Exploring the possibility of wobbling motion in ^{129}Ba

S. Chakraborty [1,*] S. Bhattacharyya [1,2] G. Mukherjee [1,2] and C. Majumder [3]
[1] Variable Energy Cyclotron Centre, Kolkata 700064, India
[2] Homi Bhabha National Institute, Mumbai 400094, India
[3] Department of Physics, Indian Institute of Technology Bombay, Mumbai 400076, India

(Received 30 April 2024; revised 26 July 2024; accepted 15 August 2024; published 28 August 2024)

The structure of the negative parity bands, based on the $\nu h_{11/2}$ orbital, in ^{129}Ba is revisited to search for the wobbling mode in this nucleus. Existing angular distribution/correlation and linear polarization results are indicative of a large $E2$ admixture in the 365 keV interconnecting $\Delta I = 1$ γ transition between yrast bands. This provides an indication of the existence of a wobbling mode in this nucleus. Theoretical calculations performed in this work using the quasiparticle plus triaxial rotor model have effectively replicated the available experimental results.

DOI: 10.1103/PhysRevC.110.024324

The emergence of collective behavior of atomic nuclei in terms of various geometrical shapes remains a subject of utmost importance in nuclear physics [1]. Investigation of triaxial nuclear shapes has drawn a lot of attention in recent decades as it manifested a number of structural phenomena in high as well as low angular momentum regions. Observation of a pair of chiral doublet bands or a wobbling band can serve as the experimental fingerprints of nonaxial nuclear shapes. A triaxially deformed rotating nucleus could have its rotational angular momentum precess and nutate about the principal axis of the nucleus, leading to an origin of the wobbling bands [1]. It is the quantum mechanical analog of the spinning motion of an asymmetric top. The existence of wobbling motion in atomic nuclei was predicted long ago by Bohr and Mottelson [1] in even-even nuclei, but, experimentally it was first identified in odd-A ^{163}Lu by Ødegård and co-workers in the year 2001 [2]. Subsequently, a few more wobbly Lu nuclei were discovered, expanding the list to $^{161-167}$Lu [2,3]. Outside the Lu isotopes, the wobbling band was also identified in ^{167}Ta at higher angular momentum [4,5]. However, no other wobbling band was identified in the neighboring nuclei as there was a competition between wobbling and particle hole excitations, and the latter become favored as calculated under the framework of tilted axis cranking [6]. Apart from Lu and Ta nuclei, a wobbling mode was also identified in odd-A ^{105}Pd [7], ^{125,127}Xe [8,9], ^{133}Ba [10], ^{133}La [11], ^{135}Pr [12,13], ^{151}Eu [14] and ^{183,187}Au [15,16] at relatively lower angular momentum. In most of these cases, the earlier reported unfavored signature partner of $\pi h_{9/2}$, $\pi h_{11/2}$, $\pi i_{13/2}$ or $\nu h_{11/2}$ bands have been reinterpreted in terms of wobbling excitation mostly based on the large δ value of the connecting transitions. However, in contrast to the rich abundance of chiral bands in the nuclear landscape [17], experimental evidence in favor of wobbling mode is rare, hence the global characteristics of this phenomena are not well understood.

The description of wobbling motion faced opposition when some discrepancies related to this unique mode of excitation were noticed theoretically [18]. At the same time, evidence against the wobbling nature of the low-spin bands in ^{135}Pr and ^{187}Au was also reported from experimental studies [19,20]. The scenario became more complicated when a new kind of rotational motion, namely tilted precession (TiP), was introduced [21]. The TiP bands are found to be approximately similar to the one-quasiparticle longitudinal wobbling bands at high spins in odd-A nuclei, but, such a one-quasiparticle TiP band cannot be approximated with a transverse wobbling band either at low or at high spins [21]. Interestingly, two adjacent isotopes of neodymium, ^{135,136}Nd, exhibit two different kinds of band structures. While, the odd-A ^{135}Nd shows a TiP band associated with $\nu h_{11/2}$ configuration [22], the $\pi h^2_{11/2}$ two-quasiparticle configuration in the *even-even* ^{136}Nd nucleus is found to be wobbly [23]. Hence, the study of triaxial deformation has drawn a considerable interest nowadays in the field of nuclear structure research and demands more experimental as well as theoretical investigations. Recent works on the odd-N Xe–Ba nuclei in the $A \approx 130$ region have added a further twist to research on the wobbling mode. For instance, ^{133}Ba (^{127}Xe) shows the existence of wobbling bands based on the $\nu h_{11/2}$ quasiparticle configuration [9,10], but no such evidence of wobbling mode is found in its isotone (isotope), ^{131}Xe ($Z = 54$, $N = 77$) [24]. It was suggested that the nonobservation of a wobbling band in ^{131}Xe can be explained in terms of the deformation of the adjacent *even-even* core. Consequently, it was also proposed that the favorable candidates for wobbling motion are the odd-A nuclei surrounded by two *even-even* nuclei with $E_{4^+}/E_{2^+} \geqslant 2.3$ and $Q_0 \geqslant 2.6$ b [24].

