PHYSICAL REVIEW C **110**, 014601 (2024)

Compound nucleus formation probability for superheavy nuclei

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(Received 22 March 2024; revised 27 May 2024; accepted 7 June 2024; published 1 July 2024;
corrected 21 October 2024)

The present work delves into compound nucleus formation probabilities in superheavy nuclei, developing empirical formulas incorporating deformation, entrance channel properties, and energy dependencies. The present formula also included shell correction. Experimental validation ensures the reliability of these formula. We have applied them to fusion reactions involved in synthesizing superheavy elements from $Z = 104$ to 118, enhancing our understanding of nuclear structure and stability. Excitingly, the present study identifies fusion reactions with higher probabilities, potentially enabling the synthesis of elements with $Z = 119$ and 120. The fusion reaction ^{58}Fe + ^{237}Np and ^{64}Ni + ^{238}U has a larger fusion probability among the attempted fusion reactions to synthesize superheavy elements $Z = 119$ and 120, respectively. Such knowledge not only advances our understanding of nuclear dynamics but also informs future experiments aimed at synthesizing and characterizing elusive superheavy elements by bridging the gap between theory and experiment. The present work offers a roadmap for navigating this frontier, guiding experimental efforts toward achieving breakthroughs in superheavy element research.

DOI: 10.1103/PhysRevC.110.014601

I. INTRODUCTION

Compound nucleus formation is essential for synthesizing superheavy elements, enabling fusion reactions, dissipating excess energy, increasing reaction probabilities, and facilitating neutron capture in neutron-rich environments. Compound nucleus (CN) formation in heavy ion fusion faces challenges such as Coulomb barriers, fusion cross-section dependence on energy, shell effects, fragmentation, angular momentum redistribution, fission competing channels, and evaporation residue yield optimization [1]. Deformation significantly influences CN formation probability in nuclear reactions. It alters the potential energy landscape experienced by colliding nuclei, impacting fusion and fission outcomes. Understanding deformation's role is crucial for predicting and interpreting nuclear reaction phenomena, benefiting nuclear physics, astrophysics, and nuclear engineering research [2–5].

The potential energy landscape experienced by colliding nuclei during reactions is influenced by nuclei deformation. Deformation alters the shape and depth of the landscape, impacting the probability of forming a CN [6,7]. CN formation probability is intricately related to the potential energy landscape experienced by colliding nuclei during nuclear reactions. The shape and depth of this landscape determine the likelihood of nuclei overcoming energy barriers to form a CN. Deformation and alignment can alter this landscape, affecting the probability of CN formation [8–10].

The CN formation probability (P_{CN}) is influenced by entrance channel parameters such as incident energy, angular momentum, and the mass and charge of colliding nuclei [2]. Higher incident energies and angular momenta generally increase P_{CN}, facilitating fusion [11]. Heavier nuclei and those with similar structures exhibit higher P_{CN} due to stronger attractive forces and enhanced stability [12]. These parameters collectively dictate the likelihood of nuclei overcoming the Coulomb barrier and forming a CN, crucial in understanding nuclear reactions and fusion processes across various scientific disciplines [3].

An accurate empirical formula for CN formation probability (P_{CN}) is essential for accurately predicting and understanding nuclear reactions. Such a formula provides a quantitative framework to relate entrance channel parameters like incident energy, angular momentum, and nuclear properties to P_{CN}. It enables researchers to model and interpret experimental data, guiding the design of nuclear reactors, astrophysical models, and fusion processes. Additionally, an accurate empirical formula aids in optimizing reaction conditions for desired outcomes, enhancing our knowledge of nuclear physics and facilitating advancements in fields reliant on nuclear reactions, including energy production and fundamental research. Hence in the present work, we have formulated deformation dependent, entrance channel dependent and energy dependent empirical formula for CN formation probability (P_{CN}).

