Elastic and inelastic proton scattering from ^{21}Ne and $T=1$ energy levels in ^{22}Na

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Excitation functions for the reactions ^{21}Ne(p,p) and ^{21}Ne(p,p') were studied at eight angles for bombarding energies in the range $E_p=0.6-2.0$ MeV. A total of 21 resonances was observed corresponding to ^{22}Na levels between 7.3 and 8.6 MeV excitation energy. The proton angular momenta are determined and spin and parity assignments are presented. Several new isobaric analog pairs in ^{22}Na and ^{22}Ne are also presented. Previous experimental and nuclear structure information is discussed.

[NUCLEAR REACTIONS ^{21}Ne(p,p), (p,p'), $E=0.6-2.0$ MeV; ^{22}Na deduced resonances, L_p, π, J, T. Isobaric pairs. Enriched target.]

I. INTRODUCTION

Extensive studies have been made of the ^{22}Na nucleus. The interest was renewed these past few years with the improved accuracy of the experimental measurements and the advent of sophisticated computer codes. The present paper is concerned with the search for states with isobaric spin $T=1$ above 7 MeV in ^{22}Na as resonances in the compound nucleus. At present, only a few $T=1$ states are known for this nucleus (see the compilation by Endt and Van der Leun[1]). The identification of the $T=1$ character of these states has sometimes been accomplished from a comparison of the cross sections with their isobaric counterparts in various channels. Thus, the ^{20}Ne-(^{3}He, p) and ^{20}Ne(^{6}Li, α) reactions[2] have been used to suggest the $T=1$ assignment to levels at 5.166 and 6.834 MeV excitation energies in ^{22}Na. A possibly more rigorous approach to identifying isobaric analog states in the ^{22}Na nucleus is to compare the spectroscopic factors obtained from (p,d), (d,t), and (^{3}He, α) reactions on the ^{23}Na target nucleus (Refs. 3–5, respectively) with those deduced from the ^{23}Na$(p,2p)$ (Ref. 6) and the ^{23}Na$(d,^3$He) reaction.[6,7] This has been done[5,8] for the ^{22}Na states at $E_x=0.657$, 1.952, 4.071, 5.166, and 5.957 MeV. Similar results concerning the 0^+ isobaric pairs at $E_x=0.657$ and 6.834 MeV have been obtained from the two-nucleon transfer reactions ^{20}Ne(t,p) and ^{20}Ne(^{3}He, p) (Ref. 9). The one-nucleon transfers ^{21}Ne(^{3}He, d) (Ref. 10) and ^{21}Ne(d,p) (see the references in Ref. 1) give confirmation[10] of the first four $T=1$ states. Studies of the ^{21}Ne(p,γ) reaction[11–14] have confirmed and suggested the $T=1$ assignment respectively for the 4.071 MeV level and for a level at 7.47 MeV. However, we note that the radiative capture of a proton has not been employed to investigate the analog states in the successful manner employed for heavier nuclei in the $2s$-$1d$ shell.[15,16] More recently,[17] a comparison of the (t,p) angular distributions and absolute cross sections with the corresponding (^{3}He, p) data has led to the confirmation of all $T=1$ levels quoted above (except the 7.47 MeV level) as well as to the identification of two new $T=1$ candidates at $E_x=6.19$ and 6.56 MeV in ^{22}Na.

Therefore, it appears that the known $T=1$ assignments to ^{22}Na levels have been made for excitation energies below 7 MeV. In the energy range between 7 and 9 MeV, although many energy values are known, and some γ-ray decay schemes exist in the literature, only a few spin and parity assignments have been hitherto reported[1] as results of experiments on transfer reactions or on γ rays following proton bombardments. It was thus felt that further knowledge of spin-parity assignments and a search for levels with isobaric spin $T=1$ were desirable above 7 MeV in ^{22}Na. The investigation of isobaric analog resonances in the compound nucleus ^{22}Na excited by elastic and inelastic scattering of 0.6–2.0 MeV protons from ^{21}Ne was therefore undertaken. This region of excitation contains analog states of the parent nucleus ^{22}Ne above 6.8 MeV where the level density permits the avoidance of using a high resolution apparatus.

From the theoretical point of view, it has long been known that the ^{22}Na nucleus lies in a region of important stable deformations. It thus has been successfully described in the collective rotational model framework (see, for example Refs. 5, 10, and 18). On the other hand, extensive shell-model calculations have been performed on the $A=22$

mass system (see Refs. 19 and 20 and references therein). In both approaches, the theoretical predictions have not been fully tested mainly because of the lack of experimental information; for example, in their shell-model calculation[19] based on a realistic two-body interaction, Preedom and Wildenthal quoted 29 calculated levels with isobaric spin $T=1$, of which only one-third have been related[19,8] to corresponding experimental levels. In the present paper, the emphasis is placed on a discussion of our results in terms of nuclear structure predictions by both models.

The experimental setup and the obtained excitation functions are presented in Sec. II and the method of analysis in Sec. III. In Sec. IV, the results concerning 21 levels lying between 7.3 and 8.6 MeV in ^{22}Na are given. A detailed comparison with previous data is also presented. In Sec. V, the data from the present experiment, in conjunction with data from ^{21}Ne(p,γ) (Refs. 12 and 14) and ^{21}Ne(d,p) (Refs. 21 and 22) reaction studies are discussed, and isospin assignments in ^{22}Na and isobaric analog pair states in the ^{22}Na-^{22}Ne nuclei are proposed.

II. EXPERIMENTAL PROCEDURE AND EXCITATION FUNCTIONS

The experiment was carried out with the 4 MeV Van de Graaff accelerator at Lyon. The energy of the proton beam was varied within the range 0.6–2.0 MeV. The target chamber, described in detail elsewhere[23] and used in previous measurements contained neon enriched to 91 mole% in ^{21}Ne, the rest being ^{20}Ne (3.3%) and ^{22}Ne (5.7%). To limit the leak rate, the interior microchamber was replaced by a faceted cylinder with Formvar windows. The pressure, measured with a differential manometer, was kept around 10 Torr. The gas was pumped off when the contaminants concentration (mainly ^{16}O and ^{12}C) exceeded about 1%. The entrance window was a 0.15 μm thick nickel foil. The energy dispersion in the beam was estimated to be 8.5 keV at the center of the target and was mainly due to straggling in the entrance window and in the gas.

The energy calibration of the accelerator was obtained by observing the known resonances in the ^{14}N(p,p) and ^{15}N$(p,\alpha\gamma)$ reactions at $E_p=1.743$ and 0.898 MeV, respectively, and in the ^{19}F$(p,\alpha\gamma)$ reaction at $E_p=1.091$, 1.348, and 1.373 MeV. The energy loss in the entrance window and in the gas was taken from the tables by Williamson, Boujot, and Picard.[24] The rareness of the gas obliged us to limit the number of measurements; we adopted 7 keV energy steps except when the resonance structure was so narrow as to require more detail, the energy steps then being 2 keV.

The reaction particles were detected simultaneously by eight surface barrier detectors, placed at angles between $\simeq 40°$ and $\simeq 160°$ in the laboratory system. The information was interfaced by a multiplexer into a Hewlett-Packard 2116C computer which allowed the identification and the summation of the peaks after each counting run. The total experimental energy resolution increased from about 30 keV up to 50 keV with the energy of the detected proton groups decreasing from 2 to 0.5 MeV. A typical pulse-height spectrum is shown in Fig. 1.

