is a decomposition of the Hamiltonian into two parts:

$$H = H_\alpha + V_\alpha = H_\beta + V_\beta = \cdots . \tag{1}$$

H_α contains the kinetic energy and the intracluster interactions, while V_α is the sum of the intercluster interactions for partition α.

The transition operator for scattering from a partition-α configuration to a partition-β configuration is

$$T_{\beta\alpha} = V_\beta + V_\beta \mathcal{G} V_\alpha , \tag{2}$$

where

$$\mathcal{G} = (E + i\epsilon - H)^{-1} \tag{3}$$

is the system Green's function operator. The partition γ Green's function operator is defined to be

$$G_\gamma = (E + i\epsilon - H_\gamma)^{-1} = (\mathcal{G}^{-1} + V_\gamma)^{-1} . \tag{4}$$

It follows that the relationship between the two kinds of Green's function operators may be written as

$$\mathcal{G} = G_\gamma + G_\gamma V_\gamma \mathcal{G} \tag{5}$$

or

$$\mathcal{G} = G_\gamma + \mathcal{G} V_\gamma G_\gamma . \tag{6}$$

Substitution of Eq. (5) into Eq. (2) leads to the relationship

$$\begin{aligned} T_{\alpha\beta} &= V_\alpha + V_\alpha G_\gamma (V_\beta - V_\gamma + T_{\gamma\beta}) \\ &= V_\alpha G_\gamma (G_\beta^{-1} + T_{\gamma\beta}) , \end{aligned} \tag{7}$$

while substitution of Eq. (6) into Eq. (2) gives

$$T_{\alpha\beta}=V_\alpha+T_{\alpha\gamma}G_\gamma V_\beta . \tag{8}$$

For the choice $\gamma=\beta$ these become the Lippmann-Schwinger equations for the transition operators. The CEF results from a different choice. A set of values for α and β in Eq. (7), or for β and γ in Eq. (8), are chosen so that the result is a set of coupled integral equations for all the elements in one column or one row of the transition operator matrix.

The coupled integral equations for the elements of the transition operator matrix can be formulated compactly in terms of an array of numbers $W_{\alpha\beta}$ such that

$$\sum_\alpha W_{\alpha\beta}=\sum_\beta W_{\alpha\beta}=1 . \tag{9}$$

Multiplying both sides of Eq. (7) by $W_{\alpha\gamma}$ and summing on γ gives

$$T_{\alpha\beta}=\sum_\gamma V_\alpha W_{\alpha\gamma}G_\gamma(G_\beta^{-1}+T_{\gamma\beta}) , \tag{10}$$

while multiplying both sides of Eq. (8) by $W_{\alpha\gamma}$ and summing on γ gives

$$T_{\alpha\beta}=V_\alpha+\sum_\gamma T_{\alpha\gamma}G_\gamma W_{\gamma\beta}V_\beta . \tag{11}$$

Equation (11) is the Baer-Kouri[3] version of the CEF, while Eq. (10) is the Kouri-Levin[4] version. It is possible to choose the partition coupling array $W_{\alpha\beta}$ in a manner that causes the iterated kernel of the set of integral equations to be connected.[1]

III. CONSEQUENCES OF EXCHANGE SYMMETRY FOR SCATTERING

The channels associated with partition α are identified with the various possible unit incident current eigenstates $\Phi_a\equiv\Phi_{\alpha i}$ of H_α having eigenvalue E:

$$(E-H_\alpha)\Phi_a=0 . \tag{12}$$

The cross section for scattering from channel a to channel b is then

$$\sigma_{ba}=\frac{\pi}{k_b^2}|\mathcal{T}_{ba}|^2 , \tag{13}$$

where

$$\mathcal{T}_{ba}=\langle\Phi_b|T_{\beta\alpha}|\Phi_a\rangle . \tag{14}$$

Now let us suppose that the N fermions comprising our system are in fact indistinguishable. That means that the channels in our scattering formalism now can be arranged into groups that represent physically indistinguishable configurations. In particular, consider the partition α with n_α particles in one cluster and $N-n_\alpha$ particles in the other. There are

$$N_\alpha=\frac{N!}{n_\alpha!(N-n_\alpha)!}(1-\tfrac{1}{2}\delta_{n_\alpha N/2}) \tag{15}$$

partitions in the family of partitions related to α merely by the interchange of pairs of identical fermions. Let us label the partitions in this family by the indices $\alpha(1),\alpha(2),\ldots,\alpha(N_\alpha)$. The associated channels must be similarly distinguished. So we have the channel indices $a(1),a(2),\ldots,a(N_\alpha)$.

