Trineutron resonances in the SS-HORSE extension of the no-core shell model

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The SS-HORSE–NCSM method is generalized to the case of democratic decay into an odd number of fragments. This method is applied to the search for resonances in three-neutron system (trineutron) using *ab initio* no-core shell model calculations with realistic nucleon-nucleon (NN) potentials. The $3/2^-$ and $1/2^-$ strongly overlapping resonances are predicted when softened NN interactions are used and are preferred over the case where bare NN interactions of the chiral effective field theory are used with no resonance obtained.

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I. INTRODUCTION

In this paper, we develop and apply an *ab initio* method of calculating the democratic decay of light nuclei into an odd number of fragments within the no-core shell model (NCSM) [1]. Such an approach is of a current interest for the studies of neutron-excess light nuclei and, in particular, Borromean neutron-excess nuclei near and beyond the neutron drip line.

We apply this method to the search for resonant states in the three-neutron system (trineutron). There is an increasing interest in theoretical and experimental investigations of multineutron systems following the experimental observation of the tetraneutron resonance [2,3]. By studying the multineutron systems we can sensitively probe the interaction between neutrons for details that are not available from neutron-neutron scattering experiments.

The first experimental investigations of the three-neutron system were published in the 1960s. In particular, the bound trineutron search failed in the studies of the $^3\mathrm{H}(n,p)^3n$ reaction in Ref. [4]. A comprehensive description of the history of trineutron experimental searches can be found in reviews of Refs. [5,6]. The main conclusion of all experiments is the exclusion of the bound trineutron. At the same time, the existence of a resonant trineutron state is not ruled out.

References [5,6] present also the history of theoretical investigations of the three-neutron system. Among those we note the recent studies based on realistic NN interactions [7–10]. The resonant trineutron has not been found in Refs. [7,9]. The binding energy of three neutrons confined by an external potential (trap) has been extrapolated in Ref. [8] to the case of the vanishing trap to estimate the trineutron resonance energy (without any estimation for the resonance width). The obtained resonance energy of $E_r = 1.11(21)$ MeV is close to the result of Ref. [10] where the trineutron resonance is predicted by the calculations in the *ab initio* no-core Gamow shell model at the energy of $E_r = 1.29$ MeV with the width of $\Gamma = 0.91$ MeV.

In this work we will extend our SS-HORSE–NCSM approach [11–17] that generalizes the NCSM to the description of continuum spectrum states. The advantage of the SS-HORSE–NCSM is that the scattering phase shifts are computed by simple analytical expressions at the NCSM eigenenergies and there is no need in additional numerical challenges for no-core systems as compared to other continuum generalizations of NCSM like the NCSM with continuum (NCSMC) [18] or the no-core Gamow shell model [10,19]. Next the S matrix is parametrized and the resonant energies and widths are obtained by a numerical location of the S-matrix poles. Recently this method has been successfully applied to the description of resonant states in ^{5}He [11], ^{5}Li [15], ^{7}He [20], and ^{9}Li [21] with JISP16 [22] and Daejeon16 [23] realistic NN interactions in the channels of elastic scattering of protons in the case of ^{5}Li or neutrons in all other nuclei by the remaining nuclear fragment in the ground and sometimes in excited states. This method has been also generalized to the case of four-body democratic decays and applied to the description of resonances in the tetraneutron [16,17] and in the ^{7}He nucleus in the channel of four-body decay into ^{4}He and three neutrons [20]. In short, we have previously applied the SS-HORSE–NCSM approach up to now only to the decay channels with an even number of fragments.

On the other hand, the SS-HORSE–NCSM has been applied to the hypernuclear system Λnn in Ref. [24], a three-body decay, for the first time. The distinction from Refs. [11–17] is that Ref. [24] did not search for the S-matrix poles but extracted the resonance parameters from the slope of the phase shifts of the true three-body ($3 \to 3$) scattering.

In this paper we generalize the technique of locating the S-matrix poles proposed in Refs. [11–17] to the case of democratic decay into an odd number of fragments. We construct a family of parametrizations of the $3 \to 3$ scattering S matrix in a minimal approximation to enable the possibility of the S-matrix pole search.