It was impossible to separate the ^{21}Ne proton peaks from those of ^{20}Ne and ^{22}Ne. An evaluation of the contribution by both these isotopes was made assuming potential and Coulomb scattering. The

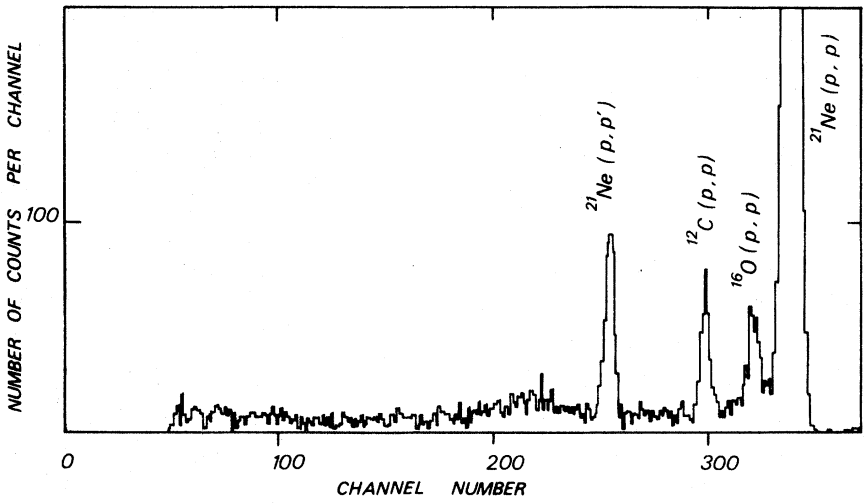

FIG. 1. Typical proton spectrum taken at $\theta=159°$ and $E_p=1.480$ MeV (in the laboratory system).

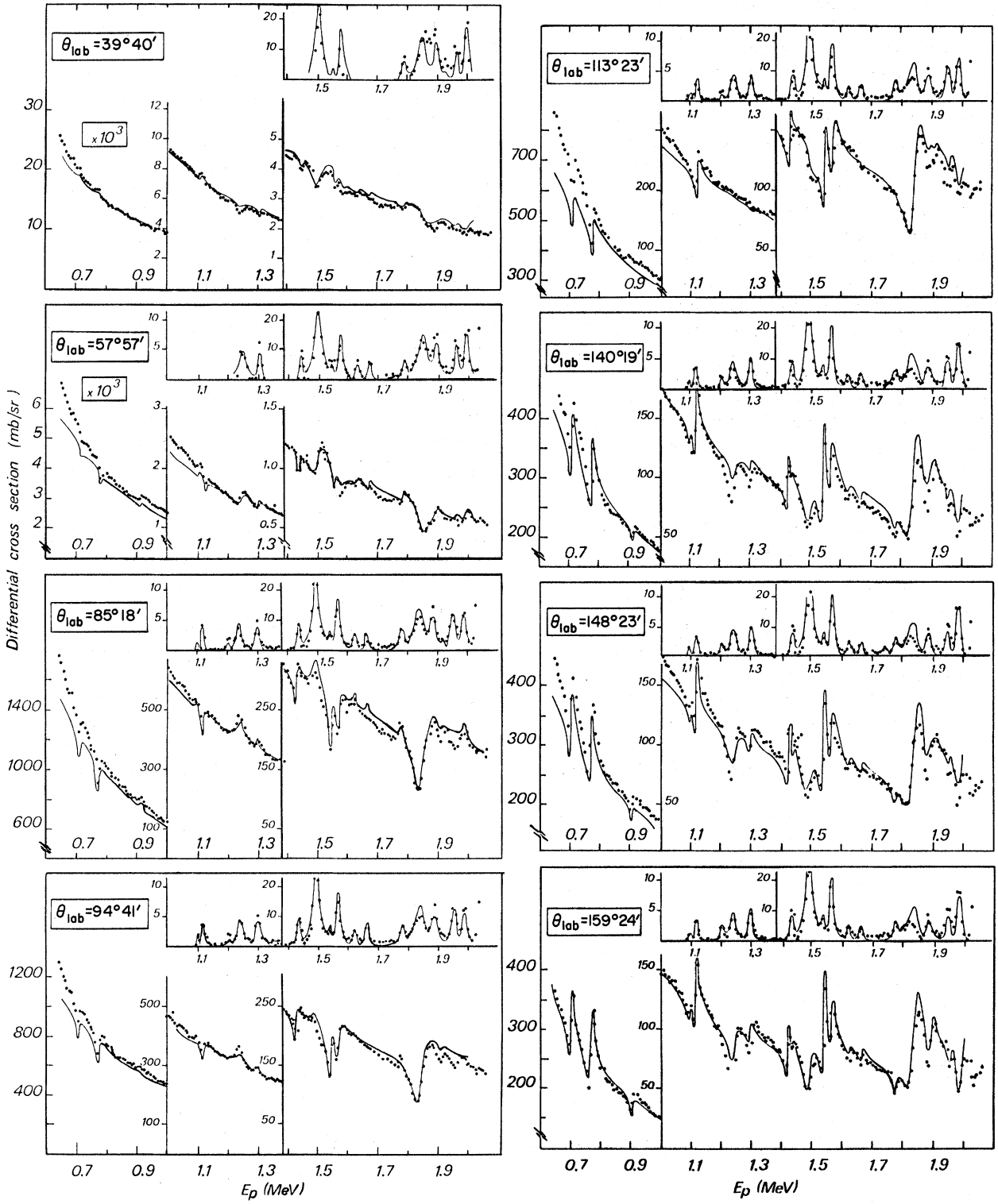

FIG. 2. Excitation functions. For each angle, the inelastic excitation function is shown on the upper right corner.

inelastic peak was not summed when mixed with the elastic peak tail, i.e., when $E_p < 1.4$ MeV at the forward angles and $E_p < 1.1$ MeV at the backward angles. The absolute differential cross section was obtained by a comparison with the Coulomb scattering cross section of low energy protons from krypton. This calibration was performed at several energies in the range 0.6–2.0 MeV to take into account the incident beam fraction that was uncollected after scattering in the gas and in the exit window. The elastic and inelastic excitation functions are shown in Fig. 2 for the eight angles which are investigated here. Most resonances seen on the experimental curves correspond to levels previously reported in the studies of the ^{21}Ne(p, γ) and ^{21}Ne$(p, p'\gamma)$ reactions (Refs. 14 and 25, respectively).

III. ANALYSIS

At the proton energies considered here, the ^{21}Ne(p, p) and ^{21}Ne(p, p') reactions proceed by compound nucleus formation as described by the Breit and Wigner theory. Accordingly, the theoretical excitation functions were calculated within the formulation of Ref. 26. In the present experiment, the Wigner criterion for the reduced widths allowed us to consider only the resonating partial waves with angular momentum $l \leq 2$. The potential scattering was described by the corresponding hard-sphere phase shifts because of the lack of energy regions unperturbed by the resonances. For each detector, the energy dispersion due to the beam and to the target thickness was taken into account by using a normal distribution as resolution function[27] (the width at half-maximum was estimated to be 9 keV).

