Resonating-group calculation of $n + {}^3\mathrm{H}$ scattering*

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The single-channel resonating-group method with a central nucleon-nucleon potential is used to study the $n + {}^3\mathrm{H}$ system up to a c.m. energy of about 17 MeV. The specific distortion of the triton in this system is investigated, and it is found that its effect on the $n + {}^3\mathrm{H}$ phase shifts is small. A phenomenological imaginary potential is included in the calculation in order to account approximately for the effects of reactions on the elastic channel. The strength of this potential is chosen to reproduce measured total reaction cross sections for the charge-conjugate system $p + {}^3\mathrm{He}$. The results of the calculation are in satisfactory agreement with measured $n + {}^3\mathrm{H}$ elastic differential cross sections.

[NUCLEAR REACTIONS $^3\mathrm{H}(n,n)$, $E_{\mathrm{c.m.}} = 4.5$–17.25 MeV; calculated phase shifts and $\sigma(\theta)$; studied specific distortion. Resonating-group method with imaginary potential.]

I. INTRODUCTION

In previous studies of specific distortion in the $d+\alpha$ system[1] and in the $\alpha+\alpha$ system,[2] the wave functions were constructed by adding to the usual single-channel resonating-group wave functions a sum of square-integrable functions. These additional distortion functions allow greater flexibility in the wave function in the region where the nucleons interact strongly. In the $d+\alpha$ study,[1] the distortion functions were given a $d+\alpha$ cluster structure with a relative-motion function of a harmonic-oscillator type, with the deuteron cluster having an rms matter radius either larger or smaller than that of the free deuteron, and with the α cluster having an rms matter radius equal to that of the free α particle. For this system the effects of deuteron distortion were found to be quite large. In the $\alpha+\alpha$ study[2] the distortion functions contained an $\alpha+\alpha$ cluster structure, and both α particles were allowed to undergo distortion. For this system the effects of α distortion were found to be small. Thus, as expected, effects of specific distortion are large when the system contains a loosely bound cluster and are small when the system consists of tightly bound clusters. The $n + {}^3\mathrm{H}$ system, which we investigate here, is a system intermediate between the two in which specific distortion has already been studied, in the sense that ${}^3\mathrm{H}$ is more tightly bound than the deuteron, but not so tightly bound as the α particle.

II. FORMULATION

The formulation of the calculation for the $n + {}^3\mathrm{H}$ system with distortion is quite similar to that of Refs. 1 and 2. The wave function is taken to be

$$\Psi = \mathcal{A}\left[\phi_t F(\vec{r})\xi + \sum_{i=1}^{N} \phi_i G_i(\vec{r})\xi\right], \tag{1}$$

where $\mathcal{A}$ is an antisymmetrization operator, ϕ_t is the spatial part of the free triton wave function, the ϕ_i are spatial parts of distorted triton wave functions, $\vec{r}$ is the vector from the center of mass (c.m.) of the triton to the neutron, the relative-motion function $F(\vec{r})$ contains the proper asymptotic boundary condition, the $G_i(\vec{r})$ are relative-motion parts of the N distortion functions, and ξ is a spin-isospin function. The free triton function ϕ_t is taken as

$$\phi_t = \sum_{i=1}^{3} A_i \exp\left[-\tfrac{1}{2}\alpha_i \sum_{j=1}^{3} (\vec{r}_j - \vec{R}_t)^2\right], \tag{2}$$

where $\vec{R}_t$ is the c.m. coordinate of the triton, and the values for A_i and α_i are chosen by minimizing the expectation value of the triton Hamiltonian operator with a central nucleon-nucleon potential which is given by Eqs. (4), (5), and (6) of Ref. 2 and which fits[3] reasonably well the low-energy nucleon-nucleon data. Such a minimization is necessary if the distortion functions are to represent the polarizing influence of the incident neutron on the triton rather than the contraction of the triton under the influence of the nonsaturating nucleon-nucleon force which we use. This minimization yields the values

$$\begin{aligned} A_1 &= 1.00, \quad \alpha_1 = 0.172\ \mathrm{fm}^{-2}, \\ A_2 &= 9.11, \quad \alpha_2 = 0.510\ \mathrm{fm}^{-2}, \\ A_3 &= 16.60, \quad \alpha_3 = 1.320\ \mathrm{fm}^{-2}, \end{aligned} \tag{3}$$

and a binding energy and rms matter radius for the triton of 7.94 MeV and 1.52 fm, respectively.

