PHYSICAL REVIEW C **110**, L021001 (2024)

Letter

Tritium β decay and proton-proton fusion in pionless effective field theory

Ha S. Nguyen [*]
Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA

Jared Vanasse [†]
Fitchburg State University, Fitchburg, Massachusetts 01420, USA

(Received 15 May 2024; accepted 2 August 2024; published 27 August 2024)

The Gamow-Teller and Fermi matrix elements, $\langle \mathbf{GT} \rangle$ and $\langle \mathbf{F} \rangle$, respectively, for tritium β decay are calculated to next-to-leading order (NLO) in pionless effective field theory in the absence of Coulomb interactions and isospin violation giving the leading order predictions $\langle \mathbf{GT} \rangle_0 = 0.9807$ and $\langle \mathbf{F} \rangle_0 = 1$. Using an experimentally determined value for the tritium β decay GT matrix element, the two-body axial current low energy constant is fixed at NLO yielding $L_{1,A} = 6.01 \pm 2.08$ fm^3 at the renormalization scale of the physical pion mass, which agrees with predictions based on naive dimensional analysis. The impact of $L_{1,A}$ on proton-proton fusion is also discussed. Finally, the consequences of Wigner-SU(4) spin-isospin symmetry are considered for the Gamow-Teller matrix element.

DOI: 10.1103/PhysRevC.110.L021001

Introduction. The simplest nuclear system that offers an experimentally clean probe of the axial current is tritium β decay. Due to the small difference in the ^{3}H-^{3}He binding energies ($\sim$764 keV), tritium β decay near threshold can be described quite naturally in pionless effective field theory [EFT($\not\pi$)], which has been used to great success in the description of two- and three-body nuclear systems (see Refs. [1,2] for reviews). Kong and Ravndal [3,4] used EFT($\not\pi$) to calculate the $pp \to de^+\nu_e$ cross section to next-to-leading order (NLO), but were constrained by the unknown two-body axial current low energy constant (LEC) $L_{1,A}$. To next-to-next-to-leading order (NNLO), Ref. [5] showed that $\nu_x d \to np\nu_x$ and $\bar{\nu}_e d \to nne^+$ were also limited by the uncertainty in $L_{1,A}$. Precise knowledge of these processes is important for determining the total flux of neutrinos from the sun in the Sudbury Neutrino Observatory experiment. $L_{1,A}$ also appears in the process $\mu^- d \to nn\nu_\mu$, which will be measured in the upcoming MuSUN experiment [6,7].

Tritium β decay is a superallowed process that is predominantly given by the Gamow-Teller (GT) and Fermi (F) matrix elements. It is also of inherent interest as a detailed analysis of the tail end of its spectrum can give the mass of the antineutrino [8] and potentially give evidence for sterile neutrinos [9–11].

*Contact author: ha.s.nguyen@duke.edu
†Contact author: jvanass3@fitchburgstate.edu

Using the formalism of Ref. [12], this letter calculates the GT and F matrix elements of tritium β decay to NLO in EFT($\not\pi$). Isospin invariance is assumed and Coulomb interactions are neglected but their contribution is estimated to be small. Such effects can be treated as perturbative corrections and making these assumptions removes the need for an additional Coulomb dependent isospin breaking three-body force correction [13,14]. The experimentally determined GT matrix element is used to fit $L_{1,A}$ at NLO. In addition, the consequences of Wigner-SU(4) spin-isospin symmetry [15] on the GT matrix element are explored, the order at which three-nucleon effects will become important for the axial current is discussed, and the impact of $L_{1,A}$ on proton-proton (pp) fusion is given.