Elastic scattering

A visual examination and the comparison of each resonance shape with that predicted by a single-level Breit-Wigner formula allowed us to determine approximative values for the energy of each resonance, for the total width (Γ), for the proton angular momentum (l_p) and to estimate the other parameters (J, S, Γ_p) that were then used in a multilevel analysis. The level-level interference was found not to destroy the l_p dependence of the resonances, but their amplitudes, leading to the spin and partial widths, were found to be very sensitive to the interference effects. In the preliminary fits, it was possible to divide the region from 0.6 to 2.0 MeV into separate spans, but the maximum of levels (as many as 21) had to be included in the final fits. This greatly complicated the parameter search.

Inelastic scattering

As for the elastic case, a first approach was made assuming no interference. The expansion into Legendre polynomials of the experimental angular distributions allowed us to deduce the total inelastic cross section σ_T which satisfies the Breit-Wigner formula

$$\sigma_T = \frac{2J+1}{2} \pi \lambda^2 \frac{\Gamma_p \Gamma_{p'}}{\Gamma^2} .$$

If only the elastic and inelastic channels are open (i.e., in our case, up to $E_p = 1.85$ MeV, where the exit channel $\alpha + {}^{18}$F opens), the above formula, together with the relation $\Gamma = \Gamma_p + \Gamma_{p'}$, sets a correspondence between assumed J and Γ_p/Γ values. Thus some J, S assignments were eliminated and approximate values were given for Γ_p/Γ. The refinements were made through the multilevel analysis. Finally, the inelastic scattering analysis was both an intermediate stage and an additional check of the elastic scattering analysis.

IV. EXPERIMENTAL RESULTS

The fits of the excitation functions at eight angles are displayed in Fig. 2 for both elastic and inelastic scattering. The quality of the fits is satisfactory, except at the extreme part of the energy range because the unknown upper levels, which certainly interfere, are not included in the calculations. The results concerning the characteristics of the analyzed levels are listed in Table I. The resonance energies, given with a 4 keV uncertainty, agree reasonably well with previous works.[1] We found three resonances at $E_p = 1438$, 1545, and 1915 keV which do not seem to have been previously reported by means of the ^{21}Ne$(p, p'\gamma)$ (Ref. 25). The proton orbital angular momentum (and thus the parity) is, of course, always well determined by the shape of the resonance, but spin ambiguities persist for 15 over 21 levels due to the uncertainty in the actual amplitude of the resonance. This amplitude is damped by the energy dispersion of the beam, especially when the total width is weak. Now, the small amount of experimental measurements available for each resonance makes precise determination of the width impossible as soon as it becomes smaller than the experimental resolution. Unfortunately, this occurs for most levels in the investigated energy range and gives uncertainties on Γ_p up to ~50%.

Up to here, the spin assignments resulting from our analysis may be discussed on the basis of previous works which eventually allow the selection among J values. (i) First, an upper value of the

TABLE I. Summary of the resonance parameters discussed in Sec. IV. The energies of resonance E_p are measured in the present work with a 4 keV uncertainty. They correspond to the excitation energies E_x in the ^{22}Na nucleus which are taken from Ref. 1 for the first six levels and are our own determinations for the other levels. The unambiguous proton orbital angular momenta are labeled by l_p. Among the J^π assignments of the present analysis, the proposed values are those consistent with previous results of Refs. 12, 14, and 25. The corresponding proton widths Γ_p are listed in the last column (the uncertainties are discussed in the text).

E_p (keV)	E_x (keV)	l_p	J^π assignments Present work	Compatibility with previous work a	b	c	Summary	Proposed Γ_p (keV)
706	7410.7 ± 1.2	0	$(1,2)^+$				$(1,2)^+$	<2 [d]
772	7473.9 ± 1	0	$(1,2)^+$	2^+		2^+ if $\Gamma<2.5$	2^+	2
909	7607 ± 2	1	$(1-3)^-$	$(2)^-$			$(2)^-$	2.5
1100	7780 ± 2	1	$(1-3)^-$	$1^-, 2^-$			$(1,2)^-$	<2 [d]
1120	7803 ± 2	0	$(1,2)^+$	$1^+, 2^+$	$1^+, 2^+$	2^+ if $\Gamma<2.5$	2^+	1.8
1205	7890 ± 2	2	$(0-4)^+$	$3^+, 4^+$	4^+		4^+	<0.5 [e]
1240	7920 [f]	1	1^-		1^-		1^-	23
1300	7980	1	$(1-3)^-$	$1^-, 2^-$	2^- $(S=2)$		2^-	2
1428	8105	0	$(1,2)^+$				$(1,2)^+$	<4 [d]
1438	8117	1	$2^-, 3^-$		$2^-_{(S=1)}, 3^-$		$(2,3)^-$	0.6
1496	8169	1	3^-		3^-		3^-	15
1545	8210	0	2^+				2^+	4.5
1570	8239	0	2^+		2^+		2^+	4.4
1626	8292	1	$1^-, 2^-, 3^-$		$1^-, 2^-$		$(1,2)^-$	5
1666	8331	1	$1^-, 2^-$		2^- $(S=1)$		2^-	2
1780	8441	1	$1^-, 2^-, 3^-$		$1^-, 2^-$		$(1,2)^-$	8
1840	8500	0	2^+		2^+		2^+	29
1888	8540	1	$(1,2,3)^-$		$1^-_{(S=2)}, 2^-$		$(1,2)^-$	<8 [d]
1915	8570	2	$(0-4)^+$				$(0-4)^+$	<4.5 [d]
1954	8607	1	$(1^-, 2^-, 3^-)$		2^- $(S=1)$		2^-	2.1
1988	8641	1	3^-		3^-		3^-	5

[a] Antilla *et al.* (Ref. 14).
[b] Storizhko *et al.* (Ref. 25).
[c] Arnell *et al.* (Ref. 12).
[d] The upper limit corresponds to the upper proposed spin value.
[e] For weak widths only an upper limit may be given.
[f] Possible doublet (see the text).

total width Γ has been found to be 2.5 keV by Arnell and Wernbom-Selin.[12] (ii) Second, we use the γ decay data, mainly those by Antilla, Bister, and Arminen.[14] We normalize their results of relative yields with the value 50 eV estimated by Bröstrom, Huus, and Koch[28] for the γ-ray width Γ_γ for the $E_p = 772$ keV resonance. For lack of a knowledge of the multipole mixing ratios, one may assume pure (unmixed) multipoles, extract the corresponding transition strengths, and rule out the J assumptions which give values in total disagreement (by an order of magnitude) with those compiled by Skorka, Hertel, and Retz-Schmidt[29] or more recently by Endt and Van der Leun.[30] This method for evaluating the partial widths may lead only to approximate values and has significance only for the most important branchings. (iii) Finally, we consider the angular distribution of the 350 keV γ ray issuing from the inelastic scattering (hereafter denoted as γ 350). For all the J^π assumptions of a given resonance, we calculate the range of the Legendre expansion coefficients and eliminate those J^π values in disagreement with the experimental coefficients of Storizhko *et al.*[25]

Resonance at $E_p = 706$ keV. The shape corresponds to the s-wave formation of $J^\pi = 1^+$ or 2^+. The γ 350 peak is not pronounced,[14] justifying our approximative $\Gamma_p/\Gamma \simeq 1$. The resonance is then well analyzed with $J^\pi = 1^+$ if $\Gamma \sim 3$ keV and $J^\pi = 2^+$ if $\Gamma < 1.5$ keV. Accordingly, if this resonance has $\Gamma < 2.5$ keV, as suggested by Arnell and Wernbom-Selin,[12] the level at $E_x = 7408$ keV would be a candidate for the 2^+ assignment, but this reasoning does not constitute a reliable determination of the spin.