The functions ϕ_i and G_i of Eq. (1) are given the forms

$$\phi_i = \exp\left[-\tfrac{1}{2}\tilde{\alpha}_i \sum_{j=1}^{3} (\vec{r}_j - \vec{R}_t)^2\right] \tag{4}$$

and

$$G_i(\vec{r}) = \sum_l \tilde{A}_{li}\, \frac{g_{li}(r)}{r}\, P_l(\cos\theta), \tag{5}$$

where

$$g_{li}(r) = r^{n+1} \exp(-\tfrac{3}{8}\tilde{\beta}_i r^2), \tag{6}$$

with

$$\begin{aligned} &n=2 \text{ for } l=0, \\ &n=l \text{ for } l\geq 1. \end{aligned} \tag{7}$$

The values for n expressed by Eq. (7) result from simple shell-model arguments similar to those of Ref. 4 for the mass-6 system.

The choice of the nonlinear parameters $\tilde{\alpha}_i$ [Eq. (4)] and $\tilde{\beta}_i$ [Eq. (6)] will be discussed in Sec. III. The linear parameters $\tilde{A}_{li}$ [Eq. (5)] and the relative-motion function $F(\vec{r})$ [Eq. (1)] are obtained by solving, for fixed $\tilde{\alpha}_i$ and $\tilde{\beta}_i$, the variational equation

$$\langle \delta\Psi | (H - E') | \Psi \rangle = 0, \tag{8}$$

where the variation is carried out over the $\tilde{A}_{li}$ and $F(\vec{r})$. Here E' is the total energy in the c.m. system, and the Hamiltonian operator H is given by

$$H = -\frac{\hbar^2}{2M} \sum_{i=1}^{4} \nabla_i^2 + \sum_{i<j=1}^{4} V_{ij} - T_{\text{c.m.}}, \tag{9}$$

where V_{ij} is the nucleon-nucleon potential mentioned immediately after Eq. (2) and $T_{\text{c.m.}}$ is the c.m. kinetic-energy operator.

III. NONLINEAR PARAMETERS AND RESULTS

The procedure we employ to select the values of the nonlinear parameters $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ to be used in the distortion calculations is quite similar to that of Refs. 1 and 2, and therefore we describe this procedure only briefly here. Because it is simpler to observe effects of varying the distortion parameters for bound states rather than for scattering states, an artificial bound ${}^3P$ state of ${}^4\mathrm{H}$ was created by the use of a nucleon-nucleon potential containing no space-exchange component.[5] Such a force yields a bound state which, when no distortion functions are included in the calculation, has a neutron separation energy of 0.532 MeV. With one distortion function in the calculation [$N=1$ in Eq. (1)], a grid search on $\tilde{\alpha}_1$ and $\tilde{\beta}_1$ was carried out to determine values which yield local maxima in the neutron separation energy. As in previous studies,[1,2] two such one-distortion local maxima were found. Both maxima have $\tilde{\beta}_1$ values near 1.0 fm^{-2}, and the values of α_1 for the two maxima are 0.22 and 0.95 fm^{-2}, respectively. These $\tilde{\alpha}_1$ values yield triton rms matter radii of about 2.1 and 1.0 fm, respectively, which bracket the value 1.52 fm associated with ϕ_t. In our scattering calculations we will use a more realistic, less attractive nucleon-nucleon potential than we use for the bound-state study. Therefore, from past experience we expect that the range of the relative-motion part of a one-distortion function which produces a stationary behavior of the phase shifts for scattering states with this less attractive potential will be larger than the range obtained above, which produces a relative maximum in the bound-state separation energy. This suggests that values for $\tilde{\beta}_i$ smaller than 1.0 fm^{-2} should be used in the $n+{}^3\mathrm{H}$ scattering calculations. To investigate this point, one-distortion calculations were made of some $n+{}^3\mathrm{H}$ phase shifts for a range of values of $\tilde{\beta}_1$ from 0.13 to 0.67 fm^{-2}. These calculations were made at c.m. energies of 3 and 9 MeV, for values of $\tilde{\alpha}_1$ of 0.2 and 0.95 fm^{-2}, and with a pure Serber nucleon-nucleon potential.[6] It was observed in these calculations that most of the phase shifts displayed a stationary behavior as a function of $\tilde{\beta}_1$; for example, the ${}^1S_0$ phase was stationary for $\tilde{\beta}_1$ close to a value of 0.4 fm^{-2} at both energies and at both values of $\tilde{\alpha}_1$ investigated.