Now it can be shown[5] that the consequence of exchange symmetry is the replacement of Eq. (14) by

$$\mathcal{T}_{\tilde{b}a}=N_\alpha^{1/2}N_\beta^{-1/2}\sum_{n=1}^{N_\beta}(-1)^{\sigma_\beta(n)}\langle\Phi_{b(n)}|T_{\beta(n)\alpha(1)}|\Phi_{a(1)}\rangle \tag{16}$$

or by

$$\mathcal{T}_{b\tilde{a}}=N_\beta^{1/2}N_\alpha^{-1/2}\sum_{n=1}^{N_\alpha}(-1)^{\sigma_\alpha(n)}\langle\Phi_{b(1)}|T_{\beta(1)\alpha(n)}|\Phi_{a(n)}\rangle , \tag{17}$$

where $\sigma_\alpha(n)$ is the parity of the permutation $P_\alpha(n)$ that transforms partition $\alpha(1)$ into $\alpha(n)$. We will rewrite Eqs. (16) and (17) to read

$$\mathcal{T}_{\tilde{b}a}=\langle\Phi_{b(1)}|T_{\tilde{\beta}(1)\alpha(1)}|\Phi_{a(1)}\rangle \tag{18}$$

and

$$\mathcal{T}_{b\tilde{a}}=\langle\Phi_{b(1)}|T_{\beta(1)\tilde{\alpha}(1)}|\Phi_{a(1)}\rangle , \tag{19}$$

where

$$T_{\tilde{\beta}(1)\alpha(1)}=N_\alpha^{1/2}N_\beta^{-1/2}\sum_{n=1}^{N_\beta}(-1)^{\sigma_\beta(n)}P_\beta(n)^\dagger T_{\beta(n)\alpha(1)} \tag{20}$$

and

$$T_{\beta(1)\tilde{\alpha}(1)}=N_\beta^{1/2}N_\alpha^{-1/2}\sum_{n=1}^{N_\alpha}(-1)^{\sigma_\alpha(n)}T_{\beta(1)\alpha(n)}P_\alpha(n) . \tag{21}$$

IV. ANTISYMMETRIZED COUPLED INTEGRAL EQUATIONS

The task will now be to find a set of dynamical equations for the antisymmetrized transition operators $T_{\tilde{\beta}(1)\alpha(1)}$ and $T_{\beta(1)\tilde{\alpha}(1)}$. Start by combining Eq. (8) with Eq. (21):

$$T_{\beta(1)\tilde{\alpha}(1)}=N_\beta^{1/2}N_\alpha^{-1/2}\sum_{n=1}^{N_\alpha}[V_{\beta(1)}+T_{\beta(1)\gamma(j)}G_{\gamma(j)}V_{\alpha(n)}](-1)^{\sigma_\alpha(n)}P_\alpha(n)$$

$$=N_\beta^{1/2}[V_{\beta(1)}P_\alpha+T_{\beta(1)\gamma(j)}G_{\gamma(j)}P_\alpha V_{\alpha(1)}]\,, \tag{22}$$

where we have defined

$$P_\alpha=N_\alpha^{-1/2}\sum_{n=1}^{N_\alpha}(-1)^{\sigma_\alpha(n)}P_\alpha(n)\,. \tag{23}$$