Resonance at $E_p = 772$ keV. Krone and Singh[11] and Tanner[13] proposed $J^\pi = 2^-$ for this level. However, the elastic scattering analysis gives the unambiguous value for the proton momentum $l_p = 0$. The inelastic channel is weakly fed, which implies $\Gamma_p \sim \Gamma$. This resonance is well fitted with $J = 1^+$ if

$\Gamma \sim 6$ keV and with $J^\pi = 2^+$ if $\Gamma = 2.5$ keV. Using the remarks of Ref. 12 on the resonance widths, we propose the 2^+ assignment. To enforce this point of view, we know that the γ decay of this level proceeds mainly (77%) to the 3^+ ground state. The 1^+ assignment would lead to an $E2$ transition strength of several 100 W.u. (Weisskopf unit), which is very unlikely. On the contrary, the 2^+ assignment prescribes an $M1$ transition strength of some W.u. in agreement with the compilation of Ref. 29. Furthermore, the angular distribution of the 7.47 MeV γ ray exhibits[13] a small anisotropy $W_{\theta=0^\circ}/W_{\theta=90^\circ} = 1.15 \pm 0.08$. Considering the experimental uncertainty this might be interpreted as a confirmation of the 2^+ hypothesis. In this case, the angular distribution would be isotropic, if the mixing parameters are neglected.

Resonance at $E_p = 909$ keV. The shape of this resonance predicts the p-wave formation corresponding to $J^\pi = (0, 1, 2, 3)^-$. Its amplitude rules out the 0^- hypothesis independently of the value of Γ. The $J^\pi = 1^-$ hypothesis is also unlikely because the 16% γ branching to the ground state would give $\Gamma_\gamma(M2) \sim 10^3$ W.u. The $J^\pi = 3^-$ assumption for the 10% branching to the 0^+ level at $E_x = 657$ keV would give $\Gamma_\gamma(E3) \sim 10^4$ W.u. Unhappily, the last possibility $J^\pi = 2^-$ leads to $\Gamma_\gamma(M2) \simeq 160$ W.u. for that branching, which is a rather large value even taking into account the experimental uncertainties. A solution of this problem (developed in the following section) is to assume the existence of a doublet.

Resonance at $E_p = 1100$ keV. This resonance has a weak amplitude in both exit channels. Its particle width is about 1 keV and it is hampered by the energy resolution effect and by the proximity of the anomaly at $E_p = 1120$ keV. It indicates a p-wave formation. The $M2$ transitions, leading to the 1^+ state at $E_x = 1937$ keV and to the 2^+ state at $E_x = 3059$ keV, have transition strengths >100 W.u., which excludes $J^\pi = 0^-$ and 3^-.

Resonance at $E_p = 1120$ keV. This is a very pronounced resonance, the shape of which corresponds to the s-wave formation. Data on γ decay[14] for this level do not allow us to choose between $J^\pi = 1^+$ and 2^+. The isotropy of the angular distribution for the γ 350 does not remove the ambiguity. However, if $\Gamma \leq 2.5$ keV,[12] only the 2^+ assignment is possible.

Resonances at $E_p = 1205$ and 1240 keV. The broad resonance at $E_p = 1240$ keV is assigned unambiguously to a p wave. The best fit occurs for $J = 1$ with $S = 2$, in agreement with a small anisotropy in the angular distribution of the γ 350. However, the fit does not account for the amplitude of the resonance at any angle. This could be due to the existence of a doublet which is not separated experimentally and which may explain the exceptional broadening of the observed anomaly. However, this assumption would imply a weak γ width (Γ_γ) for both levels because no corresponding transition has been reported by Antilla *et al.*[16] The fit is a little better if one includes a resonance at 1205 keV encountered in the inelastic channel. The humps observed in elastic scattering at $\simeq 39^\circ$ and at $\simeq 159^\circ$ characterize a d-wave formation. The γ decay data for this level with positive parity are consistent only with $J = 3$ and 4, which gives for the main transition (48%) to the 5^+ state, $E2$ and $M1$ transition strengths close to 100 and 1.5 W.u., respectively. The experimental value of the "a_2" coefficient (defined as in Ref. 17) obtained from the γ 350 angular distribution rules out the 3^+ assignment, allowing only the 4^+ possibility.

Resonance at $E_p = 1300$ keV. If the observed energy of this resonance is close to the value given by Storizhko *et al*,[25] it differs by 13 keV from that given by Antilla *et al.*[14] As suggested by Arnell *et al.*,[12] it is probably a doublet with a fine component nonfed by elastic and inelastic scattering. The component at 1300 keV corresponds to p waves. Depending on the resonance width, $J^\pi = (1, 2, 3)^-$ assignments may be proposed, but only $J^\pi = 2^-$ with $S = 2$ is consistent with the results of Ref. 25.

Resonances at $E_p = 1428$ and 1438 keV. A new resonance is observed only in the elastic channel at $E_p = 1428$ keV, with the characteristic s-wave shape giving $J^\pi = (1, 2)^+$. Another resonance at 1438 keV is found to correspond to p waves following the elastic scattering analysis. The angular distribution of the γ 350 allows us to select $J^\pi = (2, 3)^-$.

Resonance at 1496 keV. It is a broad resonance ($\Gamma \sim 20$ keV) with $l_p = 1$. Its amplitude in the inelastic channel implies $J = 3^-$. The expansion in Legendre polynomials of the angular distribution of the γ 350 gives, for an initial $J^\pi = 3^-$, $-0.2 < a_2 < -0.15$ consistent with the experimental value $a_2 = -0.184 \pm 0.014$ measured by Storizhko *et al.*[25] This confirms the 3^- assignment.

Resonances at 1545 and 1570 keV. Only the second resonance, corresponding to the s-wave formation, has been previously reported.[1] The inelastic channel yield for the 1570 keV resonance indicates $J \geq 2$, and consequently, the only possibility for J^π is 2^+. This is confirmed by the γ 350 angular distribution which exhibits a small anisotropy ($a_2 = -0.047 \pm 0.009$) not too far from the theoretical isotropic distribution. The resonance at 1545 keV is narrower; it interferes with two resonances located on each side and with $S = 2$ in the entrance channel. Thus only $J = 2$ is suitable.

Resonances at 1626 and 1666 keV. These narrow and well isolated resonances both correspond

to a p-wave resonance formation. As concerns the first one, the elastic data analysis eliminates $J^\pi = 0^-$ and the small anisotropy of the γ 350 ($a_2 = 0.039 \pm 0.019$)[24] eliminates $J = 3^-$. For the second resonance, a strong anisotropy of the γ 350 has been reported[24] leading to the unique 2^- assignment.