Lagrangian and two-body system. The Lagrangian for EFT($\not\pi$) including the weak axial and vector currents in the dibaryon formalism is given by

$$\begin{aligned}\mathcal{L} = {} & \hat{N}^\dagger\left(i\partial_0 + \frac{\vec{\nabla}^2}{2M_N}\right)\hat{N} + \frac{g_A}{\sqrt{2}}\hat{N}^\dagger\sigma_i\tau^+\hat{N}\hat{A}_i^- + g_V\hat{N}^\dagger\tau^+\hat{N}\hat{V}_0^- \\ & + \hat{t}_i^\dagger\left[\Delta_t - c_{0t}\left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} + \frac{\gamma_t^2}{M_N}\right)\right]\hat{t}_i \\ & + \hat{s}_a^\dagger\left[\Delta_s - c_{0s}\left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} + \frac{\gamma_s^2}{M_N}\right)\right]\hat{s}_a \\ & + y\left[\hat{t}_i^\dagger\hat{N}^T P_i\hat{N} + \hat{s}_a^\dagger\hat{N}^T\bar{P}_a\hat{N} + \text{H.c.}\right] \\ & + \hat{\psi}^\dagger\Omega\hat{\psi} + \sum_{n=0}^{1}\left[\omega_{t0}^{(n)}\hat{\psi}^\dagger\sigma_i\hat{N}\hat{t}_i - \omega_{s0}^{(n)}\hat{\psi}^\dagger\tau_a\hat{N}\hat{s}_a + \text{H.c.}\right] \\ & + l_{1,A}\hat{t}_k^\dagger\hat{s}_-\hat{A}_k^- + l_{1,V}\hat{s}_3^\dagger\hat{s}_-\hat{V}_0^- + \text{H.c.},\end{aligned} \tag{1}$$

where $\hat{t}_i$ ($\hat{s}_a$) is the spin-triplet (spin-singlet) dibaryon, $\hat{\psi}$ is the isodoublet three-nucleon field, $\hat{A}_k^-$ is the leptonic axial

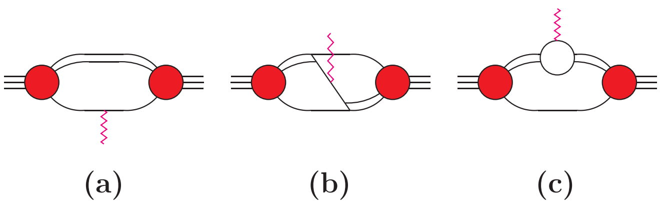

FIG. 1. Diagrams for the LO three-nucleon axial and vector form factors. The zig-zag represents the one-body axial or vector current.

current, and $\hat{V}_0^-$ is the temporal component of the leptonic vector current. The strong interaction parameters are fit using the Z parametrization [16], which at leading order (LO) fits to the nucleon-nucleon (NN) scattering ${}^3S_1$ and ${}^1S_0$ poles and their residues at NLO, yielding

$$y^2 = \frac{4\pi}{M_N}, \quad \Delta_t = \gamma_t - \mu, \quad c_{0t} = (Z_t - 1)\frac{M_N}{2\gamma_t},$$
$$\Delta_s = \gamma_s - \mu, \quad c_{0s} = (Z_s - 1)\frac{M_N}{2\gamma_s}, \tag{2}$$

where $\gamma_t = 45.7025$ MeV ($\gamma_s = -7.890$ MeV) is the ${}^3S_1$ bound state (${}^1S_0$ virtual bound state) momentum and $Z_t = 1.6908$ ($Z_s = 0.9015$) is the residue about the ${}^3S_1$ (${}^1S_0$) pole. Three-body parameters Ω and $\omega^{(n)}_{\{t,s\}0}$ are fit to the ^{3}H binding energy [17] and $P_i = \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2$ ($\bar{P}_a = \frac{1}{\sqrt{8}}\tau_2\tau_a\sigma_2$) projects out the spin-triplet isosinglet (spin-singlet isotriplet) combination of nucleons. The Pauli matrix τ^+ is normalized such that $\tau^+ = -(\tau_1 + i\tau_2)/\sqrt{2}$ and the one-nucleon axial (vector) coupling is $g_A = 1.26$ ($g_V = 1$). The two-body axial current LEC $l_{1,A}$ in the dibaryon formalism is related to the traditional $L_{1,A}$ coupling in Ref. [18] via

$$l_{1,A} = -\frac{M_N}{4\pi}\Delta_t\Delta_s L_{1,A} + \frac{g_A}{2}\left(c_{0t}\frac{\Delta_s}{\Delta_t} + c_{0s}\frac{\Delta_t}{\Delta_s}\right),$$
$$l_{1,V} = g_V c_{0s}. \tag{3}$$

The additional terms in $l_{1,A}$ and $l_{1,V}$ are induced by the coordinate transformation relating the nucleon formalism to the dibaryon formalism. $l_{1,V}$ is entirely predicted by other known LECs and contains no new two-body vector current LEC.