Let us next consider the partition-$\gamma(j)$ Green's function operator appearing in Eq. (22). It is related to the partition-$\gamma(1)$ Green's function operator by

$$G_{\gamma(j)}=P_\gamma(j)G_{\gamma(1)}P_\gamma(j)^\dagger\,. \tag{24}$$

The operator $P_\gamma(j)^\dagger$ will be operating to the right on the state $P_\alpha V_{\alpha(1)}|\Phi_{a(1)}\rangle$. By virtue of the definition of P_α and the fact that $\Phi_{a(1)}$ is to be chosen antisymmetric with respect to intracluster permutations, the state $P_\alpha V_{\alpha(i)}|\Phi_{a(i)}\rangle$ is completely antisymmetric with respect to fermion exchange. Thus, $P_\gamma(j)^\dagger$ acting on this state will have the same effect as its inverse $P_\gamma(j)$, namely, it simply produces a factor of $(-1)^{\sigma_\gamma(j)}$. It follows that Eq. (22) may be replaced by

$$T_{\beta(1)\tilde{\alpha}(1)}=N_\beta^{1/2}[V_{\beta(1)}P_\alpha+T_{\beta(1)\gamma(j)}(-1)^{\sigma_\gamma(j)}P_\gamma(j)G_{\gamma(1)}P_\alpha V_{\alpha(1)}]\,. \tag{25}$$

Equation (25) is valid for any choice of $\gamma_{(j)}$. Let us then average the right-hand side with respect to the N_γ possible values of j:

$$T_{\beta(1)\tilde{\alpha}(1)}=N_\beta^{1/2}N_\gamma^{-1}\sum_{j=1}^{N_\gamma}[V_{\beta(1)}P_\alpha+T_{\beta(1)\gamma(j)}(-1)^{\sigma_\gamma(j)}P_\gamma(j)G_{\gamma(1)}P_\alpha V_{\alpha(1)}]$$

$$=N_\beta^{1/2}V_{\beta(1)}P_\alpha+T_{\beta(1)\tilde{\gamma}(1)}G_{\gamma(1)}N_\gamma^{1/2}P_\alpha V_{\alpha(1)}\,. \tag{26}$$

Here we have a set of integral equations for the elements of the antisymmetrized transition operator matrix. The same procedure applied to Eqs. (7) and (20) leads to the alternative equations,

$$T_{\tilde{\beta}(1)\alpha(1)}=V_{\beta(1)}P_\beta^\dagger N_\alpha^{1/2}+V_{\beta(1)}P_\beta^\dagger N_\gamma^{-1/2}G_\gamma(1)[(P_\gamma^\dagger V_{\alpha(1)}N_\alpha^{1/2}-V_{\tilde{\gamma}(1)\alpha(1)})+T_{\tilde{\gamma}(1)\alpha(1)}]$$

$$=V_{\beta(1)}P_\beta^\dagger N_\gamma^{-1/2}G_{\gamma(1)}[P_\gamma^\dagger G_{\alpha(1)}{}^{-1}N_\alpha^{1/2}+T_{\tilde{\gamma}(1)\alpha(1)}]\,, \tag{27}$$

where

$$V_{\tilde{\gamma}(1)\alpha(1)}=N_\alpha^{1/2}N_\gamma^{-1/2}\sum_{n=1}^{N_\gamma}(-1)^{\sigma_\gamma(n)}P_\gamma(n)^\dagger V_{\gamma(n)}\,. \tag{28}$$

In Eqs. (26) and (27) we have dynamical equations for the antisymmetrized transition operators which are similar in structure to Eqs. (7) and (8) for the nonantisymmetrized transition operators. The new equations refer only to partitions $\alpha(1),\beta(1),\ldots,$ thus eliminating the physically equivalent channels $\alpha(j>1)$, $\beta(j>1),\ldots$. The price paid for this simplification is the appearance of the permutation operators P multiplying the residual interactions V. The critical element in our dynamical equations, Eqs. (7) and (8), that made it possible to perform the antisymmetrization is the freedom of choice of the subscript γ.