Resonance at 1780 keV. The amplitude of this p-wave narrow resonance is modified by the resolution effect and by a neighboring resonance, not clearly seen in the elastic channel, but located in the inelastic channel at $E_p = 1748$ keV. The inelastic yield is consistent with $J^\pi = (1, 2, 3)^-$. The small anisotropy of the γ 350 rules out the 3^- assignment.

Resonance at 1840 keV. This is a broad resonance formed by s waves, and with an amplitude practically unperturbed by the resolution effects. The 2^+ assignment is proposed. The strong anisotropy of the inelastic proton is not consistent with the interference of the reported resonances, thus an unreported level probably lies in the vicinity.

Resonance at 1888 keV. Formed by p waves, the amplitudes of the resonance may be accounted for in the elastic and inelastic channels only with $J^\pi = (1, 2, 3)^-$. The experimental value of the a_2 coefficient in the angular distribution of the γ 350 forbids $J^\pi = 3^-$.

Resonance at 1915 keV. This level, which is not found by the ^{21}Ne$(p, p'\gamma)$ reaction has been reported by Garrett *et al.*[2] studying the ^{20}Ne(^{3}He, p) (Ref. 1). The analysis yields $l_p = 2$. The total width being unknown no choice among the $J^\pi = (0, 1, 2, 3, 4)^+$ values is possible.

Resonance at 1954 keV. This resonance is very pronounced in the inelastic channel and corresponds to the d-wave formation in the elastic channel. For these reasons, there should not be confusion with the ^{20}Ne resonance seen at the same energy by Bloch, Knellwolf, and Pixley[31] with $l_p = 2$, in spite of our target composition (3.3% ^{20}Ne, see above). The strong anistropy of the γ 350 is consistent only with $J^\pi = 2^-$.

Resonance at 1988 keV. It is formed by p waves. The inelastic scattering assigns $J = 3^-$, consistent with the results of Storizhko *et al.*[25]

V. DISCUSSION OF ANALOG STATES

A. Criteria

For states in ^{22}Na above 7 MeV excitation energy, we use the present and previous experimental data to identify isobaric analog pairs on the basis of the following criteria:

(a) A Coulomb energy shift ΔE_C within the range 4.1–4.4 MeV. This value is consistent with the experimental energies of the known 10 isobaric states (see, for examples, Table 4 of Ref. 17) and with the theoretical evaluations [see for example Ref. 32, except for the first $(J^\pi, T) = 0^+$, 1 level].

(b) The structure of the analog $|T_>\rangle$ and antianalog $|T_<\rangle$ states for ^{22}Na is

$$|T_>\rangle = \frac{1}{(2T_0+1)^{1/2}}\left(|pC\rangle + \sqrt{2T_0}\ |nA\rangle\right)$$

and

$$|T_<\rangle = \frac{1}{(2T_0+1)^{1/2}}\left(\sqrt{2T_0}\ |pC\rangle - |nA\rangle\right),$$

where $|pC\rangle = |p\rangle|^{21}\text{Ne}\rangle$ and $|nA\rangle = |n\rangle|^{21}\text{Na}\rangle$. This leads to the following relations between proton and neutron partial reduced widths (γ_p^2 and γ_n^2, respectively)

$$\gamma_p^2(T_>) = \frac{1}{2T_0+1}\,\gamma_n^2$$

and

$$\gamma_p^2(T_<) = \frac{2T_0}{2T_0+1}\,\gamma_n^2 .$$

In our case, the ^{21}Ne target has an isospin $T_0 = \frac{1}{2}$; thus

$$\gamma_n^2 = 2\gamma_p^2(T_>) = 2\gamma_p^2(T_<) .$$

Of course, the validity of this criterion depends on the isospin conservation and the splitting of analog and antianalog states complicates this simple picture. However, and this is an attractive feature of s-d shell nuclei, the isospin mixing is expected to be either absent or occurring between only a few neighboring levels.

Standard methods are used to calculate the particle widths for a given orbital angular momentum (see, for example, Ref. 33): The γ_p^2 reduced width is obtained from the corresponding proton partial width Γ_p and penetration factor [the channel radius in fm being $1.33\,(A^{1/3}+1)$ where A is the mass number of the target]. The γ_n^2 reduced width is obtained from the corresponding spectroscopic factor of the (d, p) reaction[21,22] together with the radial part of the bound single particle wave function for the captured neutron. A real Woods-Saxon well with a spin orbit term (defined in Ref. 33) is assumed (the radius is $r = 1.25A^{1/3}$ fm), the diffuseness parameter is $a = 0.65$ fm, and the well depth is adjusted to reproduce the binding energy, respecting the characteristics of the wave functions: normalization and number of nodes.

(c) In Sec. IV, general upper limits for the γ-ray widths taken from the experimental compilations of Refs. 29 and 30 have been used to eliminate J values without any isospin consideration.

TABLE II. Selection of γ-ray decay scheme results for the levels analyzed in the present work. The energies (in keV), final spin and isospin values are from Ref. 1 except where otherwise specified. The method for calculating the γ transition strengths $|M(\sigma L)|^2$ from results of Refs. 14 and 28 is given in Sec. IV and the choice of the initial spin and isospin values is justified in Sec. V B.

E_{x_i}	J_i^{π} [a]	T_i	E_{x_f}	J_f^{π} [a]	T_f	(%)	$\|M(M1)\|^2$	$\|M(E2)\|^2$	$\|M(E1)\|^2$	$\|M(M2)\|^2$
7410	$2^+(1)^+$	(1)	583	1^+	0	10	0.29	43	...	...
			1937	1^+	0	14	0.78	180	...	...
			2571	2^-	0	6	...	...	1.9×10^{-2}	3840
			3060	2^+	0	24	2.7	990	...	...
			3944	1^+	0	18	4.0	2300	...	...
			4360	$2^+(1)^+$	0	28	9.1	7680	...	...
7473	2^+	(1)	0	3^+	0	77	7.0	890	...	...
			583	1^+	0	2	0.23	34	...	...
			1937	1^+	0	9	2.0	480	...	...
			1983	3^+ [b]	0	3	0.70	160	...	...
			3060	2^+	0	6	2.7	960	...	...
			4360	$2^+(1)^+$	0	1	1.3	920	...	...
			4778	$(0-4)^+$	0	2	3.8	3680	...	...
7607	$(4)^-$	(1)	0	3^+	0	16	...	...	0.08×10^{-2}	80
			1983	3^+ [b]	0	16	...	...	0.23×10^{-2}	360
	$(1)^-$	(0)	583	1^+	0	13	...	...	0.34×10^{-2}	320
			657	0^+	1	10	...	...	0.27×10^{-2}	260
			1952	2^+	1	45	...	...	2.0×10^{-2}	3300
7780	$1^-(2)^-$	(1)	1937	1^+	0	24	...	...	2.4×10^{-2}	3360
			2111	1^-	0	54	15.6	354	...	...
			3060	2^+	0	22	...	...	4.2×10^{-2}	9000
7803	2^+	(0)	0	3^+	0	20	0.11	11	...	...
			1952	2^+	1	80	0.89	198	...	...
7890	4^+	(1)	0	3^+	0	3	0.028	2.2	...	...
			891	4^+	0	29	0.37	55	...	...
			1528	5^+	0	48	0.83	163	...	...
			1983	3^+ [b]	0	17	0.35	80	...	...
			2969	3^+	0	3	0.11	33	...	...