The structure of the paper is the following. We discuss the $3 \to 3$ scattering, the structure of the respective S matrix and the generalization of the SS-HORSE–NCSM approach to the case of the democratic decay into an odd number of fragments using minimal approximations in Sec. II. We apply the developed method to the search of resonances in the three-neutron system based on the NCSM calculations with various realistic NN interactions in Sec. III. The conclusions are presented in Sec. IV.

II. SS-HORSE–NCSM METHOD FOR DEMOCRATIC DECAY INTO ODD NUMBER OF FRAGMENTS

We make use of the version of the J-matrix formalism [25,26] in scattering theory utilizing the harmonic oscillator basis, which is also known as HORSE [27], for the generalization of the NCSM to the case of the continuum spectrum. The essence of the HORSE formalism is the division of the many-body Hilbert space into a finite-dimensional oscillator subspace where both the potential energy of the interactions between particles and their kinetic energy are taken into account (P space) and the remaining infinite-dimensional subspace where only the kinetic energy is retained and the interaction is neglected (Q space). The P space conventionally includes all many-body states with oscillator excitation quanta, which do not exceed some certain number $N_{\max}$. This definition is well matched with the NCSM where $N_{\max}$ is used to restrict the model space.

We use a generalization of the HORSE formalism to the case of the true many-body ($A \to A$) scattering developed in Ref. [28] to describe states in the many-body continuum. The version of HORSE for $A \to A$ scattering utilizes the ideas of the method of hyperspherical harmonics (HH) (see, e.g., Refs. [29,30]), which was widely used in studies of various atomic and nuclear systems, in particular, of the trineutron [31–35].

In the case of continuum states, the HH method is an adequate tool for the description of the so-called democratic decays of an A-body system when no subgroup of the A particles has a bound state. This condition appears to be satisfied for the trineutron or tetraneutron. The wave function dependence on the democratic hyperradius,

$$\rho = \sqrt{\sum_{i=1}^{A} (\mathbf{r}_i - \mathbf{R})^2}, \tag{1}$$

is of a primary importance within the HH approach. Here $\mathbf{r}_i$ are the individual neutron coordinates and $\mathbf{R}$ is the center-of-mass coordinate. The remaining degrees of freedom are described by hyperspherical functions depending on some set of $3A-4$ angles Ω_i on the $(3A-3)$-dimensional sphere coupled with neutron spins and a function describing the center-of-mass motion. Both the hyperspherical and the hyperradial functions are characterized by the hypermomentum K and some other quantum numbers α distinguishing different states with the same hypermomentum, which are of no interest for us in this research. For the states of a definite total angular momentum J and parity, $K = K_{\min}, K_{\min}+2, \ldots$, where generally $K_{\min} \geqslant 0$ is integer, and $K_{\min} = 1$ in the case of trineutron natural (negative) parity states with $J = 3/2$ or $1/2$.

In the HH approach, the Schrödinger equation takes the form of a set of coupled equations, which is equivalent to a set of equations describing a multichannel scattering with the same threshold in all channels. Each of the equations includes a centrifugal term $\mathcal{L}(\mathcal{L}+1)/\rho^2$, where the effective orbital momentum [28]

$$\mathcal{L} = K + \frac{3A-6}{2}. \tag{2}$$

We note that the NCSM calculations performed in the P space utilize a complete set of HH with $K \leqslant N_{\max} + N_{\min}$. However, in the Q space, which is associated with the long-range behavior of the wave functions, the HH with $K > K_{\min}$ are suppressed by the high centrifugal barrier. Therefore, we utilize the democratic decay minimal approximation that implies retaining only one HH with $K = K_{\min}$ in the Q space. So, the wave function is characterized by a single phase shift δ of $A \to A$ scattering. This phase shift can be calculated using NCSM eigenenergies E_d obtained with given values of $N_{\max}$ and the NCSM oscillator basis parameter $\hbar\omega_d$ within the SS-HORSE–NCSM approach as [16]