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II. THEORY

Effective fissility $\chi_{\rm eff}$, representing a nucleus's propensity for fission, directly influences the likelihood of forming compound nuclei. Higher effective fissility correlates with increased compound nuclear formation probability, as nuclei with greater fission tendencies are more likely to form compound nuclei during nuclear reactions. Armbruster [13] has suggested the following relation:

$$P_{CN}(E,l) = 0.5\ \exp[-c(\chi_{\rm eff}-\chi_{\rm thr})]. \tag{1}$$

Terms used in the above equation have the following definitions:

$$\chi_{\rm eff} = \left[\frac{(Z^2/A)}{(Z^2/A)_{\rm crit}}\right][1-\alpha+\alpha f(k)] \tag{2}$$

with $(Z^2/A)_{\rm crit}$, $f(k)$ and k given by

$$(Z^2/A)_{\rm crit} = 50.883\left[1-1.7286\left(\frac{(N-Z)^2}{A}\right)\right], \tag{3}$$

$$f(k) = \frac{4}{k^2+k+\frac{1}{k}+\frac{1}{k^2}}, \tag{4}$$

$$k = (A_1+A_2)^{1/3}. \tag{5}$$

Equations (3) and (5) should be used in Eqs. (8) and (10) for the calculation of $\chi_{\rm eff}$. Fusion probability depends on the energy which is ignored in the Armbruster [13] equation. Later, energy dependence was considered and suggested empirical relations for P_{CN}. This behavior could be also approximated by the very simple Fermi function [14]

$$P_{CN}^0 = \frac{1}{1+\exp\left(\frac{Z_1Z_2-\zeta}{\tau}\right)}. \tag{6}$$

However, the above equation will not produce accurate P_{CN} due to not considering other entrance channel parameters and deformations. The modified energy dependence of the fusion probability may be approximated by the previous researcher [14]:

$$P_{CN}(E^*,l) = \frac{P_{CN}^0}{1+\exp\left(\frac{E_B^*-E_{\rm int}^*(l)}{\Delta}\right)}. \tag{7}$$

Here, E_B^* is the excitation energy of compound nuclei at the center-of-mass beam energy equal to the Bass barrier [15]. $E_{\rm int}^*(l) = E_{\rm c.m.}+Q-E_{\rm rot}(l)$ is the "internal" excitation energy, Δ is the adjustable parameter of about 4 MeV. Later on, beam energy dependence for the fusion probability was considered by previous workers [14,16–18]:

$$P_{CN}^1(E,l) = \frac{\exp[-c(\chi_{\rm eff}-\chi_{\rm thr})]}{1+\exp\left[\frac{E_B^*-E^*}{\Delta}\right]}, \tag{8}$$

where E^* is the excitation energy of the CN, E_B^* denotes the excitation energy of the CN when the center-of-mass beam energy is equal to the Coulomb and proximity barrier. For the best fit to the cold fusion reaction, the values of c and $\chi_{\rm thr}$ are 136.5 and 0.79, respectively. For the hot fusion reaction, the best fit for $\chi_{\rm eff} \leqslant 0.8$ is $c = 104$ and $\chi_{\rm thr} = 0.69$; whereas for $\chi_{\rm eff} > 0.8$, the values are $c = 82$ and $\chi_{\rm thr} = 0.69$. These constants are suggested by Loveland [16].

The empirical formula defined in Eq. (8) with the fitting parameters c, $\chi_{\rm thr}$, and $\chi_{\rm eff}$ of Loveland [16] will not produce P_{CN} values in the superheavy region. The outcomes generated by this formula are inconsistent with the results obtained from the experiments. This may be due to the reason that the empirical formula defined in Eq. (8) does not include deformation effects and information on the entrance channels. Hence, we improved Eq. (8) by including the quadrupole and hexadecapole deformations effects in terms of the factor f_β and it is as follows:

$$f_\beta = \exp(\beta_{2P}+\beta_{2T}) + \exp(\beta_{4P}+\beta_{4T}). \tag{9}$$

There is also chance to improve the formula by including the entrance channel parameter such as A_1, A_2, Z_1, and Z_2, this can be included through the ζ parameter ($\zeta = Z_1Z_2[(A_1A_2)/(A_1+A_2)]^{1/2}$). The modified deformation, entrance channel-dependent, and energy-dependent CN formation probability can be written as

$$P_{CN}(\beta,\zeta,\chi_{\rm eff}) = \frac{\exp[-c\ (\chi_{\rm eff}-\chi_{\rm thr})]}{\alpha f_\beta \zeta + \exp\left(\frac{E_B^*-E^*}{\Delta}\right)}. \tag{10}$$