GT and F matrix elements. The half-life of ^{3}H β decay $t_{1/2}$ is given by [19]

$$\frac{(1+\delta_R)f_V}{K/G_V^2}t_{1/2} = \frac{1}{\langle \mathbf{F}\rangle^2 + (f_A/f_V)g_A^2\langle \mathbf{GT}\rangle^2}, \tag{4}$$

where $\langle \mathbf{F}\rangle$ ($\langle \mathbf{GT}\rangle$) is the Fermi (Gamow-Teller) matrix element and other parameters are given in Ref. [19].

In the absence of Coulomb interactions and assuming isospin invariance, the axial and vector form factor can be calculated using Ref. [12]. In the limit $Q^2 = 0$, the axial (vector) form factor gives the GT (F) matrix element. The axial (vector) form factor is given by the sum of diagrams in Fig. 1 where single lines are nucleons, double lines dibaryons, triple lines three-nucleon systems, circles three-nucleon vertex functions, and zig-zag lines the axial or vector current. Details of how these diagrams and the three-nucleon vertex functions are calculated can be found in Refs. [12,17]. The LO GT (F) matrix element is given by

TABLE I. Values of coefficients for the LO weak form factor used in Eq. (6).

Form factor	a_{11}	a_{22}	b_{11}	b_{12}	b_{21}	b_{22}	c_{11}	c_{12}	c_{21}	c_{22}
$F_W^{\rm GT}(Q^2)$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	0	$\frac{2}{3}$	$\frac{2}{3}$	0
$F_W^{\rm F}(Q^2)$	1	$-\frac{1}{3}$	1	1	1	$-\frac{5}{3}$	0	0	0	$\frac{4}{3}$

$$F^{\chi}_{W,0}(Q^2 = 0) = 2\pi M_N\left(\widetilde{\boldsymbol{\Gamma}}_0(q)\right)^T \otimes f_0(q,\ell) \otimes \widetilde{\boldsymbol{\Gamma}}_0(\ell), \tag{5}$$

where

$$f_0(q,\ell) = 2\pi M_N\left\{g(q,\ell)\begin{pmatrix} c_{11}+a_{11} & c_{12} \\ c_{21} & c_{22}+a_{22}\end{pmatrix} + h(q,\ell) \times \begin{pmatrix} b_{11}-2a_{11} & b_{12}+3(a_{11}+a_{22}) \\ b_{21}+3(a_{11}+a_{22}) & b_{22}-2a_{22}\end{pmatrix}\right\}, \tag{6}$$

$$g(q,\ell) = \frac{\pi}{2}\frac{\delta(q-\ell)}{q^2\sqrt{\frac{3}{4}q^2 - M_N B}}, \tag{7}$$

and

$$h(q,\ell) = \frac{1}{q^2\ell^2 - (q^2+\ell^2-M_N B)^2}. \tag{8}$$

χ is GT (F) for the axial (vector) form factor. The function $\widetilde{\boldsymbol{\Gamma}}_0(q)$ is related to the three-nucleon vertex function [17]. Coefficients a_{11}, b_{11}, c_{11}, etc., in Eq. (6) come from projecting out the one-body axial or vector current for each of the diagrams in the doublet S-wave channel giving the values in Table I. $B = 8.48$ MeV is the triton binding energy.