The Xe–Ba isotopes around $A = 130$ are located between the spherical Sn region and the well-deformed Ce region. As a result, these nuclei exhibit various structural features that are expected in transitional nuclei. The nucleus ^{129}Ba is surrounded by *even-even* ^{128,130}Ba nuclei, both of which satisfy the conditions proposed in Ref. [24] in the context

*Contact author: dr.saikat@yahoo.com

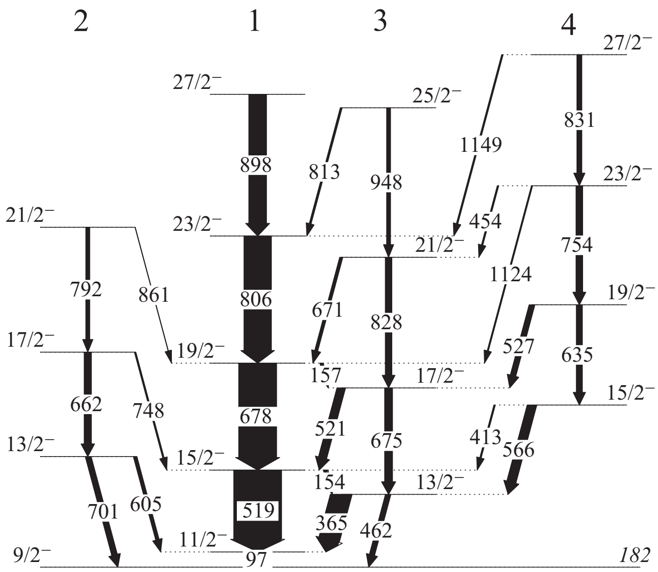

FIG. 1. The negative parity bands, as reported in Ref. [25], based on the $\nu h_{11/2}$ orbitals in ^{129}Ba.

of wobbling motion. A family of four negative parity bands based on the $\nu h_{11/2}$ quasi-particle configuration was reported in ^{127}Xe and ^{129}Ba. Since evidence of wobbling motion was already found in ^{127}Xe$_{73}$ [9], an attempt was made in this work to reveal whether or not the triaxially deformed ^{129}Ba$_{73}$ nucleus also exhibits the same phenomenon.

A few low-spin negative parity states in ^{129}Ba were identified through deuteron (^{2}H) induced reactions [28,29]. However, a majority of the higher spin states in this nucleus were studied via heavy-ion induced fusion-evaporation reactions [25–27,30,31]. Gizon and co-workers first attempted to explore the structure of the high spin states in this nucleus using the ^{120}Sn(^{12}C, $3n\gamma$) ^{129}Ba reaction at a beam energy of 52 MeV [26]. Along with detailed $\gamma\gamma$-coincidence and angular distribution measurements, the linear polarization measurement was also carried out for a few γ transitions [27]. From this investigation, they managed to identify two sets of rotational bands associated with $\nu h_{11/2}$ and $\nu g_{7/2}$ orbitals. A further major upgrade of the level scheme of ^{129}Ba was done by Byrne and co-workers via the ^{116}Cd(^{18}O, $5n\gamma$) ^{129}Ba reaction at 86 MeV beam energy [25]. In this work, the earlier reported $\nu g_{7/2}$ ($\nu h_{11/2}$) band was extended up to $\frac{55}{2}\hbar$ ($\frac{45}{2}\hbar$) tentatively along with several other newly identified band structures. In the present work, however, the spectroscopic information reported mainly in Refs. [25–27] is critically revisited.