$$\tan\delta(E_d) = -\frac{S_{N_{\max}+N_{\min}+2,\mathcal{L}}(E_d)}{C_{N_{\max}+N_{\min}+2,\mathcal{L}}(E_d)}, \tag{3}$$

where $N_{\min}$ is the minimal number of oscillator quanta allowed by the Pauli principle, $S_{n\mathcal{L}}(E)$ and $C_{n\mathcal{L}}(E)$ are regular and irregular solutions for a free motion in the HORSE formalism, which explicit analytical expressions can be found in Ref. [28]. Note, $S_{n\mathcal{L}}(E)$ and $C_{n\mathcal{L}}(E)$ depend on the oscillator parameter $\hbar\omega$.

The accuracy of the approximation retaining the single lowest HH in the Q space was confirmed in the studies of three-body democratic decays in Refs. [36–39]. We also used the minimal approximation for the democratic decay in investigations of the four-neutron system [16,17].

The $A \to A$ S matrix is related to the phase shift δ,

$$S(k) = e^{2i\delta(E)}. \tag{4}$$

To study the S matrix analytical properties, it is more convenient to analyze it as a function of the momentum k instead of the energy E,

$$E = \frac{\hbar^2 k^2}{2M}, \tag{5}$$

where M is total mass of the system.

In the case of even A, $\mathcal{L}$ is integer, and the $A \to A$ S matrix analytical properties are similar to those of two-body scattering. In particular [40,41],

$$S(-k) = S^{-1}(k) \tag{6}$$

and

$$S^*(k) = \frac{1}{S(k^*)}, \tag{7}$$

which are crucial for the S-matrix parametrization. The parameterized S matrix can be analytically continued to the complex k plane for the search of its poles associated with resonant and bound states. This technique has been used to estimate the energy and width of the resonant state in the tetraneutron [16,17].

Analytical properties of the $A \to A$ S matrix become more complicated in case of an odd A due to a half-integer value of the effective angular momentum $\mathcal{L}$ as follows from Eq. (2). The S-matrix properties in the case of arbitrary noninteger angular momentum are discussed in Ref. [41]. In this case Eq. (6) is generalized to

$$S(ke^{i\pi}) = e^{2\pi i\mathcal{L}} S^{-1}(k) + 1 - e^{2\pi i\mathcal{L}}, \tag{8}$$

which holds for any complex value of k. As a result, for a half-integer $\mathcal{L}$ we have

$$S(ke^{i\pi}) = -S^{-1}(k) + 2. \tag{9}$$

Note that Eq. (7) is valid for any real value of angular momentum. We attribute properties (7) and (9) to the $A \to AS$ matrix in the case of an odd A.

The S matrix has multiple sheets and its properties are complicated in the case of a noninteger angular momentum. The S matrix can be expressed as [42]

$$S(k) = \frac{Z(k) - ik^{2\mathcal{L}+1}e^{i\pi(2\mathcal{L}+1)}}{Z(k) - ik^{2\mathcal{L}+1}}, \tag{10}$$

where $Z(k)$ has the following property:

$$Z(ke^{i\pi}) = Z(k). \tag{11}$$

Equation (10) cannot be used directly in the case of a half-integer $\mathcal{L}$: according to Ref. [42], in this case we have an uncertainty of the 0/0 type that should be resolved using the L'Hôpital's theorem considering $\mathcal{L}$ as a continuous variable and investigate the limit $\mathcal{L} \to K + (3A-6)/2$ to obtain

$$S(k) = 1 + \frac{2\pi k^{2\mathcal{L}+1}}{Y(k) - 2ik^{2\mathcal{L}+1}\ln(k/q_0)}, \tag{12}$$

where $Y(k) = \frac{\partial Z(k)}{\partial \mathcal{L}}|_{\mathcal{L}=K+(3A-6)/2}$ and q_0 is a real-valued momentum needed to make dimensionless the argument of ln in the denominator. We note that our final results for the S-matrix poles are independent of q_0. Using Eqs. (4) and (12), it is easy to deduce

$$\tan\delta = \frac{\pi k^{2\mathcal{L}+1}}{2k^{2\mathcal{L}+1}\ln(k/q_0) + i[Y(k) + \pi k^{2\mathcal{L}+1}]}. \tag{13}$$