As before, we can use the partition coupling array to impose a particular coupling scheme on our set of coupled integral equations.

$$T_{\beta(1)\tilde{\alpha}(1)}=N_\beta^{1/2}V_{\beta(1)}P_\alpha+\sum_{\gamma(1)}T_{\beta(1)\tilde{\gamma}(1)}N_\gamma^{-1/2}G_{\gamma(1)}W_{\gamma(1)\alpha(1)}P_\alpha V_{\alpha(1)}\,, \tag{29}$$

$$T_{\tilde{\beta}(1)\alpha(1)}=\sum_{\gamma(1)}V_{\beta(1)}P_\beta^\dagger W_{\beta(1)\gamma(1)}N_\gamma^{-1/2}G_{\gamma(1)}[P_\gamma^\dagger G_{\alpha(1)}{}^{-1}N_\alpha^{1/2}+T_{\tilde{\gamma}(1)\alpha(1)}]\,. \tag{30}$$

V. ANTISYMMETRIZED COUPLED DIFFERENTIAL EQUATIONS

Hahn, Kouri, and Levin[6] have shown how the coupled integral equations for the transition operator may be transformed into a set of coupled differential equations. We will here apply that procedure to our equations for the antisymmetrized transition operators and thus arrive at a coupled-differential-equations formalism for scattering that includes the effects of exchange symmetry.

Let us first rewrite Eqs. (29) and (30) with deletion of the arguments of the subscripts:

$$T_{\beta\tilde{\alpha}} = N_\beta^{1/2} V_\beta P_\alpha + \sum_\gamma T_{\beta\tilde{\gamma}} N_\gamma^{-1/2} G_\gamma W_{\gamma\alpha} P_\alpha V_\alpha , \tag{31}$$

$$T_{\tilde{\beta}\alpha} = \sum_\gamma V_\beta P_\beta^\dagger W_{\beta\gamma} N_\gamma^{-1/2} G_\gamma \langle P_\gamma^\dagger G_\alpha^{-1} N_\alpha^{1/2} + T_{\tilde{\gamma}\alpha} \rangle . \tag{32}$$

It is now understood that only those partitions which cannot be transformed into each other by permutations of identical particles are to be included. We next define a vector Ψ in partition space by

$$T_{\beta\tilde{\alpha}} \Phi_a = V_\beta \Psi_{\beta\tilde{a}} , \tag{33}$$

and another such vector by

$$T_{\tilde{\beta}\alpha} \Phi_a = V_\beta \Psi_{\tilde{\beta}a} . \tag{34}$$

We first consider the antisymmetrized Baer-Kouri equations Eq. (31). Let us write Eqs. (31) and (33) as matrix equations in channel space:

$$\tilde{T} = N^{1/2} V U P + \tilde{T} N^{-1/2} G W P V , \tag{35}$$

$$\tilde{T} \phi_a = V \psi_a , \tag{36}$$

where

$$U_{\alpha\beta} = 1 , \tag{37}$$

$$(G)_{\alpha\beta} = G_\alpha \delta_{\alpha\beta} , \tag{37a}$$

$$(\phi_a)_\beta = \Phi_{\beta a} = \delta_{\alpha\beta} \Phi_a . \tag{38}$$

The formal solution of Eq. (35) is

$$\tilde{T} = V N^{1/2} U P (1 - N^{-1/2} G W P V)^{-1} . \tag{39}$$

Combining this with Eq. (36) gives

$$\psi_a = N^{1/2} U P (1 - N^{-1/2} G W P V)^{-1} \phi_a . \tag{40}$$

Define

$$\chi_a = (1 - G N^{-1/2} W P V)^{-1} \phi_a = \phi_a + G N^{-1/2} W P V \chi_a . \tag{41}$$

Then operating on both sides of Eq. (41) with G^{-1} and using the fact that

$$G^{-1} \phi_a = 0 \tag{42}$$

leads to the relationship

$$G^{-1} \chi_a = N^{-1/2} W P V \chi_a . \tag{43}$$

In component form this equation reads

$$G_\beta^{-1} \chi_{\beta a} = \sum_\gamma N_\beta^{-1/2} W_{\beta\gamma} P_\gamma V_\gamma \chi_{\gamma a} . \tag{44}$$