The negative parity bands of present interest in ^{129}Ba, as reported by Byrne *et al.* [25], are shown in Fig. 1. The spin-parity assignment and the decay pattern of the states belonging to bands 2 and 3 are found to be similar, making both of them suitable candidates for the unfavored signature partner of the $\nu h_{11/2}$ band. Therefore, it is important to study the electromagnetic properties of these $\Delta I = 1$ transitions and their decomposition in the $E2$ and $M1$ amplitudes. The A_{22} and $R_{\rm DCO}$ values, as reported in Ref. [25], of the pertinent $\Delta I = 1$ transitions indicate that the magnitudes of these two quantities for the transitions between the negative parity bands

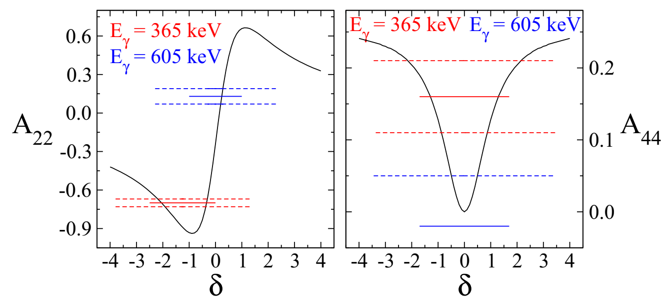

FIG. 2. Variation of theoretical angular distribution coefficients (black line), A_{22} and A_{44}, as a function of the multipole mixing ratio (δ). The experimental values (error) of A_{22} and A_{44} are shown as red and blue solid (dashed) lines [26,27].

$3 \rightarrow 1$ (e.g., 365 keV) and $4 \rightarrow 3$ (e.g., 566 keV) are different than that of the negative parity bands $2 \rightarrow 1$ (e.g., 605 keV). This is perhaps owing to the different amounts of multipole mixing in these transitions. The large negative values of A_{22} and correspondingly small magnitudes of $R_{\rm DCO}$ are indicative of a negative $\delta = \langle E2\rangle/\langle M1\rangle$ mixing ratio for the $\Delta I = 1$ transitions of bands $3 \rightarrow 1$ and $4 \rightarrow 3$ [25]. To explore this quantitatively, the available experimental results are revisited.

The angular distribution coefficients, A_{22} and A_{44}, of $13/2^- \rightarrow 11/2^-$ transition have been calculated for different values of δ and have been plotted along with their experimentally measured values as shown in Fig. 2. Figure 3 shows the contour plot of these angular distribution coefficients and the corresponding χ^2 analysis. From these figures, it is evident that the analysis of the angular distribution results for $E_\gamma = 605$ keV transition (band $2 \rightarrow 1$) yields a low value of δ, hence they are indicative of having low $E2$ admixture. Similarly, the $E_\gamma = 748$ keV transition also has a low $E2$ fraction, as shown in Fig. 4. Thus, band 2 is more likely the unfavored signature partner of the $\nu h_{11/2}$ band. In this context it is worth noting that, in the recent studies on ^{125,127}Xe and ^{133}Ba, the bands above yrare $I^\pi = 13/2^-$ states were also identified as the unfavoured signature partner of the $\nu h_{11/2}$ band [8–10].

Gizon and co-workers also measured the angular distribution along with the linear polarization of some of the γ rays, decaying from band 3 to band 1, in ^{129}Ba [27]. In particular, for the 365 keV γ-transition, the following spectroscopic

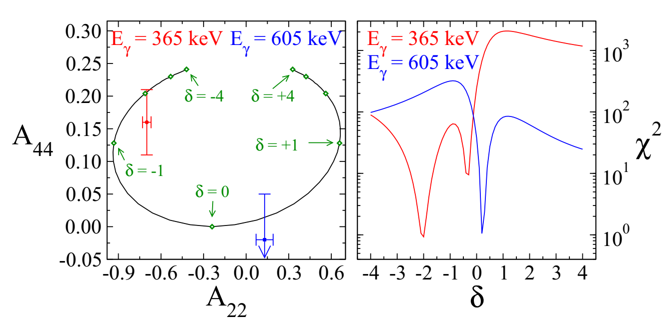

FIG. 3. Left: Contour plot of the angular distribution coefficients, A_{22} versus A_{44}, for different mixing ratios (δ, marked in green). The experimental data points are red and blue. Right: Plot of the χ^2 of A_{22} and A_{44} as a function of δ.