The NLO correction to the axial (vector) form factor is given by the diagrams in Fig. 2 where diagram (d) is the $l_{1,A}$ ($l_{1,V}$) term for the axial (vector) form factor and diagram (e) is subtracted to avoid double counting from diagram-(a) and its time reversed version. In the limit $Q^2 = 0$, these

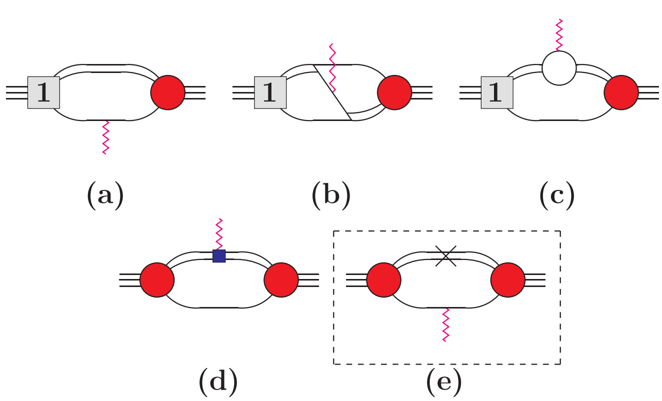

FIG. 2. NLO corrections to the axial and vector form factor. The dashed-boxed diagram is subtracted to avoid double counting and the box with "1" is the NLO correction to the three-nucleon vertex function. Diagram (d) for the axial (vector) form factor comes from the two-body axial (vector) current term $l_{1,A}$ ($l_{1,V}$). Diagrams related by time reversal symmetry are not shown.

TABLE II. Values of coefficients for the NLO correction to the axial (vector) form factor used in Eq. (10).

Form factor	d_{11}	d_{12}	d_{21}	d_{22}
$F_W^{GT}(Q^2)$	0	$\frac{1}{3}l_{1,A}/M_N$	$\frac{1}{3}l_{1,A}/M_N$	0
$F_W^F(Q^2)$	0	0	0	$\frac{4}{3}c_{0s}/M_N$

contributions give the NLO correction to the GT (F) matrix element given by

$$F_{W,1}^X(Q^2=0) = 2\pi M_N\big(\widetilde{\boldsymbol{\Gamma}}_1(q)\big)^T \otimes f_0(q,\ell)\otimes \widetilde{\boldsymbol{\Gamma}}_0(\ell) + 2\pi M_N\big(\widetilde{\boldsymbol{\Gamma}}_0(q)\big)^T \otimes f_0(q,\ell)\otimes \widetilde{\boldsymbol{\Gamma}}_1(\ell) - 4\pi M_N\big(\widetilde{\boldsymbol{\Gamma}}_0(q)\big)^T \otimes f_1(q,\ell)\otimes \widetilde{\boldsymbol{\Gamma}}_0(\ell), \quad (9)$$

where

$$f_1(q,\ell) = \frac{\pi}{2}\frac{\delta(q-\ell)}{q^2}\begin{pmatrix} \frac{c_{0t}}{M_N}a_{11}+d_{11} & d_{12} \\ d_{21} & \frac{c_{0s}}{M_N}a_{22}+d_{22}\end{pmatrix}. \quad (10)$$

Coefficients d_{11}, d_{12}, d_{21}, and d_{22} are given in Table II. $\widetilde{\boldsymbol{\Gamma}}_1(q)$ is related to the NLO correction to the three-nucleon vertex function [17].

Results. The LO GT matrix element is $\langle \mathbf{GT}\rangle_0 = \sqrt{3}\times 0.9807$, while the NLO GT matrix element depends on $L_{1,A}$.[1] Taking $L_{1,A}=0$ gives $\langle \mathbf{GT}\rangle_{0+1} = \sqrt{3}\times 0.8777$. Choosing the renormalization scale $\mu = m_\pi$, a fit of $L_{1,A}$ to the value extracted from experiment $\langle \mathbf{GT}\rangle_{\exp} = \sqrt{3}\times 0.9511(13)$ [20] yields

$$L_{1,A}(\mu = m_\pi) = 6.01 \pm 2.08\ \mathrm{fm}^3. \quad (11)$$

This is compatible with naturalness expectations, which predicts a value of [4]

$$|L_{1,A}(\mu=m_\pi)| \approx \frac{1}{m_\pi(m_\pi-\gamma_t)^2} = 6.5\ \mathrm{fm}^3. \quad (12)$$