In our final calculation of the effects of distortion on $n+{}^3\mathrm{H}$ scattering, we employed nine distortion functions [$N=9$ in Eq. (1)] with the parameters $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ listed in Table I. These parameters were chosen as a result of our preliminary one-distortion calculations.[7] A pure Serber nucleon-nucleon potential was used. The $n+{}^3\mathrm{H}$ phase shifts from the nine-distortion calculation were compared with those from a no-distortion calculation over the energy range 0 to 50 MeV. It was found that the inclusion of distortion has only a small effect on the phase shifts; in particular, both the position and shape of the low-energy $l=1$ resonances are very similar in the two calculations. Our conclusion is that effects of specific distortion can be neglected in the $n+{}^3\mathrm{H}$ system and, therefore, also in the mirror system $p+{}^3\mathrm{He}$.

TABLE I. Values (fm^{-2}) of $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ used to study the effect of distortion on $n+{}^3\mathrm{H}$ scattering.

i	1	2	3	4	5	6	7	8	9
$\tilde{\alpha}_i$	0.2	0.2	0.2	0.4	0.4	0.4	1.0	1.0	1.0
$\tilde{\beta}_i$	0.21	0.40	0.67	0.21	0.40	0.67	0.21	0.40	0.67

IV. GENERAL DISCUSSION OF DISTORTION CALCULATIONS

Several systems[1,2,8,9] have now been studied for the effects of distortion. To compare these effects it is useful to list for each system a quantity Δu which is a measure of the strength of the distortion. This quantity Δu is an increment in u, the space-exchange parameter of the nucleon-nucleon potential,[2] and is defined as the increase in the value of u which one must make in a no-distortion calculation for a system in order to obtain agreement with the phase shifts obtained from a calculation with distortion for the same system. As was discussed in Ref. 1, such a quantity is angular-momentum and energy dependent. In Table II we list values for Δu at low energies and for the lowest l values of the "Pauli-favored" states.[10]

Some general comments, based on our experience with distortion calculations, are now made. First, a nucleus is more easily distorted the higher its compressibility. This is indicated in Table II by the increase in the value of Δu as one proceeds from the system $\alpha+\alpha$ to $^3\mathrm{He}+\alpha$ to $d+\alpha$. Second, a nucleus is more distorted the larger the number of nucleons in the other nucleus which causes the distortion. This is indicated in Table II by Δu being larger for the $\alpha+{}^3\mathrm{He}$ system than for the $n+{}^3\mathrm{H}$ system. Third, except near energies where resonances occur, distortion effects decrease with increasing energy. This is shown, for example, by Table III of Ref. 1, and agrees with the intuitive feeling that distortion effects should decrease with decreasing reaction time. Fourth, except for resonance effects and odd-even effects,[10] the influence of distortion decreases with increasing orbital angular momentum. This agrees with the expectation that distortion effects should decrease with increasing cluster-cluster average separation during the interaction. Fifth, distortion appears to be stronger in "Pauli-favored" states.[10] An example of this effect is seen in Figs. 4 and 5 of Ref. 1.

TABLE II. Low-energy values of Δu in states of orbital angular momentum l for systems in which distortion has been studied.

System	l	Δu	Ref.
$d+\alpha$	0	0.2	1
$^3\mathrm{He}+\alpha$	1	0.03	8
$\alpha+\alpha$	0	0.02	2
$n+{}^3\mathrm{H}$	1	0.01	Present work

V. CALCULATIONS WITH IMAGINARY POTENTIAL

We have seen above that effects of specific distortion are small for $n+{}^3\mathrm{H}$ scattering, and therefore it is reasonable to perform no-distortion calculations to compare with experimental data. To do this, two changes are made in the formulation of Sec. II. One change is that for the triton wave function we replace Eq. (3) by

$$A_1=1.00,\quad \alpha_1=0.25\ \mathrm{fm}^{-2},$$
$$A_2=3.17,\quad \alpha_2=0.71\ \mathrm{fm}^{-2}, \tag{10}$$
$$A_3=0.$$