The phase shift is a real-valued function for real $k > 0$. Therefore it is convenient to introduce a real-valued at real k function

$$X(k) = i[Y(k) + \pi k^{2\mathcal{L}+1}]. \tag{14}$$

It is easy to show that $Y(ke^{i\pi}) = Y(k)$, that leads to the following symmetry property of the function $X(k)$:

$$X(ke^{i\pi}) = X(k). \tag{15}$$

The $A \to A$ S matrix and phase shift are expressed in terms of $X(k)$ as

$$S(k) = \frac{X(k) + 2k^{2\mathcal{L}+1}\ln(k/q_0) + i\pi k^{2\mathcal{L}+1}}{X(k) + 2k^{2\mathcal{L}+1}\ln(k/q_0) - i\pi k^{2\mathcal{L}+1}}, \tag{16}$$

$$\tan\delta = \frac{\pi k^{2\mathcal{L}+1}}{2k^{2\mathcal{L}+1}\ln(k/q_0) + X(k)}. \tag{17}$$

The expression (16) satisfies the properties of Eqs. (7) and (9).

Due to Eq. (15), the function $X(k)$ can be parameterized as a series expansion in even powers of k,

$$X(k) = \sum_{i=0}^{W} w_i k^{2i}. \tag{18}$$

We note that the value of q_0 is arbitrary. Redefining q_0 results in a redefinition of parameters w_i ($i = 0, \dots, W$) in Eq. (18) such that the S matrix defined by Eq. (16) remains unchanged.

The parametrization (18) provides for an estimation of the phase-shift behavior in the limit $k \to 0$. For example, for the three-body problem ($A = 3$), supposing that $X(k) \xrightarrow{k\to 0} w_0$, we obtain from Eq. (17):

$$\tan\delta \sim \delta \sim k^{2K+4} \sim k^{2\mathcal{L}+1} \sim E^{K+2}. \tag{19}$$

This behavior is in line with the analysis presented in Ref. [29] justifying the parameterization (18).

Following the ideas of the SS-HORSE–NCSM approach [11–17], we can obtain the parameters w_i of the expansion (18) by calculating a set of the $A \to A$ phase shifts $\delta(E_d)$ using Eq. (3) at the NCSM eigenenergies E_d obtained with a chosen $N_{\max}$ and a set of the $\hbar\omega_d$ values, and next parameterize this set of $\delta(E_d)$ by means of Eqs. (17) and (18) (see the next section for more details). To calculate energies and widths of resonances, we locate the S-matrix poles by searching for zeros of the denominator in the right-hand side of Eq. (16), which is equivalent to solving numerically in the complex k plane $[-\pi < \arg(k) < \pi]$ equation

$$X(k) + 2k^{2\mathcal{L}+1}\ln(k/q_0) - i\pi k^{2\mathcal{L}+1} = 0 \tag{20}$$

using the technique suggested in Ref. [15] or the Newton-Raphson method (see, e.g., Ref. [43]).

III. TRINEUTRON

The above method is applied to the search of resonances in the three-neutron system. We use various realistic NN