The pp-fusion rate is given by the matrix element [4]

$$|\langle d; j|A_k^-|pp\rangle| = g_A C_\eta \sqrt{\frac{32\pi}{\gamma_t^3}}\Lambda(p)\delta_k^j, \quad (13)$$

where C_η is the Sommerfeld factor in Coulomb scattering and $\Lambda(p)$ at threshold to NLO in the Z parametrization is given by [4][2]

$$\Lambda(0) = \frac{1}{2}(1+Z_t)\{e^\eta - \gamma_t a_{pp}[1-\eta e^\eta\Gamma(0,\eta)]\} - \gamma_t^2 a_{pp}\frac{\gamma_t-\mu}{M_N C_{0,-1}^{(pp)}}\left[\frac{L_{1,A}}{g_A} - \frac{M_N}{2}\left(C_{2,-2}^{(pp)} + C_{2,-2}^{(d)}\right)\right]. \quad (14)$$

The value $\eta = \alpha M_n/\gamma_t$, α is the fine structure constant from quantum electrodynamics (QED), and $\Gamma(0,\eta)$ an incomplete gamma function all arising from Coulomb corrections. The LECs $C_{0,-1}^{(pp)}$, $C_{2,-2}^{(pp)}$, and $C_{2,-2}^{(d)}$ are given in Ref. [4]. Plugging in physical values, the NLO Z-parametrization prediction for $\Lambda(0)$ is

$$\Lambda(0) = 2.72 + 0.0087\left(\frac{L_{1,A}}{1\ \mathrm{fm}^3}\right) + \mathcal{O}(12\%). \quad (15)$$

Using the value for $L_{1,A}$ from Eq. (11) gives $\Lambda(0) = 2.77(33)$. Within errors, this prediction agrees with the phenomenological value $\Lambda(0) = 2.65(1)$ [22].

In the isospin limit and ignoring higher partial waves, the GT matrix element is given by

$$\langle \mathbf{GT}\rangle = \sqrt{3}\times(P_S - P_{S'}/3), \quad (16)$$

where P_S is the probability of the triton wave function being in the symmetric S state and $P_{S'}$ is the probability of the mixed symmetry S' state [19]. In the Wigner-SU(4) limit ($\gamma_t = \gamma_s$, $Z_t = Z_s$), $P_{S'} = 0$ and therefore $\langle \mathbf{GT}\rangle = \sqrt{3}$ up to NLO, which is verified numerically. In order to get $\langle \mathbf{GT}\rangle = \sqrt{3}$ at NLO, $l_{1,A}$ is defined by Eq. (3) with $L_{1,A} = 0$. In the Wigner-SU(4) limit, the non $L_{1,A}$ term in Eq. (3) becomes μ independent. Similarly, since there is no isospin breaking, it is found at LO and NLO that the F matrix element reproduces the wave function renormalization expression and therefore $\langle \mathbf{F}\rangle_0 = 1$ and $\langle \mathbf{F}\rangle_{0+1} = 1$, in agreement with the Ademollo-Gatto theorem [23].

The GT matrix element has been calculated previously in EFT($\not\pi$) with and without Coulomb interactions in Ref. [24]. However, Ref. [24] did not include Coulomb corrections either nonperturbatively or strictly perturbatively. Rather Ref. [24], building upon the work of Ref. [13], treated Coulomb nonperturbatively in the two-body pp subsector and iterated all three-nucleon diagrams with a single Coulomb photon exchange. Although correct to $\mathcal{O}(\alpha)$ this method contains an infinite subset of higher order Coulomb corrections and therefore impedes proper error estimates. To include Coulomb corrections strictly perturbatively (nonperturbatively) the approach in Ref. [14] (Refs. [25,26]) can be used. In addition, when Ref. [24] dropped Coulomb and isospin breaking terms, they found $\langle \mathbf{GT}\rangle_0 = \sqrt{3}$. However, this result is only true in the Wigner-SU(4) limit and Ref. [24] did not appear to be in the Wigner-SU(4) limit as indicated by their choice of parameters.