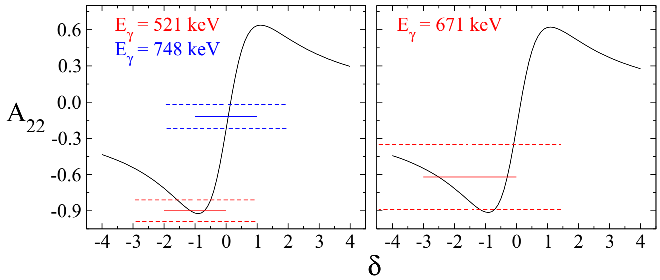

FIG. 4. Variation of theoretical angular distribution coefficient A_{22} (black line) as a function of the multipole mixing ratio (δ). Corresponding experimental values (error) are shown as red and blue solid (dashed) lines [25].

results were reported:

$$A_{22} = -0.70(3), \quad A_{44} = 0.16(5), \quad P = 0.072(68).$$

Based on the angular distribution and linear polarization results, the values of multipole mixing ratio $-0.97 \leqslant \delta \leqslant -0.79$ and $-0.84 \leqslant \delta \leqslant -0.31$ were determined respectively for the 365 keV γ ray and, finally, an effective value of $-0.84 \leqslant \delta \leqslant -0.79$ was adopted. The experimentally measured linear polarization also satisfies a higher magnitude of δ, as shown in Fig. 2 of Ref. [27]. Therefore, the reported angular distribution results are reexamined to estimate the multipole mixing ratio (δ). Figure 2 shows that the reported values of angular distribution coefficient A_{22} and A_{44} are possible for two different values of multipole mixing ratios $\delta \approx -2.0(2)$, $-0.33(3)$ and $\delta \approx \pm 1.3\ (7)$, respectively. However, the contour plot of A_{22} versus A_{44}, as presented in Fig. 3, clearly shows that these values are simultaneously possible only for the higher magnitude of δ, within the limit of uncertainty. Accordingly, the χ^2 of A_{22} and A_{44} is found to be minimum at $\delta \approx -2.0$ (Fig. 3), which is equivalent to 80(4)% of $E2$ contribution in the 365 keV γ ray. Thus, based on the present analysis, the 365 keV γ ray is found to be a predominant $\Delta I = 1$, $E2$ transition. For the other higher lying $\Delta I = 1$ transitions, viz., 521 and 671 keV, similarly large (negative) A_{22} coefficients were reported [25]. Therefore, as shown in Fig. 4, these transitions may also contain a significant $E2$ component. However, due to the lack of A_{44} values, complete information cannot be provided. Interestingly, similarly to the cases of ^{125,127}Xe and ^{133}Ba, the DCO ratios of these transitions between bands 3 and 1 show a smaller value than that expected for a pure or nearly pure $M1$ transition [25]. Perhaps this also indicates highly negative $M1/E2$ mixing ratios.

Thus, the present analysis of the available experimental data concludes that the $E_\gamma = 365$ keV transition and also the other higher lying $\Delta I = 1$ transitions of bands $3 \rightarrow 1$ and $4 \rightarrow 3$ in ^{129}Ba have high $E2$ admixture. In this context it is worth mentioning that the similar $13/2^- \rightarrow 11/2^-$ γ transition ($E_\gamma = 483$ keV) in ^{127}Xe ($N = 73$) is also found to be highly mixed ($\delta \approx -2.1$) with about 81% $E2$ fraction [9]. In fact, the measured $E2$ fractions in the connecting $\Delta I = 1$ transitions between wobbling bands in any odd-A nuclei are found to be quite large, approximately 60% to 95% (see Fig. 4 in Ref. [9]). On the other hand, the $E2$ mixing in the linking