[a] In the case of spin ambiguity, the most probable value has been taken in the calculations of the transition strengths.
[b] Haas *et al.* (Ref. 18).

Here, particular limits are obtained taking into account the isospin character which is expected to exert a great influence on the γ-ray widths in a self-conjugate nucleus. As a summary, we quote below data from Ref. 30 concerning the upper limits for the γ strengths

$$|M(\sigma L)|^2 = \Gamma_\gamma(\sigma L)/\Gamma_W(\sigma L) ,$$

where Γ_γ is the γ-ray width for a magnetic ($\sigma = M$) or an electric ($\sigma = E$) transition with a L-multipole order (Γ_W is the corresponding Weisskopf estimation):

	$E1$	$M1$	$E2$	$M2$
$\Delta T = 0$	0.003	0.03	100	...
$\Delta T = 1$	0.03	10	3	3

In a self-conjugate nucleus, such as ^{22}Na, it is noteworthy that ($E1$, $\Delta T = 0$) transitions are either forbidden (for pure $T = 0$ or $T = 1$ isospin states) or strongly retarded (for mixed isospin states). In addition (for useful references see Ref. 34), according to Morpurgo's rule, the ($M1$, $\Delta T = 0$) transitions are hindered and an upper limit may be found from the sum rule of Gell-Mann and Telegdi in terms of the reduced transitions rates:

$$\sum_f B_{i\to f}(M1,\ \Delta T = 0) < \frac{3}{16\pi}(\mu - \tfrac{1}{2})^2 a(a+2)\mu_N^2 ,$$

where $\mu = 0.88$, $\mu_N = e\hbar/2Mc$, and "a" may be approximated by the number of active nucleons in the transition. One may assume here that only one nucleon is active because ^{22}Na is satisfactorily accounted for by the Nilsson model which forbids two-nucleon transitions; therefore,

$$B(M1; \Delta T = 0) < 2 \times 10^{-2} \mu_N^2 .$$

It is interesting to remark that this rule derived from quite rough considerations, is actually verified by all $M1$ transitions (about 50) calculated

TABLE III. Proposed isospin assignments for ^{22}Na levels, studied in the present (p,p) experiment, and isobaric analog states in ^{22}Ne (see Sec. V). All ^{22}Ne data are from Ref. 17. The particle reduced widths are calculated for the more probable spin assignment of the corresponding ^{22}Na level. We report here the first six lower levels for which γ-ray data are available. The higher levels are reported only when it has been possible to propose an analog state. Energies and reduced particle widths are in keV.

			Proposed analog states		γ_n^2		$2\gamma_p^2$	
E_x (^{22}Na)	J^π (^{22}Na)	T (^{22}Na)	E_x (^{22}Ne)	J^π (^{22}Ne)	a	b	c	ΔE_C
7410	$2^+(1)^+$ [d]	(1) [e]	6819	2^+	160	152		4216
			or				<192	
			6855	$(1,2)^+$	246	280		4180
7473	2^+	(1)	6819	2^+	160	152		4279
			or				116	
			6855	$(1,2)^+$	246	280		4243
7607	$(1)^-$ [d] / $(4)^-$ [d]	(0) / (1)	... / ?	...	...	...	260	...
7780	$1^-(2)^-$ [d]	(1)	7052	$(1,3)^-$		18	<91	4353
7803	2^+	(0) [e]	...	...	...	...	25	...
7890	4^+	(1)	7336	$(4)^+$		21	<139	4179
8210	2^+						18	4203
or		(1)	7632	$(1,2)^+$		21		
8239	2^+						17	4232
8500	2^+	(1)	7922	2^+		114	77	4203

[a] Calculated from Chambon *et al.* (Ref. 21).
[b] Calculated from Neogy *et al.* (Ref. 22).
[c] Present work.
[d] This J assignment, more precise than in Table I, is discussed in Sec. V B.
[e] A possible isospin mixing between the 7410 and 7803 keV levels is considered in the text.

in the shell-model framework of Ref. 19. The corresponding γ strength is

$$|M(M1,\ \Delta T=0)|^2 < 0.01\ ,$$

which is more selective than the value quoted above.

B. Discussion

In this subsection, we discuss the isospin assignment consistent with the above criteria. A selection of available γ-ray data for the levels investigated here is presented in Table II. Our main conclusions are listed in Table III and in Figs. 3–5.

$2^+(1)^+$ level at $E_x = 7410$ keV in ^{22}Na. The $M1$ strengths exceed 0.1 for all branchings (see Table II), thus suggesting $\Delta T = 1$ transitions and a $T = 1$ character for this level. But, in the $J^\pi = 1^+$ hypothesis some of them are not consistent with the upper limit of $M1$ strengths. A tentative assignment then is $J^\pi = 2^+(1)^+$ for this level. To confirm the $T = 1$ character, we note that this state decays mainly to $T = 0$, 2^+, and 1^+ states which are characterized by important spectroscopic factors [measured in the ^{21}Ne(^{3}He, d) reaction[10]]. On the other hand, the main branching [28% → $2^+(1)^+$, 4360 keV] leads to a large $M1$ strength (see Table II), which may be the signature of an analog to antianalog transition. Furthermore, we note that the 7410 keV level is well excited by s wave and that the (^{3}He, d) spectroscopic factor is[10] then 0.66 for the 4360 keV level. Thus, we propose that this last state is the antianalog state of the $2^+(1)^+$ 7410 keV level. In this interpretation, the 7410 keV level may be seen as a member of a $T = 1$, $K^\pi = 2(1)^+$ rotational band based on the $\frac{3}{2}^+[211] - \frac{1}{2}^+[200]$ Nilsson configuration outside a ^{20}Ne core (see Fig. 3 and see Ref. 10 for the interpretation of the 4360 keV level). Finally, the Coulomb energy and the particle reduced widths (Table III) suggest that the 7410 keV state is the isobaric analog state of one of the two levels: 2^+, $E_x = 6819$ keV and $(1,2)^+$, $E_x = 6855$ keV in ^{22}Ne. However, if we take into account the energy uncertainties, a 6^- state is seen at the same energy by the reactions: ^{20}Ne$(\alpha, d)^{22}$Na, ^{10}B(^{16}O, α)^{22}Na (see Refs. 35 and 36, respectively), thus proving the existence of a doublet with $J^\pi = 6^-$ and probably a $T = 0$ component unseen in the present (p,p) experiment. Other reactions which lead mainly to $T = 0$ states in ^{22}Na: ^{24}Mg$(d, \alpha)^{22}$Na, ^{20}Ne(^{6}Li, α)^{22}Na, ^{12}C + ^{14}N → α + ^{22}Na (Refs. 37, 2, and 38, respectively) excite one of the members of this doublet and it is just possible that the level seen in (p,p) and (p,γ) reactions is concerned. In that case, the isospin character of