We have considered experimental P_{CN} values in the superheavy region [19] and fitted the constants using least square fitting and these constants are $c = 150$, $\chi_{\rm thr} = -0.0256$, $\Delta = 2.5$, and $\alpha = 0.0005$. However, this does not include the shell effects. The potential barrier for heavier superheavy nuclei is probably a double-humped potential barrier: the external barrier leading at least to quasifission reactions and an internal barrier leading to real compound almost spherical nuclei. The shell effects play the main role in the inner barrier. Hence we have included the shell correction to the formula by including the factor $f_{sh} = \exp[E_{sh}(p)+E_{sh}(t)]$ and it is

$$P_{CN}(\beta,\zeta,\chi_{\rm eff},E_{sh}) = \frac{\exp[-c\ (\chi_{\rm eff}-\chi_{\rm thr})]+\psi_{sh}\sqrt{f_{sh}}}{\alpha f_\beta \zeta^{0.1} + \exp\left(\frac{E_B^*-E^*}{\Delta}\right)}. \tag{11}$$

The fitting parameters for this formula are $c = 0.88$, $\chi_{\rm thr} = 0.059$, $\Delta = 3.78$, $\alpha = 0.225$. Further, $\psi_{sh} = -0.956$ when $f_{sh} \leqslant 1$ and $\psi_{sh} = -0.296$ when $f_{sh} > 1$. Further, $E_{sh}(p)$ and $E_{sh}(t)$ are the shell correction terms of a projectile and target nuclei. These shell corrections were taken as the difference between experimental nuclear and theoretical mass value [20] is as follows:

$$E_{sh}(p/t) = M_{\rm exp} - M_{th}. \tag{12}$$

Here, $M_{\rm exp}$, and M_{th} nuclear mass were taken from the finite range droplet model (FRDM) [21].

III. RESULTS AND DISCUSSION

The constructed semiempirical formula includes the information on the CN, projectile-target, and its deformations. Comparison of the present formula with experiments [19] is also shown in Fig. 1. To validate this formula for other projectile-target combinations and compound nuclei, we have compared the P_{CN} produced by the present formula [Eq. (11)] with that of the experiments in the superheavy region and it is

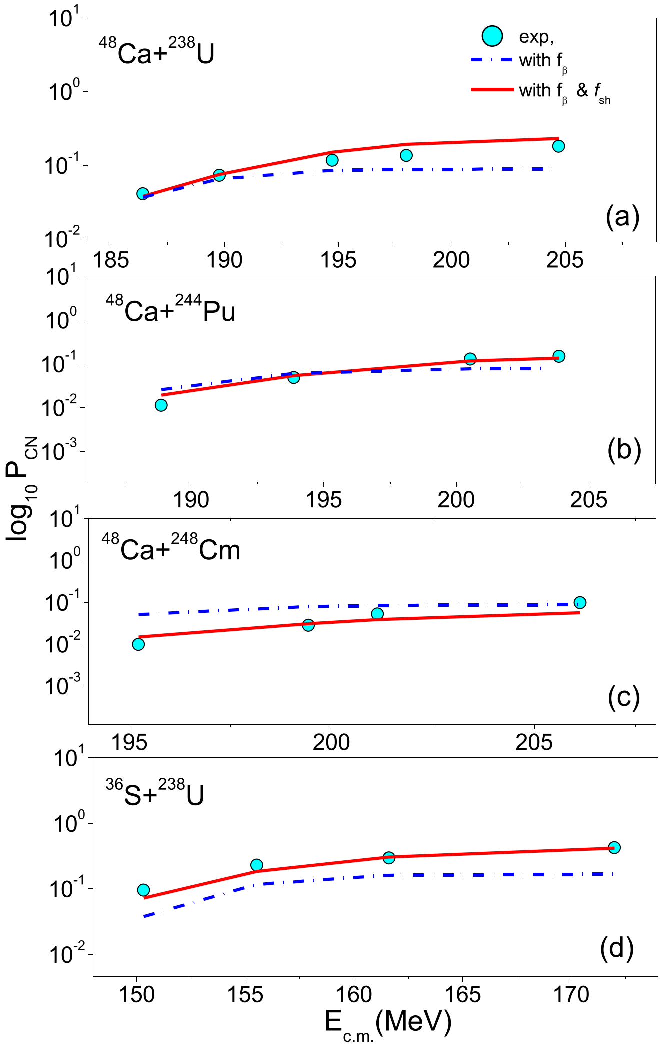

FIG. 1. Comparison of the present formula with experiments [19].