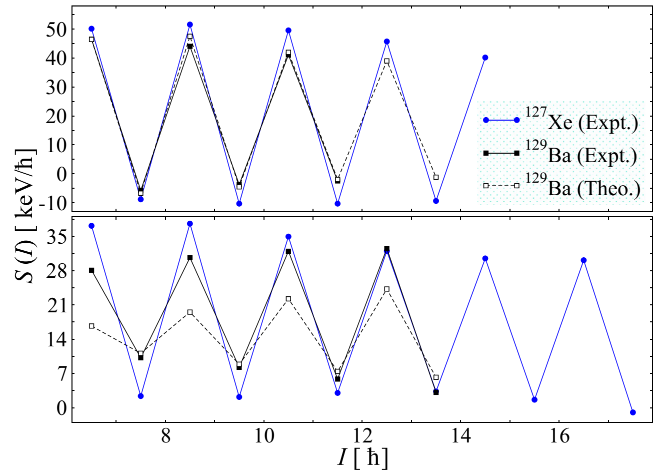

FIG. 5. Plot of the energy staggering, $S(I) = [E(I) - E(I-1)]/2I$, as a function of angular momentum (I), between bands 1 and 2 (top) and bands 1 and 3 (bottom) in ^{127}Xe [9] and ^{129}Ba [25]. Theoretical TPRM calculation for bands 1 and 2 was carried out in this work. For bands 1 and 3, the theoretical $S(I)$ plot was generated from the results reported in Ref. [30].

transitions of tilted precession (TiP) bands in ^{135}Nd is found to be no more than 20% [22], which is significantly lower than that estimated for the present case of ^{129}Ba. This apparently excludes the possibility of these bands having a TiP nature. In addition to these, the energy staggering between bands 1 and 3 in ^{129}Ba is found to be remarkably similar to that reported for the $n_\omega = 0, 1$ wobbling bands in ^{127}Xe (Fig. 5). Based on these features, band 3 (band 4) in ^{129}Ba can be considered as the first (second) phonon wobbling band associated with the $\nu h_{11/2}$ quasiparticle configuration. In contrast, as discussed earlier, band 2 in ^{129}Ba is found to be the most suitable candidate for the unfavored signature partner of the $\nu h_{11/2}$ band. To infer the microscopic structure of this negative parity band further, theoretical investigation has been carried out under the framework of the particle rotor model.

The rotational features of the triaxially deformed odd-A Xe–Ba nuclei in the $A \approx 130$ region were studied systematically by Gelberg and co-workers under the framework of the triaxial particle-rotor model (TPRM) [32]. Later, Stuch *et al.* studied the structure of the negative parity bands in ^{129}Ba using almost the same input parameters as those used by Gelberg *et al.* in their TPRM calculations [30]. However, in both of these cases, the band above the yrast $I^\pi = 13/2^-$ state, i.e., band 3, was considered as the unfavored signature partner of the $\nu h_{11/2}$ band. A comparison between experimental and calculated (TPRM [30]) energy staggering between bands 1 and 3 is shown in the bottom panel of Fig. 5. From this figure it is clearly seen that the TPRM calculation reported in Ref. [30] was not very successful in reproducing the energy staggering, particularly at lower spin. Moreover, this calculation failed to estimate the relative excitation energies of the states (Fig. 6). In fact, the estimated excitation energies even deviate more from the corresponding experimental values with increasing spin, as shown in Fig. 6. The present work reveals that the band above the yrare $I^\pi = 13/2^-$ state

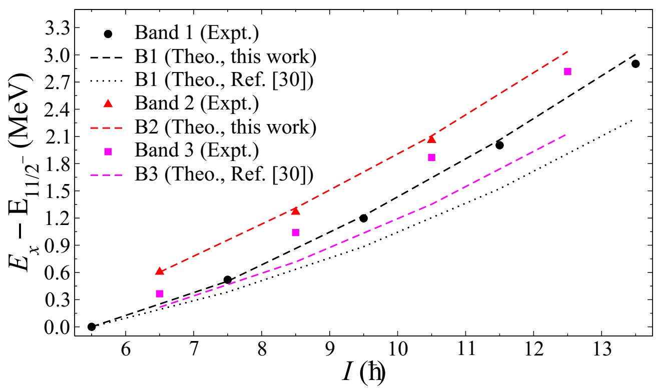

FIG. 6. Comparison of the experimental and calculated excitation energies, after subtracting the energy of the $I^{\pi} = 11/2^{-}$ state, for negative parity sequences in ^{129}Ba.

