Comparison of the angular velocity and the angular momentum expansions for yrast band level energies in even-even nuclei

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The results from the three- and the four-parameter expressions of the angular velocity expansion (AVE) and the angular momentum expansion (AME) are compared with the available experimental information on transition energies for yrast band levels in even-even nuclei. It is found that from the point of view of range of applicability, degree of fit to experimental transition energies, and high spin behavior, AME gives distinctly better results than AVE. The critical spin for backbending calculated from three-parameter AME is found to be in a fair agreement with the experimental data.

[NUCLEAR STRUCTURE Even-even rare earth nuclei, calculated yrast band transition energies, three- and four-parameter expansions in angular velocity and angular momentum, compared with experimental data.]

With the availability of precise excitation energies for an ever increasingly large number of levels in yrast bands of even-even nuclei in various regions of the Periodic Table, the earlier classification of such nuclei as nonspherical or spherical has lost its meaning, and the usual energy expressions in terms of the rotational bands picture are no longer appropriate for their description. The inadequacy of the usual power series expansion in $I(I+1)$ with a reasonably small number of terms (and free parameters) for describing high spin states in well deformed nuclei and even for moderately high spin states in transitional nuclei was pointed out by Sood,[1] who went on to sum up the infinite power series in $I(I+1)$ taking guidance from molecular spectra to obtain a two-parameter expression which proved to be unexpectedly successful[2] for describing the rotational energies in deformed nuclei with $E(4^+)/E(2^+) \gtrsim 2.90$. However, since then many such bands have been populated in nuclei wherein this energy ratio is well below the above limit. In such cases the angular velocity expansion (AVE) in power series of ω^2 first proposed by Harris[3] through consideration of the higher order cranking corrections proved to be quite successful. The celebrated variable moment of inertia (VMI) model of Mariscotti, Scharff-Goldhaber, and Buck[4] has been shown to be mathematically equivalent to the two-parameter Harris expression. Recently a tabulation of the up-to-date precisely determined excitation energies of levels in yrast bands of even-even nuclei between Ce and Pt has been provided by the Stockholm group.[5] They also used the AVE with two, three, and four parameters with a view to describe the experimental transition energies as correctly as possible and concluded that the four-parameter fit gives good agreement with the experiment up to about 10 units of angular momentum for most of the nuclei in the rare earth region. In view of the established[1] nonconvergence of the $I(I+1)$ expansion, and the possible interplay of the rotational and the vibrational aspects in the not-so-well-deformed nuclei,[6] consideration of a power series expansion in angular momentum I (hereafter referred to as AME) gave the result that the three-parameter cubic polynomial (CP) in I yields[7,8,10] agreement with the experiment which is on the whole better than that achieved in any other phenomenological description and is distinctly better than that obtained with the VMI model[4] or its three-parameter extension.[9] Here we compare the results of the three- and the four-parameter expressions from angular velocity expansion (AVE) and similar expressions from angular momentum expansion (AME) for describing the experimental transition energies and other features for levels in yrast bands of even-even nuclei.

Since the results of detailed computations with AVE are available in the literature,[5] we follow the same procedure for evaluating the parameters of AME. Briefly the procedure and the scope of these calculations is as follows. The quantities to be fitted are the experimental transition energies, rather than the level excitation energies, thus setting up a more stringent requirement than usually adopted in phenomenological approaches. Further, instead of including all the known energies, the evaluation of respective parameters is under-

taken with only one degree of freedom, i.e., for three-parameter fits the lowest four transition energies are used as input data. This normally means that a minimum of four transition energies known with an accuracy better than 2 keV are needed. Within this constraint the study includes 58 nuclei from ^{128}Ce to ^{194}Pt, the transition energies in more than two-thirds of the cases being known to within a fraction of a keV.

As mentioned above, the respective parameters are determined from the experimental transition energies using one degree of freedom. These parameters are then used to calculate the transition energies for higher spin levels and root mean square deviation of the predictions from the experiment is calculated. For consistency in comparison, calculations for AME have included the same experimental data as used by Saethre *et al.*[5] for AVE. No newer data have been included. The maximum spin level up to which rms deviations are calculated are the same as in their case or correspond to the value where back-bending anomalies become evident.