shown in Table I. From this table it is clear that the values produced by this present formula agree well with the experiments in the superheavy region. Further, to study the effect of P_{CN} on shell correction δ_{sh}, ζ, and f_β parameter, P_{CN} is evaluated by varying δ_{sh}, ζ, and f_β arbitrarily or the fusion reaction ^{24}Mg + ^{208}Pb and it is shown in Fig. 2. This figure illustrates that P_{CN} decreases with increasing shell correction δ_{sh}, ζ, and f_β.

The agreement between P_{CN} values obtained by present formula, i.e., Eq. (11) with the inclusion of shell correction, ζ, and f_β with that of experiments are tested using average percentage of deviation as follows:

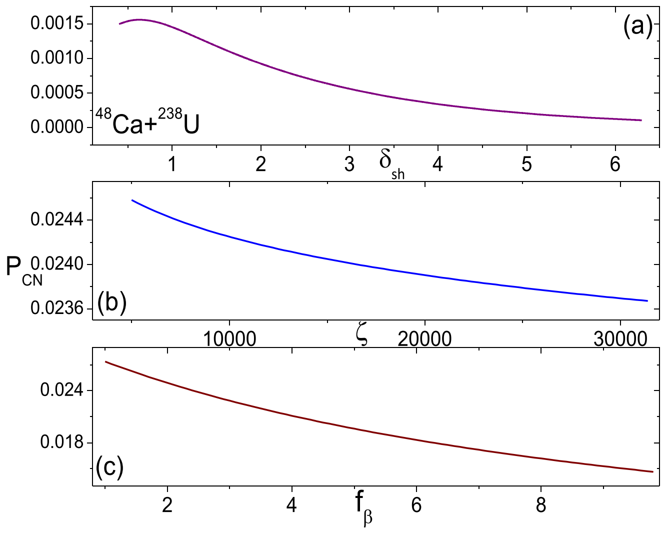

FIG. 2. Effect of P_{CN} on (a) shell correction, (b) ζ parameter, (c) f_β for the fusion reaction ^{48}Ca + ^{238}U.

$$\sigma = \frac{1}{n}\sum_{i=1}^{n}\frac{\left|\left(P_{CN}^{\exp} - P_{CN}^{PF}\right)\right|}{P_{CN}^{\exp}} \times 100. \tag{13}$$

Here, n, $P_{CN}^{\exp}$, and P_{CN}^{PF} are the number of experimental values considered, P_{CN} values corresponding to experiments and present formula [using Eq. (11)]. The σ values produced from the above equation is tabulated in Table II. Here, we noticed ±7.2% deviation of values produced from the present formula with that of experiments.

We have also evaluated the CN formation probability for the cold and hot fusion reactions used in the synthesis of superheavy elements $Z = 104$–118 using the present formula. The variation of P_{CN} produced by the present formula with E^* for cold fusion reactions and hot fusion reactions used in the synthesis of superheavy elements 104–118 is shown in Fig. 3. From this figure, it is observed that the increasing trend of P_{CN} with energy for both cold and hot fusion reactions. The CN formation probability of cold fusion reactions is comparably larger than that of hot fusion reactions.

Synthesizing superheavy elements, particularly those with atomic numbers like 119 and 120, involves highly complex and experimental procedures due to their extreme rarity

TABLE I. Comparison of CN formation probability produced by the present formula with that of the experiments.

Reactions	V_B (MeV)	$E_{c.m.}$ (MeV)	P_{CN}	P_{CN}[Ref.]
^{26}Mg + ^{248}Cm	127.749	129.4	0.764	0.827 [22]
^{48}Ti + ^{238}U	221.019	215.209	0.0364	0.042 [23]
^{50}Ti + ^{208}Pb	199.577	183.7–202.3	0.0204–0.28	0.02–0.19 [24]

TABLE II. Tabulation of average percentage deviation obtained using a present formula with that of experiments for the fusion reactions of ^{48}Ca + ^{238}U, ^{48}Ca + ^{244}Pu, ^{48}Ca + ^{248}Cm, and ^{36}S + ^{238}U [19].