(band 2) is more suitable for the unfavored signature partner of the $\nu h_{11/2}$ band. Consequently, in this work a further theoretical investigation was carried out using the same formalism outlined in Refs. [33,34], but with different sets of deformation parameters. Within this formalism, the total particle-rotor Hamiltonian is expressed as $H = H_{\rm core} + H_{\rm sp} + H_{\rm pair}$. Details of the formalism are given in Refs. [33,34]. The present calculation nicely reproduces both the energy staggering (see the top panel of Fig. 5) as well as the excitation energies (Fig. 6) of the states belonging to bands 1 and 2, with deformation parameters $(\epsilon_2, \gamma) = (0.22, 33.5^{\circ})$. Thus, it seems that the present TPRM calculation also supports the $\alpha = \pm 1/2$ signature relationship of $\nu h_{11/2}$ bands 1 and 2 in ^{129}Ba. The experimental signature splitting in $\nu h_{11/2}$ band (bands 1 and 2) of ^{127}Xe and ^{129}Ba $N = 73$ isotones was also found to be quite similar, as shown in Fig. 5.

To study the microscopic structure of band 3, semiclassical quasiparticle triaxial rotor (QTR) model calculations with the frozen alignment (FA) approximation were performed following the prescription of Frauendorf and Dönau [35]. Under this framework, the wobbling frequency can be expressed as

$$\hbar\omega_w = E_{\rm wob} = \frac{j}{\mathcal{J}_3}\sqrt{\left[1 + \frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_1} - 1\right)\right]\left[1 + \frac{J}{j}\left(\frac{\mathcal{J}_3}{\mathcal{J}_2} - 1\right)\right]},$$

where the angular momentum $J = \sqrt{I(I+1)}$.

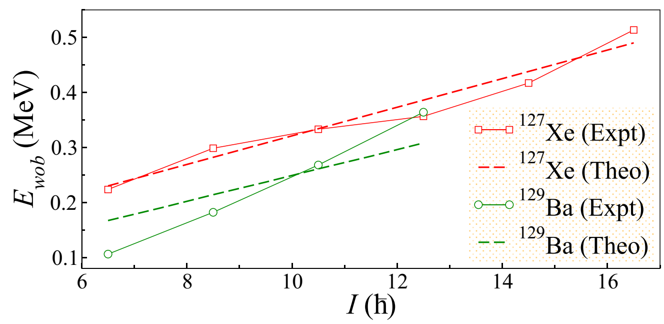

FIG. 7. Variation of wobbling energy as a function of spin for the longitudinal wobbling bands in $N = 73$ Xe–Ba nuclei.

The negative parity bands in ^{129}Ba are supposed to originate from the mid-Ω $h_{11/2}$ orbital according to the present TPRM calculation. Accordingly, a longitudinal coupling of the particle-like odd quasineutron with the triaxial rotor is expected to occur in this nucleus [35]. The present calculation successfully reproduces the increasing trend of the experimental wobbling energy as a function of spin (Fig. 7) with $\mathcal{J}_3 > \mathcal{J}_1, \mathcal{J}_2$, hence it supports the longitudinal nature of the wobbling band in both ^{127}Xe and ^{129}Ba $N = 73$ isotones.

To summarize, the intruder negative parity bands in ^{129}Ba were examined from both experimental and theoretical perspectives. The results of the investigation indicate that the yrast sequence above the $I^{\pi} = 13/2^{-}$ state is not the signature partner of the $\nu h_{11/2}$ band, as previously thought. Instead, it is likely caused by the excitation of the first wobbling phonon. The band above the yrare $I^{\pi} = 13/2^{-}$ state is found to be a better candidate for the unfavored signature partner of the $\nu h_{11/2}$ band. Theoretical calculations carried out in the present work closely mirrored the experimental findings. This reinterpretation of the bands in ^{129}Ba is important for better understanding the nature of collective triaxial properties of atomic nuclei. However, further measurements of angular distribution/correlation and linear polarization are required to unequivocally establish this exotic mode of excitation in this nucleus.