The values in Eq. (10) are taken from Ref. 11 and yield an rms matter radius of 1.67 fm, a value which is close to mass-3 radii deduced[12] from electron scattering data and which yield a good fit to the mass-3 body form factor. In addition, they yield a triton binding energy of 6.91 MeV. For scattering calculations, however, it is much more important that the cluster internal wave functions yield proper cluster sizes than that they yield proper binding energies (for example, 8.48 MeV for the triton). Thus, in order to compare our calculations with experiment, we use the triton function given by Eqs. (2) and (10), although we stress again that it was necessary in the distortion calculations to use the parameters given by Eq. (3) in order that the changes which occur in the $n+{}^3\mathrm{H}$ phase shifts when distortion functions are included represent the polarizing influence of the neutron on the triton. The other change in the formulation of Sec. II is that for calculations at energies above the reaction threshold of 6.26 MeV (c.m.), a phenomenological imaginary potential is included in order to account approximately for the influence of reaction channels on the elastic channel. The imaginary potential we use here has the form of Eq. (10) of Ref. 13. This potential is a Woods-Saxon type having a common strength W_0 for the volume and surface parts. The diffuseness of this potential is taken as $a=0.5$ fm, a value which we have used in calculations for other systems, and the radius of this potential is taken as $R=2.5$ fm, a value which is about equal to the rms radius of the direct part of the nuclear potential for the $n+{}^3\mathrm{H}$ system.[11] The space exchange parameter u of the nucleon-nucleon potential[11] is set equal to 1.0. This value corresponds to a pure Serber force and is close to values used in calculations for other systems. For example, the value $u=0.99$ was used in a calculation[14] of $p+\alpha$ scattering in which no nucleon-nucleon spin-orbit force was included.

Figure 1 shows a comparison of calculated $n+{}^3\mathrm{H}$ differential cross sections with experimental

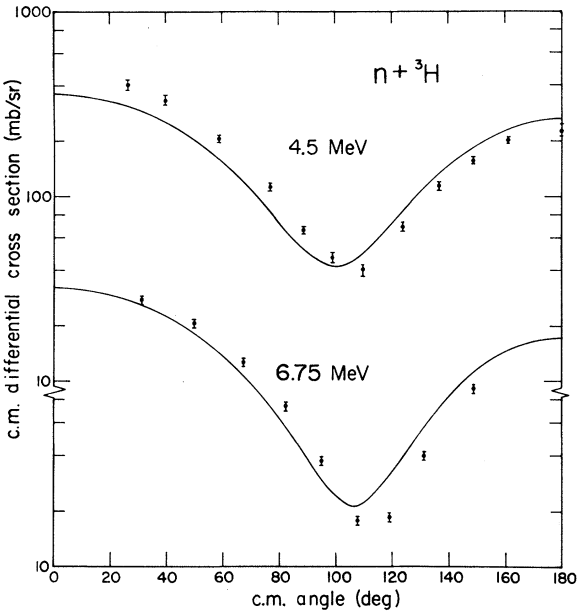

FIG. 1. Comparison of calculated cross sections (curves) for $n+{}^3H$ scattering with experimental data (points) at indicated c.m. energies. The 4.5-MeV data are from Ref. 16 and the 6.75-MeV data are from Ref. 17.

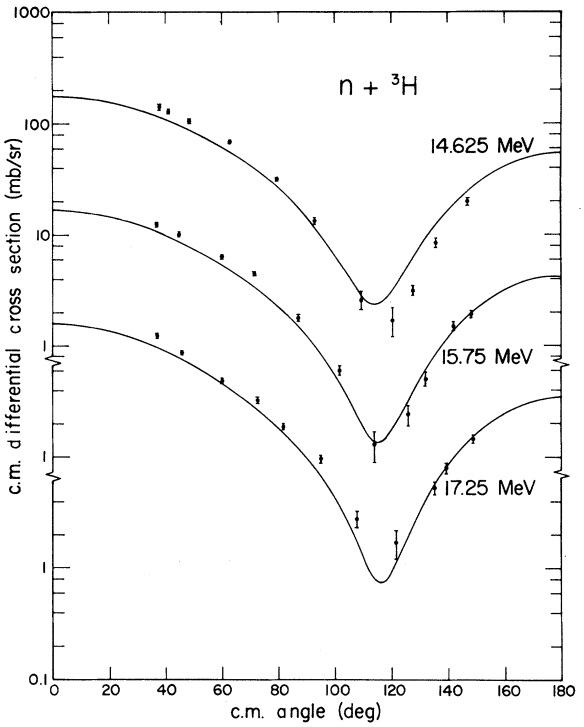

FIG. 2. Comparison of calculated cross sections (curves) for $n+{}^3H$ scattering with the experimental data (points) of Ref. 17 at the indicated c.m. energies.

data at c.m. energies[15] of 4.5 MeV[16] and 6.75 MeV.[17] The strength W_0 of the imaginary potential was set equal to zero for both energies because 6.75 MeV is only slightly above the reaction threshold. The general shapes of the calculated cross sections are satisfactory; however, they are somewhat too small at forward angles and somewhat too large at backward angles. In Fig. 2 we compare our calculated cross sections with experimental data[17] at three higher c.m. energies. For these calculations the strength W_0 of the imaginary potential at each energy was chosen so that the calculated total reaction cross section σ_R agreed with measured values[18] of σ_R for the $p+{}^3He$ system at the same energy. Values for W_0 and σ_R are listed in Table III. It can be seen in Fig. 2 that the agreement of the calculated differential cross section with the experimental one becomes better as the energy increases, especially at backward angles.