Now by Eqs. (40) and (41)

$$\Psi_{\beta a} = N_\beta^{1/2} \sum_\gamma U_{\beta\gamma} P_\gamma \chi_{\gamma a} = N_\beta^{1/2} \sum_\gamma P_\gamma \chi_{\gamma a} , \tag{45}$$

so that by Eq. (33)

$$T_{\beta\tilde{\alpha}} \Phi_a = V_\beta N_\beta^{1/2} \sum_\gamma P_\gamma \chi_{\gamma a} . \tag{46}$$

Equation (44) is our set of coupled differential equations and Eq. (46) shows how the solutions are related to the antisymmetrized transition operators.

Next let us derive a set of coupled differential equations from the antisymmetrized Kouri-Levin equations, Eq. (32). We write Eqs. (32) and (34) in matrix form:

$$\tilde{T} = V P^\dagger W N^{-1/2} G (P^\dagger U G^{-1} N^{1/2} + \tilde{T}) , \tag{47}$$

$$\tilde{T} \phi_a = V \psi_a . \tag{48}$$

Combining Eqs. (47) and (48) gives

$$\psi_a = P^\dagger W N^{-1/2} G P^\dagger U N^{1/2} G^{-1} \phi_a + P^\dagger W N^{-1/2} G V \psi_a . \tag{49}$$

Introduce

$$\theta_a = G P^\dagger U N^{1/2} G^{-1} \phi_a + G V P^\dagger W N^{-1/2} \theta_a , \tag{50}$$

so that

$$\psi_a = P^\dagger W N^{-1/2} \theta_a . \tag{51}$$

Then, by virtue of Eq. (42),

$$G^{-1} \theta_a = V P^\dagger W N^{-1/2} \theta_a . \tag{52}$$

In component form this equation reads

$$G_\beta^{-1} \theta_{\beta a} = \sum_\gamma V_\beta P_\beta^\dagger W_{\beta\gamma} N_\gamma^{-1/2} \theta_{\gamma a} . \tag{53}$$

Finally, combining Eqs. (48) and (51) we find

$$T_{\tilde{\beta}\alpha} \Phi_a = V_\beta P_\beta^\dagger \sum_\gamma W_{\beta\gamma} N_\gamma^{-1/2} \theta_{\gamma a} . \tag{54}$$

Equation (53) is our alternative set of coupled differential equations, and Eq. (54) shows how the solutions are related to the antisymmetrized transition operators.

VI. ANTISYMMETRIZED COUPLED INTEGRAL EQUATIONS FOR THE K MATRIX

In the nonantisymmetrized coupled-integral-equations formalism the transition operator can be assumed to fulfill any sort of homogeneous asymptotic boundary conditions. It all depends on the asymptotic behavior chosen for the Green's function operators. In Eqs. (3) and (4) we specified outgoing-wave boundary conditions for G and G_γ. As a result, T is the T-matrix transition operator. This choice is necessary because, as shown in Eqs. (13), (14), (16), and (17), the consequences of exchange symmetry are expressed most naturally in terms of the T matrix. Thus our antisymmetrized coupled-integral-equations formalism is a T-matrix theory.

On the other hand, a K-matrix version of the formalism is preferable because it lends itself to approximations that preserve unitarity and only requires the evaluation of real quantities. The translation of Eqs. (13), (14), (16), and (17) into statements about the K-matrix operator appears to be a very difficult task. It would appear to be preferable to find a K-matrix operator that could be related to the antisymmetrized T-matrix operator by a Heitler equation.