	Reactions			
	^{48}Ca + ^{238}U	^{48}Ca + ^{244}Pu	^{48}Ca + ^{248}Cm	^{36}S + ^{238}U
Average percentage deviation	7.1	7.2	2.01	7.1

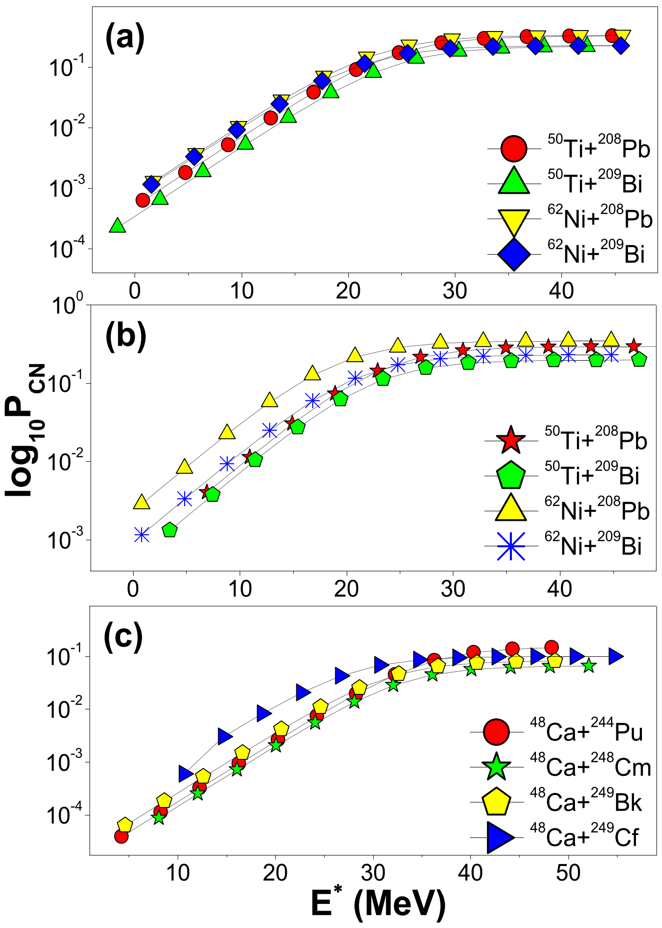

FIG. 3. CN formation probability produced by the present formula with E^* for (a) and (b) cold fusion reactions and (c) hot fusion reactions used in the synthesis of superheavy elements 104–118.

and instability. The synthesis of superheavy elements faces challenges due to low fusion probability, short half-lives, scarce target materials, experimental constraints, background interference, and unknown reaction mechanisms. High atomic numbers lead to weaker fusion probabilities, while rapid decay complicates detection. More than 14 fusion reactions were attempted to synthesize superheavy elements at institutions such as Joint Institute for Nuclear Research (JINR) in Russia, Lawrence Berkeley National Laboratory (LBNL) in the USA, Gesellschaft für Schwerionenforschung (GSI) in Germany, RIKEN in Japan, and Oak Ridge National Laboratory (ORNL) in the USA. We have evaluated the P_{CN} values of those 14 fusion reactions that were attempted to synthesize superheavy elements 119 and 120 using the present formula and it is shown in Fig. 4. This figure depicts that the fusion reaction ^{58}Fe + ^{237}Np has a larger fusion probability among the attempted fusion reaction to synthesize superheavy element $Z = 119$. Likewise, the fusion reaction ^{64}Ni + ^{238}U has a larger fusion probability among the attempted fusion reactions to synthesize superheavy element $Z = 120$.