In the following we compare the results from the AVE and the AME approaches on three scores—the region of applicability, the quantitative fits to experimental transition energies, and the high spin behavior. As regards the fits to transition energies, the results are viewed on three considerations—the transition energies as a function of spin, the $\mathcal{I}$-ω^2 plots, and the percentage rms deviations of the calculated transition energies from the observed ones for individual nuclei and also for the whole range of nuclei considered. As a matter of notation we add the number 3 or 4 denoting the number of parameters following the abbreviations AVE and AME.

About the region of applicability of the two ap-

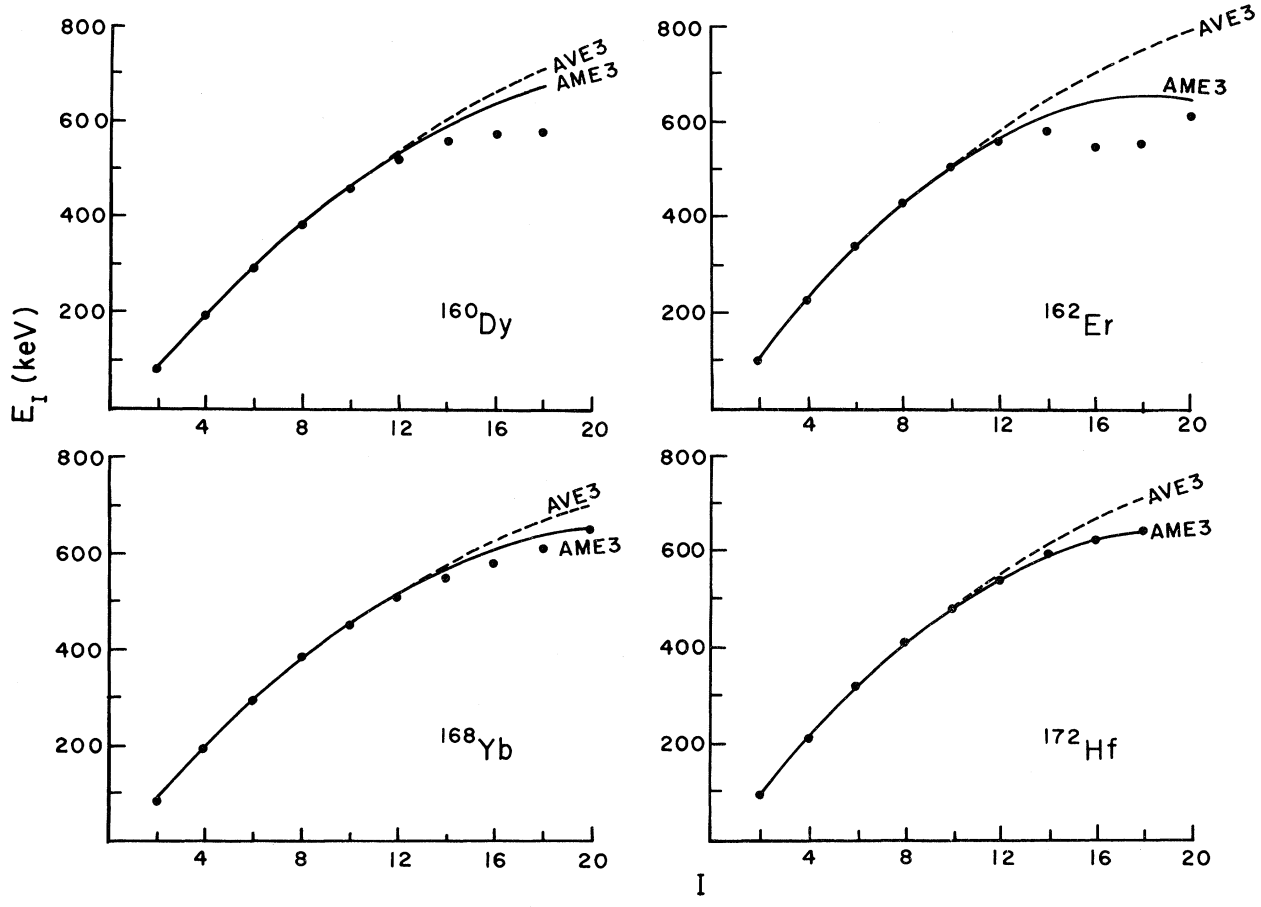

FIG. 1. The experimental and calculated transition energies in four selected doubly even nuclei plotted as a function of the level spin. The dots indicate the experimental values; the solid curves correspond to fits with the three-parameter angular momentum expansion (AME3) and the dashed curves represent the fits with the three-parameter angular velocity expansion (AVE3). For clarity we represent the calculated values in this and both the following figures by smooth curves obtained by joining the points at even spin values.