Suppose we write the antisymmetrized Kouri-Levin equations, Eq. (47), in the form

$$\tilde{T} = \tau G P^\dagger U G^{-1} N^{1/2}, \quad \tilde{T}_{\beta\alpha} = T_{\tilde{\beta}\tilde{\alpha}} \tag{55}$$

$$\tau = V P^\dagger W N^{-1/2}(1 + G\tau). \tag{56}$$

Let

$$\Gamma = G + i\pi\Delta \tag{57}$$

be the standing-wave Green's function operator. Then if we define the K-matrix operator to be the solution of

$$\kappa = V P^\dagger W N^{-1/2}(1 + \Gamma\kappa), \tag{58}$$

the operators κ and τ will be related by

$$\begin{aligned} V P^\dagger W N^{-1/2} &= \tau(1 + G\tau)^{-1} \\ &= \kappa(1 + \Gamma\kappa)^{-1} \\ &= (1 + \kappa\Gamma)^{-1}\kappa . \end{aligned} \tag{59}$$

This leads to a Heitler equation

$$\tau = \kappa - i\pi\kappa\Delta\tau . \tag{60}$$

Thus we can regard Eq. (58) as the antisymmetized coupled integral equations for the K matrix. By means of Eqs. (60) and (55) the resulting K matrix can be used to calculate the antisymmetrized T matrix.

A similar analysis can be applied to the antisymmetrized Baer-Kouri equations, Eq. (35). They can be given the form

$$\begin{aligned} \tilde{T} &= N^{1/2} V P U (N^{-1/2} W P V)^{-1}\tau \\ &= N^{1/2} V P U V^{-1} P^\dagger W^{-1} N^{1/2}\tau, \quad \tilde{T}_{\beta\alpha} = T_{\tilde{\beta}\tilde{\alpha}} \end{aligned} \tag{61}$$

$$\tau = (1 + \tau G) N^{-1/2} W P V . \tag{62}$$

Now κ is defined by

$$\kappa = (1 + \kappa\Gamma) N^{-1/2} W P V . \tag{63}$$

Again κ and τ are related by the Heitler equation, Eq. (60). This form of the theory is rather questionable since the operator V^{-1} does not always exist.

VII. SUMMARY

To summarize our results we can state that the CEF integral equations (in matrix form)

$$T = VWG(UG^{-1} + T) \tag{64}$$

or

$$T = VU + TGWV \tag{65}$$

can be antisymmetrized, and the result is

$$\tilde{T} = V P^\dagger W N^{-1/2} G(P^\dagger U G^{-1} N^{1/2} + \tilde{T}) \tag{66}$$

or

$$\tilde{T} = N^{1/2} V U P + \tilde{T} N^{-1/2} G W P V , \tag{67}$$

where U is defined by Eq. (37), and N and P are diagonal matrices whose elements are defined by Eqs. (15) and (23), respectively. Equations (57) and (58) differ from Eqs. (55) and (56) not only by the presence of the factors $N^{1/2}$ and P but also by the reduction of the dimension of the partition space due to the elimination of physically indistinguishable partitions. The CEF differential equations

$$G^{-1}\chi_a = WV\chi_a, \quad T\phi_a = VU\chi_a \tag{68}$$

or

$$G^{-1}\theta_a = VW\theta_a, \quad T\phi_a = VW\theta_a \tag{69}$$

can be antisymmetrized, with the result

$$G^{-1}\tilde{\chi}_a = N^{-1/2} W P V \tilde{\chi}_a, \quad \tilde{T}\phi_a = V N^{1/2} U P \tilde{\chi}_a \tag{70}$$

or

$$G^{-1}\tilde{\theta}_a = V P^\dagger W N^{-1/2}\tilde{\theta}_a, \quad \tilde{T}\phi_a = V P^\dagger W N^{-1/2}\tilde{\theta}_a . \tag{71}$$

As with the integral equations, the dimensionality of the partition-space matrices in the antisymmetrized differential equations is considerably reduced.

A K-matrix version of the antisymmetrized CEF integral equations is provided by the coupled equation

$$\kappa = V P^\dagger W N^{-1/2}(1 + \Gamma\kappa) .$$

A reduced T-matrix operator τ is calculated from the K-matrix operator κ by the Heitler equation

$$\tau = \kappa - i\pi\kappa\Delta\tau .$$

Then the antisymmetrized T-matrix operator is given by

$$\tilde{T} = \tau G P^{\dagger} U G^{-1} N^{1/2} .$$

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