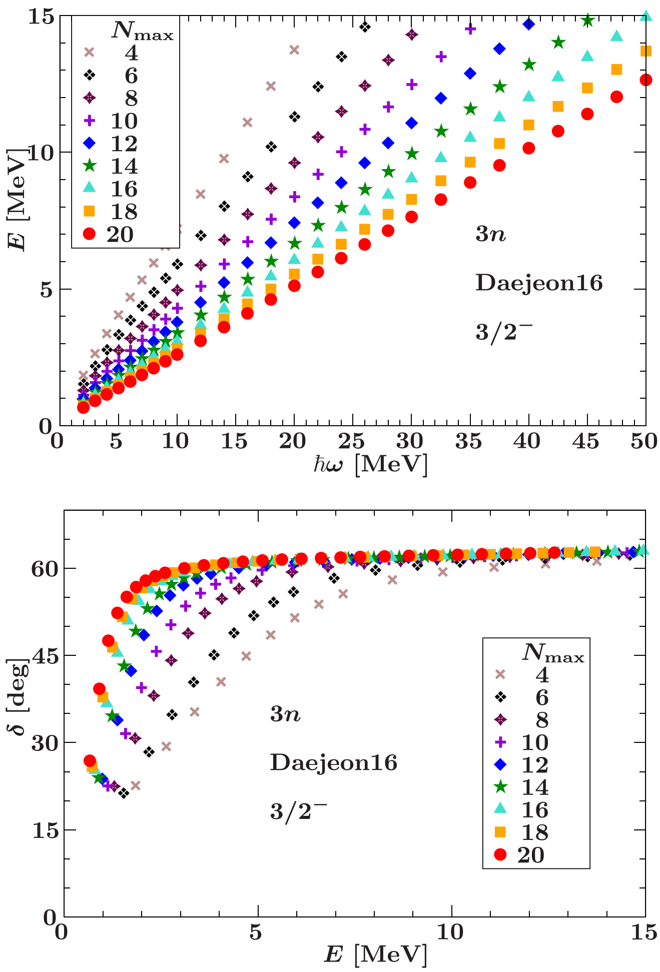

FIG. 1. Top: NCSM results for the trineutron $3/2^-$ ground-state energy obtained with Daejeon16 NN interaction with various $N_{\max}$ plotted as functions of $\hbar\omega$. Bottom: $3 \to 3$ phase shifts at the NCSM eigenenergies obtained using Eq. (3).

interactions, the same as employed in our analysis of the tetraneutron [16,17]. We utilize the MFDn code [44,45] to perform the NCSM calculations with $N_{\max}$ ranging from 4–20 and $\hbar\omega$ spanning from 2–50 MeV.

The results for the $3/2^-$ ground-state energy obtained with the Daejeon16 [23] interaction are shown in the top panel of Fig. 1. The $3 \to 3$ phase shifts at the NCSM eigenenergies obtained using Eq. (3) are presented in the bottom panel. It is seen that the phase shifts tend to the same smooth resonance-like curve as $N_{\max}$ is increasing demonstrating a convergence of the $3 \to 3$ phase-shift calculations.

We parametrize the function $X(k)$ for each individual value of $N_{\max} \geqslant 16$ used in the NCSM calculations of the trineutron ground-state energies. For a given $N_{\max}$, we use a set of parameters w_i defining $X(k)$ to find the energies $\mathcal{E}_d$ by solving the equation

$$-\frac{S_{N_{\max}+3,\mathcal{L}}(\mathcal{E}_d)}{C_{N_{\max}+3,\mathcal{L}}(\mathcal{E}_d)} = \frac{\pi\kappa_d^6}{2\kappa_d^6\ln(\kappa_d/q_0) + X(\kappa_d)}, \tag{21}$$

TABLE I. Convergence ofenergy E_r and width Γ of the trineutron $3/2^-$ resonant state obtained with NN interaction Daejeon16 with increasing $N_{\max}$. ξ is the r.m.s. deviation defined by Eq. (23).

$N_{\max}$ W	16 4	18 4	20 4	16 5	18 5	20 5
E_r, MeV	0.560	0.508	0.483	0.607	0.537	0.481
Γ, MeV	1.458	1.152	0.924	1.524	1.176	0.963
ξ, keV	3.3	3.9	2.5	2.7	2.0	1.8

for each value of $\hbar\omega_d$ used in the respective NCSM calculations. Here $\kappa_d = \frac{\sqrt{2M\mathcal{E}_d}}{\hbar}$ and $\mathcal{L} = K_{\min} + \frac{3}{2} = \frac{5}{2}$. To find the optimal values of w_i, we minimize the function