proaches we find from Saethre *et al.*[5] that both AVE3 and AVE4 probably do not yield results worth reporting for nuclei with $N \leq 88$ or $N \geq 114$ in the rare earth region, i.e. the region of meaningful applicability in rare earth nuclei includes only $90 \leq N \leq 112$. Even in this restricted region, AVE3 results are reported only for spin values up to 8^+ for platinum isotopes with $A = 182$, 184, and 186 and up to 10^+ for all nuclei with $N = 90$ and also for ^{186}Os, ^{188}Os, ^{188}Pt, ^{130}Ce, and ^{134}Ce. Thus out of a total of 58 nuclei for which they list a confirmed 8^+ identification AVE3 has a restricted applicability for about one-third of the cases. In contrast with this, AME3 as well as AME4 yield meaningful results for all these nuclei.

Let us now look at the quantitative fits to the experimental transition energies obtainable in the two approaches. We discuss in somewhat detail the comparison of the results from the three-parameter expressions and then summarize those from the four-parameter fits. Having evaluated the respective parameters with one degree of freedom in each case, the transition energies were calculated for comparison with the experiment. In Fig. 1 we present our results from AME3 in comparison with those from AVE3 and from the experiment for nuclei included in Fig. 1 of Saethre *et al.*[5] The nature of agreement in each case is quite evident and is typical of most of the cases. In Fig. 2 we plot the percentage rms deviations of the calculated values from the experimental energies for a few sets of nuclei which typically represent the