We compared the P_{CN} values obtained from the current formula for the fusion reactions of ^{48}Ca + ^{249}Cf and ^{50}Ti + ^{249}Cf, as illustrated in Fig. 5. Significantly higher P_{CN} values were observed for the successful fusion reaction ^{48}Ca + ^{249}Cf compared to the unsuccessful fusion reaction ^{50}Ti + ^{249}Cf, which was an attempt to synthesize the superheavy element with atomic number 120. This indicates that

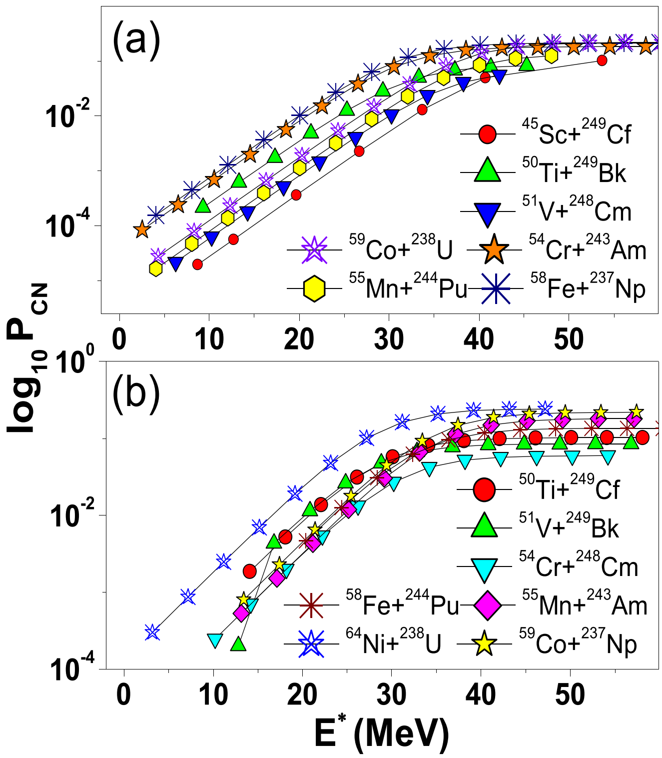

FIG. 4. Compound nuclear fusion probability for the fusion reactions using in the synthesis of superheavy elements $Z = 119$ and $Z = 120$.

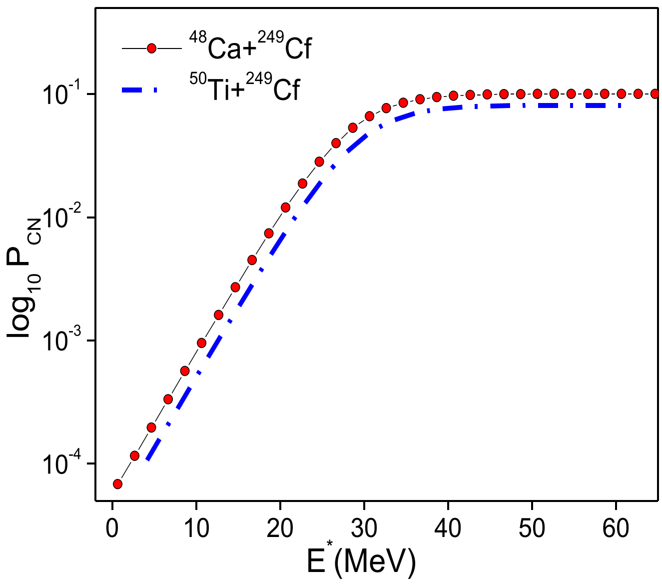

FIG. 5. A plot of CN formation probability as a function of excitation energy for the fusion reaction of ^{48}Ca + ^{249}Cf and ^{50}Ti + ^{249}Cf.

the ^{48}Ca + ^{249}Cf combination has a higher probability of compound nucleus formation, leading to a more favorable fusion process.

IV. SUMMARY

We have formulated deformation, entrance channel, and energy-dependent empirical formula for CN formation probability in the superheavy nuclei region. The present formula also included shell correction. The constructed empirical formula is validated with the experiments. The CN formation probability is also studied for the fusion reactions which were employed in the synthesis of superheavy elements $Z = 104$–118. The possible fusion reaction of larger fusion probability is identified among the attempted fusion reactions to synthesize superheavy elements $Z = 119$ and 120.

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Correction: Equations (3) and (5) contained errors and have been fixed. A corresponding statement has been added after Eq. (5).