$$\Xi = \sqrt{\frac{1}{D}\sum_{d=1}^{D}\left[(\mathcal{E}_d - E_d)^2\left(\frac{\hbar\omega_M}{\hbar\omega_d}\right)^2\right]}, \tag{22}$$

where D is the number of the NCSM energies E_d obtained with the same $N_{\max}$ and the same $\hbar\omega_d$ as each of the respective energies $\mathcal{E}_d$, $\hbar\omega_M = \max_{d=1,\dots,D}\hbar\omega_d$, and $(\hbar\omega_M/\hbar\omega_d)$ is the weight increasing the importance of states with smaller $\hbar\omega_d$ corresponding to energies closer to the resonance region.

The quality of the fits can be estimated by the r.m.s. deviation

$$\xi = \sqrt{\frac{1}{D}\sum_{d=1}^{D}(\mathcal{E}_d - E_d)^2}. \tag{23}$$

In our case we get approximately the same r.m.s. deviations ξ obtained with five ($W = 4$) or six ($W = 5$) parameters w_i in Eq. (18), which are, however, significantly smaller than the r.m.s. deviations obtained with four parameters ($W = 3$). Energies and widths obtained by locating the S-matrix poles using Eq. (20) and the NCSM results from calculations with $N_{\max} = 16$, 18, 20 and parametrizations with $W = 4$ and 5 together with the respective ξ values are presented in Table I.

Fits of the $3 \to 3$ phase shifts in the $3/2^-$ state with six parameters w_i in Eq. (18) ($W = 5$) are presented by solid curves in Fig. 2. The $3 \to 3$ phase shifts at the NCSM eigenenergies obtained by Eq. (3) and used for the fitting are shown by symbols in Fig. 2. The trineutron resonance energy and width obtained by locating the S-matrix pole based on the NCSM calculation with $N_{\max} = 20$ and fit with $W = 5$ are adopted as the final result presented in the Table II together with their uncertainties estimated as deviations of results obtained with $N_{\max} = 18, 20$ and $W = 4, 5$ from the final result.

It is interesting that we obtain in the trineutron NCSM calculations the $1/2^-$ state very close to the lowest $3/2^-$ state. We perform the same analysis of the $1/2^-$ trineutron resonance. The $3 \to 3$ $1/2^-$ phase shifts are very close to those in the $3/2^-$ state and the obtained $1/2^-$ resonance energy and width are presented in Table II. It is seen that the $3/2^-$ and $1/2^-$ resonance energies and widths are the same within the uncertainty estimations and these resonances completely overlap.

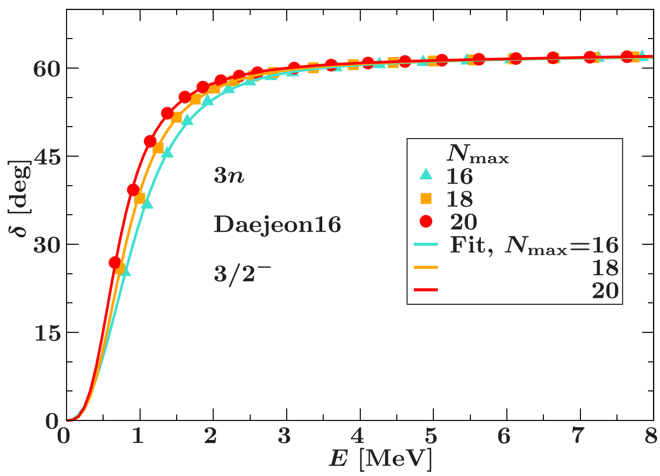

FIG. 2. Fits of $3 \to 3$ phase shifts in the $3/2^-$ trineutron state obtained with NN interaction DAEJEON16 and $W = 5$ in the $X(k)$ expansion (18).

We employ the same technique to search for resonances with other soft NN interactions, in particular, with Idaho N^{3}LO [46] softened by the similarity renormalization group (SRG) transformation [47,48] with the flow parameter $\Lambda = 2$ fm^{-1} and JISP16 [22]. The respective results are also presented in the Table II. Note, in these cases the $3/2^-$ and $1/2^-$ resonance also degenerate and strongly overlap.