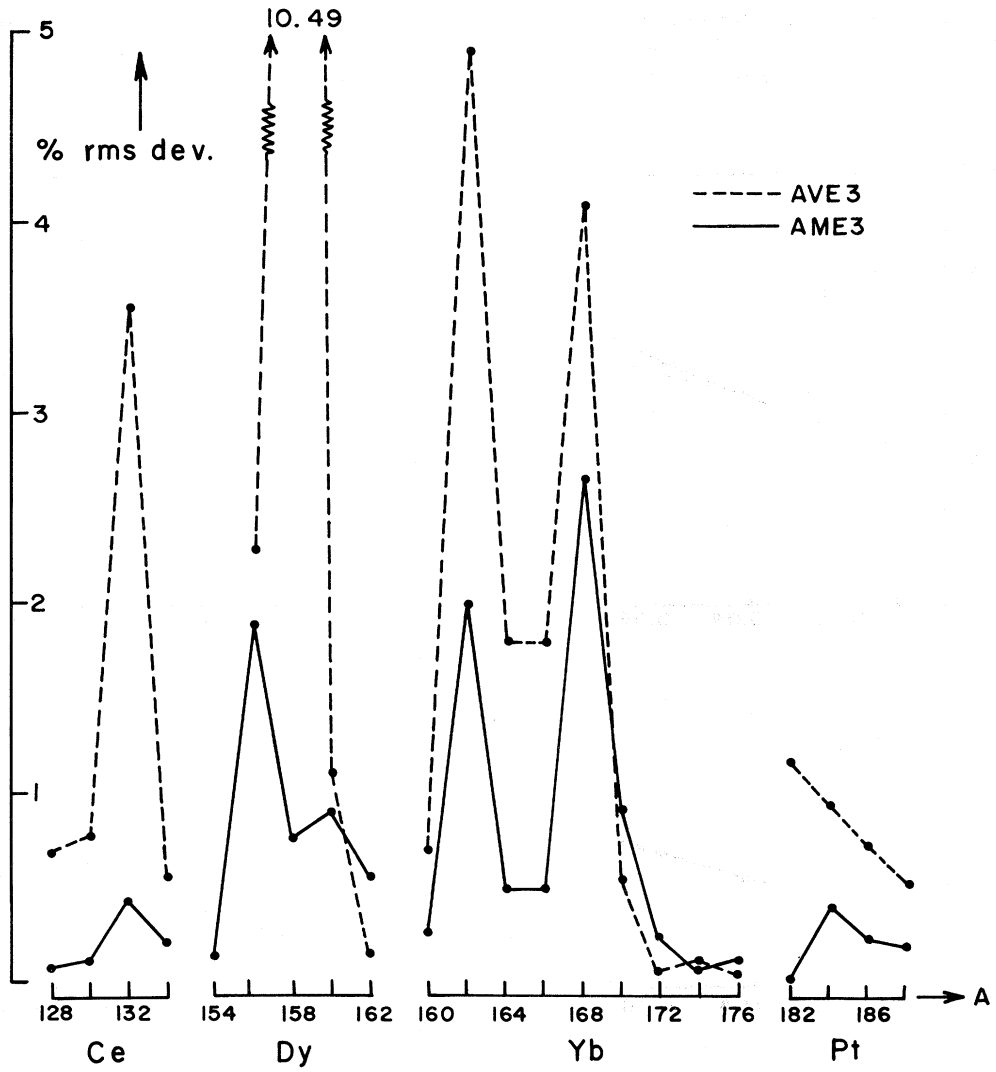

FIG. 2. Percentage rms deviations of those calculated from experimental transition energies for four sets of nuclei. Notation is the same as in Fig. 1.

over-all trends. Specifically we find that the fits from AME3 are distinctly better than or comparable to those from AVE3 for 47 out of the 58 nuclei available for analysis, i.e. for over 80% of the cases. Leaving aside the 6 nuclei with $N \leq 88$ or $N \geq 114$, for which AVE3 does not even attempt a fit, the over-all rms deviation for 52 nuclei is 0.83% from AME3 and 1.35% from AVE3.

The results for four-parameter description are not too much different. As remarked by Saethre *et al.*[5] in the case of AVE a significant improvement is noticed in going from three to four parameters. However for AME the three-parameter description is already unexpectedly good and the improvement obtained in going from three to four parameters is just that expected from adding one more free parameter. Quantitatively we have 39 nuclei with a confirmed 10^+ identification, out of which AME4 gives a fit better than or comparable to that with AVE4 for 28 cases. Leaving aside the three cases for which AVE4 results are not listed, we find that for 36 nuclei the over-all rms deviation is 0.59% from AME4 as compared to 0.72% from AVE4.

Next we examine the $\mathcal{I}$-ω^2 plots for which a few typical illustrations are presented in Fig. 3. The nuclear moments of inertia $\mathcal{I}$ and the squared angular velocity ω^2 are derived from the respective transition energies with the usual prescription.[11] The superiority of AME3 over AVE3 in this representation is evident and is further discussed below.

The main utility of the phenomenological energy expressions from the experimentalist's point of view is to provide guidance about the extension of the known domains by extrapolation, etc. It is rapidly becoming evident that in most of the even-even nuclei, regardless of their being well-deformed transitional or nearly spherical in the commonly used classification, the energies of levels in yrast bands are expected to exhibit the so-called backbending behavior[11] at some critical spin value I_c.