S.C. is thankful to the Science and Engineering Research Board, (SERB, India) for financial support under National Post-Doctoral Fellowship (N-PDF) scheme, *vide* reference no. PDF/2022/001829.

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[2] S. W. Ødegård *et al.*, Phys. Rev. Lett. **86**, 5866 (2001).
[3] D. R. Jensen *et al.*, Phys. Rev. Lett. **89**, 142503 (2002).
[4] D. J. Hartley, R. V. F. Janssens, L. L. Riedinger, M. A. Riley, A. Aguilar, M. P. Carpenter, C. J. Chiara, P. Chowdhury, I. G. Darby, U. Garg *et al.*, Phys. Rev. C **80**, 041304(R) (2009).
[5] D. J. Hartley *et al.*, Phys. Rev. C **83**, 064307 (2011).
[6] N. S. Pattabiraman *et al.*, Phys. Lett. B **647**, 243 (2007).
[7] J. Timár *et al.*, Phys. Rev. Lett. **122**, 062501 (2019).
[8] M. Prajapati *et al.*, Phys. Rev. C **109**, 034301 (2024).
[9] S. Chakraborty *et al.*, Phys. Lett. B **811**, 135854 (2020).
[10] K. Rojeeta Devi *et al.*, Phys. Lett. B **823**, 136756 (2021).
[11] S. Biswas *et al.*, Eur. Phys. J. A **55**, 159 (2019).
[12] J. T. Matta *et al.*, Phys. Rev. Lett. **114**, 082501 (2015).
[13] N. Sensharma *et al.*, Phys. Lett. B **792**, 170 (2019).
[14] A. Mukherjee *et al.*, Phys. Rev. C **107**, 054310 (2023).
[15] S. Nandi *et al.*, Phys. Rev. Lett. **125**, 132501 (2020).

[16] N. Sensharma *et al.*, Phys. Rev. Lett. **124**, 052501 (2020).
[17] A. A. Raduta, Prog. Part. Nucl. Phys. **90**, 241 (2016).
[18] K. Nomura and C. M. Petrache, Phys. Rev. C **105**, 024320 (2022).
[19] B. F. Lv *et al.*, Phys. Lett. B **824**, 136840 (2022).
[20] S. Guo *et al.*, Phys. Lett. B **828**, 137010 (2022).
[21] E. A. Lawrie, O. Shirinda, and C. M. Petrache, Phys. Rev. C **101**, 034306 (2020).
[22] B. F. Lv *et al.*, Phys. Rev. C **103**, 044308 (2021).
[23] B. F. Lv *et al.*, Phys. Rev. C **105**, 034302 (2022).
[24] S. Chakraborty *et al.*, Phys. Rev. C **107**, 064318 (2023).
[25] A. Byrne *et al.*, Nucl. Phys. A **548**, 131 (1992).
[26] J. Gizon *et al.*, Nucl. Phys. A **277**, 464 (1977).
[27] J. Gizon, A. Gizon, Y. K. Lee, and D. O. Elliott, Phys. Rev. C **17**, 596 (1978).
[28] R. D. Griffioen and R. K. Sheline, Phys. Rev. C **10**, 624 (1974).
[29] D. Bucurescu *et al.*, Nucl. Phys. A **630**, 643 (1998).
[30] O. Stuch *et al.*, Phys. Rev. C **61**, 044325 (2000).
[31] J. Kaur, A. K. Bhati, N. Bansal, V. Kumar, V. R. Sharma, H. Kumar, R. Kumar, and R. K. Bhowmik, Phys. Rev. C **87**, 064312 (2013).
[32] A. Gelberg *et al.*, Nucl. Phys. A **557**, 439 (1993).
[33] S. E. Larsson, Phys. Scr. **8**, 17 (1973).
[34] S. E. Larsson, G. Leander, and I. Ragnarsson, Nucl. Phys. A **307**, 189 (1978).
[35] S. Frauendorf and F. Dönau, Phys. Rev. C **89**, 014322 (2014).