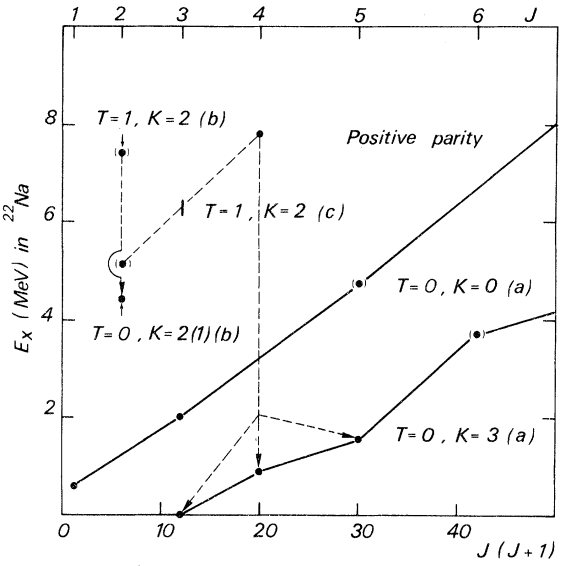

FIG. 3. Band structure of ^{22}Na positive parity levels. On this figure, we show only the two lowest bands (data taken from Ref. 35) and the bands which are quoted or discussed in Sec. V B. The bands labeled by (a), (b), and (c) are constructed, respectively, on the Nilsson configurations: $(\frac{3}{2}^+[211])^2$, $\frac{3}{2}^+[211]-\frac{1}{2}^+[200]$, and $\frac{3}{2}^+[211]-\frac{1}{2}^+[211]$. The dotted lines correspond to the strong $M1$ γ branchings discussed in the text. The dots ● [and (●)] correspond to levels without (and with) spin ambiguity.

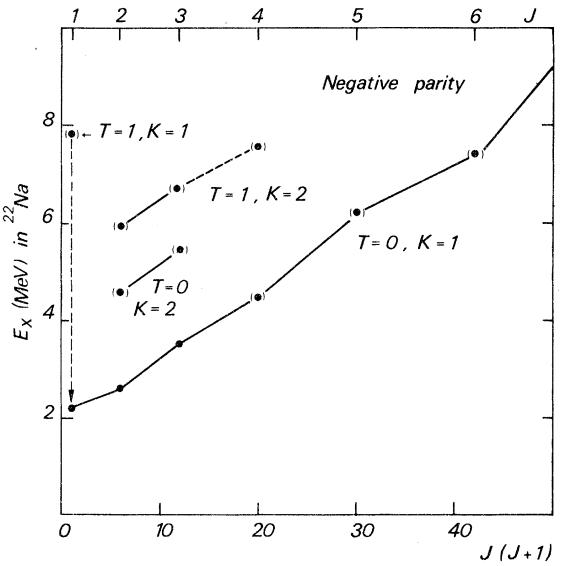

FIG. 4. Band structure of ^{22}Na negative parity levels. The two lowest band data are from Refs. 36 and 5. The other band assignments are those quoted or discussed in Sec. V B. All the bands are constructed on the same $\frac{1}{2}^-[101]-\frac{3}{2}^+[211]$ Nilsson configuration. See also caption of Fig. 3.

the $J^\pi = 2^+(1)^+$ level at 7410 keV should not be pure which would be consistent with a $T=0$ impurity coming from the 7803 keV level (see below).

2^+ level at $E_x = 7473$ keV. The $M1$ transition strengths (see Table II) exhibit $\Delta T = 1$ and thus indicate a $T=1$ character for this level. The isobaric analog would be one of the two levels at $E_x = 6819$ and 6855 keV in ^{22}Ne as suggested by the Coulomb energy and the particle reduced width (see Table III).

Level at $E_x = 7607$ keV. In Sec. IV, it was suggested that this level should be a doublet to account for its γ decay. One component of this doublet may be a $T=1$, $J^\pi = 4^-$ state, member of the rotational band $K^\pi = 2^-$, $T=1$ constructed on the $\frac{1}{2}^-[101]-\frac{3}{2}^+[211]$ Nilsson configuration (see Fig. 4). The $(2)^-$, $E_x = 5957$ keV and $3^-(0, 1, 2)^-$, $E_x = 6715$ keV states have previously been thought to belong to this band (Refs. 8 and 5, respectively), and the $J^\pi = 4^-$, $T=1$ member is expected at $E_x \sim 7.6$ MeV (see Fig. 4). This interpretation is consistent with the allowed ($E1$, $\Delta T = 1$) branchings to the 3^+ states at $E_x = 0$ and 1983 keV. The three other branchings to the $T=0$ state: 1^+ at $E_x = 582$ keV and to the $T=1$ states: 0^+ at $E_x = 657$ keV and 2^+ at $E_x = 1951$ keV are then due to an other $T=0$ component of

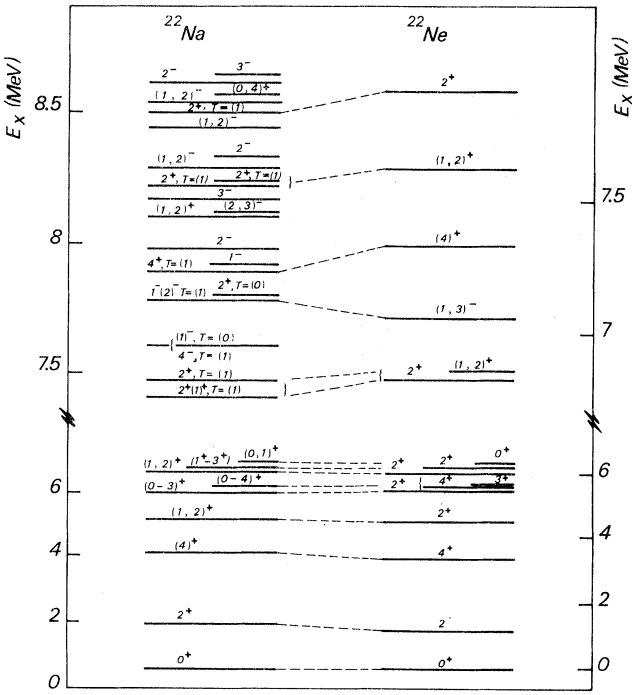

FIG. 5. The lower part of the figure shows the analog isobaric pairs in ^{22}Na and ^{22}Ne as given in Ref. 17 (^{22}Na spin and parity assignments are from Ref. 1). The upper part shows the parity, spin, isospin assignments, and ^{22}Ne isobaric analogs proposed for the ^{22}Na levels investigated here.

the doublet. This component might be the state seen in $^{12}C + ^{14}N \rightarrow \alpha + ^{22}Na$ and $^{10}B(^{16}O, \alpha)^{22}Na$ reaction studies (Refs. 37 and 35, respectively). A tentative spin assignment would be $J^\pi = 1^-$ in order to account for the $l_p = 1$ resonance formation observed in elastic scattering and which cannot be very much perturbed by the 4^- state (to participate in the process, the 4^- state involves $l_p = 3$ and a corresponding Wigner width less than 500 eV). The above $J^\pi = 4^-$ $T = 1$ state in ^{22}Na cannot be considered as a good candidate as analog to a state in ^{22}Ne. We note that the situation is the same for the $3^-(0, 1, 2)^-$, 6715 keV state belonging to the same rotational band.