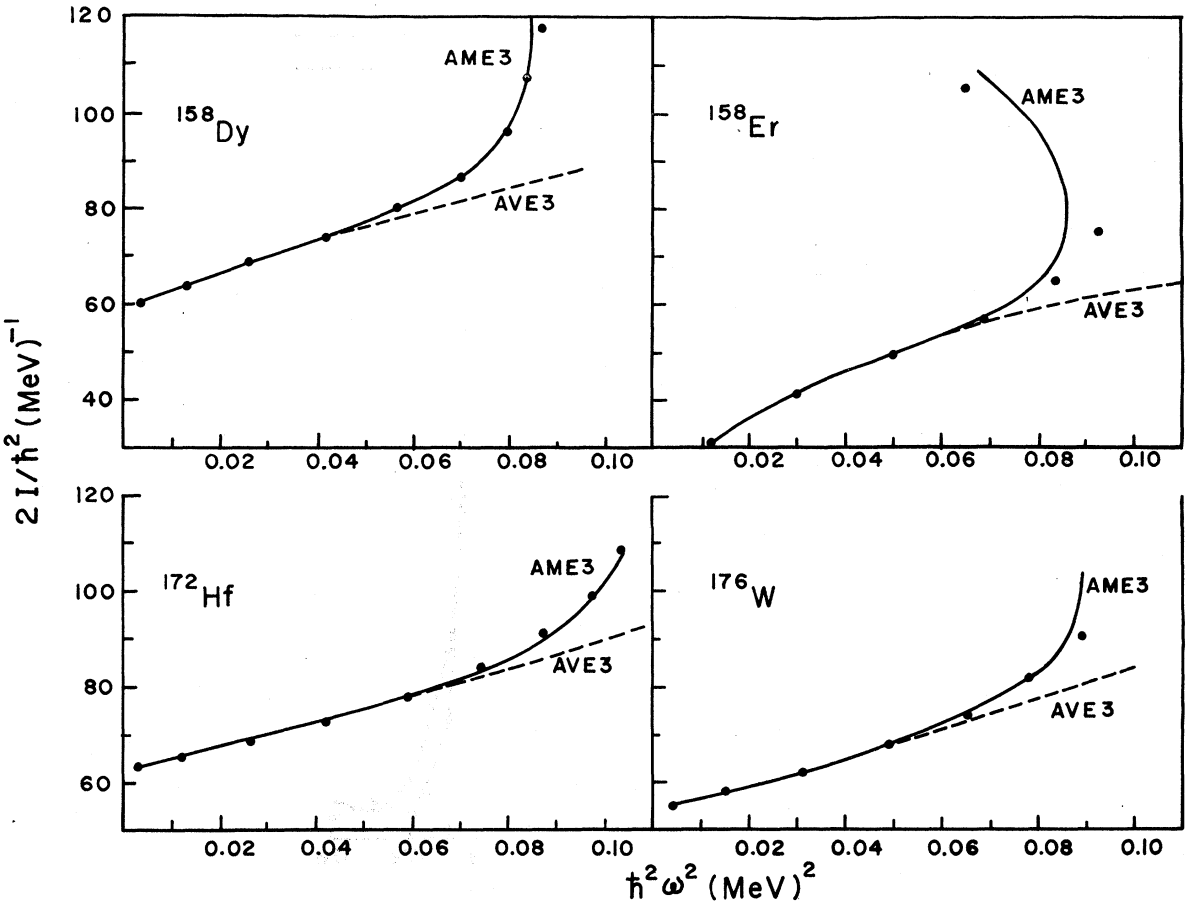

FIG. 3. Nuclear moments of inertia $2\mathcal{I}/\hbar^2$ plotted as a function of $(\hbar\omega)^2$ for four selected even-even nuclei. Notation is the same as in Fig. 1.

TABLE I. Predicted values of critical spin I_c at which back bending is expected from AME3 expressions compared with the available experimental data from Refs. 13 and 14.

Nucleus	I_c (Expt.)	I_c (Calc.)	Nucleus	I_c (Expt.)	I_c (Calc.)
^{130}Ce	12	12	^{162}Er	14	18
^{132}Ce	12	10	^{164}Er	14	24
^{134}Ce	12	12	^{164}Yb	12	14
^{154}Gd	16	14	^{166}Yb	14	18
^{156}Dy	14	12	^{168}Yb	12	22
^{158}Dy	12	16	^{170}Yb	14	32
^{160}Dy	14	24	^{168}Hf	12	14
^{156}Er	12	16	^{170}Hf	14	14
^{158}Er	12	12	^{172}Hf	16	20
^{160}Er	14	14	^{182}Os	12	14

Considering the approaches under review here, it is seen that AVE does not predict such a behavior at any spin for two-, three-, or even four-parameter expressions where AME3 and AME4 predict such a behavior in practically all cases. This has been demonstrated earlier[8,10,12] for AME3. Considering the energy expression

$$E(I) = aI + bI^2 + cI^3$$

we obtain the critical spin I_c as the next nearest even integer to the $b/3C$. In Table I we compare the values of I_c obtained from AME3 with the available experimental results.[13,14] It is seen that our predictions are within two steps of the experimental values for 16 out of the 20 cases. This by itself is quite a success of the adopted expression since no other phenomenological approach has yielded an evaluation of I_c so far.

We conclude that from the points of view of range of applicability, degree of fits to transition energies, and the high spin behavior, straightforward angular momentum expansion gives better results than expansion in terms of squared angular velocity. For all practical purposes a three-parameter expansion in I is quite sufficient. The physical interpretation of the parameters of this expansion is presently being examined.

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