Conclusions. In this work, the Gamow-Teller ($\langle \mathbf{GT}\rangle$) and Fermi ($\langle \mathbf{F}\rangle$) matrix elements of ^{3}H β decay have been calculated to NLO in EFT($\not\pi$) in the Z parametrization ignoring Coulomb and isospin effects. The omitted Coulomb effects are perturbative corrections approximately of the size $\alpha M_n/p_* \approx 6\%$, where $p_* = \sqrt{(4/3)(M_N(B_{^3\mathrm{H}} + \gamma_t^2)} \approx 112$ MeV is the approximate three-nucleon binding momentum.[3] Although Coulomb corrections to the amplitude are $\approx 6\%$, their effect on $L_{1,A}$ can be more sizable and is given

[1]The factor of $\sqrt{3}$ comes from a Clebsch-Gordan coefficient.

[2]pp fusion in the Z parametrization can be obtained from calculations in the ERE parametrization by replacing all occurrences of the effective range ρ_t with $(Z_t-1)/\gamma_t$. Differences between ERE and Z parametrization in the $^1S_0$ channel are $\sim 1\%$ effects [21] and can be neglected at NLO.

[3]This estimate for the binding momentum differs from previous estimates that did not include the binding of the deuteron [27]. Previous estimates found Coulomb effects to be slightly larger.

naively by

$$\frac{\Delta L_{1,A}}{L_{1,A}} \approx \left| \frac{\frac{\alpha M_N}{p^*}\langle \mathbf{GT}\rangle_0}{\langle \mathbf{GT}\rangle_{\text{exp}} - \langle \mathbf{GT}\rangle_{0+1}|_{L_{1,A}=0}} \right| = 0.80\ . \qquad (17)$$

Thus, Coulomb corrections are potentially important for $L_{1,A}$ and can be included perturbatively as in Ref. [14] or in a nonperturbative fashion as in Refs. [25,26]. Propagating the uncertainty due to Coulomb corrections for $L_{1,A}$, Eq. (17), through our expression for *pp* fusion, Eq. (15), leads to a ≈1.5% effect.

At NLO, the two-body axial current LEC $L_{1,A} = 6.01 \pm 2.08$ fm^3 was fit to reproduce $\langle \mathbf{GT}\rangle_{\text{exp}}$. In addition, it was also found that in the Wigner-SU(4) limit, $\langle \mathbf{GT}\rangle = \sqrt{3}$ at LO and NLO (with values for $l_{1,A}$ solely predicted from two-body physics), in agreement with analytical predictions and is a nontrivial check on the calculation. The value for $\langle \mathbf{F}\rangle$ at LO and NLO was found to be 1, which is expected due to the lack of isospin breaking up to NLO and the Adellamo-Gatto theorem [23]. Finally, using the value for $L_{1,A}$ determined from ^{3}H β decay, the threshold value for the *pp*-fusion reduced matrix element is $\Lambda(0) = 2.77(33)$ with a 12% NLO EFT($\not\pi$) error estimate.

Outlook. Our calculation in the Wigner-SU(4) limit provides an essential benchmark for any calculation of ^{3}H β-decay. $L_{1,A}$ is the only unknown two-body axial current LEC up to NNLO in EFT($\not\pi$). Thus, in principle with a prediction of $L_{1,A}$, the *pp*-fusion cross section could be determined to 3% with a NNLO EFT($\not\pi$) calculation including Coulomb corrections. However, our predicted value of $L_{1,A}$ relied on a fit to ^{3}H β decay at NLO in EFT($\not\pi$). A NNLO EFT($\not\pi$) $L_{1,A}$ calculation of ^{3}H β decay would necessitate refitting $L_{1,A}$ or adding a perturbative correction. Reference [28] demonstrated that a NNLO EFT($\not\pi$) calculation of the three-nucleon magnetic moments requires the insertion of a new three-body current counterterm. This would imply there is a three-body axial current counterterm. Therefore, a NNLO calculation of ^{3}H β decay is not possible without fitting this new three-body axial current counterterm to a new three-body datum. In the case of χEFT, this would manifest as a three-nucleon meson exchange current that would give a sizable contribution at low energies. This also implies that any calculation including meson exchange must include three-body meson exchange currents at low energies to make accurate comparisons between *pp* fusion and ^{3}H β decay.