We also have analyzed the trineutron resonance with bare realistic NN interactions derived in chiral effective field theory: Idaho N^{3}LO and LENPIC N^{4}LO semilocal coordinate space interaction [49] with regulator $R = 0.9$ fm. In these cases, the $3 \to 3$ phase shifts do not demonstrate a resonant behavior. Due to the almost complete degeneracy of the $3/2^-$ and $1/2^-$ states, these interactions also do not support the $1/2^-$ resonance.

We obtained [17] a $4 \to 4$ S-matrix pole at negative imaginary momentum in the tetraneutron calculations with the Idaho N^{3}LO interaction, which corresponds to a virtual state at the energy of $E_v = -15.2$ keV. It is easy to prove that the $A \to A$ S-matrix for an odd number of fragments A does not allow S-matrix poles at the negative imaginary half-axis of momentum. Therefore, a virtual state is prohibited in the trineutron treated as a democratic decaying system.

TABLE II. Energies E_r and widths Γ of trineutron resonant states obtained with soft NN interactions Daejeon16 [23], JISP16 [22], and SRG-evolved Idaho N^{3}LO [46]. Uncertainties are presented in parentheses. All values are in MeV.

	$3/2^-$		$1/2^-$	
Interaction	E_r	Γ	E_r	Γ
Daejeon16	0.48(6)	0.96(21)	0.48(8)	0.96(17)
JISP16	0.35(8)	0.70(9)	0.35(11)	0.67(22)
N^{3}LO, SRG, $\Lambda = 2$ fm^{-1}	0.34(8)	0.70(19)	0.35(9)	0.68(16)

IV. SUMMARY AND CONCLUSIONS

We suggest an extension of the SS-HORSE–NCSM method to a democratic decay into odd number of fragments. The first application of this method is the analysis of the resonant trineutron state.

We conclude that the soft NN interactions that we investigated predict two low-lying nearly degenerate overlapping trineutron resonances with spin parities $3/2^-$ and $1/2^-$. On the other hand, these resonances are not supported by bare NN interactions of chiral effective field theory. We do not include NNN interaction in our calculations, which has yet to be designed for three-nucleon systems with isospin $T = 3/2$.

We argue that the Daejeon16 NN interaction is preferable for the trineutron studies since it originates from the chiral effective field theory and is fitted to stable light nuclei up to ^{16}O by phase-equivalent off-shell variations, which effectively mimic effects of NNN forces. The JISP16 NN interaction, which was also fitted to light nuclei by off-shell variations, leads to similar trineutron results as well as the SRG-evolved Idaho N^{3}LO NN interaction.

We predict two overlapping trineutron resonances with spin parities $3/2^-$ and $1/2^-$ with nearly exactly the same energies E_r and widths Γ: $E_r \simeq 0.5$ MeV and $\Gamma \simeq 1$ MeV obtained in calculations with the Daejeon16 and $E_r \simeq 0.35$ MeV and $\Gamma \simeq 0.7$ MeV obtained in calculations with the JISP16 and SRG-evolved N^{3}LO NN interactions.

Our results are in line with the conclusions of Refs. [8,10] predicting the trineutron resonance at lower energy than the tetraneutron resonance [16,17]. However, in our calculations we obtain the trineutron resonance at lower energies as compared to $E_r = 1.29$ MeV in Ref. [10] and $E_r = 1.11(21)$ MeV in Ref. [8]. Meanwhile, its width obtained with the Daejeon16 NN interaction is in agreement with $\Gamma = 0.91$ MeV proposed in Ref. [10]. We note that Refs. [8,10] do not specify the spin parity of the predicted trineutron resonance.

Note added in proof. Recently, a new unsuccessful experimental attempt to find the trineutron and triproton resonances in the reactions ^{3}H(t, ^{3}He)3n and ^{3}He(^{3}He, t)3p was published in Ref. [50].

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