An interesting feature of both the experimental and calculated cross sections, which can be noted in Figs. 1 and 2, is that the angular position of the minimum in the cross section moves toward a more backward angle as the energy increases. This behavior with energy is opposite to that expected for a diffraction-type minimum. Here the minimum is generated by the interference between a forward-peaked direct amplitude and a backward-peaked exchange amplitude. The change with energy of the angular position of the cross-section minimum is caused by the fact that, as the energy increases, the exchange amplitude becomes weaker in relation to the direct amplitude.

In Table IV we give the phase shifts for the calculations of Figs. 1 and 2. These phases may be useful as starting values for phase-shift analyses of $n+{}^3H$ scattering data.

VI. CONCLUSION

We have examined the effects of specific distortion in the $n+{}^3H$ system and, by considering these results along with those of distortion calculations for other systems, we have been able to determine some general characteristics of specific distortion effects in nuclear systems. For example, it was found that a nucleus under-

TABLE III. Imaginary potential strength W_0 (MeV) and calculated total reaction cross section σ_R (mb) at c.m. energies E (MeV) for the calculations shown in Fig. 2.

E	W_0	σ_R
14.625	0.25	43.4
15.75	0.45	61.3
17.25	0.65	76.4

TABLE IV. Calculated phase shifts $^{2s+1}\delta_l$ (deg) for the $n+{}^3$H system at c.m. energies E (MeV).

E	4.5	6.75	14.625	15.75	17.25
$^1\delta_0$	108.52	96.98	$69.79+i5.19$	$64.61+i5.11$	$60.31+i4.75$
$^1\delta_1$	35.56	38.19	$35.38+i1.20$	$34.80+i2.13$	$34.04+i3.01$
$^1\delta_2$	−1.88	−2.71	$-1.09+i0.45$	$-0.56+i0.92$	$0.17+i1.51$
$^1\delta_3$	0.29	0.71	$2.32+i0.08$	$2.52+i0.16$	$2.79+i0.29$
$^1\delta_4$	−0.02	−0.06	$-0.17+i0.01$	$-0.15+i0.02$	$-0.12+i0.05$
$^1\delta_5$	0	0.01	$0.09+i0$	$0.10+i0$	$0.12+i0.01$
$^3\delta_0$	112.81	102.01	$76.47+i4.49$	$71.86+i4.51$	$67.90+i4.35$
$^3\delta_1$	43.62	48.86	$48.19+i1.28$	$47.56+i2.23$	$46.67+i3.07$
$^3\delta_2$	−1.50	−2.12	$-0.59+i0.45$	$-0.13+i0.90$	$0.52+i1.49$
$^3\delta_3$	0.24	−0.59	$2.11+i0.08$	$2.34+i0.17$	$2.65+i0.30$
$^3\delta_4$	−0.02	−0.05	$-0.12+i0.01$	$-0.11+i0.02$	$-0.07+i0.05$
$^3\delta_5$	0	0.01	$0.07+i0$	$0.08+i0$	$0.10+i0.01$

goes distortion more easily the more compressible it is, and a nucleus causes distortion more readily the more nucleons it contains.

An important conclusion drawn from the $n+{}^3$H distortion calculation is that distortion effects are weak enough to be neglected in studying $n+{}^3$H scattering. Therefore, to calculate differential cross sections to compare with experiment a no-distortion calculation was performed, which included a phenomenological imaginary potential whose strength was chosen at each energy to reproduce the experimental total reaction cross section for the charge-conjugate system $p+{}^3$He. The comparison of the calculated differential cross section with experiment is satisfactory, with better agreement being obtained at the higher energies. In particular, the change toward a larger angle with increasing energy of the angular position of the cross section minimum is explained by the calculation as being due to a decrease in the magnitude of the exchange amplitude relative to the direct amplitude as the energy increases.

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[5]This potential is obtained by setting the exchange parameter u equal to 2 in Eq. (4) of Ref. 2.

[6]This corresponds to setting the exchange parameter u equal to 1 in Eq. (4) of Ref. 2. This type of potential is much more realistic than the potential we use to generate a ^{4}H bound state. We use a Serber potential in all the present scattering calculations.

[7]The value $\tilde{\alpha}_i = 0.4$ fm^{-2} corresponds to a triton rms matter radius about midway between those corresponding to $\tilde{\alpha}_i = 0.2$ fm^{-2} and $\tilde{\alpha}_i = 1.0$ fm^{-2}.

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