$1^-(2)^-$ level at $E_x = 7780$ keV. The $J^\pi = 1^-$ and 2^- assignments lead to important $M1$ transition strengths associated with $\Delta T = 1$ (see Table II). This level has thus probably a $T = 1$ character and the final state $T = 0$, $J^\pi = 1^-$, $E_x = 2211$ keV obtained through the main branching with a strong $M1$ transition may be interpreted as its antianalog state. Then, a tentative J^π assignment is $J^\pi = 1^-(2)^-$ for the 7780 keV level. Further evidence is provided by the fact that both levels have a small particle width: for the 7780 keV level $\gamma_p^2 \sim 50$ keV (from the present work) and the 2211 keV level is weakly excited in the $^{21}Ne(^3He, d)^{22}Na$ reaction.[10] This last level has been regarded as the head of the $T = 0$, $K^\pi = 1^-$ band constructed on the $\frac{1}{2}^-[101] - \frac{3}{2}^+[211]$ Nilsson configuration (see Fig. 4). Then, the 7780 keV level would be the head of the corresponding $T = 1$ band in agreement with Garrett *et al.*[5] who predicted this bandhead at about 8 MeV excitation energy.

2^+ level at $E_x = 7803$ keV. This level exhibits a strong $M1$ transition to the $T = 1$, $J^\pi = 2^+$, $E_x = 1951$ keV level (see Table II). Thus the transition probably occurs with $\Delta T = 1$ and the 7803 keV level has a $T = 0$ character. However, the other branching (20% to the ground state with J^π, $T = 3^+$, 1) shows a $M1$ transition stronger than the $\Delta T = 0$ upper limit. Thus we assume a $T = 1$ isospin impurity which may come from the isospin mixing already considered with the $2^+(1)^+$ level at $E_x = 7410$ keV.

4^+ level at 7890 keV. For all branchings, the $M1$ strengths exceed the upper limit for $\Delta T = 0$ (see Table II), thus indicating a $T = 1$ isospin for the initial state. This 7890 keV state decays mainly to the ground state rotational band (3^+, 4^+, 5^+ levels) but this cannot indicate a splitting of the antianalog character because a $T = 1$, $K^\pi = 3^+$ band cannot correspond to the $T = 0$, $K^\pi = 3^+$ ground state band which is based on the symmetric $(\frac{3}{2}^+[211])^2$ Nilsson configuration. We therefore propose that the 4^+ level at 7890 keV is the third member of the rotational band $T = 1$, $K^\pi = 2^+$ constructed on the $\frac{3}{2}^+[211] - \frac{1}{2}^+[211]$ Nilsson configuration (see Fig. 3). The attribution for this first member $[(1, 2)^+$ at $E_x = 5165$ keV] has been proposed in Ref. 10, and the 3^+ level is to be expected at $E_x \sim 6.2$ MeV [possibly the $(0\text{-}4)^+$ level at $E_x = 6189$ keV] from the comparison with the known corresponding rotational band in ^{22}Ne. To justify the $K = 2$ value for the 7890 keV level, we recall that in the simple Nilsson model, the $M1$ transition matrix elements between one initial and several final states which are members of the same rotational band have the reduced matrix element between the two intrinsic configurations, as a common factor. From the Wigner-Eckart theorem, it follows that

$$B(M1, I_i \rightarrow I_f)/\langle J_i 1 K_i, K_f - K_i | J_f K_f \rangle$$

must be the same for the three transitions involved here, i.e., $(J_i = 4,\ K_i^\pi = 2^+) \rightarrow (J_f = 3, 4, 5,\ K_f^\pi = 3^+)$. One then readily obtains the relative contributions of these transitions: 2.8, 35, and 62.2%, respectively. These are to be compared with the values evaluated from the experimental data (see Table II): 2.2, 30.6, and 67.2%, in the same order. The agreement is satisfying, providing a good justification of the $K^\pi = 2^+$ band defined above for the 7890 keV level. The level at 7572 keV also decays to the ground state rotational band (3^+, 4^+, 5^+ members) and is thus another candidate as the third member of this $T = 1$, $K^\pi = 2^+$ band. However, the branchings are not consistent with the preceding discussion and especially the $J^\pi = 4^+$ hypothesis would yield $M1$ transition only consistent with $T = 0$. As concerns this level, it is worth noting that a 4^- hypothesis would yield the same isospin value thereby not disturbing our discussion of the J^π, $T = (4)^-$, (1) member of the 7607 keV doublet. However, we note that the 17% branching from the 7890 keV level to the 3^+, 1983 keV level member of the $K = 0$ ground state band violates the $\Delta K = 2$ selection rule. We also note that the 7890 keV is populated by $(^6Li, d)$ and (d, α) reactions (Refs. 2 and 36, respectively) thus suggesting a $T = 0$ isospin impurity, but it is not surprising in such an energy range and the same argument applies to both $T = 1$ levels[17] at $E_x = 6.19$ and 6.56 MeV. The Coulomb energy shift and particle widths criteria suggest that the $(4)^+$ level, $E_x = 7336$ is a good candidate for the analog state.

Levels above $E_x = 7900$ keV. To our knowledge, no γ-ray data are available for these levels. Therefore, the only criteria which may be used to identify isobaric analog pairs are the Coulomb energy shift and the particle reduced width [(a) and (b) of Sec. V A]. Our conclusions are shown in Table III and concern 2^+ levels at 8210, 8239, and 8500 keV. The first and second levels are

both good candidates for the isobaric analogs of the 7632 keV state in ^{22}Ne, the third level at 8500 keV is a good candidate for the isobaric analog at 7922 keV in ^{22}Ne. These three levels are not populated by reactions involving mainly $T=0$ states [(^{6}Li, α), (d, α), (^{16}O, α), etc., ...]. $T=1$ assignments for other levels would be more speculative.

VI. CONCLUSION

The present elastic and inelastic proton scattering experiment at low energy has led to an unambiguous proton angular momentum determination for 21 resonances corresponding to ^{22}Na states between 7.3 and 8.6 MeV excitation energy. This study has proved to be a useful tool to assign or to confirm the spin values for all these states and to allow the proposition of isospin character for eight of them.

However, in spite of these satisfactory results, more was expected as concerns the $T=1$ states (at least those having $J \leq 4$) on the basis of the shell model predictions of Ref. 19. Table III shows that $J=2$ is the most-probable assignment for all positive parity states found with an isospin $T=1$, except the state at 7.89 MeV to which $J^\pi = 4^+$ is assigned. Unfortunately, all theoretical states quoted in Ref. 19 with $T=1$ and $J^\pi = 2^+$ have already been related to corresponding experimental states.[19,8,17] Thus, we just may propose a correspondence between the 4^+ experimental state at 7.89 MeV and the fourth theoretical state with $T=1$, $J^\pi = 4$ expected at 7.50 MeV, the third theoretical state with $T=1$, $J^\pi = 4^+$ being presently unseen experimentally.

Finally, it would be interesting to dispose of extended shell-model results (for example more states calculated from the model of Ref. 19) to discuss the $J^\pi = 2^+$ states and the negative parity states seen in the present experiment with a $T=1$ isospin. On the other side, new measurements (for example, with γ-γ angular correlations) are needed to relate the remaining theoretical states[19] to corresponding experimental states (curiously, any $T=1$ calculated state[19] with an odd spin value have been reported in the previous and present experiments). Further evidence of this assertion is provided by (i) the fact that some levels seen in (p, γ) measurements, particularly those unseen in the present proton scattering study, have still unknown spin and parity values (ii) and that our discussion on the isospin character of the upper energy resonances suffered from the lack of γ data.

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