Acknowledgments. We would like to thank R. P. Springer, X. Lin, B. C. Tiburzi, and D. R. Phillips for useful discussions during the course of this work. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Awards No. DE-FG02-93ER40756 (J.V.) and No. DE-FG02-05ER41368 (H.S.N.).

[1] S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips, and M. J. Savage, From hadrons to nuclei: Crossing the border, in *At The Frontier of Particle Physics: Handbook of QCD* (World Scientific, Singapore, 2001), pp. 133–269.
[2] J. Vanasse, Int. J. Mod. Phys. E **25**, 1641002 (2016).
[3] X. Kong and F. Ravndal, Phys. Rev. C **64**, 044002 (2001).
[4] M. Butler and J.-W. Chen, Phys. Lett. B **520**, 87 (2001).
[5] M. Butler, J.-W. Chen, and X. Kong, Phys. Rev. C **63**, 035501 (2001).
[6] V. A. Andreev *et al.*, arXiv:1004.1754.
[7] P. Kammel (MuSun Collaboration), SciPost Phys. Proc. **5**, 018 (2021).
[8] G. Drexlin, V. Hannen, S. Mertens, and C. Weinheimer, Adv. High Energy Phys. **2013**, 293986 (2013).
[9] S. Mertens, T. Lasserre, S. Groh, G. Drexlin, F. Glueck, A. Huber, A. W. P. Poon, M. Steidl, N. Steinbrink, and C. Weinheimer, J. Cosmol. Astropart. Phys. 02 (2015) 020.
[10] J. Barry, J. Heeck, and W. Rodejohann, J. High Energy Phys. 07 (2014) 081.
[11] M. Aker *et al.* (KATRIN Collaboration), J. Phys. G: Nucl. Part. Phys. **49**, 100501 (2022).
[12] J. Vanasse, Phys. Rev. C **98**, 034003 (2018).
[13] J. Vanasse, D. A. Egolf, J. Kerin, S. König, and R. P. Springer, Phys. Rev. C **89**, 064003 (2014).
[14] S. König, H. W. Grießhammer, H.-W. Hammer, and U. van Kolck, J. Phys. G **43**, 055106 (2016).
[15] E. Wigner, Phys. Rev. **51**, 106 (1937).
[16] D. R. Phillips, G. Rupak, and M. J. Savage, Phys. Lett. B **473**, 209 (2000).
[17] J. Vanasse, Phys. Rev. C **95**, 024002 (2017).
[18] M. Butler and J.-W. Chen, Nucl. Phys. A **675**, 575 (2000).
[19] R. Schiavilla *et al.*, Phys. Rev. C **58**, 1263 (1998).
[20] A. Baroni, L. Girlanda, A. Kievsky, L. E. Marcucci, R. Schiavilla, and M. Viviani, Phys. Rev. C **94**, 024003 (2016); **95**, 059902(E) (2017).
[21] H. W. Grießhammer, Nucl. Phys. A **744**, 192 (2004).
[22] E. G. Adelberger *et al.*, Rev. Mod. Phys. **83**, 195 (2011).
[23] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).
[24] H. De-Leon, L. Platter, and D. Gazit, Phys. Rev. C **100**, 055502 (2019).
[25] S.-I. Ando and M. C. Birse, J. Phys. G **37**, 105108 (2010).
[26] S. König, H. W. Grießhammer, and H.-W. Hammer, J. Phys. G **42**, 045101 (2015).
[27] J. Vanasse and D. R. Phillips, Few Body Syst. **58**, 26 (2017).
[28] X. Lin, H. Singh, R. P. Springer, and J. Vanasse, Phys. Rev. C **108**, 044001 (2023).